# INFLUENCE OF ELASTIC DEFORMATION OF THE STRIP ON THE ROLL FORCE AND TORQUE IN COLD ROLLING

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الخلاصـة :

تـمَّ في هذا البحث وضـع حل عددي كامل للسحب على البارد لشرائح التشغيل المراد تقويتها والتي تقع تحت تأثير الشد في الأمام والخلف مع الأخذ في الاعتبار أقواس الاتصال المرنـة . وطبقت معادلة ( ثان كارمان ) المعدلة والتي اقترحها ( أوروان ) لحساب توزيع ضغط السحب على قوس الاتصال البلاستيكي . وتم ايجاد امتداد المناطق المرنة من معادلات الاتزان وعلاقات الاجهاد والانفعال في هذه المناطق وشروط الاستمرارية للاجهاد عند الانتقال من الحالة المرنة إلى الحالة البلاستيكية .

وتم تمثيل النتائج بالأشكال ليتضح تأثير التغير المرن والتقوية بالانفعال ومعامل الاحتكال والانخفاض الكسري على توزيع ضغط السحب وقوة السحب وعزم الدوران .

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## ABSTRACT

A complete numerical solution is developed for the cold rolling of work-hardening strips with applied front and back tensions, taking account of the elastic arcs of contact in a rigorous manner. The modified form of the von Karman equation proposed by Orowan is employed for the computation of the roll pressure distribution over the plastic arc of contact. The extents of the elastic zones are determined from the equilibrium equations and stress–strain relations applicable to these regions, and the conditions of continuity of the stress at the elastic–plastic interfaces. Hitchcock's formula is used to estimate the radius of the deformed arc of contact. The results are presented graphically to show the effects of elastic deformation, strain-hardening, coefficient of friction and fractional reduction on the roll pressure distribution, roll force and torque. No attempt has been made to compared the numerical solution with the elementary theories of rolling, since an assessment of the accuracy of these theories is already available in the literature.

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## NOTATION

$A$, $B$ = Coefficients of differential equation for the elastic regions.
$E$ = Young's modulus for the material of the strip.
$E_r$ = Young's modulus for the material of the roll.
$F$, $H$ = Coefficients of differential equation for the plastic region.
$G$ = Plastic part of the roll torque per unit width.
$G_e$ = Elastic part of the roll torque per unit width.
$L$ = Length of the arc of contact.
$P$ = Plastic part of the roll force per unit width.
$P_e$ = Elastic part of the roll force per unit width.
$R$ = Undeformed roll radius.
$R'$ = Radius of the deformed arc of contact.
$c$ = Constant parameter equal to $\pi E_r/16(1-\nu_r^2)$.
$e$ = Local fractional reduction in thickness.
$h$ = Local strip thickness.
$h_0$ = Strip thickness at the minimum section.
$h_1$ = Entry thickness of the strip.
$h_2$ = Exit thickness of the strip.
$h_e$ = Strip thickness at the plastic boundary near the entry.
$k$ = Local yield stress of the strip in pure shear.
$\bar{k}$ = Mean shear yield stress over the arc of contact.
$k_1$ = Initial yield stress in pure shear.
$k_2$ = Final yield stress in pure shear.
$m$, $n$ = Empirical constants for the strain-hardening law.
$p$ = Horizontal pressure over a vertical section.
$q$ = Vertical compressive stress acting on a slice.
$r$ = Overall fractional reduction in thickness.
$s$ = Local roll pressure.
$s_e$ = Roll pressure at the plastic boundary near the entry.
$s_0$ = Roll pressure at the minimum section.
$t_1$ = Back tension per unit area of cross-section of the strip.
$t_2$ = Front tension per unit area of cross-section of the strip.
$\alpha$ = Overall angle of contact.
$\beta$ = Elastic angle of contact at the exit.
$\gamma$ = Elastic angle of contact at the entry.
$\phi$, $\theta$ = Angular distances from the minimum section.
$\mu$ = Coefficient of friction between roll and strip.
$\phi_n$ = Angular distance of the neutral point from the exit.
$\nu$ = Poisson's ratio for the strip material.
$\nu_r$ = Poisson's ratio for the roll material.

## 1. INTRODUCTION

The basic theory of cold rolling, formulated by von Karman [1] sixty-five years ago, has been examined by many investigators, who discussed the calculation of roll pressure distribution, the roll separating force and the roll torque. In a comprehensive review of the subject, Orowan [2] considered certain modifications of the von Karman theory, and indicated how, in exceptional circumstances, slipping friction could give way to sticking friction over a part of the arc of contact. Orowan also discussed the inhomogeneity of the deformation of the strip in the roll gap, and proposed a correction factor to take account of this effect in an approximate manner. These refinements are only of minor importance, for usual rolling geometries and coefficients of friction [3], since they involve approximations of the same order of magnitude as those inherent in the von Karman theory.

From the practical point of view, an approximate solution developed by Bland and Ford [4], and modified by Bland and Sims [5], is considered useful for predicting the relevant rolling parameters. Numerical results based on this theory for a range of values of front and back tensions have been reported by Ford, Ellis and Bland [6]. The effects of various modifications and simplifications of the von Karman theory have been examined by Alexander [7], who developed a computer method of solving the basic equations in cold and hot rolling processes. The contributions of the elastic arcs of contact suggested by Bland and Ford [8], and the inhomogeneity of deformation proposed by Orowan can be easily incorporated in the numerical framework [9]. More recently, a finite element solution for the rolling problem has been presented by Li and Kobayashi [10]. For a useful experimental investigation, reference should be made to Lai-Seng and Lenard [11].

