THE DYNAMIC LOT-SIZING PROBLEM (DLSP): REVIEW, EXTENSIONS, AND COMPARISON

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الخلاصـــة :

المسألة المعتبرة في هذا المقال هي مسألة تحديد حجم الدفعة لمنتج وحيد ذي طلب ثابت بينما هو متغير مع الزمن بصورة منفصلة ولفترة تخطيط محددة . وعلى الرغم من أن هناك العديد من الطرق التنقيبيّـة قد اقترحت ؛ فإن الجهد المبذول في مراجعة ومقارنة هذه الطرق يظل معتدلاً نسبيا . في هذا المقال نـقِّـدم مراجعة شاملة للمقالات الحالية للمسألة المذكورة ، ونقترح طرقَ حـلٍّ جديدة وسهلة مبنية على تعديلات مباشرة للطرق التنقيبيّـة الموجودة ، كما نقوم بعمل تقييم واسع وشامل للفعالية ، من حيث انحراف التكلفة عن الأفضل ومن حيث الوقت المحسوب لثلاث عشرة طريقة تنقيبية . ونبيِّـن نتائج ماجموعه (٥٤٠٠) مسألة مكونة عشوائيا تحت أنماط مختلفة من الطلب وبنيات التكلفة . وأخيراً ، نوصي بطريقتين تنقيبيتين فعالتين لتحديد جداول إنتاج قريبة من الأفضل .

ABSTRACT

The problem considered in this paper is the single product deterministic and discrete time-varying demand inventory lot-sizing problem with finite horizon. Although several heuristics have been proposed, the effort devoted to reviewing and comparing these heuristics is still relatively modest. In this paper we present an extensive review of the current literature related to the above problem and propose new simple heuristics that are based on straightforward modifications of existing heuristics. We conduct an extensive assessment of the effectiveness, in terms of the cost deviation from optimality and computational time, of thirteen heuristics. We also report the results of 5400 randomly generated problems under different demand patterns and cost structures. Finally, we recommend two computational effective heuristics for determining near-optimal ordering schedules.

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THE DYNAMIC LOT-SIZING PROBLEM (DLSP): REVIEW, EXTENSION, AND COMPARISONS

1. INTRODUCTION

Over the past years, several authors have considered the problem of determining the ordering quantities over a known and fixed planning horizon for a single product when its demand is deterministic, time-varying, and occurs at discrete equally spaced points in time. The relaxation of the constant demand approximation, which is encountered in most classical inventory models, allows the problem to cover a wide variety of practical industrial engineering situations. In inventory systems with dependent demand-items (material requirement planning systems: MRP), lot-sizing techniques have played a major role in setting the reorder points and the ordering/production quantities for materials, parts, components, and subassemblies which form the product structure of the end item. Moreover, most MRP softwares include a module for the selection of the lot-sizing technique to be used. In a production to contract situation, where certain quantities for each final product have to be delivered to the customer on specified dates, it is required to determine the least-cost production schedule (date and production quantities) to meet the customer requirement. Finally, lot-sizing techniques can also be used in preventive maintenance situations to determine the parts purchasing schedules when the maintenance timings are accurately known. Hereafter, we refer to this problem as the dynamic lot-sizing problem, DLSP.

After the pioneering work of Wagner and Whitin [1], WW, numerous papers have appeared in the literature focusing on heuristic methods. Although WW provided an optimal solution to the problem, their algorithm was not used extensively in practice because its mathematical complexity made it difficult to grasp for practitioners. Other reasons for the unpopularity of the WW algorithm are its computer implementation cost in terms of time and storage, besides its sensitivity to the length of the planning horizon. As a result, a large number of heuristic procedures have been proposed to overcome the shortcomings of the WW algorithm. Some of these authors [2–7] have additionally compared their heuristics to existing ones in terms of cost deviation from optimality. Other authors [8, 9] have devoted their entire papers to the review and comparison of various dynamic lot-sizing heuristics. However, we believe that the effort expended in this respect is still relatively modest since the reviews were not up-to-date and the comparisons were not exhaustive. All the comparisons were based on the demand patterns and cost structures provided by Berry [9]. Therefore, it would be misleading to draw conclusions based on five demand patterns over a fixed planning horizon of 12 periods.

In this paper, we will provide an up-to-date review of the DLSP literature and will propose new heuristics. Moreover, we will conduct an extensive comparison based on a more general framework of demand patterns and cost structures. In the next Section, we will review the current status of DLSP literature. In the third Section, we will propose new heuristics for DLSP. Section 4 describes the experiments undertaken and summarizes the results. Finally, Section 5 concludes the paper.

Before presenting the mathematical model, we will make the following assumptions that are commonly used in the DLSP literature:

The demand is known and has a discrete time-varying features.
The planning horizon has a known and fixed length.
The periods that subdivide the planning horizon have equal duration.
The initial and final inventories are set equal to zero.
The production rate is infinite in each period.
The orders are not allowed to be split.
The holding cost is calculated for the quantity left at the end of the period.
The holding and ordering costs are known and constant over time.
The unit purchasing cost is time invariant and is not included in the model below.
The demand quantity for each period must be available at the beginning of the period so that shortage is avoided.
The delivery lead time is set equal to zero.
The supplier does not offer quantity discounts.

Furthermore, in deriving the mathematical model, the following notation is used:

N = length of the planning horizon,
A = ordering cost which is independent of the quantity ordered,

h = cost of holding one unit of the product for one period,
D_i = demand at period i, $i = 1,\dots, N$,
I_i = amount in stock by the end of period i, $i = 1,\dots, N$,
Q_i = quantity ordered at period i, $i = 1,\dots, N$.

Given tha above stated assumptions and notation, the DLSP can be presented mathematically as follows:

$$\text{Min} \sum_{i=1}^{N} (\alpha_i A + h\, I_i) .$$

Subject to:

$$I_{i-1} + Q_i - I_i = D_i \qquad i = 1,\dots, N,$$

$$I_i \geq 0,\ Q_i \geq 0 \qquad i = 1,\dots, N,$$

$$\alpha_i = \begin{cases} 0 \text{ if } Q_i = 0 \\ 1 \text{ if } Q_i > 0 \end{cases} \qquad i = 1,\dots, N,$$

$$I_0 = 0.$$

The first type of constraint represents the inventory balance equation and guarantees that the demand quantity for each period is satisfied without shortage.

2. REVIEW OF DYNAMIC LOT-SIZING TECHNIQUES

Since the paper of Wagner and Whitin [1], the area of inventory theory with dynamic lot-sizing and discrete demand has witnessed an explosive growth in the number of heuristics that claim to be computationally efficient and assure near-optimal solutions. It would therefore be impossible to review all the literature of DLSP. However, our purpose is to present the best known and most recent heuristics. We begin our review with the Wagner and Whitin algorithm and then present the most cited heuristics in the literature.

2.1. Wagner and Whitin Algorithm

The WW algorithm is basically a dynamic programming procedure that uses the following recursion equation in a forward fashion [10]:

$$F_k = \min_{0 \leq j \leq k} [F_j + M_{jk}] ,$$

with

$$F_0 = 0,$$

and where F_k is the minimum total inventory costs (ordering plus holding) for periods $1, 2,\dots, k$, when the ending inventory of period k is zero, and M_{jk} is the total inventory costs incurred in periods j through k, *i.e.*,

$$M_{jk} = A + h \sum_{i=j}^{k} (i-j)\, D_i .$$

The main drawback of the algorithm is its computational complexity since it requires lengthy calculations to be performed. However, recently Evans [11] and Bahl and Taj [12] developed efficient computer programs that can solve large inventory dynamic problems in only a few seconds. Bahl and Taj included the setup cost horizon theorem [13] in their code and showed empirically that it is faster by a factor of $N/4$ in the best case and slower by only 1–2% in the worst case than Evans code. For this reason, we adopted the Bahl and Taj code in our computational comparison.

