ON *I*-COMPACT SUBSETS AND *I*C-CONTINUOUS FUNCTIONS

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الخلاصة :

يهدف هذا البحث إلى تقديم صفات وخواص جديدة لمفهوم المجموعات المصمتة من النوع (*I*) التي أدخلها نيوكمب Newcomb عامً ١٩٦٧ والتي تَمَّ دراستها أخيرا في عام ١٩٩٠ بواسطة كلٍّ من : هاملت Hamlett وروز Rose وجانكوفيش Janković واعتماداً على هذا المفهوم السابق استنبط الباحثون أحد أنواع الدوال المتصلة والمسماة *IC-continuous* ، والتي تشمل صفاً من الاتصال موجود في مايُسمىٰ *H*-continuity وتم استنتاج العديد من خواص وصفات هذا المُعرَّف الجديد بالاضافة الى العلاقات التي تربطه بمختلف الأنواع الأُخرى .

ABSTRACT

The concept of compactness modulo an ideal was first introduced by Newcomb in 1967 and investigated by Hamlett, Rose, and Janković in 1990. In this paper we give some new characterizations and properties of *I*-compact subsets. By using this notion, we introduce a new class of functions, called *I*C-continuous functions, which contains the class of continuity and is contained in the class of *H*-continuity. Some characterizations and several properties of this new type of function are presented. Relationships between *I*C-continuity and other corresponding notions are studied.

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1. INTRODUCTION AND PRELIMINARIES

Throughout the present paper (X, τ) and (Y, σ) (or simply X and Y) are topological spaces on which no separation axioms are assumed unless explicitly stated. The closure (resp. interior) of any subset S of X will be denoted by $Cl(S)$ (resp. $Int(S)$). First recall that a set $S \subseteq (X, \tau)$ is regular open [1] if $S = Int(Cl(S))$. A subset S of (X, τ) is said to be semi-open [2] (resp. α-open [3], preopen [4]) if $S \subset Cl(Int(S))$ (resp. $S \subset Int(Cl(Int(S)))$, $S \subset Int(Cl(S))$). The complement of a semi-open set is called semi-closed [5] and the intersection of all semi-closed sets containing S is called the semi-closure [5] of S and denoted by $s\text{-}Cl(S)$. $S \subset X$ is said to be regular semi-open [6] if there exists a regular open set U such that $U \subset S \subset Cl(U)$. A subset S of a space (X, τ) is called locally closed [7] if $S = U \cap F$, where $U \in \tau$ and F is closed in (X, τ). A subset S of a space (X, τ) is said to be an H-set [8] or quasi H-closed (QHC) [9] (resp. N-closed [10], S-closed [11], s-closed [12], RS-compact [13]) relative to X if for every cover $\{V_\alpha | \alpha \in \Delta\}$ of S by open (resp. open, semi-open, semi-open, regular semi-open) sets of X there exists a finite subset Δ_o of Δ such that $S \subset \cup\{Cl(V_\alpha) | \alpha \in \Delta_o\}$ (resp. $S \subset \cup\{Int(Cl(V_\alpha)) | \alpha \in \Delta_o\}$, $S \subset \cup \{Cl(V_\alpha) | \alpha \in \Delta_o\}$, $S \subset \cup \{s\text{-}Cl(V_\alpha)) | \alpha \in \Delta_o\}$, $S \subset \cup \{Int(V_\alpha) | \alpha \in \Delta_o\}$). A space (X, τ) is said to be quasi H-closed, abbreviated QHC, iff any open cover of X has a finite subfamily, the closures of whose members cover X. A space is said to be H-closed iff it is Hausdorff and QHC. A space X is said to be extremely disconnected (abbreviated as ED) if the closure of every open set in X is open. A function $f: (X, \tau) \to (Y, \sigma)$ is called precontinuous [4] (resp. LC-continuous [14]) if the inverse image under f of each open set in Y is preopen (resp. locally closed) in X. $f: (X, \tau) \to (Y, \sigma)$ is called C-continuous [15] (resp. H-continuous [16], N-continuous [17], L-continuous [18], S-continuous [19], s-continuous [20], RS-continuous [21]) if for each $x \in X$ and each open set $V \subset Y$ containing $f(x)$ having compact (resp. H-closed, N-closed, Lindelöf, S-closed, s-closed, RS-compact) complement, there exists $U \in \tau$ containing x such that $f(U) \subset V$. Given a set X, a collection I of subsets of X is called an ideal [22] on X if the following hold:

