EFFECT OF ADDED DAMPERS ON THE SEISMIC RESPONSE OF MULTI-STORY BUILDINGS

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الخلاصـة :

تَـمَّ في هذا البحث دراسة أثر إضافة المثبطات الإضافية على الاستجابة الزلزالية في المباني المتعددة الأدوار . ويتضمن التحليل إيجاد الاستجابة وتقييم مكونات الطاقة . كما تم دراسة موثوقية الحل بواسطة التراكب الشكلي مع الأخذ في الاعتبار خاصية عدم التناسب في التثبيط . وقد تم اقتراح أربع طرق لحساب الاستجابة لمبنى يحتوي على تثبيط إضافي ، كما أُخذ في الاعتبار نماذج متعددة لكيفية تركيب المثبطات الاضافية للحصول على أعلى كفاءة لها . ولتحقيق هذه الأهداف تم استخدام مبنى ذي عشرة أدوار كمثال لتوضيح التحليل ولتوضيح الجدوى من استعمال التثبيط الإضافي في المنشآت .

ABSTRACT

The consequences of adding supplemental damping in buildings were investigated. The analysis included response determinations and energy component evaluations. The reliability of modal superposition was studied by considering the effects of damping nonproportionality. Four procedures were proposed to calculate the response of a structure having supplemental damping and alternative patterns of damper installation to increase the efficiency of the damper in reducing the response were considered. An example ten-story building was used to illustrate these analyses and the feasibility of introducing supplemental damping in structures.

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INTRODUCTION

It is recognized that damping in structures is beneficial by limiting the maximum response of the structures when subjected to earthquake ground motions. By dissipating some of the energy input to the structure, the earthquake-caused structural vibrations are reduced and the associated structural and nonstructural damage will be reduced [1–6].

Current earthquake-resistant design procedures place reliance on the ductile capabilities of the structural elements to dissipate energy while undergoing inelastic deformations. This usually assumes permanent damage, in some cases just short of collapse, but the damage may be as economically significant as total destruction of the structure [7, 8]. An alternative approach for controlling the structural response is by using supplemental mechanical damping devices. These dampers have the capability of concentrating energy dissipation in manufactured devices located throughout the structure, thereby limiting the dynamic response and required member ductile capabilities.

PROGRAM OF INVESTIGATION

Damping Modeling

The amount of damping provided by a mechanical damping device depends on the physical configuration of the device, the material characteristics, and the dissipation mechanism by which the device operates in the structure. Two common devices are the frictional device [9, 10] and the direct shear viscoelastic material device [11–13]. The former dissipates energy by Coulomb friction while the latter dissipates energy by straining the viscoelastic material in cyclic shear. The amount of damping provided by the viscoelastic material device depends on the frequency, amplitude, and temperature [13]. Although the properties change somewhat as the temperature increases, a constant damping coefficient has been shown to be acceptable for response evaluations [1]. Thus, supplemental damping for this study was considered as interstory viscous damping with constant coefficients.

It should be noted that the material damping device has an associated static stiffness; however, for the installation configuration considered here the additional stiffness of the device is negligible compared with the stiffness of the structural system [1, 14].

Structure Considered

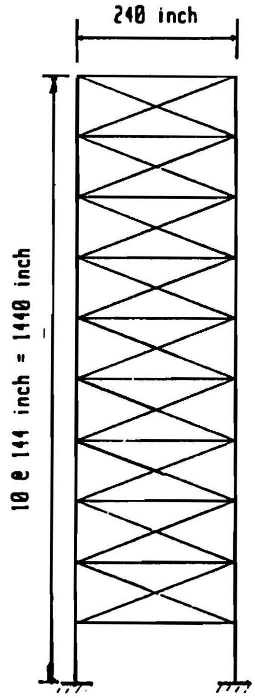
A ten-story ductile moment frame with minimum cross braces in the upper nine stories [7] will be used to demonstrate the effect of added damping. The frame was designed using two design criteria, member stress and a lateral deflection limit of 0.35 percent of the height. Bracing areas were determined using an allowable stress of 22 kips/in^2 (150 MPa) without utilizing the 33 percent increase in allowable stress. A minimum area of 2.88 in^2 (18.6 cm^2) was used. A story height of 12 feet (3.66 m) and a bay width of 20 feet (6.1 m) were used. The structure is documented in the literature [7] and is illustrated in Figure 1.

The following assumptions were made to simplify the analysis:

1. The building weights are concentrated at the floor levels.
2. The column stiffnesses are unaffected by column axial loads, however axial deformations of the columns are included.
3. Moments induced by the lateral displacement of gravity loads (P-Δ effects) are neglected.
4. The damping is assumed to be interstory viscous damping.
5. The system is fully elastic except that the bracing is effective in tension only. Elastic response is maintained by assuming one of the braces is fully effective in tension and compression and the other brace removed.

Reference Analyses

The building was subjected to the El Centro 1940 NS accelerogram and the maximum floor displacement, Figure 2*a*, and the maximum relative story displacement, Figure 2*b*, were calculated numerically using the Newmark-β method [15] for zero damping condition. Damping was added to the structure in equal amounts at each level except the first level to achieve fundamental modal damping of 2, 5, and 10 percent of critical. The modal properties of the building together with the interstory damping coeffi-



FLOOR LEVEL	Bracing Area in^2	Column Inertia in^4	Column Area in^2	Girder Inertia in^4	Floor Height kip
10	2.88	339.2	10.00	800.6	132
9	2.88	542.1	15.59	800.6	132
8	2.88	542.1	15.59	800.6	132
7	2.88	851.2	22.94	800.6	132
6	2.88	851.2	22.94	800.6	132
5	2.88	1165.8	30.26	984.0	132
4	2.94	1165.8	30.26	984.0	132
3	2.94	1373.1	34.99	984.0	132
2	3.38	1373.1	34.99	984.0	132
1		2274.8	54.07	1286.8	132

Figure 1. Structural Properties of 10 Story Building.
$1\ in^2 = 645 = \mu m^2$, $1\ in^4 = 416\ nm^4$, $1\ kip = 4.45\ kN$

cient for these cases of damping are shown in Table 1 for the first five modes.