In the present investigation, the basic equations have been developed for the elastic portions of the strip at the entry and exit by combining the equilibrium equation with the elastic stress–strain relation. The extent of the elastic regions have been established by satisfying the yield criterion at the elastic–plastic interfaces. Once the elastic arcs of contact have been determined, the stresses in the plastic part of the strip can be computed from the equilibrium equation and the yield criterion, using the conditions of continuity across the interfaces. The basic differential equation used for the plastic region in this paper is that given by Orowan [2]. The computed results have been displayed in graphical forms to indicate how the roll pressure distributions, roll force, and torque depend on such parameters as the fractional reduction, the applied tensions, the coefficient of friction, and the rate of hardening.

## 2. BASIC EQUATIONS

We begin with the usual assumption that the deformed arc of contact is a circular arc of radius $R'$, which is somewhat greater than the radius $R$ of the undeformed roll. The material in the roll gap consists of a central plastic part and a pair of elastic parts as shown diagrammatically in Figure 1. On the entry side, the material is deformed elastically to suffer a change in thickness from $h_1$ to $h_e$. On the exit side, plastic material recovers elastically from the

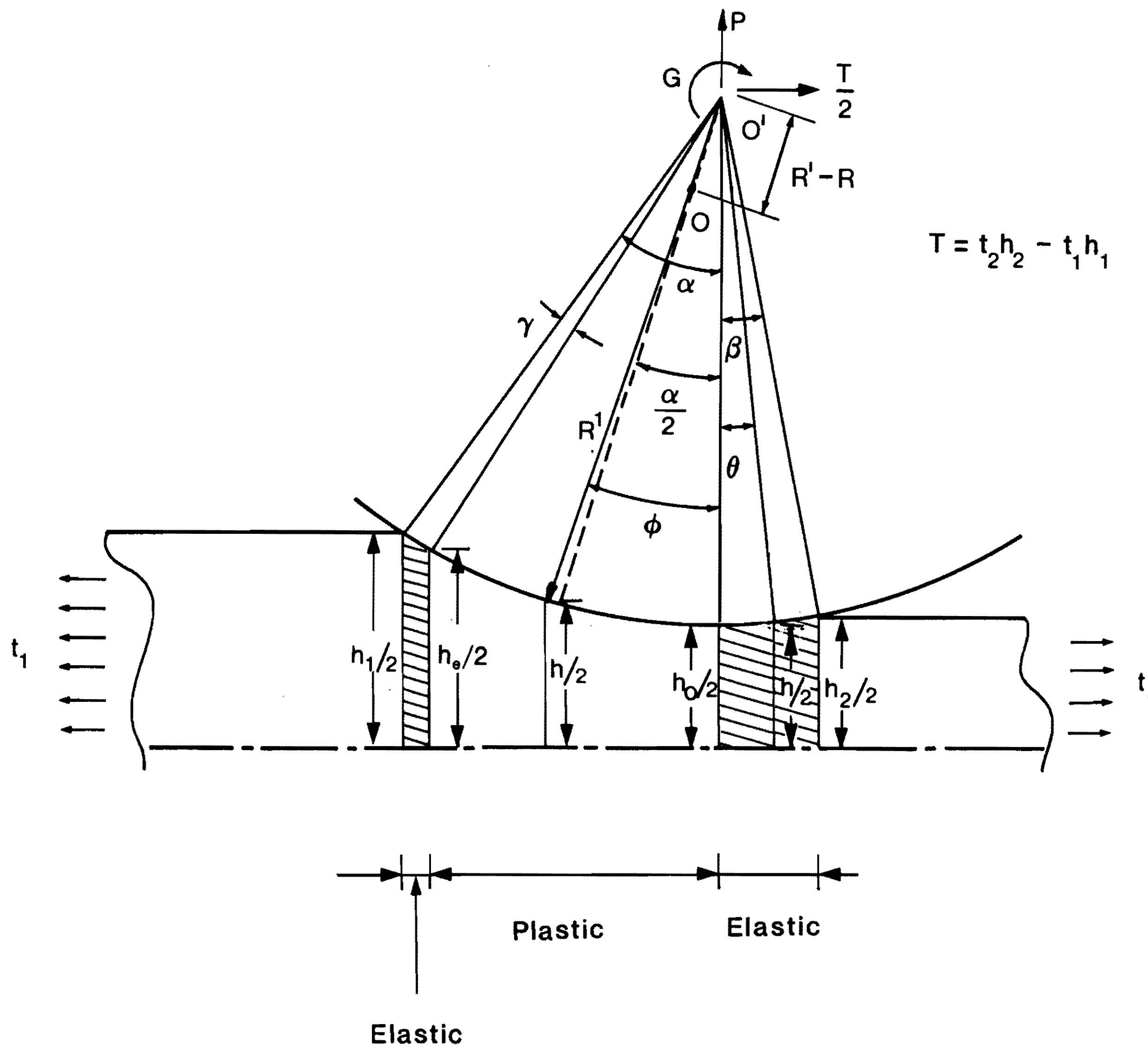


*Figure 1. Geometry of Cold Rolling, Showing the Statically Equivalent Forces and Couples on the Upper Roll. All angles are reckoned positive in the senses they are measured from the vertical plane.*

minimum thickness $h_o$ to the final thickness $h_2$. The elastic angles of contact are denoted by $\gamma$ and $\beta$ on the entry and exit sides respectively. The elastic contact on the exit side has significant effects when the reduction in thickness is sufficiently small.

Consider the longitudinal equilibrium of a vertical slice of material defined by angular distances $\phi$ and $\phi + d\phi$ from the plane of the minimum section. The equation of equilibrium is easily shown to be:

$$\frac{d}{d\phi}(ph) = 2R's(\sin\phi \pm \mu\cos\phi)\,, \qquad (1)$$

where $p$ is the horizontal pressure on a generic section, $s$ the local roll pressure, and $h$ the local sheet thickness. The upper sign refers to the exit side and the lower sign to the entry side of the neutral plane, across which the frictional stress changes sign. The coefficient of friction $\mu$ is assumed constant throughout the arc of contact. The vertical pressure $q$ acting on the element is:

$$q = s(1 \pm \mu\tan\phi)\,. \qquad (2)$$

The above relationship is based on the assumption that $q$ is a principal compressive stress, the other principal component in the considered plane being equal to $p$. Since $\mu$ is generally less than 0.1 and $\alpha$ is usually less than 0.15 radians, $q \cong s$ to a close approximation. We shall not use this approximation, however, in our present analysis.