2.2. Silver and Meal Heuristic (SM)

The SM heuristic [14] determines the ordering lot size that covers an integer number of periods of demand such that the total inventory costs per unit of time are minimized. The heuristic selects the first value of T such that

$$\frac{A + h \sum_{i=1}^{T+1} (i-1)\, D_i}{T+1} > \frac{A + h \sum_{i=1}^{T} (i-1)\, D_i}{T} ,$$

where T represents the number of periods of demand that should be satisfied by the current period's order. In our computer code we implemented the following stopping rule, which can be easily derived from the above inequality, to reduce the computational time

$$T^2 D_{T+1} - \sum_{i=1}^{T} (i-1)\, D_i > \frac{A}{h} .$$

2.3. Marginal Cost Approach (MCA)

Groff [4] developed a simple marginal cost heuristic based on a theoretically sound rule. He stated:

> The economic order quantity rule is established by increasing the lot as long as the marginal savings in ordering cost are greater than the marginal cost in inventory holding cost. Marginal costs are the effect of the final unit or dollar added. Thus, the optimal lot size is reached when the decrease in marginal cost just equals the marginal increase.

Using this rule and the approximation

$$\sum_{i=2}^{T} (i-1)\, D_i \approx \frac{T}{2} \sum_{i=2}^{T} D_i \,,$$

Groff determined that the Tth period of demand will be added to the lot ordered in the current period if

$$T(T-1)\, D_T < \frac{2A}{h} \,.$$

Note that this inequality can be obtained from the SM stopping rule by using the above approximation.

2.4. Part Period Algorithm (PPA)

The PPA procedure [15] determines the ordering lot size that includes the largest number of periods of demand such that the accumulated holding cost is less than or equal to the ordering cost. Mathematically, this condition can be restated as finding the largest value of T satisfying

$$\sum_{i=1}^{T} (i-1)\, D_i \le \frac{A}{h} \,,$$

where A/h is called the economic part period, and

$$\left(\sum_{i=1}^{T} (i-1)\, D_i \right)$$

is called the accumulated part periods.

2.5. Incremental Part Period Algorithm (IPPA)

In contrast to PPA which increases an order as long as the accumulated holding cost is not greater than the ordering cost, the incremental part period algorithm [5] includes the demand for period T in the current order if the incremental holding cost of that period's demand does not exceed the ordering cost. Therefore, the demand for period T will be covered by the current ordering lot size if

$$(T-1)\, D_T \le \frac{A}{h} \,,$$

where $(T-1)\, D_T$ is called the incremental part periods.

2.6. Least Unit Cost (LUC)

As the name of the heuristic implies, the objective is to determine the ordering lot size that covers an integer number of periods of demand such that the total inventory costs per unit demand are minimized [16]. This local-optimal number of periods is obtained by finding the first value of T such that

$$\frac{A + h \sum_{i=1}^{T+1} (i-1)\, D_i}{\sum_{i=1}^{T+1} D_i} > \frac{A + h \sum_{i=1}^{T} (i-1)\, D_i}{\sum_{i=1}^{T} D_i} \,.$$

To reduce the computational time of our computer code, we used another stopping rule that can be easily derived from the above inequality. This stopping rule is as follows:

$$\sum_{i=1}^{T} (T+1-i)\, D_i > \frac{A}{h} \,.$$

2.7. Economic Order Quantity (EOQ)

This heuristic is based on the static economic order quantity formula

$$\text{EOQ} = \sqrt{\frac{2A\bar{D}}{h}} \,,$$

using the average demand over the planning horizon. In this heuristic, the demand is accumulated until it gets close to the EOQ value. Then, an order of

$$Q_t = \sum_{i=t}^{T} D_i$$

units is placed, where T is the number of the period that gave the closest accumulated demand to EOQ and t is the number of the current period. The order quantity in any period is selected in much the same way as WW algorithm since it is equal to zero or to the exact demand of one or more of its succeeding periods, including that period [3].

2.8. Periodic Order Quantity (POQ)

POQ is also based on the EOQ formula since the fixed number of periods of demand, T, to be covered by each ordering lot size is computed by dividing the EOQ value by the average demand over the planning horizon and rounding the result to the nearest integer [17], *i.e.*,

$$T = \left[\sqrt{\frac{2A}{h\bar{D}}} \right] ,$$

where $[x]$ is the nearest integer to x.

2.9. Lot For Lot (LFL)

The LFL heuristic is the simplest procedure for determining a solution to the dynamic lot sizing problem [18]. Orders are placed at each period in the exact quantity to satisfy the demand. As a consequence, no inventory will be held at any period and hence inventory holding cost is zero. The total costs are equal to the ordering cost multiplied by the number of periods with non-zero demand.

2.10. Bookbinder and Tan Heuristics (H1, H2)

Two lot sizing heuristics have been proposed by Bookbinder and Tan [2]. The first (H1) simplifies the stopping rule of SM heuristic whereas the second (H2) combines SM and LUC stopping rules into one complicated criterion function.

After examining the SM stopping heuristic for some difficult cases of sharply decreasing demand, they noticed that if the term

$$\left(h \sum_{i=1}^{T} (i-1)\, D_i\right)$$

is discarded from the SM stopping rule, the new heuristic will give better results for the special case of a sharply decreasing demand pattern. Moreover, using only the periods with non-zero demand, they set the stopping rule for H1 as

$$T\, Z_T\, D_{T+1} > \frac{A}{h}\,(Z_{T+1} - Z_T)\,,$$

where Z_T is the number of non-zero demand periods in the interval $[1,\ T]$.

Previous research studies on DLSP have shown that the SM heuristic does not perform well when demand is sharply decreasing. For this case, the LUC heuristic is preferred. The objective of the Bookbinder and Tan heuristic H2 is to take advantage of the merits of both the SM and LUC heuristics and to eliminate some drawbacks of each heuristic. To this end, they proposed a complicated criterion function $F(T)$ composed of two portions. The first portion retains the benefits of SM and the second one retains the benefits of LUC.

$$F(T) = \frac{A}{Z_T} + \frac{h \sum_{i=2}^{T} \left[\frac{(i-1)}{Z_i}\, D_i \sum_{j=1}^{i} D_j\right]}{\sum_{i=1}^{T} D_i}$$

3. EXTENSIONS

In this section, we propose four new simple heuristics that are based on straightforward modifications of some existing heuristics. We developed these heuristics in the hope of eliminating the deficiencies of the original ones under certain severe demand conditions. The first heuristic we suggest is based on the POQ heuristic. The second one is a straight modification of the SM heuristic by considering only the periods with positive demand. The last two heuristics combine SM and LUC in a much simpler way than Bookbinder and Tan.

3.1. Modified Periodic Order Quantity (MPOQ)

In this heuristic, we devised a new simple approach to find the fixed number of periods of demand, T, to be covered by each ordering lot size. Using the average demand over the planning horizon, the average total inventory cost per unit time can be written as

$$f(T) = \frac{A}{T} + ½\, h \bar{D} T\,.$$

Since T can take only discrete values, the value of T that minimizes $f(T)$ can be found by the first difference approach, *i.e.*, T should satisfy:

$$f(T-1) \le f(T) < f(T+1)\,.$$

Substituting $f(T)$ by its expression yields:

$$T(T-1) \le \frac{2A}{h\bar{D}} < T(T+1)\,.$$

3.2. Modified Silver and Meal Heuristic (MSM)

This heuristic is based on a simple modification of SM procedure by considering only non-zero demand periods. The heuristic determines the integer number of periods of demand to be covered by the ordering lot size such that the total inventory costs per period of positive demand are minimized. Under this new criterion function, the stopping rule of SM is modified to be:

$$Z_T T D_{T+1} - \sum_{i=1}^{T} (i-1)\, D_i > \frac{A}{h}\,,$$

where Z_T is the number of periods with non-zero demand in the first T periods.

3.3. Combined SM and LUC Heuristic 1 (CSMLUC1)

The SM and LUC heuristics share the same procedure with slight differences. In this suggested combined SM and LUC heuristic, an order is placed to cover the demand for an integer number of periods,

T, corresponding to the maximum reorder intervals of both heuristics. If T_1 and T_2 are the reorder intervals that satisfy the stopping rules of SM and LUC, respectively, then $T = \text{Max}\{T_1, T_2\}$. Hopefully, at the expense of more computational time, some of the drawbacks of SM and LUC heuristics will be eliminated.