The significance of adding damping to the building can be seen clearly in Figure 2. The floor displacement was reduced by 60 percent at the top of the building and the maximum relative story displacement is reduced by 77 percent when damping increased from zero to 10 percent. Keeping the same damping of 10 percent for the fundamental mode, while adding damping and stiffness in the first story, the response is less than for the open first story.

In the dynamic analysis of a multistory building the examination of the energy components is an effective way to understand the behavior of the system. These components involve the input energy, the dissipated energy, and the stored energy, which is expressed as the sum of the strain and kinetic energy. The energy balance also serves as a check on the accuracy of the method of analysis and results obtained therefrom. The total energy input to the structure at a given time can be computed by integrating the product of base shear and ground velocity. This must equal the sum of the dissipated energy up to that time plus the recoverable strain energy and the kinetic energy present in the structure at that time. The total input energy is equal to

$$EI = \int_0^t \dot{z}\{1\}^T [M]\{\ddot{x} + \ddot{z}\}dt \qquad (1)$$

where $\{1\}^T$ is the transpose unit vector, $\dot{z}$ and $\ddot{z}$ are the ground velocity and acceleration, $\ddot{x}$ is the acceleration of the masses relative to the ground and $[M]$ is the matrix of building masses. The total energy dissipated by viscous damping is given by

$$E_D = \int_0^t \{\dot{x}\}^T [C]\{\dot{x}\}dt \qquad (2)$$

where $\dot{x}$ is the velocity of the masses relative to the ground and $[C]$ is the matrix of viscous damping coefficients. The kinetic energy at time t is given by

$$KE(t) = \tfrac{1}{2}\{\dot{x} + \dot{z}\}^T [M]\{\dot{x} + \dot{z}\} \qquad (3)$$

and the strain energy at time t is given by

$$PE(t) = \tfrac{1}{2}\{x\}^T [K]\{x\} \qquad (4)$$

where $[K]$ is the stiffness matrix of the structural system and $\{x\}$ is the displacement relative to the ground.

Table 1. Modal Period and Damping of a Ten-Story Building.

Damping Coefficient c (kips-sec/inch)	Mode 1 (T_n = 1.845 sec) ζ %	Mode 2 (T_n = 0.603 sec) ζ %	Mode 3 (T_n = 0.323 sec) ζ %	Mode 4 (T_n = 0.225 sec) ζ %	Mode 5 (T_n = 0.172 sec) ζ %
1.952	2	4.77	6.75	9.16	11.42
4.881	5	11.92	16.87	22.89	28.55
9.761	10	23.84	33.74	45.78	57.09
9.0283[a]	10	25.87	36.52	47.65	57.32
8.6998[b]	10	24.48	34.47	45.07	54.49
	T_n = 1.788[b]	T_n = 0.567[b]	T_n = 0.307[b]	T_n = 0.215[b]	T_n = 0.167[b]

[a] Including damping in the first story.

[b] Including damping and stiffness in the first story.

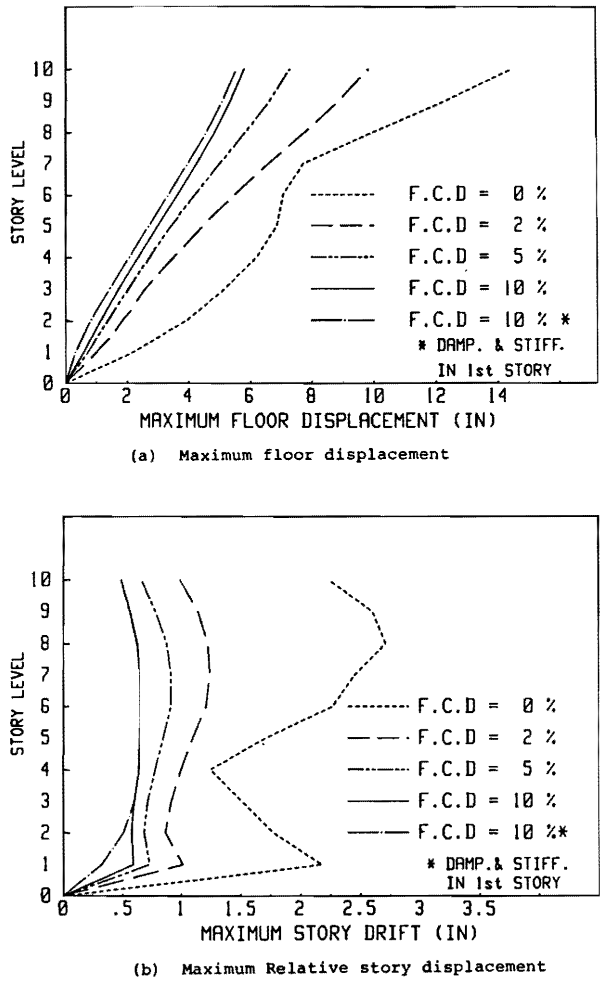


Figure 2. Response of Ten Story Building to El Centro Accelerogram.
1 in = 25.4 mm