In the elastic regions, the equilibrium equation must be supplemented by the stress–strain relation, the deformaton being assumed to occur under conditions of plane strain. Since the vertical compressive strain at any section exceeds that at $\phi = \alpha$ by the amount $(h_1 - h)/h_1$, it follows from Hooke's law that:

$$\frac{h_1 - h}{h_1} = \frac{1+\nu}{E}\left[(1-\nu)q - \nu(p + t_1)\right]\,,$$

where $E$ is Young's modulus and $\nu$ is Poisson's ratio for the strip material, and $t_1$ is the applied back tension. In view of the geometrical relations

$$\left.\begin{aligned} h &= h_o + 2R'(1-\cos\phi) \\ h_1 &= h_o + 2R'(1-\cos\alpha)\,, \end{aligned}\right\} \qquad (3)$$

the stress–strain relation furnishes:

$$\nu(p+t_1) = (1-\nu)q - \frac{2ER'}{(1+\nu)h_1}(\cos\phi - \cos\alpha). \quad (4)$$

Eliminating $s$ between (1) and (2), and using (3) and (4), there results

$$\frac{dq}{d\phi} = Aq + B\sin\phi \qquad (5)$$

where

$$A = \frac{2R'}{h}\left[\frac{\nu}{1-\nu}\left(\frac{\sin\phi - \mu\cos\phi}{1+\mu\tan\phi}\right) - \sin\phi\right] \qquad (6)$$

$$B = \frac{2ER'}{(1-\nu^2)h_1} \times \left[\frac{2R'}{h}(\cos\phi - \cos\alpha) - 1 + \nu(1+\nu)\frac{t_1}{E}\right]. \qquad (7)$$

Equation (5) must be solved under the boundary condition $q = 0$ at $\phi = \alpha$. The elastic angle of contact on the entry side can be determined from the condition that the yield criterion $q - p = 2k_1$ must be satisfied at $\phi = \alpha - \gamma$, where $k_1$ is the initial shear yield stress of the material.

On passing through the minimum roll gap, the plastically deformed strip becomes elastic due to unloading. The elastic part of the vertical compressive strain at an angular distance $\theta$ from the minimum section exceeds that at the exit plane by the amount $(h_2 - h)/h_2$. By Hooke's law,

$$\frac{h_2 - h}{h_2} = \frac{1+\nu}{E}\left[(1-\nu)q - \nu(p + t_2)\right]\,,$$

where $t_2$ is the front tension. Using the relation $h_2 - h = 2R'(\cos\phi - \cos\beta)$, the above equation may be rearranged to give

$$\nu(p+t_2) = (1-\nu)q - \frac{2ER'}{(1+\nu)h_2}(\cos\theta - \cos\beta). \quad (8)$$

where $\beta$ is the elastic angle of contact on the exit side. The governing equations in this region are again (5) through (7), except that $\phi$, $\alpha$, and $t_1$ must be replaced by $\theta$, $\beta$, and $t_2$ respectively. The boundary condition is $q = 0$ at $\theta = \beta$. The exit elastic angle of contact is obtained from the yield condition $q - p = 2k_2$ at $\theta = 0$, where $k_2$ is the final shear yield stress of the material.

Due to the applied front and back tensions, $h_1$ and $h_2$ are marginally smaller than the undeformed and rolled sheet thickness $h_1'$ and $h_2'$ respectively. The relationship between these quantities being

$$h_1 = h_1'\left[1-(1+\nu)\frac{t_1}{E}\right], \quad h_2 = h_2'\left[1-(1+\nu)\frac{t_2}{E}\right]. \quad (9)$$

Except for the estimation of roll flattening under extremely light reductions, the approximations $h_1' \cong h_1$ and $h_2' \cong h_2$ are perfectly justified.

In the plastic region, which extends from $\phi = 0$ to $\phi = \alpha - \gamma$, the quantities $p$ and $q$ must satisfy the condition of equilibrium given by (1) and (2), and the yield criterion

$$q - p = 2k \quad (10)$$

where $k$ is the local yield stress in pure shear. From (1), (2), and (10), the governing differential equation is obtained as

$$\frac{\mathrm{d}}{\mathrm{d}\phi}\{h[s(1 \pm \mu \tan\phi) - 2k]\} = 2R's(\sin\phi \pm \mu\cos\phi)$$

which may be reduced to the form

$$\frac{\mathrm{d}s}{\mathrm{d}\phi} = Fs + H\sin\phi \quad (11)$$

where

$$\left.\begin{aligned} F &= \pm\,\mu\sec\phi\left(\frac{2R'}{h} + \sec\phi\right)\Big/(1 \pm \mu\tan\phi) \\ H &= \frac{4kR'}{h}\left(1 + \frac{h}{k}\frac{\mathrm{d}k}{\mathrm{d}h}\right)\Big/(1 \pm \mu\tan\phi) \end{aligned}\right\}. \quad (12)$$

In the above expressions, the upper sign must be used for the integration of (12) along the exit side and the lower sign along the entry side of the neutral plane.