3.4. Combined SM and LUC Heuristic 2 (CSMLUC2)

In this second combined heuristic, the ordering lot size will cover an integer number of periods of demand equal to the minimum reorder intervals of SM and LUC, *i.e.*, $T = \text{Min}\{T_1, T_2\}$.

4. COMPARATIVE STUDY

A comparative study was designed to investigate the effectiveness of the different lot sizing techniques introduced in the two previous sections. We consider earlier comparative studies to be inadequate because they are limited to a few lot sizing techniques and are based on restricted demand patterns and cost structures. Furthermore, the performance of each heuristic was evaluated only by the cost deviation from optimality. Using five demand patterns over a planning horizon of 12 periods, Berry [9] studied the cost performance of the EOQ, POQ, and PPA heuristics. His study was based on two experimental factors: the coefficient of variation which measures the degree of period to period demand variation, and the ratio of the ordering cost to the inventory holding cost per unit of time. Groff [4] evaluated the cost performance of EOQ, POQ, PPA, and MCA heuristics based on the same demand patterns and cost structure of Berry. Karni [7] added a new demand pattern to Berry's data and compared EOQ and the uniform order quantity, UOQ, heuristics. We did not include the UOQ technique in our comparisons because it is time consuming, since it evaluates many ordering plans. Moreover, for each plan it has to time phase the UOQ value, where the UOQ of plan k is equal to the total demand over the planning horizon divided by k. Using only the four demand patterns of Berry's data, Freeland and Golley [6] evaluated the cost effectiveness of the LFL, EOQ, POQ, PPA, IPPA, and SM heuristics. Mitra *et al.* [3] proposed two heuristics that are modifications to the standard economic order quantity and LUC techniques. Then, they compared their heuristics to the corresponding ones using the same demand patterns of Berry. Following the same experimental design framework as Karni, Boe, and Yilmaz [5] investigated the cost performance of UOQ and IPPA. Bookbinder and Tan [2] compared their two heuristics with the SM, MCA, LUC, POQ, and EOQ techniques, relying also on the same demand patterns of Berry. Finally, Ritchie and Tsado [8] used the demand patterns that are generated from three normal distributions each with a mean of 18 and standard deviations of 4.14, 20.52, and 46.44, respectively. They also used three other types of demand that exhibit life cycle patterns of growth, stationary, and decline phases. This last groups of demand patterns each have a mean of 53 and coefficients of variation of 0.5, 1.53, and 2.11, respectively. The planning horizon for both cases has a length of 156 periods. In their experiments they used the average time between orders, TBO, as an experimental factor instead of A/h. TBO is defined as the EOQ value divided by the average demand over the planning horizon and it measures the number of period demands covered by each order. They considered seven values of TBO totalling forty two problems to simulate for each of the lot sizing techniques (SM, MCA, PPA, IPPA, EOQ, the modified LUC proposed by Mitra *et al.*, and the modified IPPA suggested by Gaither [19]). The modified IPPA was not used in our experiment since, as noticed by Ritchie and Tsado, it increases the complexity of using the original technique by introducing a correction factor that depends on the coefficient variation and A/h.

4.1. Experimental Framework

In our comparative study, all techniques discussed in the previous sections were assessed in terms of two performance measures:

The percentage increase in total cost above the optimal cost of the WW algorithm. The cost efficiency of the kth lot sizing procedures is then measured by:

$$\text{CINC}(k) = 100\ \frac{\text{Total Inventory Costs}(k) - \text{Total Inventory Costs (WW)}}{\text{Total Inventory Costs (WW)}}.$$

The computation time, which is the CPU time required to compute the ordering plan and its associated costs.

A BASIC code was written and implemented on an 80386 IBM compatible machine running at 25 MHz without a math co-processor. Our experiments were carried out by varying five experimental design factors to generate different problem settings on which the thirteen heuristics could be compared in terms of the above two performance measures. The five factors were:

1. The coefficient of variation of demand (CV).
2. The A/h ratio (A/h).
3. The percentage number of orders with zero demand over the planning horizon (PZ).
4. The length of the planning horizon (N).
5. The demand pattern (DP).

All these factors have been cited in the literature as having an effect on the lot sizing cost efficiency. However, they have not been used together in any research work to show their impact on lot sizing technique performances. The values of the experimental design factors used in our simulation are shown in Table 1. The seven demand patterns employed in our experiments deserve some explanations which will be provided later.

Table 1. Values of the Experimental Factors.

Factors	Values						
CV	0.1	0.5	1.5	2	3		
A/h	10	50	100	200	300	500	
PZ	0	10	20	50	80	90	
N	12	52	104	156	366		
DP	LN	U	LI	EI	LD	ED	S	TS

4.2. Experimental Design

Three different experiments were carried out with ten replications for each problem setting. The generation of the demand patterns differed from one experiment to another. In the first experiment we considered only demand patterns with positive values over the planning horizon. For this reason, we generated the demand for each period from a lognormal distribution LN with median 100 and the square of the shape parameter equal to $\log(1+CV^2)$. The ordering cost A was also generated randomly from a uniform distribution in the interval $[1, 10\,A/h]$. Then, based on this generated value, we computed the unit inventory holding cost per unit of time, h. The experimental factor settings (CV, A/h, and N) selected for this first experiment resulted in 180 experimental problems. Each problem was replicated 10 times totalling 1800 runs for each heuristic.

For the second experiment, the values of PZ, A/h, and N shown in Table 1 were used. Each problem setting was replicated ten times resulting also in 1800 runs for each technique. For periods with positive demand, the demand was generated from a discrete uniform distribution, U, between 100 and 1000. The periods with zero demand were selected randomly in such a way that their total number was equal to $(N\,PZ)/100$. A and h were generated in the same way as in the first experiment.

Finally, in the last experiment different demand patterns that are often encountered in real-life inventory and production planning environment were tried. These demand patterns were:

1. Linearly increasing demand, LI:
$$D_i = 10 + 10i + x, \quad i = 1, 2, \ldots, N \text{ and } x \approx u\ (0, 5).$$
2. Linearly decreasing demand, LD:
$$D_i = 15N + 10 - 10i + x, \quad i = 1, 2, \ldots, N \text{ and } x \approx u\ (-10, 5).$$
3. Exponentially increasing demand, EI:
$$D_i = 100\ e^{0.01\,i} + x, \quad i = 1, 2, \ldots, N \text{ and } x \approx u\ (0, 20).$$
4. Exponentially decreasing demand, DI:
$$D_i = 5 + 3N\ e^{-0.05\,i} + x, \quad i = 1, 2, \ldots, N \text{ and } x \approx u\ (-5, 0).$$
5. Seasonal demand, S:
$$D_i = 1000\left(1 + \sin\left(\frac{2\pi N}{i}\right)\right) + x, \quad i = 1, 2, \ldots, N \text{ and } x \approx u\ (0, 10).$$
6. Trend–Seasonal demand, TS:
$$D_i = 100\ (1+i)\left(2 + \sin\left(\frac{2\pi N}{i}\right)\right) + x, \quad i = 1, 2, \ldots, N \text{ and } x \approx u\ (0, 10).$$

A and h were also generated in the same manner as in the previous two experiments.

In order to find mathematical models explaining the behavior of the performance measures (CINC and CPU) as a function of the experimental factors, we combined the results of the three experiments (5400 runs) in one data file to use it as input to the REG procedure of the Statistical Analysis System (SAS). Several regression models were tried for each heuristic and we selected the one with the most significant coefficient estimates at the 5% level and with the highest coefficient of determination, R^2. We did not use the experimental factors CV and PZ together in any regression model because they will obviously be correlated. Instead, in one of the models, we used a two-stage linear regression where in the first stage we regress CV as a function of PZ, then we used the predicted CV as an independent variable in the next stage.

4.3. Experimental Results

In this section we present the cost deviation and CPU time results of 5400 experimental runs. Tables 2 to 7 give the maximum, average, and standard deviation of the percentage increase in the total inventory costs above the optimal solution. Tables 2 to 6 show the variation of these measurements as a function of each experimental factor, whereas in Table 7 they are calculated over the entire set of problems. To provide some further indicators about the relative cost performance of each heuristic, Figure 1 gives the frequency distribution of CINC in pie diagrams. These pie diagrams show that MCA, MSM, and H2 solved 3227, 4243, and 3260 of the 5400 experimental runs to optimality, respectively.

