COMPARISON BETWEEN THEORY AND EXPERIMENT OF WATER PARTICLE VELOCITY AND ACCELERATION UNDER LARGE AMPLITUDE WAVES

Mohammed Habib Ullah*

Research of Flood Control & Drainage Research
Bangladesh University of Engineering & Technology, Dhaka

and

Bruce B. Sharp

Reader, Department of Civil Engineering
University of Melbourne, Parkville
Victoria 3052, Australia

الخلاصـــة :

اجريت دراسة تجريبية للموجات ذات الاتساع الكبير فى المياه الضحلة وذلك باستخدام قنوات الموج المخبرية . ومن ثم تم تسجيل سرعة الذرات الافقية والرأسية المكونة للمياه وكذلك التسارع الناجم . وقد ساعد محدد قياس السرعة لسلك التعليق على اجراء القياسات الاولرية (Eulerian) وتم استخدام التصوير فى القياسات اللاجرانجية (Lagrangian) . واستخدمت نظرية الموج الثالثة لنظام ستوكس ونظرية الموج الكنويدلية (Cnoidal) للمقارنة النظرية . وكان التوافق بين المعلومات والقيم النظرية جيداً فيما يتعلق بالموجات ذات المدى القصير والاتساع المنخفض . ووجد ان النظريات تبالغ كثيرا فى تقدير البيانات عن طريق زيادة الحواف مع زيادة طول الموجات وارتفاعها النسبى .

ABSTRACT

An experimental investigation of large amplitude waves in intermediate depth water was performed in a laboratory wave channel. Horizontal and vertical components of water particle velocity and the derived acceleration are reported. The suspension wire velocity gauge enabled Eulerian measurements to be made and photography was used for the Lagrangian measurements. The third order Stokes' wave theory and cnoidal wave theory were used for theoretical comparison. The agreement between data and theoretical values was very good for waves having relatively short wavelength and low amplitudes and the theories were found to overestimate the data by increasing margins with the increase of the relative wavelength and relative wave height.

*Address for correspondence :
Institute of Flood Control
and Drainage Research
Bangladesh University of Engineering
and Technology
Dhaka-2, Bangladesh

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1. NOTATION

The following notations are used in this paper:

a	Wave amplitude from mean water level
cn, dn, sn	Jacobian elliptic functions
C	Wave celerity
$E(k)$	Complete elliptic integral of the second kind
f_3	Function of h/L in the third order Stokes' wave theory
g	Gravitational constant
h	Still water depth
H	Crest to trough wave height
k	Wave number $(2\pi/L)$, modulus of the elliptic functions
$K(k)$	Complete elliptic integral of the first kind
L	Wave-length
S	Distance from the bottom
t	Time or duration
T	Wave period
u, w	Horizontal and vertical components of water particle velocity respectively
x	Horizontal coordinate in the direction of wave propogation
z	Vertical coordinate positive upwards from the SWL (for cnoidal wave, from the bottom)
x_0, z_0	Horizontal and vertical coordinate respectively of an individual particle at still water
z_t	Distance between the wave trough and the wave channel bottom
η	Elevation of water surface above the still water level
θ	$= k(x-Ct)$
σ	Wave angular frequency $= 2\pi/T$
( )	$= 2K(k)\left(\frac{x}{L}-\frac{t}{T}\right)$, argument of elliptic functions.

2. INTRODUCTION

The water particle orbital velocity induced by a surface wave is an important factor in understanding the mechanism of wave breaking, littoral drift, suspension and diffusion of materials by waves and sediment transport, and its value is essential in computations of wave forces when applying Morison's equation.

There are more than a dozen theories available in the literature for finding orbital velocities. Le Mehaute and others [1] indicated that the choice of selecting a theory for a particular wave is not easily made. This necessitates experimental determination of the range of validity and suitability of different theories.

Iwagaki and Sakai [2] indicated that there had been very little experimental data because of the difficulty of the measurements. Tsuchiya and Yamaguchi [3] suggested that there had been very little work on the vertical water particle velocity of regular waves. It seems that most of the previous experimental work had been directed to measuring horizontal water particle velocities under regular waves in deep and intermediate depth water. In most of these measurements, velocities were measured at a particular phase of the wave i.e. under the crest only. Le Mehaute and others [1] regarded the horizontal particle velocity under the crest as the single most important feature, since, in applications, the velocity is generally most critical and under the crest it is the greatest attained at any depth.

Morison and Crooke [4] indicated that little experimental data existed on the paths and velocities of water particles within waves in very shallow depths and at breaking. The experimental work involving measurement of acceleration is extremely rare. The reason is probably that "a theory which prescribes the velocity field well is constrained to be good for other features, such as accelerations and pressure field, *a priori*" [1].

Chakrabarti [5] indicated that before using laboratory model test data for designing off-shore structures it was imperative that the kinematic and dynamic properties of waves generated mechanically in a wave tank should be known *a priori.* He suggested that more corroboration was needed between theories and those actually experienced in wave tanks covering a large range of waves from deep to shallow waters.

From the discussion above it is clear that there is scope for further experimentation. The paper concentrates on horizontal and vertical components of water particle velocity (of both the Lagrangian and Eulerian descriptions) and local acceleration under large amplitude waves in intermediate depth water.