The variation of the yield stress along the plastic arc of contact may be represented with sufficient accuracy by the empirical equation

$$k = k_1\left(1 + \frac{2m}{\sqrt{3}}\ln\frac{h_e}{h}\right)^n \quad (13)$$

where $h_e$ is the strip thickness at $\phi = \alpha - \gamma$. The constants $m$ and $n$ are given by the uniaxial stress–plastic strain curve expressed by the equation

$$\sigma = \sqrt{3}k_1(1 + m\varepsilon^p)^n. \quad (14)$$

The differentiation of (13) with respect to $h$ gives

$$\frac{h}{k}\frac{\mathrm{d}k}{\mathrm{d}h} = -\frac{2}{\sqrt{3}}mn\left(1 + \frac{2m}{\sqrt{3}}\ln\frac{h_e}{h}\right)^{-1}. \quad (15)$$

With the help of (13) and (15), the integration of equation (11) can be carried out from $\phi = 0$ and $\phi = \alpha - \gamma$, using the conditions of continuity of $s$ across these interfaces. The section $\phi = \phi_n$ where the two solutions match defines the neutral plane. Once the roll pressure distribution and the neutral point are known, the roll force and torque can be calculated by integration along the arc of contact.

## 3. METHOD OF ANALYSIS

Except for extremely light reductions, the elastic arcs of contact are small compared to the plastic arc, and considerations of overall equilibrium of the elastic regions are generally sufficient. The basic equations indicate that to a first approximation, the elastic roll pressure distribution is linear on the entry side and parabolic on the exit side. Figure 2 shows the forces acting on the elastic regions, in which the arcs of contact are replaced by the respective chords. Considering first the entry region, the equation of longitudinal equilibrium may be written as

$$p_e = -t_1 + \frac{R'\gamma}{h_1}\left[\mu\cos\left(\alpha - \frac{\gamma}{2}\right) - \sin\left(\alpha - \frac{\gamma}{2}\right)\right]s_e$$

to a close approximation. Eliminating $p_e$ by means of the yield criterion expressed in the form

$$s_e[1 + \mu\tan(\alpha - \gamma)] - p_e = 2k_1$$

which must be satisfied at the elastic–plastic interface, we get

$$s_e = (2k_1 - t_1)\Big/\left\{1 + \mu\tan(\alpha-\gamma) - \frac{R'\gamma}{h_1}\left[\mu\cos\left(\alpha - \frac{\gamma}{2}\right) - \sin\left(\alpha - \frac{\gamma}{2}\right)\right]\right\}. \quad (16)$$

From the stress–strain relation (4) considered at the interface $\phi = \alpha - \gamma$, and the yield criterion $q_e - p_e = 2k_1$, there results

$$\frac{2R'}{h_1}[\cos(\alpha-\gamma) - \cos\alpha] = \frac{1+\nu}{E}\{(1-2\nu)[1 + \mu\tan(\alpha-\gamma)]s_e + \nu(2k_1 - t_1)\}. \quad (17)$$

For any given values of $\alpha$, $R'/h_1$ and $t_1$, the quantity $s_e$ and $\gamma$ can be calculated by an iterative process, starting with $s_e \cong 2k_1 - t_1$ as a first approximation.

The equation of overall longitudinal equilibrium of the exit elastic region, with a parabolic distribution of roll pressure, is easily obtained with sufficient accuracy as

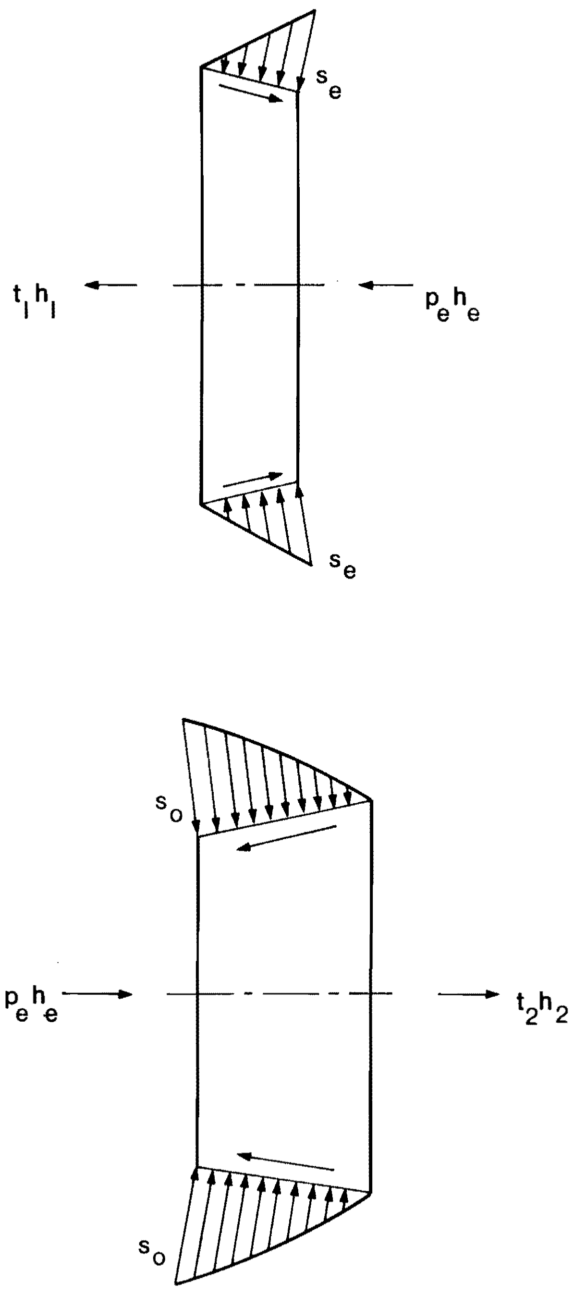


*Figure 2. Equilibrium of Forces Acting on the Elastic Zones at the Entry and Exit; (a) entry zone, (b) exit zone.*

$$p_o = -t_2 + \frac{4R'\beta}{3h_2}\left(\mu \cos\frac{\beta}{2} - \sin\frac{\beta}{2}\right)s_o.$$

A second equation connecting $p_o$ and $s_o$ is given by the yield criterion $s_o - p_o = 2k_2$. The elimination of $p_o$ then furnishes

$$s_o = (2k_2 - t_2)\Big/\left\{1 - \frac{4R'\beta}{3h_2}\left(\mu \cos\frac{\beta}{2} - \sin\frac{\beta}{2}\right)\right\}. \quad (18)$$

Setting $\theta = 0$ in the stress–strain equation (8), and using the yield criterion, we get

$$\frac{2R'}{h_2}(1-\cos\beta) = \frac{1+\nu}{E}\left[(1-2\nu)s_o + \nu(2k_2 - t_2)\right]. \quad (19)$$

Equations (18) and (19) furnish $s_o$ and $\beta$ for given values of $R'/h_2$ and $t_2$, the yield stress $k_2$ being obtained from (13) with $h = h_2$.