Table 2. Maximum, Average, and Standard Deviation of CINC as a Function of *N*.

	Length of Planning Horizon N														
	12			52			104			156			366		
	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.
LFL	238.0	24.0	38.8	161.8	19.9	31.8	193.2	20.2	34.4	228.8	21.4	38.3	321.4	24.3	49.6
SM	12.8	0.6	1.5	7.3	0.5	1.1	5.5	0.5	0.9	5.8	0.6	0.9	4.6	0.6	1.0
MSM	12.8	0.6	1.5	7.3	0.5	1.0	5.5	0.5	0.9	5.8	0.5	0.9	4.5	0.6	0.9
H2	10.2	0.6	1.6	8.6	0.6	1.2	6.5	0.6	1.1	5.4	0.6	1.0	4.8	0.6	1.0
CSMLUC1	974.1	9.2	41.3	270.1	7.5	21.4	205.9	7.5	17.4	155.1	7.3	16.7	362.3	8.0	19.8
CSMLUC2	43.8	4.4	7.4	36.6	4.5	7.0	33.5	4.3	6.8	33.8	4.3	6.7	27.4	4.3	6.8
EOQ	226.7	10.6	20.2	237.0	14.0	19.3	116.9	15.1	15.3	111.2	16.4	16.4	148.5	20.7	21.1
LUC	1134.2	23.3	70.1	387.2	25.4	48.7	489.3	23.7	46.2	627.8	24.3	50.1	480.9	23.6	47.4
POQ	129.9	6.7	13.5	89.5	9.3	14.3	126.8	10.6	15.8	118.7	11.7	17.5	121.9	14.8	25.8
MPOQ	129.9	7.0	14.4	154.3	9.6	15.5	154.3	10.7	16.3	118.7	11.9	17.7	121.9	14.9	25.7
PPA	19.2	0.9	2.3	12.9	0.9	1.7	9.6	0.8	1.6	9.3	0.9	1.7	7.9	0.9	1.6
IPPA	48.4	2.9	7.3	90.6	2.5	8.2	94.8	2.9	9.9	108.4	3.2	11.5	98.4	3.5	12.0
MCA	11.1	0.6	1.6	8.7	0.5	1.1	5.1	0.5	0.9	4.9	0.5	0.9	3.8	0.6	0.9

Table 3. Maximum, Average, and Standard Deviation of CINC as a Function of *CV*.

	Coefficient of Variation CV																	
	0.1			0.5			1.0			1.5			2.0			3.0		
	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.
LFL	92.4	29.7	32.9	117.0	31.8	33.9	119.1	38.4	34.6	120.7	40.6	34.0	128.0	42.3	33.2	151.7	45.4	33.5
SM	4.3	0.4	0.5	7.7	1.3	1.5	7.2	1.5	1.4	8.5	1.4	1.4	8.6	1.4	1.5	6.4	1.4	1.3
MSM	4.3	0.4	0.5	7.7	1.3	1.5	7.2	1.5	1.4	8.5	1.4	1.4	8.6	1.4	1.5	6.4	1.4	1.3
H2	5.4	0.4	0.6	8.8	1.4	1.6	7.2	1.7	1.6	9.5	1.7	1.6	8.6	1.7	1.7	9.5	1.7	1.7
CSMLUC1	10.7	0.7	1.2	29.7	4.5	4.8	75.2	11.7	10.8	158.0	20.2	21.7	207.1	28.7	30.2	974.1	54.6	78.1
CSMLUC2	4.3	0.7	0.9	28.5	4.6	3.7	31.1	9.5	6.5	43.8	12.7	7.8	42.9	14.8	8.5	41.5	17.2	9.3
EOQ	6.5	0.8	1.1	28.1	6.7	4.7	37.3	14.3	7.5	79.2	21.1	9.7	70.7	27.7	11.1	102.1	37.8	14.6
LUC	6.2	1.1	1.4	49.3	10.6	8.2	91.4	26.7	16.6	186.1	53.2	32.5	489.3	85.7	55.3	1134	155.9	122.9
POQ	4.5	1.0	1.2	22.3	6.2	5.1	41.4	14.3	9.9	59.3	22.9	15.5	74.5	28.4	19.2	129.9	37.7	26.4
MPOQ	5.7	1.0	1.2	30.3	6.1	5.2	41.4	14.2	9.8	59.3	22.6	15.3	128.6	28.4	19.7	154.3	37.6	27.1
PPA	3.5	0.4	0.6	9.5	2.0	2.2	17.5	2.7	3.0	19.2	2.6	2.7	17.0	2.7	2.8	11.0	2.2	2.2
IPPA	27.3	5.7	7.2	24.2	4.0	5.0	21.6	2.9	3.4	13.1	2.3	2.7	18.7	1.8	2.3	9.4	1.4	1.6
MCA	5.4	0.4	0.6	8.8	1.3	1.6	9.5	1.5	1.6	11.1	1.5	1.6	8.7	1.3	1.5	9.5	1.2	1.3

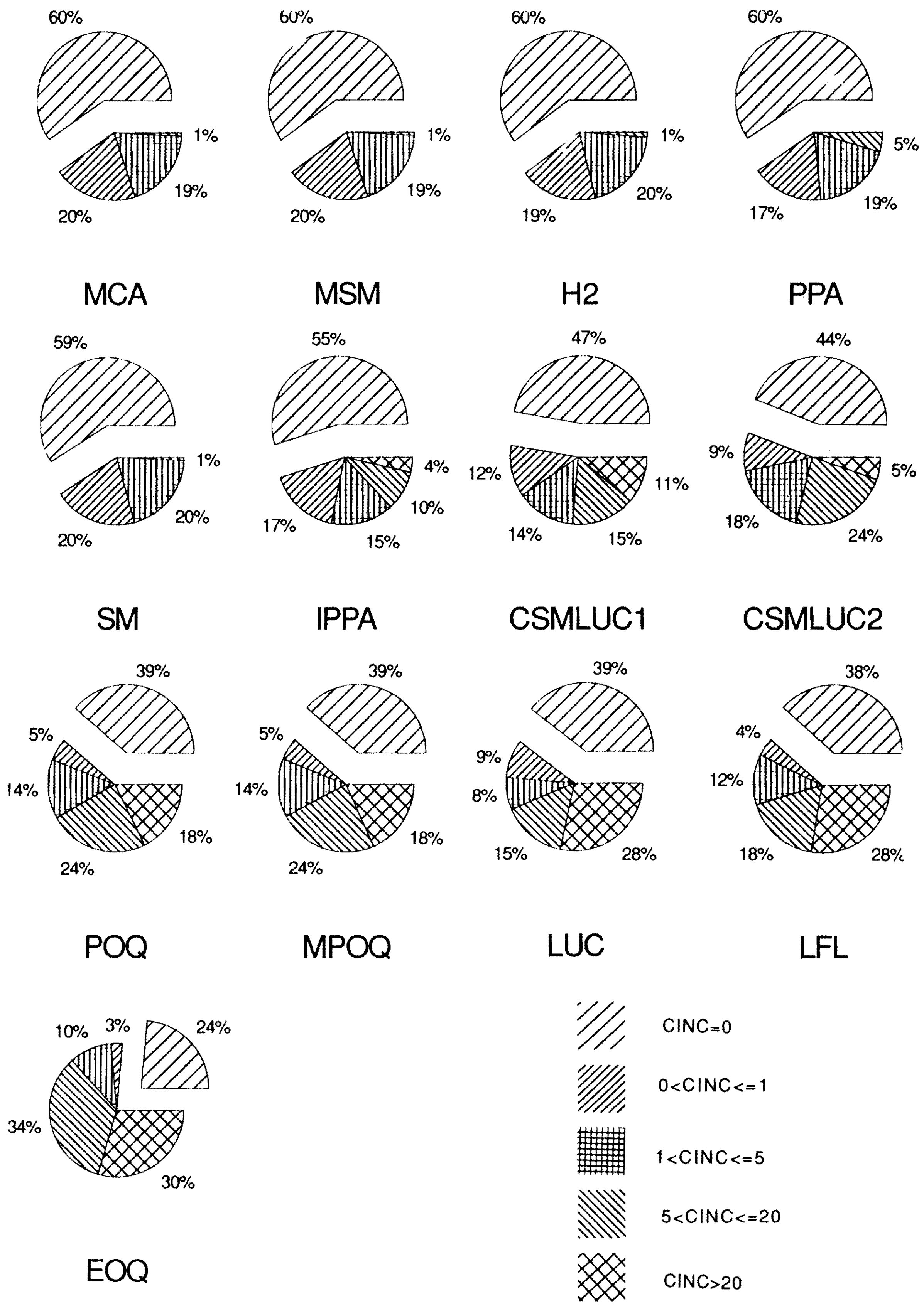

Figure 1. Pie Diagram of CINC Distribution for All Heuristics.