1. If $A \in I$ and $B \subseteq A$, then $B \in I$ (heredity), and
2. If $A \in I$ and $B \in I$, then $A \cup B \in I$ (finite additivity).

An ideal is called a σ-ideal if the following holds:

3. If $\{A_n | n = 1,2,3,......\}$ is a countable subcollection of I, then $\cup\{A_n | n = 1,2,3,......\} \in I$ (countable additivity).

If $X \notin I$ then I is said to be a proper ideal, if $X \in I$ then I is called an improper ideal. If I is a proper ideal then $\{A | X - A \in I\}$ is a filter. Hence proper ideals are sometimes called dual filters. The notation (X, τ, I) denotes a nonempty set X, a topology τ on X, and an ideal I on X. If (X, τ, I) is a space, we denote by $\tau^*(I)$ the topology on X generated by the basis $\beta(I, \tau) = \{U - E | U \in \tau, E \in I\}$ [23]. The closure operator in $\tau^*(I)$, denoted by Cl^*, can be described as follows: For $A \subseteq X$, $Cl^*(A) = A \cup A^*(I, \tau)$ where $A^*(I, \tau) = \{x \in X | U_x \cap A \notin I$ for every $U_x \in N(x)\}$, where $N(x) = \{U \in \tau, x \in U\}$. $A^*(I, \tau)$ is called the local function of I with respect to τ on A [24]. Recall that $A \subseteq (X, \tau, I)$ is $\tau^*(I)$-closed if $A^*(I) \subseteq A$ [25].

2. ON *I*-COMPACT SUBSETS

Compactness with respect to an ideal (I-compactness) has been studied in references [26–28]. In this article, we introduce some results about I-compact subsets relative to a space.

Definition 2.1. [27] A subset S of a space (X, τ, I) is said to be I-compact relative to X if for every cover $\{V_\alpha | \alpha \in \Delta\}$ of S by open sets in X there exists a finite subfamily $\{V_{\alpha_i} | i = 1,2,3,......, n\}$ such that $S\text{-}\cup\{V_{\alpha_i} | i = 1,2,3,......, n\} \in I$.

Remark 2.1. One can easily verify that for a subset S of a space (X, τ, I), the following are equivalent:

(*i*) S is a compact subset.
(*ii*) S is a $\{\phi\}$-compact subset relative to X.
(*iii*) S is an I_f-compact subset relative to X, where I_f denotes the ideal of finite subsets.

Theorem 2.1. If S_i, $i = 1, 2$ are I-compact subsets relative to a space (X, τ, I), then $S_1 \cup S_2$ is an I-compact subset relative to X.

Proof. Straightforward.

The following result is an immediate corollary.

Corollary 2.1. The intersection of two open sets having I-compact complement is also open having I-compact complement.

Theorem 2.2. A subset S of a space (X,τ,I) is I-compact relative to X iff for every cover $\{V_\alpha|\alpha\in\Delta\}$ of S by preopen and locally closed sets in X there exists a finite subfamily $\{V_{\alpha_i}|i=1,2,3,......,n\}$ such that $S\text{-}\cup\{V_{\alpha_i}|i=1,2,3,......,n\}\in I$.

Proof. Follows immediately from Definition 2.1 and Theorem (2) of [14].