The contributions $P_e$ and $G_e$ to the roll force and torque respectively (per unit width) from the elastic arcs of contact can be computed from the formulas

$$\left.\begin{aligned} P_e &= R'(\tfrac{1}{2}\gamma s_e \cos\alpha + \tfrac{2}{3}\beta s_o) \\ G_e &= \mu R R'(\tfrac{1}{2}\gamma s_e - \tfrac{2}{3}\beta s_o) \end{aligned}\right\} \quad (20)$$

which are sufficiently accurate for practical purposes. If we introduce the approximation $\sin\phi \cong \phi$ and $\cos\phi \cong 1$, and neglect higher order terms, the expression for $P_e$ and $G_e$ reduce to those given by Bland and Ford [8].

The total roll force and torque per unit width are $P + P_e$ and $G + G_e$ respectively, where $P$ and $G$ correspond to the plastic arc of contact. The change in curvature of the roll is given by

$$\frac{1}{R} - \frac{1}{R'} = \frac{P + P_e}{cL^2}, \quad c = \frac{\pi E_r}{16(1-\nu_r^2)}$$

where $L$ is the total arc of contact, and the subscript r refers to the roll material. Substituting for $L$, we obtain the modified Hitchcock formula

$$\frac{R'}{R} = 1 + \frac{P + P_e}{c(\sqrt{h_1 - h_o} + \sqrt{h_2 - h_o})^2}. \quad (21)$$

The vertical roll force per unit width over the plastic arc of contact is easily shown to be

$$P = R'\int_0^{\alpha'} s\cos\phi\,d\phi + \mu R'\left(\int_{\phi_n}^{\alpha'} s\sin\phi\,d\phi - \int_0^{\phi_n} s\sin\phi\,d\phi\right), \quad \alpha' = \alpha - \gamma. \qquad (22)$$

The moment of the normal and tangential forces per unit width acting on the surface of each roll about the center $O'$ of the deformed arc of contact is

$$G' = \mu R'^2\left(\int_{\phi_n}^{\alpha'} s\,d\phi - \int_0^{\phi_n} s\,d\phi\right). \qquad (23)$$

The roll torque $G$ per unit width is the moment of the surface forces about the roll center $O$. Since the forces acting on the roll are statically equivalent to horizontal and vertical forces $\frac{1}{2}(t_2h_2 - t_1h_1)$ and $P$ respectively acting at $O'$, together with a couple of magnitude $G'$, it follows from statics that

$$G = G' + (R' - R)\left[\tfrac{1}{2}(t_2h_2 - t_1h_1)\cos\frac{\alpha}{2} - P\sin\frac{\alpha}{2}\right].$$

The derivation of this equation involves the customary assumption that the line joining the centres $O$ and $O'$ bisects the plastic angle of contact. In view of (23), the final expression for the roll torque becomes

$$G = \mu R'^2\left(\int_{\phi_n}^{\alpha'} s\,d\phi - \int_0^{\phi_n} s\,d\phi\right) - (R' - R) \times \left[\tfrac{1}{2}T\cos\frac{\alpha}{2} + P\sin\frac{\alpha}{2}\right] \qquad (24)$$

where $T = t_1h_1 - t_2h_2$. The more complicated expression for $G$ given by Alexander [7] is completely equivalent to (24), provided $P$ is given by (22). Under the approximation $\sin\phi \cong \phi$ and $\cos\phi \cong 1$, (24) reduces to that given by Hill [12].

Since $P$ and $R'$ depend on one another in a given rolling program, they have to be evaluated simultaneously by successive iterations. For a rapid convergence of the iterative process, a suitable starting value of $R'$ must be chosen, or computed from an approximate value of $P$. Considering, for instance, the Bland and Ford theory of cold rolling, it is possible to express the roll force by the empirical formula

$$P = 2\bar{k}\sqrt{R'\delta}\left[1.02 + r(1.5 + 1.6r^2)\mu\sqrt{\frac{R'}{h_1}} - 1.9r^2\right] \qquad (25)$$

where $\delta$ is the draft $h_1 - h_2$, $r$ the fractional reduction in thickness, and $\bar{k}$ a suitable mean value of the shear yield stress over the arc of contact. The expression in the square bracket predicts the dimensionless roll force with good approximation over the relevant range. Substituting into (21), and setting $h_o \cong h_2$ and $P_e \cong 0$, we obtain the quadratic [2]

$$\left\{\frac{h_1}{\mu^2 R} - \lambda\sqrt{r}(1.5 + 1.6r^2)\right\}\left(\mu\sqrt{\frac{R'}{h_1}}\right)^2 - \lambda\left(\frac{1.02}{\sqrt{r}} - 1.9r\sqrt{r}\right)\mu\sqrt{\frac{R'}{h_1}} - 1 = 0 \qquad (26)$$

where $\lambda = 2\bar{k}/\mu c$. Equation (26) indicates that the ratio $R'/R$ depends only on the parameters $\mu\sqrt{(R/h)}$, $\lambda$ and $r$. For accuracy the mean yield stress should be estimated from the formula [13]

$$\bar{k} = k_1 + \int_{k_2}^{k_1} \sqrt{\left(1 - \frac{e}{r}\right)}\,dk, \quad e = \frac{h_1 - h}{h_1}. \qquad (27)$$

The integral in (27) can be evaluated numerically using (14), with $h_e \cong h_1$. When the value of $R'$ is computed from (26), equation (11) can be solved for the roll pressure distribution over the plastic region. A modified value of $R'$ is then obtained from (20), (22), and (21), after $s_e$, $s_o$, $\gamma$, and $\beta$ have been computed from equations (16) through (19). In general, the results should converge after only a few iterations.