Table 2 shows that the cost efficiency of most heuristics does not depend on the length of the planning horizon. This is confirmed by our regression analysis since the coefficient of *N* was not significant at the 5% level for all the regression models tested. The only exception was EOQ heuristic for which the cost performance gets worse for large *N*. In Table 3, it can be noticed that for most heuristics the cost performance is worsened as *CV* increases except for IPPA heuristic. This fact has also been confirmed by our regression analysis. Moreover, a drastic deterioration of the CSMLUC1, EOQ, LUC, POQ, and MPOQ cost performances for large *CV* can be observed from the same table. For large *PZ,* the cost deviation from optimality is improved for most heuristics except for EOQ, POQ, and MPOQ techniques. This is shown in Table 4 from which we can also observe that MSM outperforms SM for large *PZ*. The *A*/*h* factor has the same impact as the *CV* factor on most heuristics with the only exception noticed

Table 4. Maximum, Average, and Standard Deviation of CINC as a Function of *PZ*.

	Percent of Zero Demand Periods *PZ*																	
	0			10			20			50			80			90		
	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.
LFL	238.0	26.0	36.3	314.3	9.4	36.9	23.1	3.6	5.3	29.3	2.4	4.2	24.5	0.8	2.2	34.7	0.6	2.5
SM	12.8	0.7	1.2	4.5	0.3	0.7	3.4	0.2	0.5	3.4	0.2	0.5	2.9	0.1	0.4	7.1	0.1	0.6
MSM	12.8	0.7	1.2	4.5	0.3	0.6	3.4	0.2	0.4	1.5	0.1	0.3	2.7	0.0	0.2	3.4	0.0	0.2
H2	10.2	0.8	1.4	6.4	0.3	0.8	3.4	0.2	0.5	2.2	0.1	0.3	2.7	0.0	0.2	3.4	0.0	0.2
CSMLUC1	974.1	10.0	29.4	83.4	3.1	7.3	33.7	3.4	5.1	35.4	4.0	6.5	85.1	3.3	9.5	64.8	1.3	5.9
CSMLUC2	43.8	5.5	7.8	15.4	2.8	3.7	16.8	2.5	3.6	24.6	1.9	3.2	13.7	0.6	1.6	12.0	0.4	1.3
EOQ	102.1	13.6	13.7	140.6	10.0	12.9	40.3	10.9	8.6	101.5	23.3	20.2	168.0	23.2	31.3	237.0	17.6	36.2
LUC	1134	31.7	61.8	83.4	10.3	12.4	60.2	9.6	11.3	51.0	7.2	9.9	85.1	4.1	10.2	64.8	1.5	6.0
POQ	129.9	10.7	16.3	120.5	6.4	16.0	25.2	3.8	5.7	35.3	3.3	6.1	63.7	6.4	10.5	126.8	10.6	19.0
MPOQ	154.3	10.7	16.4	120.5	6.5	16.1	28.8	4.2	6.6	36.9	5.0	8.7	88.3	7.3	12.3	154.3	12.4	23.0
PPA	19.2	1.1	2.0	6.4	0.4	0.9	8.3	0.3	0.9	2.3	0.1	0.4	2.7	0.0	0.2	3.4	0.0	0.2
IPPA	94.8	3.0	8.1	95.0	1.6	10.1	3.4	0.2	0.4	1.4	0.1	0.2	1.6	0.0	0.2	3.4	0.0	0.2
MCA	11.1	0.7	1.2	5.8	0.3	0.7	3.4	0.2	0.5	3.4	0.2	0.4	2.9	0.1	0.4	6.0	0.0	0.4

Table 5. Maximum, Average, and Standard Deviation of CINC as a Function of *A*/*h*.

	A/*h* Ratio																	
	10			50			100			200			300			500		
	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.
LFL	42.6	1.1	4.7	110.5	5.9	14.1	153.2	11.8	0.9	205.6	24.4	33.4	249.5 9	35.1	43.2	321.4	53.5	59.3
SM	1.9	0.0	0.2	3.9	0.2	0.5	8.5	0.9	0.5	10.2	0.7	1.2	12.8	0.9	1.4	8.6	1.1	1.4
MSM	2.0	0.0	0.2	3.9	0.2	0.4	8.5	0.9	0.5	10.2	0.7	1.2	12.8	0.9	1.4	8.6	1.0	1.3
H2	2.3	0.0	0.2	4.7	0.2	0.5	8.5	1.0	0.5	10.2	0.8	1.4	9.5	1.0	1.5	8.6	1.1	1.5
CSMLUC1	331.9	1.5	16.6	365.9	6.1	22.2	362.3	26.7	9.1	205.6	9.2	19.8	297.8	10.1	20.7	974.1	11.4	37.4
CSMLUC2	8.1	0.4	1.0	33.3	2.3	4.2	42.9	6.5	4.1	41.5	5.6	7.9	35.5	6.4	7.8	43.8	7.6	8.3
EOQ	148.5	9.7	20.5	102.1	13.0	16.8	189.3	16.2	14.0	237.0	17.0	19.1	226.7	19.2	20.4	185.3	19.4	18.1
LUC	1134	20.8	74.4	627.8	28.8	63.0	387.2	50.2	25.3	348.8	25.1	42.0	297.8	22.3	35.1	974.1	22.0	44.2
POQ	42.6	1.1	4.7	110.5	5.5	13.9	121.9	17.7	10.5	115.7	14.3	21.0	126.8	15.4	20.7	129.9	16.8	19.6
MPOQ	42.6	1.1	4.7	110.6	5.7	14.3	121.9	17.8	10.5	154.3	14.5	21.5	126.8	16.1	21.1	154.3	17.1	20.2
PPA	4.7	0.1	0.3	7.4	0.2	0.6	9.5	1.2	0.7	17.5	1.1	1.9	17.0	1.4	2.2	19.2	1.8	2.5
IPPA	6.5	0.1	0.6	23.0	0.6	2.5	32.7	4.3	1.3	51.3	3.0	7.9	74.2	4.7	11.0	108.4	8.3	18.4
MCA	1.9	0.0	0.2	4.7	0.2	0.5	9.5	0.9	0.5	10.2	0.7	1.3	11.1	0.9	1.4	8.7	1.0	1.3

for LUC procedure. The result of the third experiment which is reported in Table 6 shows that all heuristics behaved poorly under sharply decreasing demand patterns. Finally, the best tested regression models that explain the variation of CINC as a function of the experimental factors for each heuristic are presented in Figure 2. All the coefficients of the experimental factors reported in this figure are statistically significant at the 5% level.

Based on the cost deviation from optimality, we conclude that, on the average, MCA and MSM techniques performed best. The second best heuristics are H2, SM, and PPA, then come IPPA, CSMLUC2, CSMLUC1, MPOQ, POQ, EOQ, LFL, and LUC in the order listed. This ranking is deduced from Table 7 which reports the maximum, average, and standard deviation of the cost deviation from optimality for all the 5400 runs tested.