Corollary 2.2. A subset S of a space (X,τ,I) is I-compact relative to X iff for every cover $\{V_\alpha|\alpha\in\Delta\}$ of S by α-open and locally closed sets in X there exists a finite subfamily $\{V_{\alpha_i}|i=1,2,3,......,n\}$ such that $S\text{-}\cup\{V_{\alpha_i}|i=1,2,3,......,n\}\in I$.

Theorem 2.3. Let (X,τ,I) be a space, and let $S\subseteq X$ be I-compact. Then for every cover $\{V_\alpha|\alpha\in\Delta\}$ of S by preopen sets (resp. regular open sets) of X there exists a finite subfamily $\{V_{\alpha_i}|i=1,2,3,......,n\}$ such that $S\text{-}\cup\{Int(Cl(V_{\alpha_i}))|i=1,2,3,......,n\}\in I$ (resp. $S\text{-}\cup\{V_{\alpha_i}|i=1,2,3,......,n\}\in I$.

Proof. This is obvious.

Theorem 2.4. Let (X,τ,I) be a space, and J be an ideal on X with $I\subseteq J$. If S is an I-compact subset relative to (X,τ,I), then S is a J-compact subset relative to (X,τ,J).

In [27], Newcomb defines an ideal I on a space (X,τ) to be τ-boundary if $\tau\cap I=\{\phi\}$.

Theorem 2.5. Let (X,τ,I) be a space. If $S\subseteq X$ is an I-compact subset relative to X and I is τ-boundary, then S is an H-subset relative to X.

Proof. Assume that $\{U_\alpha|\alpha\in\Delta\}$ is a cover of S by open sets of X. By hypothesis there exists a finite subfamily $\{U_{\alpha_i}|i=1,2,......,n\}$ such that $S\text{-}\cup\{U_{\alpha_i}|i=1,2,3,......,n\}=E\in I$, since I is τ-boundary. Then $Int(E)=\varnothing$ and hence S is an H-subset relative to X.

The proofs of the next two results are straightforward.

Theorem 2.6. Let (X,τ,I) be a space with I_n the ideal of nowhere dense subsets of X ($S\in I_n$ iff $Int(Cl(S))=\varnothing$). If $I_n\subseteq I$ and S is an H-subset relative to X, then S is an I-compact subset relative to X.

Theorem 2.7. A subset S of a space (X,τ) is an I_n-compact subset relative to X iff S is an H-subset relative to X.

Corollary 2.3. Let X be ED. Then for a subset S of X the following are equivalent:

(*i*) S is I_n-compact relative to X.
(*ii*) S is quasi H-closed relative to X.
(*iii*) S is S-closed relative to X.
(*iv*) S is N-closed relative to X.
(*v*) S is RS-compact relative to X.

Proof. Follows directly from Theorem 2.7 and Lemma 4.2 of [13].

Lemma 2.1. [27] Let (X,τ,I) be a space. If $S\subseteq X$, then S is an I-compact subset of (X,τ) iff S is an I-compact subset of $(X,\tau^*(I))$.

The following result is useful in studying the preservation of I-compactness by certain types of functions.

Lemma 2.2. [26] Let $f\colon(X,\tau,I)\rightarrow(Y,\sigma)$ be a function. Then

$$f(I)=\{f(E)|E\in I\} \text{ is an ideal on } Y.$$

A bijection $f\colon(X,\tau,I)\rightarrow(Y,\sigma)$ is called a *-homeomorphism [29] with respect to τ, I and σ if $f\colon(X,\tau^*(I))\rightarrow(Y,\sigma^*(f(I)))$ is a homeomorphism.

Theorem 2.8. Let $f\colon(X,\tau,I)\rightarrow(Y,\sigma,f(I))$ be a *-homeomorphism. Then $G\subseteq X$ is I-compact relative to X iff $f(G)$ is $f(I)$-compact relative to Y.

Proof. Follows directly from Lemma 2.1 and Lemma 2.2.

