θ-IRRESOLUTE MAPPINGS IN BITOPOLOGICAL SPACES

F. H. Khedr*

Department of Mathematics
Faculty of Sciences
University of Assiut
Assiut, Egypt

الخلاصــة :

يتناول هذا البحث تعريف ودراسة فصل من الرواسم في الفراغات ثنائية التوبولوجي تسمى بالرواسم مترددة الاتصال من النوع θ . وقد اشتمل على العديد من التكافؤات والخواص الاساسية لهذه الرواسم . كما تَـمَّـت دراسة العلاقة بين الرواسم مترددة الاتصال من النوع θ والأنواع الأخرى من الاتصال في الفراغات ثنائية التوبولوجي .

ABSTRACT

This paper introduces and investigates a class of mappings in bitopological spaces called pairwise θ-irresolute mappings. Several characterizations and basic properties of pairwise θ-irresolute mappings are given. The interrelation among pairwise θ-irresolute mappings and other generalizations of continuous mappings in bitopological spaces are investigated.

*Address for correspondence:
Girls' Colleges General Administration
P.O. Box 838, Dammam
Kingdom of Saudi Arabia

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1. INTRODUCTION

The purpose of this paper is to introduce and investigate a class of mappings in bitopological spaces called pairwise θ-irresolute mappings. In Section 2, we give several characterization and basic properties of pairwise θ-irresolute mappings. In Section 3, we investigate the interrelation among pairwise θ-irresolute mappings and other generalizations of continuous mappings in bitopological spaces.

Throughout the paper, by a space X we mean a bitopological space (X, τ_1, τ_2). By i-Int(A) and i-Cl(A), we shall mean the interior and the closure of a subset A of X with respect to τ_i, respectively, where $i = 1, 2$. Also, $i, j = 1, 2$ and $i \neq j$. A subset S of X is said to be ij-semiopen [1], if there exists i-open set U such that $U \subset S \subset j\text{-Cl}(U)$. S is called ij-regular open (resp. ij-regular closed) [2] if $i\text{-Int}(j\text{-Cl}(S))$ (resp. $i\text{-Cl}(j\text{-Int}(S))) = S$.

2. PAIRWISE θ-IRRESOLUTE MAPPINGS

Definition 2.1. Let S be a subset of a space X. A point x is in the ij-θ-semiclosure of S, denoted by $ij\text{-}s\text{Cl}_\theta(S)$, if $S \cap j\text{-Cl}(U) \neq \varnothing$ for every ij-semiopen set U containing x. If $S = ij\text{-}s\text{Cl}_\theta(S)$, then S is called ij-θ-semiclosed. A point x is in the ij-θ-semiinterior of S, denoted by $ij\text{-}s\text{Int}_\theta(S)$, if there exists an ij-semiopen set U containing x such that $j\text{-Cl}(U) \subset S$.

Definition 2.2. A mapping $f: X \rightarrow Y$ is said to be ij-θ-irresolute iff for every point $x \in X$ and each ij-semiopen set V, in Y, containing $f(x)$, there exists an ij-semiopen set U, in X, containing x such that $f(j\text{-Cl}(U)) \subset j\text{-Cl}(V)$. f is called pairwise θ-irresolute if it is 12-θ-irresolute and 21-θ-irresolute.

Theorem 2.1. For a mapping $f: X \rightarrow Y$, the following are equivalent:

(a) f is ij-θ-irresolute.

(b) $f^{-1}(V) \subset ij\text{-}s\text{Int}_\theta(f^{-1}(j\text{-Cl}(V)))$, for every ij-semiopen set V of Y.

(c) $f(ij\text{-}s\text{Cl}_\theta(A)) \subset ij\text{-}s\text{Cl}_\theta(f(A))$, for every subset A of X.

(d) $ij\text{-}s\text{Cl}_\theta(f^{-1}(B)) \subset f^{-1}(ij\text{-}s\text{Cl}_\theta(B))$, for every subset B of Y.

(e) $f^{-1}(ij\text{-}s\text{Int}_\theta(B)) \subset ij\text{-}s\text{Int}_\theta(f^{-1}(B))$, for every subset B of Y.

Proof. It is straightforward.

Theorem 2.2. Let $f: X \rightarrow Y$ be a mapping and $g: X \rightarrow X \times Y$ be the graph function given by $g(x) = (x, f(x))$ for every $x \in X$. If g is ij-θ-irresolute, then f is ij-θ-irresolute.

Proof. Let $x \in X$ and V be an ij- semiopen set of Y containing $f(x)$. Then $X \times V$ is ij-semiopen subset of $X \times Y$ containing $g(x)$ [1] and hence there exists an ij-semiopen set U of X containing x such that $g(j\text{-Cl}(U)) \subset j\text{-Cl}(X \times V) = X \times j\text{-Cl}(V)$. By the definition of g, we have $f(j\text{-Cl}(U)) \subset j\text{-Cl}(V)$. Therefore, f is ij-θ-irresolute.