Tables 8 to 12 give indications about the behavior of the CPU for each heuristic as a function of the experimental factors. The figures shown in Table 12 are calculated over the 5400 test runs. As it was expected, Table 8 shows that the CPU times for all procedures, including WW, increase as the length of the planning horizon increases. In Table 9, we observe that A/h has a negative impact (as A/h increases, the running time gets longer) on WW, CSMLUC2, and PPA techniques. On the other

LFL:	$CINC = 1.24\ A/h + 368.74\ CV \qquad R^2 = 0.5475$
SM:	$CINC = 0.007\ A/h + 1.525\ CV \qquad R^2 = 0.5133$
MSM:	$CINC = 0.007\ A/h + 1.54\ CV \qquad R^2 = 0.5062$
H2:	$CINC = 0.005\ A/h + 1.34\ CV \qquad R^2 = 0.5043$
CSMLUC1:	$CINC = 6.58\ CV \qquad R^2 = 0.6135$
CSMLUC2:	$CINC = 0.006\ A/h + 5.38\ CV \qquad R^2 = 0.6949$
EOQ:	$CINC = -0.0314\ A/h + 6.58\ CV \qquad R^2 = 0.6365$
LUC:	$CINC = -0.011\ A/h + 18.45\ CV \qquad R^2 = 0.6739$
POQ:	$CINC = 0.006\ A/h + 9.148\ CV \qquad R^2 = 0.6652$
MPOQ:	$CINC = 0.0128\ A/h + 7.918\ CV \qquad R^2 = 0.6609$
PPA:	$CINC = 0.005\ A/h + 1.26\ CV \qquad R^2 = 0.5182$
IPPA:	$CINC = 0.025\ A/h + 6.307\ 1/CV \qquad R^2 = 0.4880$
MCA:	$CINC = 0.005\ A/h + 0.938\ CV \qquad R^2 = 0.4887$

Figure 2. CINC Regression Models.

Table 6. Maximum, Average, and Standard Deviation of CINC for Different Demand Patterns.

	Demand Patterns																	
	LI			EI			LD			ED			S			TS		
	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.
LFL	102.6	11.4	22.3	79.8	13.9	20.6	80.7	5.5	17.0	321.4	109.3	80.4	17.2	7.4	4.5	0.1	0.0	0.0
SM	2.0	0.1	0.3	1.4	0.0	0.1	5.0	0.3	0.9	12.8	1.3	1.6	0.6	0.1	0.2	0.0	0.0	0.0
MSM	2.0	0.1	0.3	1.4	0.0	0.1	5.0	0.3	0.9	12.8	1.2	1.5	0.6	0.1	0.1	0.0	0.0	0.0
H2	3.7	0.1	0.5	1.4	0.0	0.1	4.6	0.1	0.5	10.2	1.0	1.2	1.1	0.1	0.2	0.0	0.0	0.0
CSMLUC1	5.3	0.3	0.8	1.4	0.0	0.1	5.0	0.3	0.9	12.8	1.4	1.6	4.6	0.9	1.1	1.8	0.1	0.4
CSMLUC2	6.2	0.3	1.2	1.4	0.0	0.1	9.6	0.5	1.9	21.8	2.3	3.4	12.1	4.6	3.2	0.1	0.0	0.0
EOQ	24.3	12.2	4.7	27.6	4.2	7.6	10.4	0.8	1.9	148.5	27.1	30.9	64.4	19.5	12.3	35.4	15.8	10.6
LUC	5.3	0.3	0.8	1.5	0.0	0.2	9.6	0.5	1.9	21.8	2.5	3.5	223.2	49.6	51.0	1.8	0.1	0.4
POQ	15.4	3.2	3.9	12.4	1.5	2.9	6.3	0.5	1.4	121.9	34.8	36.6	17.2	7.4	4.5	0.1	0.0	0.0
MPOQ	13.8	3.1	3.5	12.4	1.3	2.7	6.3	0.5	1.4	121.9	34.8	36.6	17.2	7.4	4.5	0.1	0.0	0.0
PPA	4.2	0.2	0.7	1.5	0.0	0.2	6.0	0.2	0.7	11.1	1.2	1.3	1.6	0.2	0.3	0.0	0.0	0.0
IPPA	20.7	1.5	3.9	17.3	1.7	3.4	29.4	1.4	5.4	108.4	30.1	28.3	1.5	0.3	0.4	0.0	0.0	0.0
MCA	2.0	0.1	0.3	1.4	0.0	0.1	4.6	0.1	0.6	10.2	1.0	1.2	0.6	0.1	0.1	0.0	0.0	0.0

hand, it has a positive effect on SM, MS, H2, POQ, MPOQ, IPPA, and MCA heuristics. Moreover, it has no significant effect on LFL, CSMLUC1, EOQ, and LUC methods. The study of the experimental results shown in Table 10 does not reveal any clear trend in the CPU time as a function of *CV*. The impact of *CV* on the CPU time can be deduced from the linear regression models given in Figure 3. From Table 11, we notice that the CPU times for WW, CSMLUC1, CSMLUC2, LUC, POQ, and MPOQ are increased by an increase of *PZ*, whereas this factor has an opposite effect on the CPU times of LFL, SM, MSM, H2, and EOQ heuristics. However, no clear trend of variation of CPU time can be observed for the remaining approaches. Finally, from Table 12, we cannot detect any significant change in CPU time with the different demand patterns tested in the third experiment. Table 13 shows that LFL is the most rapid heuristic. However, as mentioned above, the cost performance of this heuristic was one of the worst compared to other heuristics. The second fastest heuristic is IPPA.

The remaining procedures are ranked in Table 13 according to their computer time effectiveness.

Based on the above analysis, we ranked the heuristics lexicographically by cost and computer time effectiveness, *i.e.*, we ranked them first according to the average cost deviation from optimality, then in case of tie (close values of average CINC) we ranked them according to the average computer CPU time. Overall, MCA is the best heuristic followed by MSM, SM, H2, PPA, IPPA, CSMLUC2, CSMLUC1, POQ, MPOQ, LFL, and finally LUC. Therefore, we recommend to practitioners adopting MCA or MSM to obtain a cost- and computationally-effective ordering schedule for their dynamic inventory planning problems.

Table 7. Overall Cost Performance for All Heuristics.

Heuristic	Cost Increase CINC		
	Maximum	Average	Std. Dev.
MCA	11.1	0.5	1.1
MSM	12.8	0.5	1.1
H2	10.2	0.6	1.2
SM	12.8	0.6	1.1
PPA	19.2	0.9	1.8
IPPA	108.4	3.0	9.9
CSMLUC1	43.8	4.4	6.9
CSMLUC2	974.1	7.9	25.1
POQ	129.9	10.6	18.1
MPOQ	154.3	10.8	18.5
EOQ	237.0	15.4	18.9
LFL	321.4	22.0	39.1
LUC	1134.2	24.1	53.3

Table 8. Minimum, Maximum, Average, and Standard Deviation of CPU as a Function of *N*.

	Length of Planning Horizon *N*														
	12			52			104			156			366		
	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.
WW	0.116	0.072	0.021	1.16	0.446	0.259	3.03	0.971	0.648	4.83	1.49	1.03	12.2	3.66	2.61
LFL	0.028	0.009	0.006	0.049	0.029	0.007	0.067	0.057	0.004	0.106	0.085	0.009	0.022	0.193	0.012
SM	0.053	0.030	0.009	0.165	0.133	0.013	0.331	0.265	0.024	0.451	0.396	0.036	1.04	0.921	0.080
MSM	0.057	0.037	0.007	0.180	0.149	0.014	0.347	0.299	0.026	0.502	0.444	0.037	1.16	1.03	0.088
H2	0.058	0.036	0.018	0.229	0.145	0.077	0.464	0.292	0.154	0.687	0.437	0.228	1.59	1.01	0.536
CSMLUC1	0.094	0.056	0.008	0.429	0.285	0.059	0.961	0.584	0.133	1.46	0.880	0.203	3.50	2.09	0.508
CSMLUC2	0.083	0.051	0.014	0.466	0.235	0.089	1.0	0.476	0.195	1.54	0.721	0.304	3.67	1.71	0.761
EOQ	0.053	0.022	0.008	0.115	0.089	0.009	0.209	0.176	0.012	0.302	0.262	0.016	0.66	0.600	0.029
LUC	0.056	0.031	0.009	0.339	0.159	0.059	0.725	0.332	0.132	1.09	0.502	0.206	2.62	1.21	0.516
POQ	0.065	0.050	0.008	0.339	0.201	0.039	0.723	0.398	0.082	1.09	0.595	0.125	2.70	1.38	0.312
MPOQ	0.053	0.026	0.009	0.202	0.109	0.023	0.441	0.218	0.047	0.618	0.323	0.068	1.64	0.757	0.180
PPA	0.051	0.019	0.008	0.109	0.076	0.011	0.172	0.146	0.014	0.259	0.216	0.018	0.594	0.499	0.043
IPPA	0.056	0.021	0.008	0.122	0.086	0.019	0.255	0.175	0.037	0.352	0.260	0.055	0.795	0.604	0.129
MCA	0.053	0.028	0.009	0.166	0.122	0.024	0.329	0.245	0.046	0.473	0.367	0.070	1.10	0.855	0.169