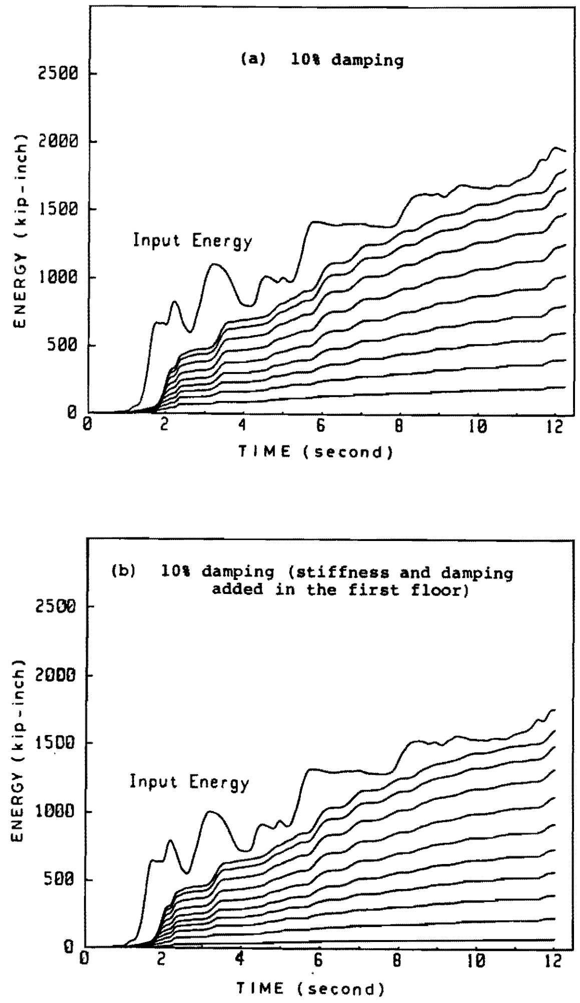


Figure 3. Input and Dissipated Energy Time History (El Centro).
1 kip-in = 113J

The energy components for 10% damping in the first mode including the history of the energy dissipated at each story are shown in Figure 3*a*. Because the frame has an open first story and damping was not added there, the dissipated energy was zero in the first story. It is interesting to note that the energy input increased as damping was increased [1]. However, the associated increases in dissipated energy resulted in a reduction in stored energy. By adding damping and stiffness (brace element of 3.38 in^2 (21.8 cm^2) area) in the first story while keeping the fraction of critical damping at 10 percent in the fundamental mode, the input energy is reduced slightly, Figure 3*b*. The cumulative dissipated energy profile at the end of the 12th second is shown in Figure 4 as a function of story level for three cases of damping and for the case of additional stiffness and damping in the first level. As can be seen, the dissipated energy increases as damping increases and for the case of additional damping and stiffness in the first story for 10 percent fundamental modal damping the dissipated energy was less than the case of open first story for the same damping.

MODAL ANALYSIS OF STRUCTURE WITH SUPPLEMENTAL DAMPING

The elastic response of a multistory building vibrating as a base excited structure can be analyzed by direct integration of the equations of motion in original coordinates. However, for large systems the solution of the simultaneous equations is a large computational task. Even with a very powerful computer, the complete dynamic analysis can be quite time consuming because the set of equations must be solved for each time increment, and it may be necessary to consider as many as 100 or 200 time increments of each second of computed dynamic response. In general the time increment should not be larger than one-tenth of the period of vibration of the highest significant vibration mode [16].

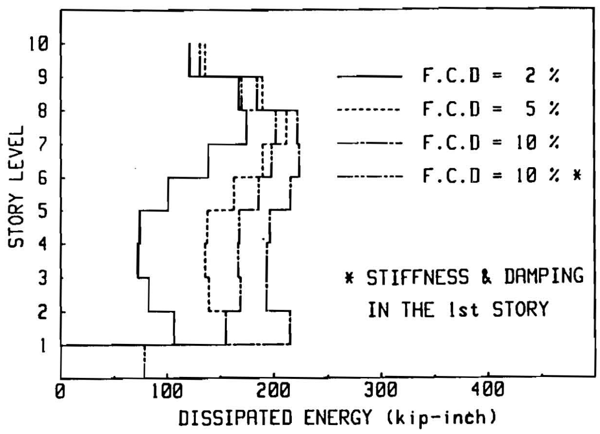


Figure 4. Dissipated Energy Profile (El Centro).
1 kip-in = 113J

A practical alternative is the method of mode superposition. The reliability of this method depends on the characteristics of the damping introduced in the equations of motion [17]. When damping is proportional to mass and stiffness it can be uncoupled with the mass and stiffness in the equations of motion and be represented as fractions in each mode. Use of proportional damping results in a series of single degree of freedom equations of motion which are easily solved. The solutions of each equation are combined by superposition. Another major advantage of modal superposition is that the essential dynamic response of a multidegree of freedom system often is associated with the lowest modal coordinates, which means that a good approximation to the response can often be obtained with a drastically reduced number of coordinates which significantly reduces the effort of computation [18].

Problem Definition

Although supplemental damping devices can provide a significant amount of damping, it is not practical to install supplemental dampers throughout the building on the basis of mass and/or stiffness proportionality. Because of the simplicity of using proportional damping for response computation, it is important to evaluate the accuracy of assuming proportional damping for the dynamic response of these buildings as a substitution to the real coupled damping distribution.

Analysis Procedure

The elastic response of multistory buildings subjected to earthquake input motion can be determined using several approaches. These approaches differ in their degree of complexity and approximation to the actual response. The selection of an approach depends on the accuracy level of evaluating the response and on the facility available to perform the mathematical computation of that approach. In this study the response of the structure was determined by using these four approaches:

1. *Direct Integration.* The general method of computing the response of any dynamic system with any pattern of damping is by integrating directly the equations of motion expressed in the physical coordinates. This method has the disadvantage of including all the coordinates in the analysis and so the computational effort increases substantially with the increase in the degrees of freedom. The equations of motion for an elastic multidegree of freedom system are given as:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = -[M]\{1\}\ddot{z}(t). \tag{5}$$

The solution can be obtained by simultaneous direct numerical integration in the original coordinates. The most common numerical integration technique is the Newmark-β method [15].

2. *Mode Superposition Using Undamped Mode Shapes.* By transforming the equations of motion from original coordinates to the modal coordinates, the modal equations are:

$$\ddot{q}_n + 2\xi_n\omega_n\dot{q}_n + \omega_n^2 q_n = -\Gamma_n\ddot{z}(t) \tag{6}$$

where

$$\{x\} = [\phi]\{q\} \tag{7}$$

ξ_n is the fraction of critical damping, ω_n is the undamped natural frequency, Γ_n is the modal participation in the nth mode and $[\phi]$ are the n mode shapes included in the solution. The response can be obtained by integrating the modal equations independently with instantaneous combinations of the contribution of each mode using Equation 7. It should be noted that the number of modal equations to be integrated does not have to equal the total number of the degrees of freedom. An acceptable solution may require only a few modes.

3. *Response Spectra.* The response spectrum approach is a powerful technique to determine an approximate response of a multidegree of freedom system to earthquake input. The generalized coordinate vector is given as:

$$\{q\} = SD\,\{\Gamma\} \tag{8}$$

where SD are the maximum relative displacement response spectral values. The total response can be approximated by using sum of the absolute or the square root of the sum of square procedures [1].