Remark 2.1. One may give an example to show that the converse of Theorem 2.2 is not true.

Definition 2.3. A space X is said to be pairwise semi-T_2' if for every two distinct points x and y in X, there exists an ij-semiopen set U and a ji-semiopen set V such that $x \in U$, $y \in V$ and $j\text{-Cl}(U) \cap i\text{-Cl}(V) = \varnothing$.

Theorem 2.3. If $f: X \rightarrow Y$ is a pairwise θ-irresolute injection and Y is pairwise semi-T_2', then X is pairwise semi-T_2'.

Proof. Let x_1 and x_2 be two distinct points of X. Since f is injective and Y is pairwise semi-T_2', then there exists an ij-semiopen set V_1 and a ji-semiopen set V_2 of Y such that $f(x_1) \in V_1$, $f(x_2) \in V_2$ and $j\text{-Cl}(V_1) \cap i\text{-Cl}(V_2) = \varnothing$. Since f is pairwise θ-irresolute, there exists an ij-semiopen set U_1 and a ji-semiopen set U_2 of X containing x_1 and x_2, respectively, such that $f(j\text{-Cl}(U_1)) \subset j\text{-Cl}(V_1)$ and $f(i\text{-Cl}(U_2)) \subset i\text{-Cl}(V_2)$. Therefore, $j\text{-Cl}(U_1) \cap i\text{-Cl}(U_2) = \varnothing$ and X is pairwise semi-T_2'.

3. COMPARISON

Definition 3.1. A mapping $f: X \rightarrow Y$ is said to be

(a) ij-irresolute [4], if the inverse image of each ij-semiopen set in Y is ij-semiopen in X.

(b) ij-semicontinuous [1], if the inverse image of each i-open set in Y is ij-semiopen in X.

(c) ij-almost continuous [2], if the inverse image of each ij-regular open set in Y is i-open in X.

Definition 3.2. [3]. A space X is called pairwise extremally disconnected if $j\text{-Cl}(A)$ is i-open for each i-open set A.

Theorem 3.1. If $f: X \to Y$ is ij-irresolute and X is pairwise extremally disconnected, then f is ij-θ-irresolute.

Proof. Let $x \in X$ and V be an ij-semiopen set of Y containing $f(x)$. Then $f^{-1}(V)$ is ij-semiopen in X and $x \in f^{-1}(V)$. Since X is pairwise extremally disconnected, $j\text{-Cl}(f^{-1}(V)) = ji\text{-}s\text{Cl}(f^{-1}(V))$ [4]. Moreover, since f is ij-irresolute, $ji\text{-}s\text{Cl}(f^{-1}(V)) \subset f^{-1}(ji\text{-}s\text{Cl}(V))$ [4]. Put $U = f^{-1}(V)$, then U is an ij- semiopen set of X containing x and $f(j\text{-Cl}(U)) \subset ji\text{-}s\text{Cl}(V) \subset j\text{-Cl}(V)$. This shows that f is ij-θ-irresolute.

Remark 3.1. In Theorem 3.1 we cannot drop the assumption that X is pairwise extremally disconnected, as can be seen by Example 3.1. Moreover, an ij-θ-irresolute mapping need not be ij-irresolute even if the domain is pairwise extremally disconnected, as Example 3.2 shows.

Example 3.1. Let $X = Y = \{a, b, c, d\}$, $\tau_1 = \{X, \varnothing, \{a\}, \{c\}, \{a, c\}\}$, $\tau_2 = \{X, \varnothing, \{b, c\}, \{a, d\}\}$, $\sigma_1 = \{Y, \varnothing, \{b\}, \{b, c\}\}$ and $\sigma_2 = \{Y, \varnothing, \{a\}, \{d\}, \{a, d\}, \{a, b\}, \{a, b, d\}\}$. Let $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ defined as follows: $f(a) = b$, $f(b) = a$, $f(c) = c$, $f(d) = d$. Then f is 12-irresolute but not 12-θ-irresolute.

Example 3.2. Let $X = Y = \{a, b, c, d\}$, $\tau_1 = \{X, \varnothing, \{b\}, \{b, c\}\}$, $\tau_2 = \{X, \varnothing, \{a\}, \{d\}, \{a, d\}, \{a, b\}, \{a, b, d\}\}$, $\sigma_1 = \{Y, \varnothing, \{a\}, \{c\}, \{a, c\}\}$ and $\sigma_2 = \{Y, \varnothing, \{a, d\}, \{b, c\}\}$. Let $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the identity mapping. Then (X, τ_1, τ_2) is pairwise extremally disconnected and f is 12-θ-irresolute but not 12-irresolute.