5. CONCLUSION

Since the paper of Wagner and Whitin, several authors have studied the single item, finite horizon, lot sizing problem. Various heuristics were developed and claimed to be simple, easy to understand, and cost effective. However the lack of adequate and exhaustive comparison of these heuristics has led us to conduct the research in this paper. We first reviewed the well known and recent heuristics dealing with this type of problem. Then, we proposed four new heuristics that are simple and based on straightforward modifications of existing methods. Finally, we conduct an adequate and extensive experimental design to compare the effectiveness of thirteen heuristics in terms of cost

Table 9. Maximum, Average, and Standard Deviation of CPU as a Function of *CV*.

	Coefficient of Variation *CV*																	
	0.1			0.5			1.0			1.5			2.0			3.0		
	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.
WW	12.2	2.59	2.96	11.3	2.43	2.76	10.0	2.19	2.45	9.18	2.01	2.23	8.09	1.85	1.98	6.82	1.66	1.73
LFL	0.209	0.077	0.065	0.208	0.077	0.067	0.203	0.074	0.064	0.215	0.077	0.068	0.207	0.077	0.067	0.203	0.075	0.067
SM	0.935	0.327	0.292	0.928	0.327	0.292	0.938	0.331	0.295	0.917	0.328	0.294	0.923	0.331	0.296	0.924	0.331	0.296
MSM	1.05	0.372	0.330	1.04	0.371	0.330	1.04	0.371	0.331	1.05	0.375	0.332	1.04	0.373	0.332	1.03	0.375	0.331
H2	1.43	0.496	0.445	1.42	0.497	0.444	1.42	0.500	0.446	1.41	0.499	0.448	1.43	0.501	0.449	1.42	0.501	0.450
CSMLUC1	3.22	0.868	0.841	3.16	0.885	0.847	3.06	0.869	0.829	2.96	0.863	0.816	2.85	0.849	0.791	2.75	0.842	0.777
CSMLUC2	3.67	0.962	0.956	3.31	0.901	0.881	3.03	0.836	0.814	2.76	0.801	0.762	2.65	0.774	0.731	2.52	0.741	0.697
EOQ	0.646	0.234	0.208	0.632	0.231	0.206	0.622	0.229	0.201	0.613	0.225	0.203	0.611	0.226	0.199	0.609	0.226	0.200
LUC	2.62	0.633	0.652	2.52	0.614	0.629	2.32	0.581	0.593	2.21	0.556	0.560	2.06	0.537	0.526	1.91	0.506	0.494
POQ	1.25	0.409	0.362	1.27	0.413	0.369	1.27	0.416	0.369	1.29	0.421	0.375	1.31	0.431	0.383	1.31	0.440	0.390
MPOQ	0.671	0.224	0.195	0.677	0.227	0.198	0.686	0.229	0.200	0.706	0.231	0.205	0.702	0.233	0.208	0.702	0.239	0.210
PPA	0.543	0.199	0.177	0.533	0.199	0.177	0.549	0.198	0.177	0.538	0.199	0.177	0.544	0.200	0.178	0.552	0.201	0.180
IPPA	0.617	0.187	0.170	0.605	0.189	0.170	0.596	0.191	0.170	0.599	0.194	0.174	0.592	0.192	0.170	0.593	0.194	0.173
MCA	0.875	0.276	0.248	0.855	0.273	0.246	0.882	0.277	0.249	0.856	0.275	0.248	0.867	0.279	0.253	0.861	0.279	0.252

Table 10. Maximum, Average, and Standard Deviation of CPU as a Function of *PZ*.

	Percent of Zero Demand Periods *PZ*																	
	0			10			20			50			80			90		
	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.
WW	1.64	0.582	0.522	1.66	0.593	0.530	1.72	0.606	0.548	1.99	0.723	0.650	3.42	1.23	1.13	6.10	2.05	1.98
LFL	0.204	0.076	0.065	0.206	0.074	0.066	0.203	0.073	0.064	0.193	0.071	0.060	0.189	0.066	0.059	0.173	0.066	0.057
SM	1.04	0.383	0.342	1.01	0.377	0.333	0.989	0.366	0.325	0.908	0.340	0.304	0.831	0.315	0.278	0.810	0.303	0.270
MSM	1.16	0.430	0.382	1.13	0.420	0.372	1.10	0.412	0.363	0.999	0.380	0.333	0.934	0.347	0.310	0.890	0.338	0.297
H2	1.59	0.590	0.526	1.55	0.576	0.514	1.52	0.563	0.502	1.37	0.516	0.460	1.25	0.468	0.416	1.19	0.448	0.400
CSMLUC1	1.69	0.638	0.570	1.71	0.644	0.576	1.73	0.650	0.582	1.89	0.709	0.634	2.51	0.919	0.836	3.50	1.23	1.16
CSMLUC2	1.13	0.371	0.333	1.18	0.394	0.354	1.26	0.423	0.378	1.45	0.528	0.475	2.15	0.792	0.721	3.24	1.12	1.06
EOQ	0.664	0.239	0.214	0.657	0.236	0.210	0.634	0.231	0.207	0.593	0.219	0.194	0.551	0.211	0.183	0.549	0.208	0.181
LUC	0.788	0.295	0.263	0.798	0.298	0.267	0.815	0.301	0.269	0.890	0.332	0.297	1.39	0.497	0.463	2.31	0.775	0.754
POQ	1.45	0.552	0.489	1.46	0.555	0.488	1.48	0.559	0.494	1.56	0.589	0.522	1.97	0.666	0.610	2.71	0.774	0.765
MPOQ	0.819	0.307	0.272	0.820	0.304	0.269	0.832	0.303	0.270	0.884	0.312	0.279	1.16	0.359	0.336	1.64	0.408	0.421
PPA	0.594	0.183	0.165	0.590	0.194	0.173	0.578	0.200	0.178	0.572	0.211	0.187	0.521	0.193	0.170	0.480	0.177	0.158
IPPA	0.794	0.285	0.255	0.759	0.273	0.244	0.712	0.262	0.233	0.605	0.224	0.200	0.495	0.186	0.161	0.458	0.173	0.150
MCA	1.09	0.395	0.354	1.05	0.381	0.337	1.00	0.365	0.322	0.839	0.314	0.276	0.688	0.260	0.229	0.648	0.241	0.214

deviation from optimality and computational time. We generated 5400 experimental problems covering 540 different settings. For each setting we computed the average, maximum, and standard deviation of the two measure of performance. Our experimental results reported that, on the average, the MCA and MSM are the best heuristics.

It is well known that the Wagner and Whitin algorithm does not provide an optimal solution in a rolling schedule environment [8]. However, our comparative study was conducted in a static framework with a fixed planning horizon. An obvious extension to our research is to study the performance of the thirteen heuristics with the demand continuing beyond the time horizon [20].

Table 11. Maximum, Average, and Standard Deviation of CPU as a Function of *A/h*.