4. *Direct Integration of the Coupled Modal Equations.* When modes are coupled due to the nonproportionality of damping, the damping can be included in the system as a truncated matrix rather than an assumed diagonal modal damping fraction [19]. For n degrees of freedom, the equations of motion in the original coordinates are given in Equation 5. In modal form Equation 6 becomes

$$[\ddot{q}] + [c_n]\{\dot{q}\} + [\omega_n^2]\{q\} = -\{\Gamma\}\ddot{z} \tag{9}$$

where

$$c_{n_{i,j}} = \frac{\{\phi_i\}^T[C]\{\phi\}_j}{\{\phi_i\}^T[M]\{\phi_i\}} \tag{10}$$

The order of the $[c_n]$ matrix and the number of equations is the same as the number of modes being used. $[c_n]$ is not a diagonal matrix so dynamic coupling of modal equations occurs. The truncated coupled modal equations, Equation 9, are solved by simultaneous direct integration and the response in the original coordinates is obtained by transformation using Equation 7. The advantage of this method is that the off-diagonal terms in the effective damping matrix are retained and the number of the equations to be integrated simultaneously is reduced, resulting in less computational effort.

Numerical Example

The structure described earlier was analyzed to determine the effects of nonproportional damping on the maximum story drift. The maximum story drifts were calculated for El Centro 1940 NS accelerogram using Newmark-β method for numerical integration of the equations of motion in the original coordinates. First mode damping is chosen to be 20 percent of critical to study the reliability of several approaches to estimate the actual response of structures with high damping. The damping distribution is nonproportional and there is a discontinuity in the damping distribution in the first story.

The maximum relative story displacements shown in Figure 5*a* demonstrate that coupled modal integration [19] represents exactly the actual response even for high damping conditions. The sum of absolute relative displacement gives an upper bound for the exact solution. When damping is added in the first story while keeping 20 percent critical damping in the fundamental mode, the uncoupled modal solution underestimates the actual solution by a maximum of 8 percent in the first level, Figure 5*b*. When damping and stiffness are added in the first story by adding a brace element with an area equal to the area of brace element in the second story, the response is more evenly distributed as shown in Figure 5*c* for 20 percent damping. It is worth to mention that only five modes were included with SRSS, sum of absolute, coupled and uncoupled modal solutions.

ALTERNATIVE PATTERNS OF DAMPING INSTALLATION

Supplemental dampers can be connected between adjacent floor levels as discussed above and illustrated in Figure 6*a*. However, there are other possible patterns for connecting dampers. [20, 21]. Figure 6*b* illustrates a two story interconnection pattern and Figure 6*c* illustrates a three story interconnection pattern. Any selected damping pattern must satisfy physical practicality and architectural considerations. These three configurations are designated as case 1, case 2, and case 3 for the following comparisons. In terms of the damping matrix, the system in case 1 is simply coupled giving a tridiagonal damping matrix as:

$$\begin{bmatrix} C_{10}+C_{21} & -C_{21} & 0 & \cdots \\ -C_{21} & C_{21}+C_{32} & -C_{32} & \cdots \\ 0 & -C_{32} & C_{32}+C_{43} & \cdots \\ \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \end{bmatrix} \qquad (11)$$

For the two story interconnection the damping system is not closely coupled:

$$\begin{bmatrix} C_{10}+C_{31} & 0 & -C_{31} & 0 & \\ 0 & C_{20}+C_{42} & 0 & -C_{42} & \\ -C_{31} & 0 & C_{53}+C_{31} & 0 & \cdots \\ 0 & -C_{42} & 0 & C_{42}+C_{64} & \\ & & \cdot & & \\ & & \cdot & & \\ & & \cdot & & \end{bmatrix} \qquad (12)$$

For the three story interconnection the damping matrix is:

$$\begin{bmatrix} C_{10}+C_{41} & 0 & 0 & -C_{41} & 0 & \\ 0 & C_{20}+C_{52} & 0 & 0 & -C_{52} & \cdots \\ 0 & 0 & C_{30}+C_{63} & 0 & 0 & \\ -C_{41} & 0 & 0 & C_{41}+C_{74} & 0 & \\ 0 & -C_{52} & 0 & 0 & C_{52}+C_{58} & \\ & & \cdot & & & \\ & & \cdot & & & \\ & & \cdot & & & \end{bmatrix} \qquad (13)$$

MAXIMUM STORY DRIFT (IN)
(a) 20% damping (open first story)

MAXIMUM STORY DRIFT (IN)
(b) 20% damping (damping in first story)

MAXIMUM STORY DRIFT (IN)
(c) 20% damping (damping and stiffness in first story)