Theorem 2.9. Let $f\colon(X,\tau,I)\rightarrow(Y,\sigma)$ be a precontinuous and LC-continuous surjection. If $G\subseteq X$ is an I-compact subset relative to X, then $f(G)$ is $f(I)$-compact relative to Y.

Proof. This follows from Theorem 4 (*iv*) of reference [14] and Theorem 2.2 of reference [26].

Lemma 2.3. [26] If $f\colon(X,\tau)\rightarrow(Y,\sigma,I)$ is an injection, then $f^{-1}(I)$ is an ideal on X.

Theorem 2.10. Let $f\colon(X,\tau)\rightarrow(Y,\sigma,I)$ be an open bijection. If G is an I-compact subset relative to Y, then $f^{-1}(G)$ is $f^{-1}(I)$-compact relative to X.

3. *I*C-CONTINUOUS FUNCTIONS

In this article, we define the class of IC-continuity as a generalization of C-continuity, H-continuity, and N-continuity. Some characterizations and properties of this concept are obtained.

Definition 3.1. A function $f\colon(X,\tau)\rightarrow(Y,\sigma,I)$ is said to be an IC-continuous function if, for each $x\in X$ and each open set $V\subset Y$ containing $f(x)$ and having

I-compact complement, there exists an open set $U \subset X$ containing x such that $f(U) \subset V$.

We now offer several characterizations of IC-continuity.

Theorem 3.1. For a function $f: (X, \tau, J) \to (Y, \sigma, I)$ the following are equivalent:

(*i*) f is IC-continuous.

(*ii*) If V is $\sigma^*(I)$-open and has an I-compact complement, then $f^{-1}(V)$ is $\tau^*(J)$-open.

(*iii*) If $F \subset Y$ is $\sigma^*(I)$-closed and an I-compact set, then $f^{-1}(F)$ is $\tau^*(J)$-closed.

(*iv*) For each $x \in X$ and each net $\{x_\alpha\}_{\alpha \in \Delta}$ which converges to x, the net $\{f(x_\alpha)\}_{\alpha \in \Delta}$ is eventually in each open set containing $f(x)$ and having an I-compact complement.

Proof. (*i*) implies (*ii*), and (*ii*) implies (*iii*) are clear from Definition 3.1. (*iii*) implies (*i*): Let $x \in X$ and $V \in \sigma$ containing $f(x)$, and $Y - V$ is I-compact, then $x \notin f^{-1}(Y - V) = X - f^{-1}(V)$, which is $\tau^*(J)$-closed. Therefore $f^{-1}(V)$ is $\tau^*(J)$-open containing x. By setting $U = f^{-1}(V)$, then $f(U) \subset V$. (*ii*) implies (*iv*): Let $\{x_\alpha\}$ be a net in X which converges to x and let $V \in \sigma$ containing $f(x)$ such that $(Y - V)$ is I-compact. Then $x \in f^{-1}(V) \in \tau^*(J)$-open and therefore $\{x_\alpha\}$ is eventually in $f^{-1}(V)$. Hence $\{f(x_\alpha)\}$ is eventually in V.

(*iv*) implies (*ii*): Let V be $\sigma^*(I)$-open having an I-compact complement. To show that $f^{-1}(V)$ is $\tau^*(J)$-open, consider the converse, *i.e.*, let $x \in f^{-1}(V)$ such that $f^{-1}(V) \notin N(x)$. Thus, there is a net $\{x_\alpha\}$ in X which converges to x and misses $f^{-1}(V)$ frequently. Then the net $\{f(x_\alpha)\}$ misses V frequently, which leads to contradiction.

Remark 3.1. Observe that if $I = \{\phi\}$ or $I = I_f$ in the previous theorem, we obtain the standard characterizations of C-continuity.

Lemma 3.1. [27] Let (X, τ, I) be a Hausdorff space. If $S \subseteq X$ is I-compact, then S is $\tau^*(I)$-closed.

Our next result follows easily from Theorem 3.1 and Lemma 3.1.