Theorem 3.2. If $f: X \to Y$ is an ij-irresolute and ij-almost continuous mapping, then f is ij-θ-irresolute.

Proof. Let $x \in X$ and V be an ij-semiopen set of Y containing $f(x)$. Since f is ij-almost continuous and $j\text{-Cl}(V)$ is ji-regular closed in Y [4], $f^{-1}(j\text{-Cl}(V))$ is j-closed in X [3]. Since f is ij-irresolute, $f^{-1}(V)$ is ij-semiopen in X containing x and $j\text{-Cl}(f^{-1}(V)) \subset f^{-1}(j\text{-Cl}(V))$. Put $U = f^{-1}(V)$, then U is an ij-semiopen in X containing x and $f(j\text{-Cl}(U)) \subset j\text{-Cl}(V)$. This shows that f is ij-θ-irresolute.

Remark 3.2. ij-almost continuous, ij-irresolute and ij-θ-irresolute are respectively independent as can be seen by Examples 3.1, 3.2, and 3.3.

Example 3.3. Let $X, Y, \tau_1, \tau_2, \sigma_1$, and σ_2 be as in Example 3.1. Consider the following cases of mapping $f: X \to Y$.

(1) $f(a) = a$, $f(b) = c$, $f(c) = b$, and $f(d) = d$. In this case, f is 12-irresolute and 12-θ-irresolute but not 12-almost continuous.

(2) $f(a) = f(b) = b$, $f(c) = f(d) = c$. Then f is 12-almost continuous but neither 12-irresolute nor 12-θ-irresolute.

Definition 3.3. A mapping $f: X \to Y$ is called:

(*a*) ij-R-map [5], if the inverse image of each ij-regular open set in Y is ij-regular open in X;

(*b*) ij-δ-continuous [6], if for every $x \in X$ and each i-open set V in Y containing $f(x)$, there exists an i-open set U of X containing x such that $f(i\text{-Int}(j\text{-Cl}(U))) \subset i\text{-Int}(j\text{-Cl}(V))$.

Remark 3.3. [5]. ij-R-map $\Longrightarrow$ ij-δ-continuous $\Longrightarrow$ ij-almost continuous but the converses are not true.

Remark 3.4. One may give examples to show that ij-δ-continuous and ij-θ-irresolute are independent of each other.

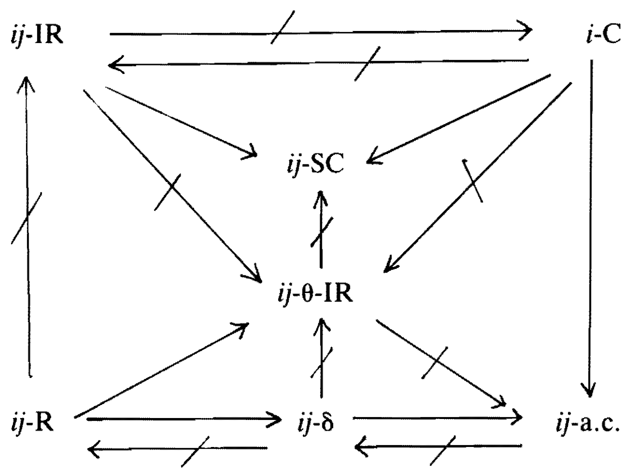
Theorem 3.3. If $f: X \to Y$ is an ij-R-map, then it is ij-θ-irresolute.

Proof. Let $x \in X$ and V be an ij-semiopen set of Y containing $f(x)$. Since $j\text{-Cl}(V)$ is ji-regular closed in Y, $f^{-1}(j\text{-Cl}(V)) = X \setminus f^{-1}(Y \setminus j\text{-Cl}(V))$ is ji-regular closed in X. Put $U = f^{-1}(j\text{-Cl}(V))$, then U is ij-semiopen in X containing x and $f(j\text{-Cl}(U)) = f(U) \subset j\text{-Cl}(V)$. This shows that f is ij-θ-irresolute.

Remark 3.5. The converse of Theorem 3.3 is not true as shown by the following example:

Example 3.4. Let $X, Y, \tau_1, \tau_2, \sigma_1$, and σ_2 be as in Example 3.1. Let $f: X \to Y$ be the identity mapping. Then f is 12-θ-irresolute but not 12-R-map.

From the above discussion we obtain the following diagram, where $A \not\rightarrow B$ indicates that A does not necessarily imply B.



In the diagram, we have abbreviated as follows: IR = irresolute; C = continuous; SC = semi-continuous; R = R-map; θ-IR = θ-irresolute; δ = δ-continuous; and a.c. = almost continuous.

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