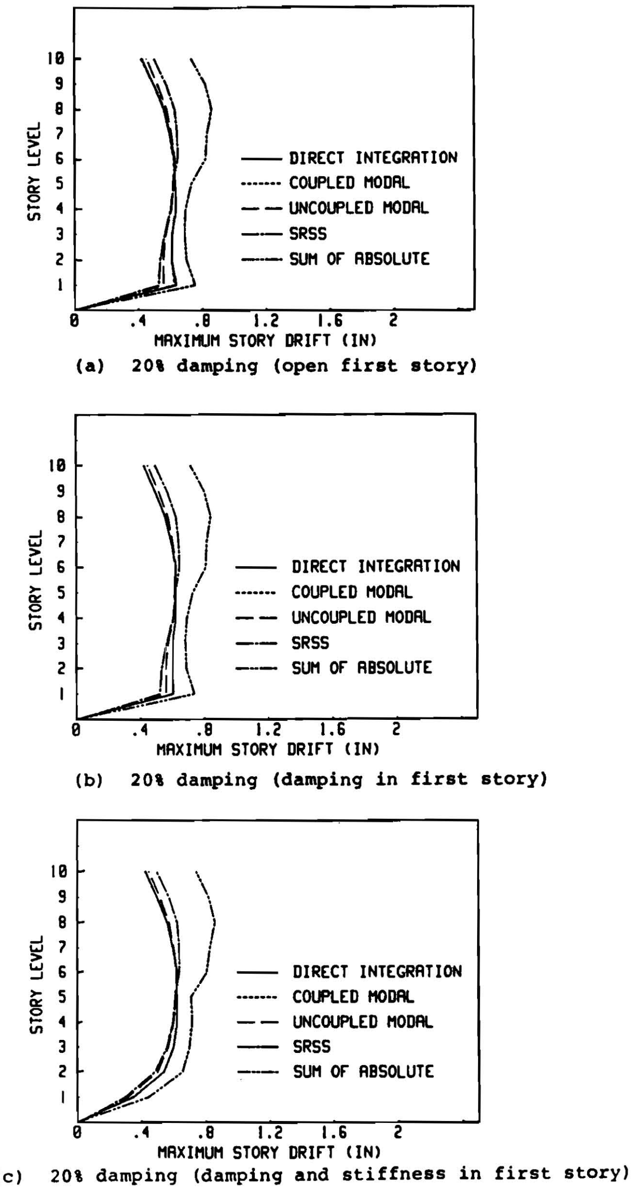


Figure 5. Relative Story Displacement by Modal Analysis.
1 in = 25.4 mm

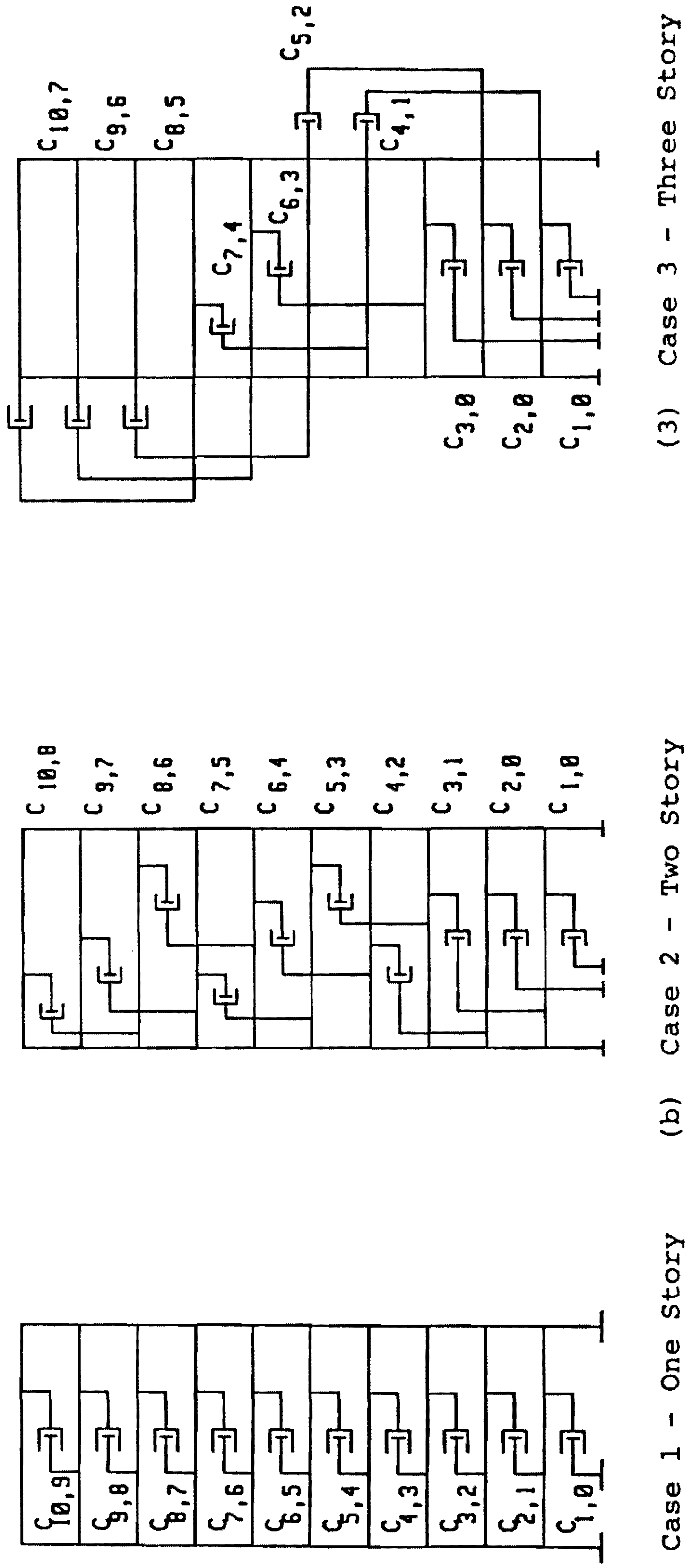


Figure 6a. Patterns of Damping Installation.