## 4. DISCUSSION OF RESULTS

Numerical results based on the preceding theory have been evaluated by using a computer program similar to that used by Alexander, and are displayed in Figures 3 to 7. The material is assumed to have an initial yield stress $2k_1 = 6.391$ tons in$^{-2}$ (98.677 MPa), the parameter $m$ of the stress–strain equation (14) being taken as 727.65. For simplicity, the strip tensions are assumed to be zero. In Figure 3, the normal roll pressure is plotted against the parameter $1 - \phi/\alpha$ for given values of $r$ and $\mu\sqrt{(R/h_1)}$, and two different values of $n$. The effect of work-hardening is to

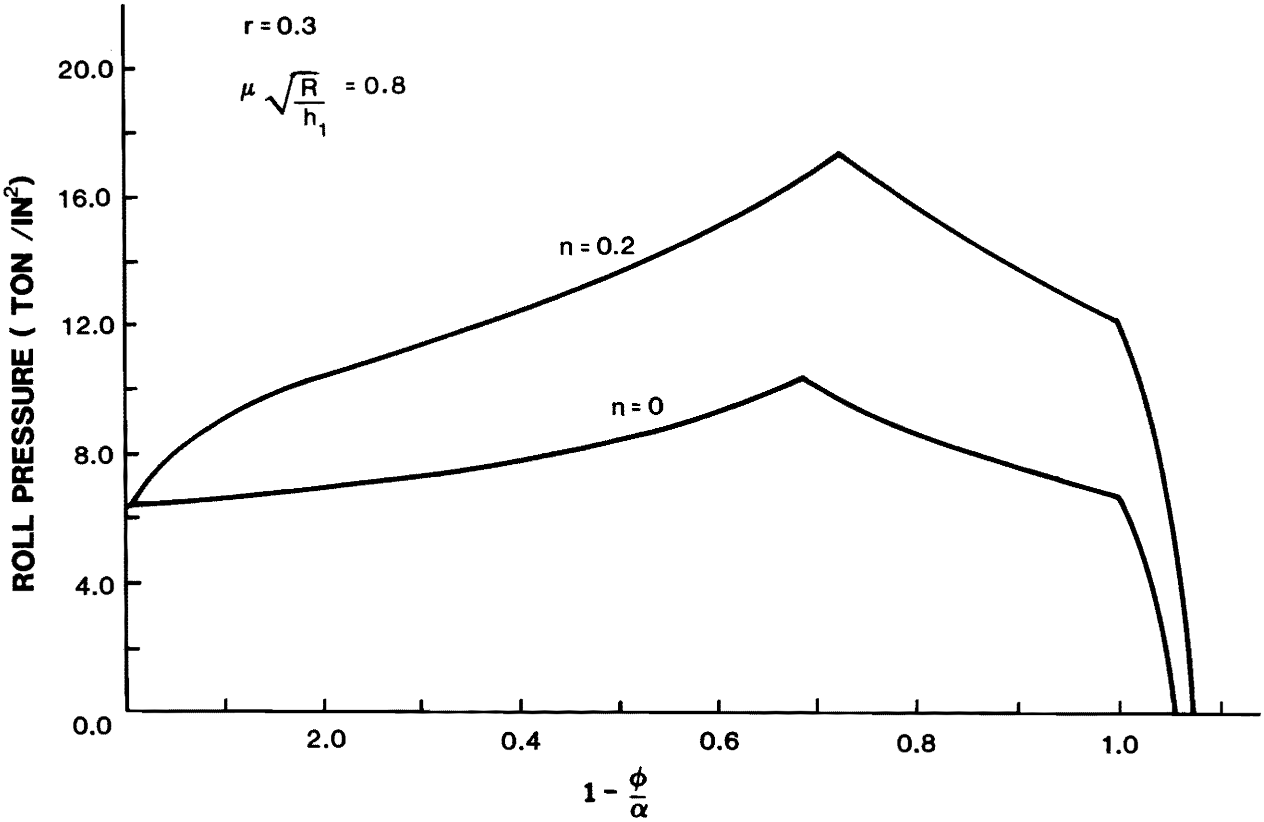


*Figure 3. Roll Pressure Distribution in the Cold Rolling of Non-Hardening and Work Hardening Strips.*