Theorem 3.2. A function f of a space (X, τ, J) into a Hausdorff space (Y, σ, I) is IC-continuous iff the inverse image of each I-compact subset of Y is $\tau^*(J)$-closed in X.

Lemma 3.2. It is easily seen that an H-subset is an I_n-compact subset, and the converse is not true as shown by the following example due to Hamlett and Janković [26].

Example 3.1. [26] Let (R, τ) denote the reals with the usual topology and let C denote the Cantor set in $[0, 1]$. Let $A = ((0, 1) \cap C) \cup [1, 2]$ and define $U_n = (1/n, 3)$. $\{U_n | n = 1,2,3,\ldots\ldots\}$ is an open cover of A and if $\{U_{n_i} | i = 1,2,\ldots\ldots,k\}$ is any finite subcollection, let $m = \min\{n_i | i = 1,2,\ldots\ldots,k\}$. Then $Cl(U_{n_i}) \subseteq [1/n_{i'}, 3] \subseteq [1/m, 3]$ for each n_i and hence $A \nsubseteq \cup \{U_{n_i}\}$. Thus A is not an H-subset. However, if $\{U_\alpha | \alpha \in \Delta\}$ is any open cover of A, there exists a finite subcover $\{U_{\alpha_i} | i = 1,2,\ldots\ldots,r\}$ of $[1, 2]$ and $A - \cup\{U_{\alpha_i}\} \subseteq C \in I_n$. Thus A is an I_n-compact subset of R which is not an H-subset.

Remark 3.2. The following implications give the connection between I_nC-continuity and other corresponding types.

$$\begin{array}{ccccc}
I_n\text{C-continuity} & \leftarrow & \text{continuity} & \rightarrow & L\text{-continuity} \\
\downarrow & & & & \downarrow \\
H\text{-continuity} & \rightarrow & N\text{-continuity} & \rightarrow & C\text{-continuity} \\
\downarrow & & & & \\
S\text{-continuity} & \rightarrow & s\text{-continuity} & \rightarrow & RS\text{-continuity}
\end{array}$$

Theorem 3.3. A function $f: (X, \tau) \to (Y, \sigma, I)$ is I_fC-continuous iff f is C-continuous.

Recall that a subset of a space is said to be meager (or of first category) if it is a countable union of nowhere dense sets. Clearly the collection of all meager subsets of a space is a σ-ideal. We denote this ideal by I_m.

Lemma 3.3. [26] Let (X, τ) be a space and let I_m denote the ideal of meager (first category) subsets of X. If $\tau \cap I_m = \{\phi\}$ and (X, τ) is Hausdorff, then (X, τ) is I_m-compact iff (X, τ) is H-closed.

Theorem 3.4. Let $f: (X, \tau) \to (Y, \sigma, I_m)$ be a function, $\sigma \cap I_m = \{\phi\}$ and (Y, σ) is Hausdorff, then f is I_mC-continuous iff f is H-continuous.

Proof. Follows from Lemma 3.3.

Theorem 3.5. Let $f: (X, \tau) \to (Y, \sigma, I)$ be a function. Then I_fC-continuity, C-continuity, N-continuity or NC-continuity [30], and continuity are equivalent if (Y, σ) is compact.

Proof. Follows directly from Corollary 3 (*iv*) of reference [31].

Lemma 3.4. [13] Let X be ED. Then for a subset W of X, the following are equivalent:

(*i*) W is RS-compact relative to X.
(*ii*) W is S-closed relative to X.
(*iii*) W is N-closed relative to X.
(*iv*) W is QHC relative to X.

Theorem 3.6. If $f: (X,\tau)\to(Y,\sigma,I)$ is a function, (Y,σ) is compact and ED, then the following are equivalent:

(*i*) f is I_fC-continuous.
(*ii*) f is continuous.
(*iii*) f is C-continuous.
(*iv*) f is H-continuous.
(*v*) f is N-continuous.
(*vi*) f is RS-continuous.
(*vii*) f is s-continuous.