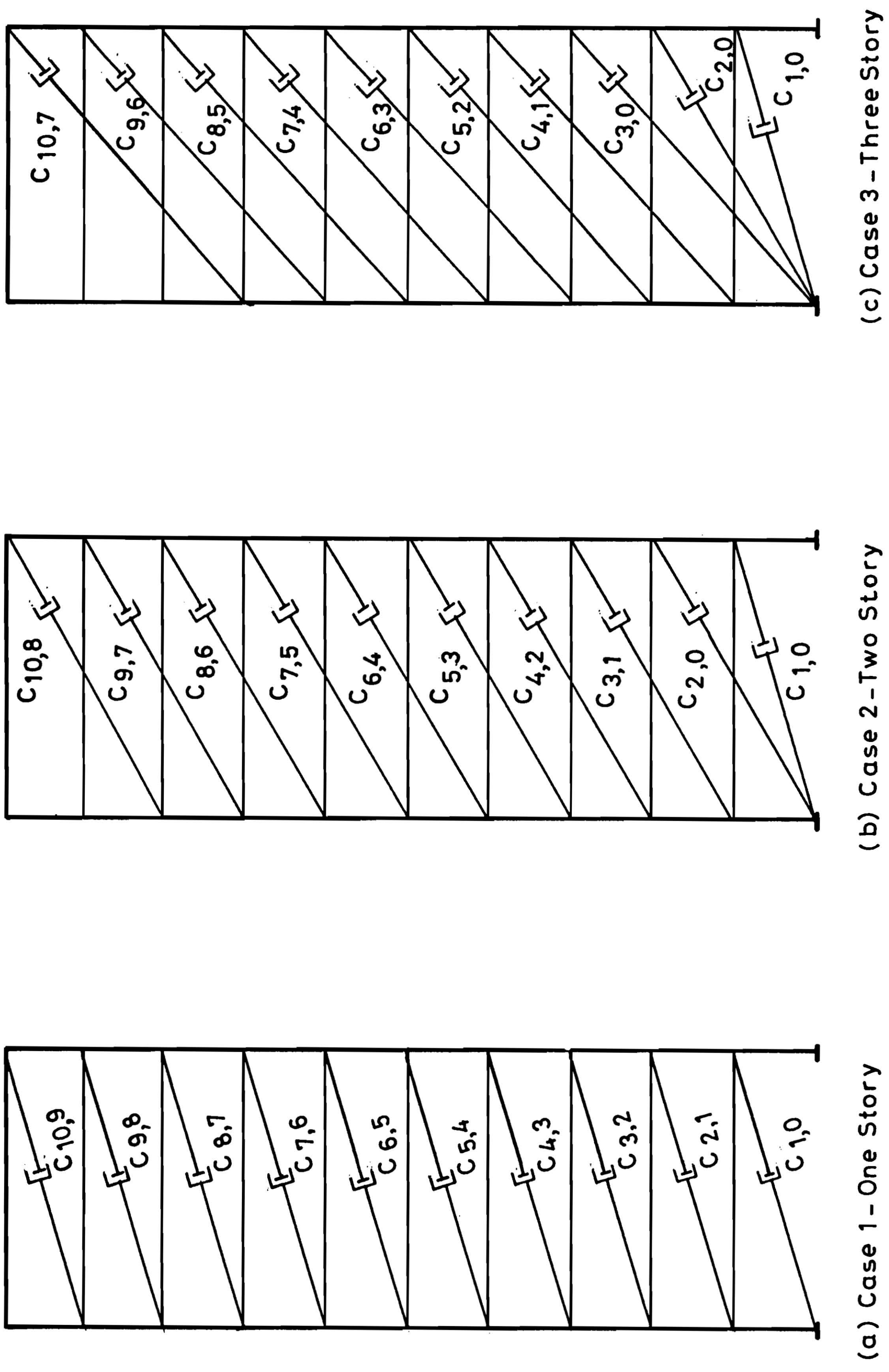
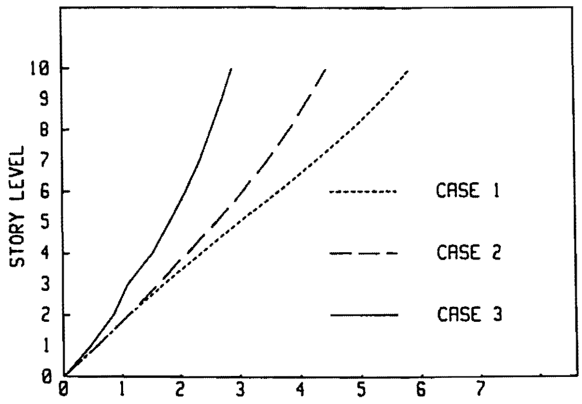


Figure 6b. Dampers as Part of the Bracing Elements.

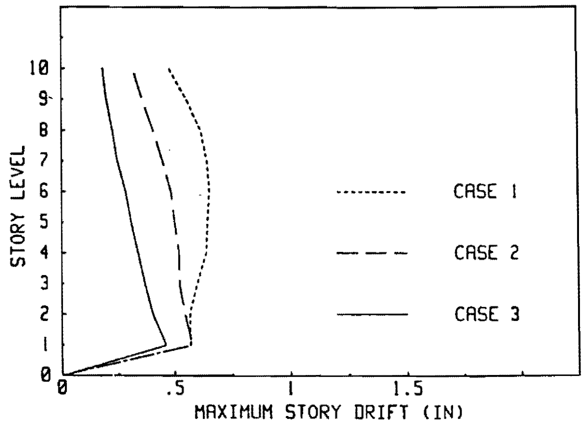
Equal damping coefficients $C = 9.03$ kip-sec/in (1580 N-sec/mm) were taken for each damper in all three cases. These values resulted in a first mode damping of 10 percent for case 1 which provided a basis for comparison, first mode damping of 37.9 percent for case 2, and first mode damping of 80 percent for case 3. The calculated maximum floor displacements and maximum relative story displacements are shown in Figure 7 for the three cases. It can be seen that the pattern of damping installation in case 3 reduces the response significantly compared with case 1. This was expected because case 3 has eight times as much first mode damping.

The time history of the input energy and dissipated energy at each level was evaluated for the three cases. The input energy for case 3 was found to be higher than for case 2 or case 1, however, the stored energy was the lowest. The dissipated energy profile at the end of the 12th second is shown in Figure 8. Much of the dissipated energy is concentrated in dampers between the base and third level for case 3. The dissipated energy profile for case 1 is much more uniform than the other cases and is largest of all cases in the upper levels.

The installations given by cases 2 and 3 subject the dampers to larger displacement than for case 1. This larger displacement is a consequence of the multistory installations. The maximum relative horizontal displacement across the dampers is shown in Figure 9. It can be seen that the damper horizontal displacement for case 1 is the lowest except at the first level and the maximum occurred in the damper connecting level three and base for case 3. Higher displacements



(a) Maximum Floor displacement.



(b) Maximum Relative story displacement.

Figure 7. Response for Three Cases of Damping Installation (El Centro).
1 in = 25.4 mm

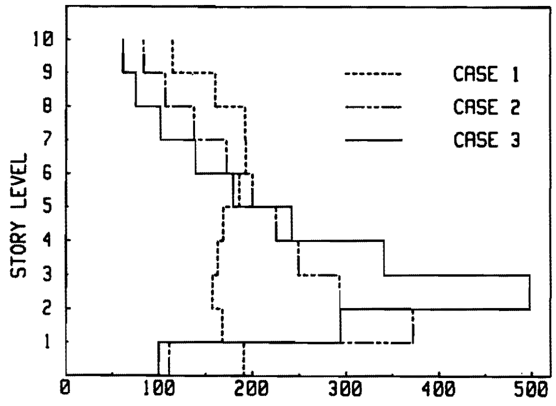


Figure 8. Dissipated Profile for Three Cases of Damping Installation (El Centro).
1 kip-in = 113J

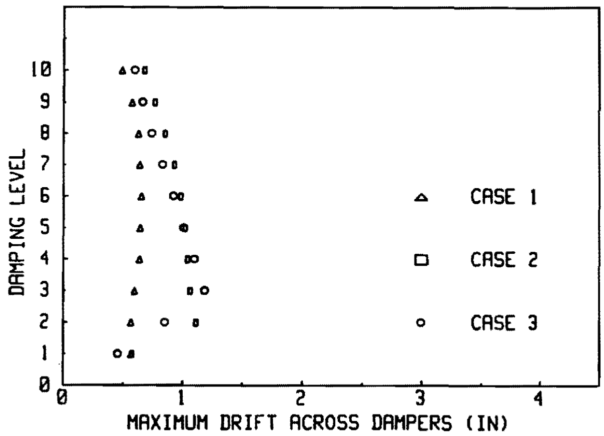


Figure 9. Maximum Displacement Across Dampers for Three Cases of Damping Installation (El Centro).
1 in = 25.4 mm

should be considered in the design of the dampers for such installations.