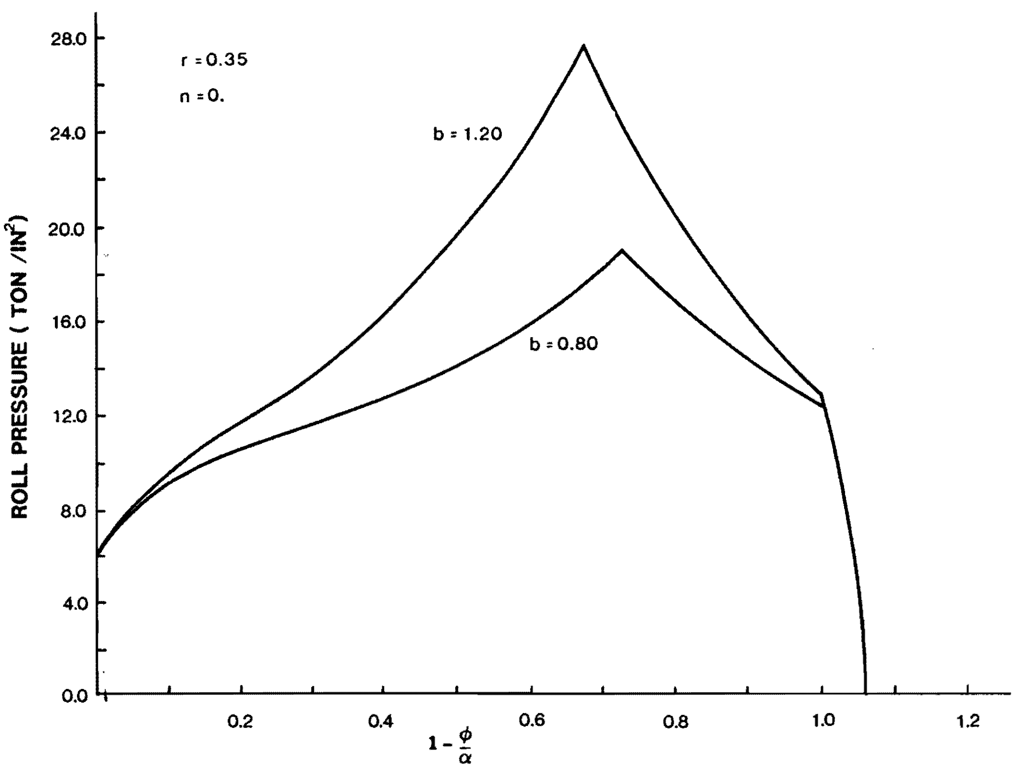


*Figure 4. Roll Pressure Distribution in the Cold Rolling for Two Different Values of* $b = \mu\sqrt{R/h_1}$.

increase the roll pressure as well as the ratio $\beta/\alpha$, representing the extent of the elastic recovery in relation to the energy angle of contact. Figure 4 shows the distribution of roll pressure for a non-hardening material, with $r = 0.35$, and indicates how the roll pressure increases with the parameter $\mu\sqrt{(R/h_1)}$. The effects of the elastic deformation are not appreciable for sufficiently large reductions, as may be seen from Figure 5. It is apparent that the elastic angle of contact on the entry side is an insignificant part of the total angle of contact even for a reduction that is as low as 3 percent.

The variation of the roll force with the fractional reduction in thickness is shown in Figure 6 for both non-hardening and work-hardening materials, using a definite value of $\mu\sqrt{(R/h_1)}$. The graphs obtained by excluding the contribution from the elastic arcs of contact are included for comparison. As expected, the percentage error involved in neglecting the elastic deformation of the strip increases as the reduction is decreased. Figure 7 indicates how the roll torque is overestimated by the neglect of the elastic arcs of contact. The disparity is seen to be considerably more for the work-hardening material, due to the augmented roll pressure on the exit side caused by work-hardening. This introduces the necessity for taking account of the elastic deformation of the strip when dealing with relatively hard materials, not only for small reductions but also for moderate and large reductions. The analysis presented in this paper should be applied to such materials for an accurate estimation of the roll force and torque in cold flat rolling.

In the elementary theories of cold rolling, which are based on various approximations of the von Karman equation, the elastic deformation of the strip is usually disregarded, and the work-hardening is allowed for by introducing a suitable mean yield