Proof. By using Theorem 3.5 and Lemma 3.4.

For any topological space (Y,σ) we may take all open sets having I_n-compact complements as a base for a new topology σ^I on Y. Likewise, the collection of all open sets having H-closed (resp. N-closed, S-closed, s-closed, compact) complements may be used as a base to generate other topologies σ^* (resp. $\bar{\sigma}$, σ_S, $\hat{\sigma}$, $\acute{\sigma}$) on Y.

Remark 3.3. One can notice that:

(*i*) $\acute{\sigma} \subset \bar{\sigma} \subset \sigma^* \subset \sigma^I \subset \sigma$.

(*ii*) $\hat{\sigma} \subset \bar{\sigma} \subset \sigma^* \subset \sigma^I \subset \sigma$.

(*iii*) $\hat{\sigma} \subset \sigma_S \subset \sigma^* \subset \sigma^I \subset \sigma$.

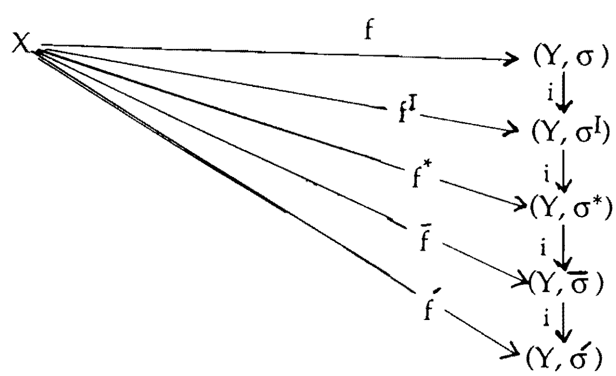

Figure 1.

Remark 3.4. It is clear that spaces (Y,σ), (Y,σ^I), (Y,σ^*), $(Y,\bar{\sigma})$, (Y,σ_S) and $(Y,\hat{\sigma})$ as shown in Figure 2 represents a complete distributive lattice.

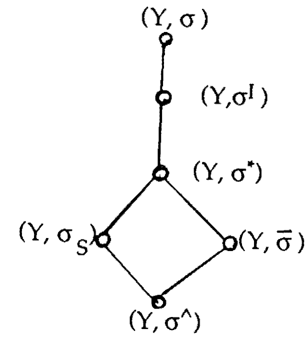

Figure 2.

The proofs of the following results are straightforward.

Theorem 3.7. A function $f: (X,\tau)\to(Y,\sigma,I)$ is IC-continuous iff $f: (X,\tau)\to(Y,\sigma^I)$ is continuous.

Theorem 3.8. Let $f: (X,\tau)\to(Y,\sigma,I)$ be IC-continuous. If $f: (X,\tau)\to(Y,\sigma^I)$ is closed (resp. open, then $f: (X,\tau)\to(Y,\sigma,I)$ is closed (resp. open).

Theorem 3.9. For any function $f: (X,\tau)\to(Y,\sigma,I)$, we have:

(*i*) If f is IC-continuous, and $A\subset X$, then $f|A$ is IC-continuous.

(*ii*) If $\{U_\alpha|\alpha\in\Delta\}$ is an open cover of X and $f_\alpha = f|U_\alpha$ is IC-continuous for each $\alpha\in\Delta$, then f is IC-continuous.

Theorem 3.10. Let $f: (X,\tau)\to(Y,\sigma,I)$ and $g: (Y,\sigma)\to(Z,\Gamma,J)$ be functions, where I and J are two ideals on Y and Z respectively, Then the following statements hold.

(*i*) If f is continuous, and g is IC-continuous, then *gof* is IC-continuous.

(*ii*) If f is surjective (open or closed) and *gof* is IC-continuous, then g is IC-continuous.

(*iii*) If f is a quotient function, then g is IC-continuous iff *gof* is IC-continuous.

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