Using the same three patterns of installation the structure was subjected to Taft 1952 N69W accelerogram. The damping was set to give 5 percent fundamental modal damping for case 1, resulting in 18.9 percent for case 2, and 40 percent for case 3. The maximum floor displacements and maximum relative story displacements are shown in Figure 10. The most effective installation is obtained by using pattern 3. The dissipated energy profile at the end of 15 seconds is shown in Figure 11 for the three cases. The same trend of energy dissipation was observed for the El Centro excitation (Figure 8). For the Taft excitation the maximum relative displacements across the damper are smaller than for the El Centro excitation for all three cases (Figure 12) and there was small difference between the three cases.

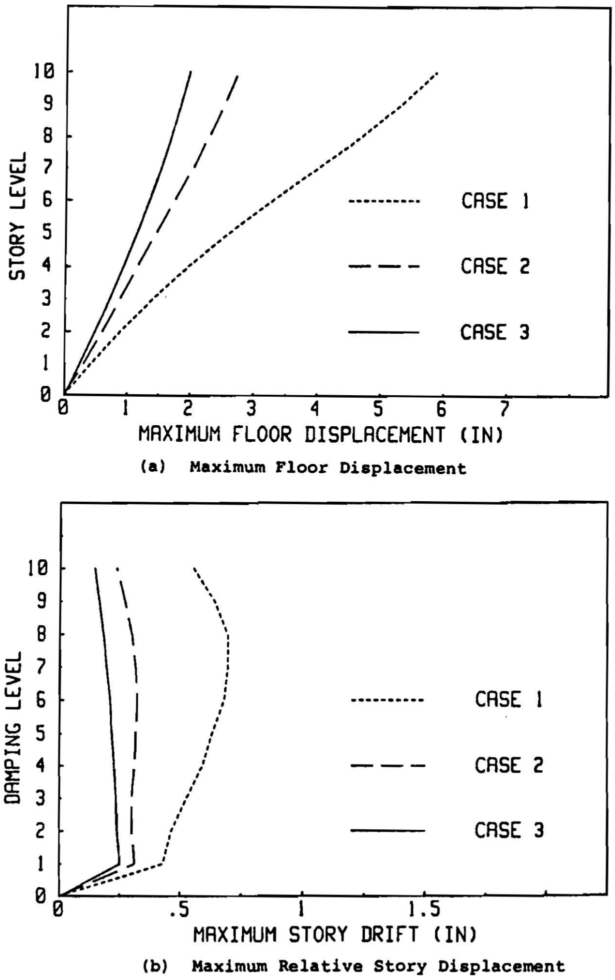


Figure 10. Response for Three Cases of Damping Installation (Taft).
1 in = 25.4 mm.

The feasibility of using the above mentioned patterns of dampers distributions depends upon story height, building configuration, slenderness ratio of the element holding the damper so that in plane and/or out of plane buckling will not be a problem.

The difference in the behavior of the building due to the two accelerograms can be explained by considering the response spectra of the two accelerograms, El Centro and Taft as shown in Figure 13 for several damping cases. Each curve represents the contour of the displacement for a constant damping. The damping was expressed in terms of normalized viscous damping coefficient, d, rather than in terms of the fraction of critical damping ξ since ξ is a function of the stiffness of the system. The mass was taken as a

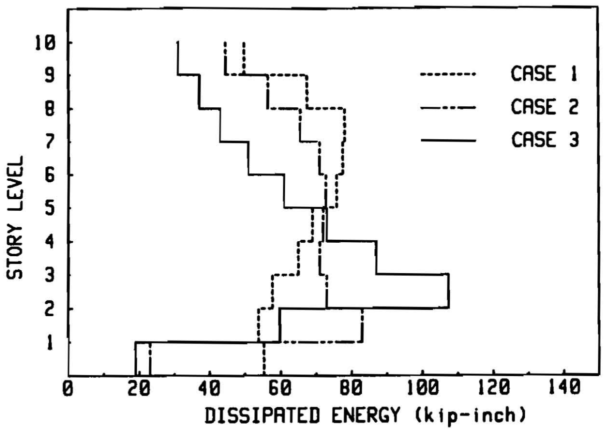


Figure 11. Dissipated Energy Profile for Three Cases of Damping Installation (Taft).
1 kip-in = 113J

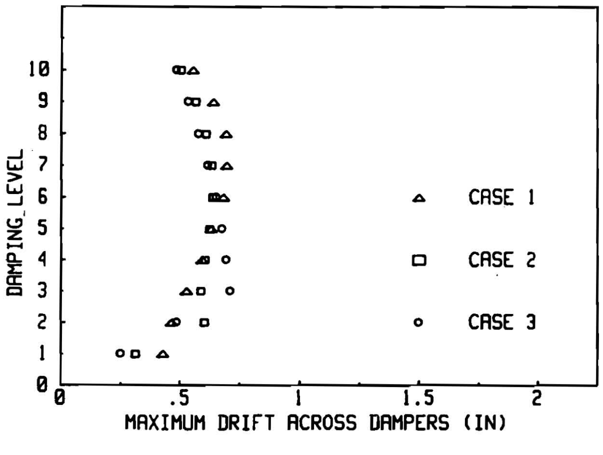


Figure 12. Maximum Displacement Across Dampers for Three Cases of Damping Installation (Taft).
1 in = 25.4 mm