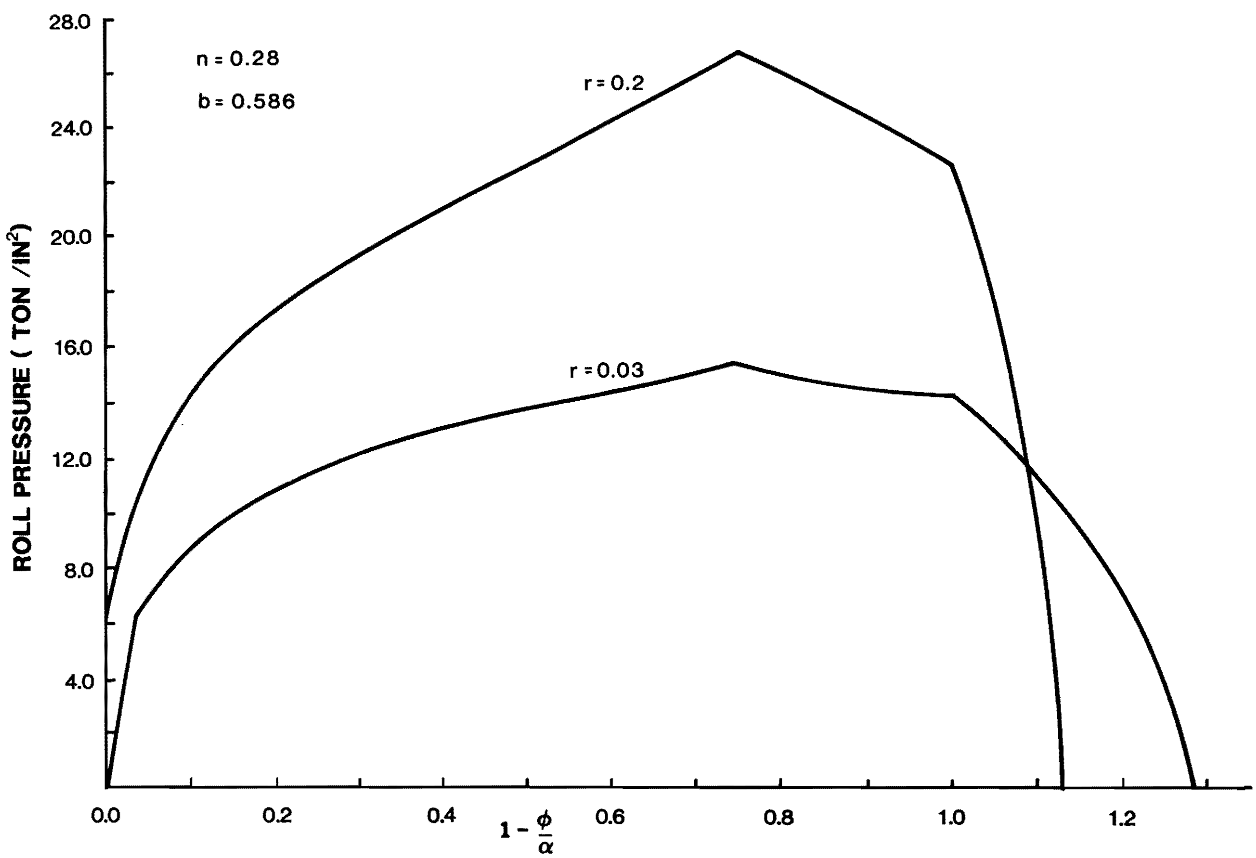


*Figure 5. Roll Pressure Distribution in the Cold Rolling for Two Different Fractional Reductions in Thickness.*

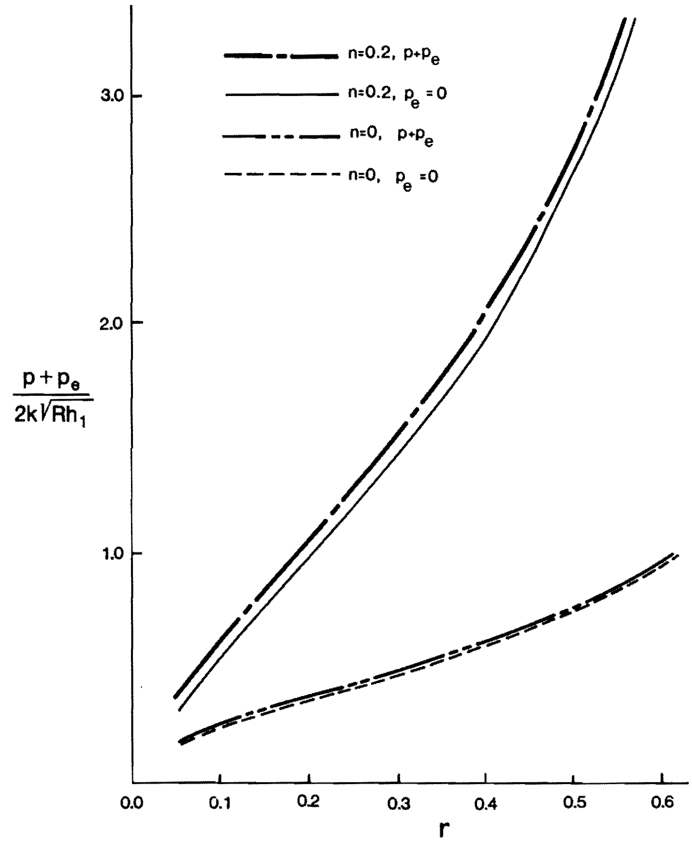


*Figure 6. Variation of Roll Force with Reduction in Cold Rolling Without Tension ($\mu\sqrt{R/h_1} = 1.2$).*

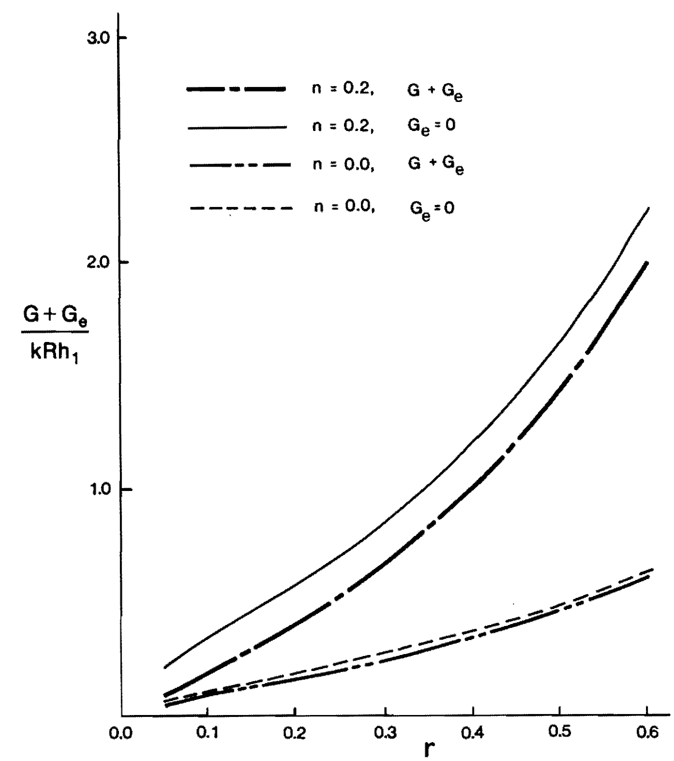


*Figure 7. Variation of Roll Torque with Reduction in Cold Rolling Without Tension ($\mu\sqrt{R/h_1} = 1.2$).*

stress over the arc of contact. In the case of thin hard metals, the elastic arcs of contact are taken into account by introducing correction terms for the estimation of roll force and torque, without considering the effects of the elastic deformation on the complete roll pressure distribution. Although this procedure seems adequate for the standard rolling problem, the determination of the minimum strip thickness, for which a reduction is possible in cold rolling, will require more rigorous mathematical formulations. The theory presented in the first part of this paper would be particularly suitable for an accurate prediction of the minimum thickness in cold rolling. Since the accuracy of the approximations based on the rigid–plastic assumption for the material response has already been discussed in the literature, no attempt has been made here to compare the present theory with the elementary theories of rolling.

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**Paper Received 4 February 1990; Revised 9 July 1990.**