	A/h Ratio																	
	10			50			100			200			300			500		
	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.
WW	6.10	0.829	0.933	5.87	1.04	1.17	6.07	1.19	1.41	8.14	1.43	1.80	9.75	1.59	2.11	12.2	1.88	2.63
LFL	0.209	0.075	0.065	0.203	0.074	0.065	0.208	0.075	0.064	0.220	0.074	0.066	0.208	0.075	0.065	0.204	0.074	0.065
SM	1.04	0.362	0.324	1.04	0.354	0.317	1.04	0.350	0.315	1.04	0.347	0.312	1.04	0.344	0.309	1.03	0.339	0.307
MSM	1.15	0.405	0.361	1.16	0.397	0.354	1.16	0.395	0.353	1.15	0.388	0.348	1.16	0.386	0.346	1.16	0.382	0.342
H2	1.59	0.400	0.457	1.59	0.391	0.444	1.59	0.386	0.438	1.56	0.382	0.435	1.52	0.377	0.428	1.48	0.371	0.420
CSMLUC1	3.50	0.706	0.675	3.43	0.730	0.693	3.32	0.754	0.714	3.25	0.789	0.752	3.28	0.821	0.795	3.38	0.872	0.869
CSMLUC2	3.24	0.509	0.542	3.15	0.574	0.602	3.03	0.616	0.646	2.97	0.667	0.709	3.17	0.706	0.760	3.67	0.765	0.840
EOQ	0.646	0.230	0.204	0.651	0.231	0.204	0.648	0.230	0.202	0.643	0.228	0.201	0.635	0.229	0.201	0.664	0.230	0.205
LUC	2.31	0.357	0.370	2.23	0.389	0.397	2.13	0.419	0.427	2.06	0.464	0.485	2.16	0.497	0.536	2.62	0.552	0.628
POQ	2.71	0.570	0.534	2.64	0.553	0.521	2.57	0.543	0.515	1.91	0.515	0.476	1.56	0.492	0.450	1.54	0.482	0.442
MPOQ	1.64	0.319	0.306	1.61	0.308	0.300	1.12	0.290	0.270	0.874	0.276	0.252	0.827	0.269	0.246	0.827	0.258	0.237
PPA	0.551	0.186	0.169	0.552	0.190	0.168	0.549	0.190	0.169	0.560	0.191	0.169	0.557	0.193	0.170	0.594	0.196	0.172
IPPA	0.793	0.250	0.227	0.794	0.237	0.220	0.790	0.232	0.215	0.784	0.225	0.211	0.776	0.219	0.208	0.786	0.213	0.201
MCA	1.10	0.353	0.320	1.09	0.335	0.308	1.09	0.326	0.306	1.09	0.317	0.297	1.10	0.310	0.290	1.09	0.301	0.283

Table 12. Maximum, Average, and Standard Deviation of CPU for Different Demand Patterns.

	Demand Patterns																	
	LI			EI			LD			ED			S			TS		
	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.	Max.	Avr.	S.D.
WW	1.650	0.602	0.045	1.660	0.670	0.036	1.660	0.724	0.057	2.003	0.509	0.025	1.901	0.532	0.003	1.890	0.340	0.012
LFL	0.223	0.005	0.027	0.223	0.005	0.027	0.223	0.005	0.027	0.223	0.005	0.027	0.223	0.006	0.028	0.223	0.005	0.027
SM	1.051	0.028	0.136	1.051	0.027	0.134	1.051	0.028	0.137	0.941	0.024	0.117	1.051	0.027	0.134	1.051	0.028	0.137
MSM	1.160	0.031	0.152	1.160	0.030	0.150	1.160	0.031	0.154	1.051	0.027	0.132	1.160	0.031	0.151	1.160	0.031	0.153
H2	1.648	0.043	0.211	1.652	0.042	0.207	1.641	0.043	0.211	1.434	0.036	0.177	1.602	0.042	0.207	1.652	0.043	0.212
CSMLUC1	1.539	0.040	0.199	1.543	0.040	0.199	1.543	0.041	0.200	2.578	0.049	0.255	1.539	0.039	0.191	1.543	0.041	0.200
CSMLUC2	0.992	0.026	0.124	1.102	0.027	0.131	0.941	0.025	0.120	2.863	0.051	0.278	1.102	0.027	0.131	0.941	0.025	0.120
EOQ	0.660	0.018	0.086	0.723	0.018	0.089	0.660	0.017	0.083	0.613	0.016	0.079	0.660	0.017	0.083	0.660	0.017	0.082
LUC	0.832	0.021	0.105	0.832	0.021	0.104	0.832	0.021	0.105	1.980	0.031	0.177	0.832	0.021	0.105	0.832	0.021	0.105
POQ	0.613	0.015	0.074	0.609	0.014	0.073	0.609	0.015	0.074	0.609	0.011	0.058	0.609	0.015	0.075	0.609	0.015	0.075
MPOQ	0.832	0.022	0.108	0.832	0.021	0.106	0.832	0.022	0.108	0.832	0.018	0.091	0.832	0.022	0.108	0.832	0.022	0.109
PPA	0.500	0.013	0.060	0.551	0.013	0.064	0.441	0.012	0.057	0.551	0.014	0.069	0.500	0.012	0.062	0.441	0.012	0.057
IPPA	0.832	0.021	0.103	0.832	0.020	0.100	0.832	0.021	0.105	0.613	0.014	0.069	0.820	0.020	0.100	0.832	0.021	0.105
MCA	1.102	0.029	0.144	1.102	0.028	0.139	1.102	0.029	0.146	0.883	0.021	0.100	1.102	0.028	0.139	1.102	0.029	0.145

WW:	$CPU = 0.011\ N + 0.002$	$A/h - 0.0378$	CV	$R^2 = 0.7528$
LFL:	$CPU = 0.001\ N + 0.00001$	$A/h - 0.0001$	PZ	$R^2 = 0.9935$
SM:	$CPU = 0.002\ N - 0.0001$	$A/h + 0.012$	CV	$R^2 = 0.9951$
MSM:	$CPU = 0.003\ N - 0.0001$	$A/h + 0.014$	CV	$R^2 = 0.9953$
H2:	$CPU = 0.004\ N - 0.0001$	$A/h + 0.02$	CV	$R^2 = 0.9938$
CSMLUC1:	$CPU = 0.006\ N + 0.0005$	$A/h - 0.08$	CV	$R^2 = 0.9546$
CSMLUC2:	$CPU = 0.005\ N + 0.001$	$A/h - 0.0124$	CV	$R^2 = 0.8950$
EOQ:	$CPU = 0.002\ N - 0.00002$	$A/h - 0.008$	CV	$R^2 = 0.9975$
LUC:	$CPU = 0.004\ N + 0.0005$	$A/h - 0.084$	CV	$R^2 = 0.8838$
POQ:	$CPU = 0.004\ N - 0.0003$	$A/h + 0.003$	PZ	$R^2 = 0.9500$
MPOQ:	$CPU = 0.002\ N - 0.0002$	$A/h + 0.002$	PZ	$R^2 = 0.9522$
PPA:	$CPU = 0.001\ N + 0.00001$	A/h		$R^2 = 0.9956$
IPPA:	$CPU = 0.001\ N - 0.001$	$A/h + 0.016$	CV	$R^2 = 0.9683$
MCA:	$CPU = 0.002\ N - 0.0001$	$A/h + 0.022$	CV	$R^2 = 0.9738$

Figure 3. CPU Regression Models.

Table 13. Overall CPU Performance for All Procedures.

Procedure	CPU Time			
	Minimum	Average	Maximum	Std. Dev.
LFL	0.00000	0.07438	0.22031	0.06501
PPA	0.00000	0.19120	0.59375	0.16936
IPPA	0.00000	0.22918	0.79453	0.21428
EOQ	0.00000	0.22977	0.66406	0.20280
MPOQ	0.00000	0.28677	1.63984	0.27070
MCA	0.00000	0.32349	1.10156	0.30135
SM	0.00000	0.34937	1.04297	0.31417
H2	0.00000	0.38448	1.59492	0.43731
MSM	0.01484	0.39208	1.15625	0.35083
LUC	0.00000	0.44645	2.62383	0.48642
POQ	0.02734	0.52563	2.70742	0.49210
CSMLUC2	0.00000	0.63949	3.66602	0.69541
CSMLUC1	0.04141	0.77869	3.50312	0.75484
WW	0.04062	1.32838	12.17695	1.80588

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