constant. Considering a multidegree of freedom system, the first mode normalized stiffness k_1 is given as:

$$k_1 = \frac{\{\phi_1\}^T[K]\{\phi_1\}}{\{\phi_1\}^T[M]\{\phi_1\}} = \omega_1^2 \tag{14}$$

or

$$k_1 = (2\pi f_1)^2 = \omega_1^2 \tag{15}$$

The first mode normalized damping d_1 can be expressed as:

$$d_1 = \frac{\{\phi_1\}^T[C]\{\phi_1\}}{\{\phi_1\}^T[M]\{\phi_1\}} = 2\omega_1\xi_1 \tag{16}$$

or

$$d_1 = 4\pi f_1 \xi_1 \tag{17}$$

where d_1 is the normalized damping in radians per sec, f_1 is first mode frequency in Hertz and ξ_1, is the first mode fraction of critical damping. The normalized damping and normalized stiffness (modal frequency squared) for the ten story building are given in Table 2. Consider the fundamental mode only, since it is the dominant mode in the response, and the response spectra in Figure 13 with the dynamic parameters (d_i and f_i). For El Centro changes in damping from case 1 to case 3 is about a factor of 2 while for Taft it is about 2.7. This is a consequence of the lower case 1 damping for Taft and of the spectral frequency and damping characteristics, Figure 13. At some frequencies and damping levels a doubling of the damping results in little change in response while at other damping and frequencies the change in response is significant. It is important to recognize that a three level connection for the dampers will not result in required damper displacement capabilities of three times the single story connection requirements.

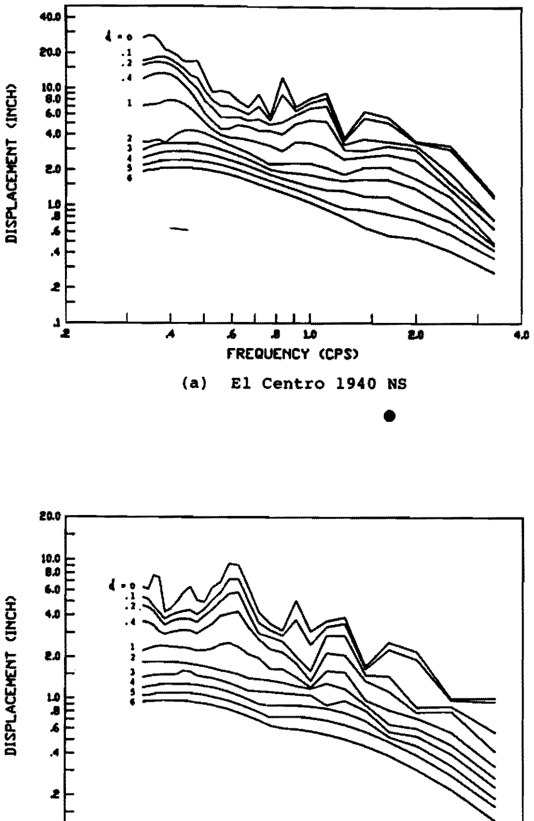


Figure 13. Response Spectra for Various Amounts of Damping.
1 in = 25.4 mm

CONCLUSIONS

Although this study focused on one structure and two earthquake excitations, other structures and excitations were included in studies reported in reference [1]. On the basis of these results the following conclusions can be made.

1. Use of supplemental damping in buildings is an effective way to limit structural response to earthquakes. This concentrates energy dissipation in manufactured damping devices rather than requiring energy to be dissipated through inelastic demands of the building elements.

2. Nonproportional damping can be treated as proportional damping in modal superposition calculations provided that the damping distribution is continuous. Integration of the reduced number of coupled modal equations is the most effective procedure when a structure has highly nonproportional damping.

3 The minimum number of coupled modal equations to be integrated simultaneously for solution depends upon the modal participation factors and continuity of damping distribution.

Table 2. Normalized Damping for Ten Story Building.

	El Centro								
	Case 1			Case 2			Case 3		
	ζ%	d_i (1/sec)	f_i (Hz)	ζ%	d_i (1/sec)	f_i (Hz)	ζ%	d_i (1/sec)	f_i (Hz)
Mode 1	10	0.681	0.54	37.9	2.58	0.54	80	5.45	0.54
Mode 2	25.8	5.39	1.66	90	18.77	1.66	168.3	35.1	1.66
Mode 3	36.5	14.22	3.1	114	44.5	3.1	175.3	68.3	3.1

	TAFT					
	Case 1		Case 2		Case 3	
	ζ%	d_i (1/sec)	ζ%	d_i (1/sec)	ζ%	d_i (1/sec)
Mode 1	5	0.341	18.9	1.28	40	2.73
Mode 2	12.9	2.69	45	9.38	84.2	17.5
Mode 3	18.26	7.16	57.1	22.2	87.6	34.1

3 The minimum number of coupled modal equations to be integrated simultaneously for solution depends upon the modal participation factors and continuity of damping distribution.

4. The required number of modes to be considered in modal superposition increases as nonproportionality of damping increases.

5. Alternative patterns of damping installation showed an increase in the efficiency of supplemental damping in a structure. All possible patterns have not been investigated. The selection of a specific pattern depends on physical practicality and architectural considerations.

6. The use of the response spectra in terms of either fraction of critical damping or normalized viscous damping coefficient provides insight to the behavior of structure and gives a better understanding of the overall response.

NOTATION

- $[C]$ Damping matrix in multidegree of freedom system.
- d_i Normalized viscous damping coefficient in mode i.
- f_i Frequency, Hertz.
- k_i Normalized mode i stiffness, ω_i^2.
- $[K]$ Stiffness matrix in multidegree of freedom system.
- $[M]$ Mass matrix in multidegree of freedom system.
- n Number of degrees of freedom or number of modes.
- q Generalized coordinates.
- SD Maximum displacement response.
- t Time variable.
- T Natural period, seconds.
- $x, \dot{x}, \ddot{x}$ Relative displacement, velocity, and acceleration.
- $\dot{z}, \ddot{z}$ Ground velocity and acceleration.
- Γ Modal participation factor.
- ξ Fraction of critical damping.
- ϕ Mode shape.
- ω_i Undamped modal natural frequency, rad/sec.

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