In this connection the difference between the Eulerian and Lagrangian methods of describing fluid motion is mentioned. The Lagrangian method is concerned with what happens to the individual fluid particles in the course of time. On the other hand, the Eulerian method describes what happens at every fixed point in space as a function of time. Lagrangian velocity of water particle under wave can be derived from Eulerian description. But the third-order Lagrangian derivation of horizontal component of water particle velocity reveals some interesting features of wave motion which the corresponding Eulerian description does not show. Stokes [6] as a result of his second-order Lagrangian derivation of water particle velocity showed that in a periodic progressive wave on a horizontal bottom, the fluid particles experience apart from their orbital motion a steady drift velocity, now generally known as mass transport velocity.

The third-order Stokes' wave theory and cnoidal wave theory by Keulegan and Patterson [7] were compared with the experimental data to check the suitability of these theories. There are different versions of Stokes' and cnoidal wave theories available in literature. Le Mehaute [8] and Dean [9] discussed approximate limits of validity for several wave theories. However, a recent finding by Fenton [10] may be mentioned in this regard. He found that some of the existing versions of Stokes' wave theory were incorrect at higher orders. This makes it very difficult to select the most suitable wave theory. As a matter of fact the above mentioned wave theories were selected because of their easiness of application.

WAVE THEORIES

This section lists the equations of water particle velocity and acceleration which were used for comparing with the experimental data. The components of Eulerian velocity at any point x, z as given by Stokes' third-order wave theory derived by Skjelbreia [11] are

$$\frac{u}{C} = F_1 \cosh kS \cos \theta + F_2 \cosh 2kS \cos 2\theta + F_3 \cosh 3kS \cos 3\theta \tag{1}$$

$$\frac{w}{C} = F_1 \sinh kS \sin \theta + F_2 \sinh 2kS \sin 2\theta + F_3 \sinh 3kS \sin 3\theta \tag{2}$$

where

$$F_1 = ka \frac{1}{\sinh kh}; \quad F_2 = \tfrac{3}{4}(ka)^2 \frac{1}{\sinh^4 kh}; \text{ and}$$

$$F_3 = \tfrac{3}{4}(ka)^3 \left[\frac{11 - 2\cosh 2kh}{\sinh^7 kh}\right] \tag{3}$$

The value of 'a' is determined from wave height by the following relation

$$H = 2a + 2\left(\frac{\pi}{L}\right)^2 a^3 f_3\left(\frac{h}{L}\right) \tag{4}$$

where

$$f_3\left(\frac{h}{L}\right) = \tfrac{3}{16}\,\frac{8\cosh^6 kh + 1}{\sinh^6 kh} \tag{5}$$

The value of the L to be used in the above equaions is given by

$$L = \frac{gT^2}{2\pi}\tanh kh\left[1 + (ka)^2\,\frac{8 + \cosh 4kh}{8\sinh^4 kh}\right] \tag{6}$$

The Lagrangian equations for the horizontal and vertical components of velocity given by Skjelbreia [11]

$$\begin{aligned}\frac{u}{C} = {} & [F_1(1 - \tfrac{3}{8}F_1^2)\cosh k(h + z_0) \\ & + \frac{F_1}{8}(F_1^2 + 10F_2)\cosh 3k(h + z_0)]\cos(kx_0 - \sigma t) \\ & + [-\tfrac{1}{2}F_1^2 + F_2\cosh 2k(h + z_0)]\cos 2(kx_0 - \sigma t) \\ & + [\tfrac{1}{4}F_1(F_1^2 - 5F_2)\cosh k(h + z_0) \\ & + F_3\cosh 3k(h + z_0)]\cos 3(kx_0 - \sigma t) \\ & + \tfrac{1}{2}F_1^2[1 - F_1\sigma t\cosh k(h + z_0) \\ & \times\sin(kx_0 - \sigma t)]\cosh 2k(h + z_0) \quad \text{and}\end{aligned} \tag{7}$$

$$\begin{aligned}\frac{w}{C} = {} & [F_1(1 - \tfrac{1}{8}F_1^2)\sinh k(h + z_0) \\ & + \frac{F_1}{8}(-F_1^2 + 6F_2)\sinh 3k(h + z_0)]\sin(kx_0 - \sigma t) \\ & + F_2\sinh 2k(h + z_0)\sin 2(kx_0 - \sigma t) \\ & + [-\tfrac{3}{4}F_1F_2\sinh k(h + z_0) \\ & + F_3\sinh 3k(h + z_0)]\sin 3(kx_0 - \sigma t) \\ & + \tfrac{1}{2}kCt\,F_1^3\sinh k(h + z_0) \\ & \times\cosh 2k(h + z_0)\cos(kx_0 - \sigma t)\end{aligned} \tag{8}$$

The components of water particle local acceleration ($\partial u/\partial t$ and $\partial w/\partial t$) at any point x, z in the fluid are:

$$\frac{\partial u}{\partial t} = \frac{2\pi C}{T}[u_1\sin\theta + 2u_2\sin 2\theta + 3u_3\sin 3\theta] \quad (9)$$

and

$$\frac{\partial w}{\partial t} = \frac{2\pi C}{T}[w_1\cos\theta + 2w_2\cos 2\theta + 3w_3\cos 3\theta] \; ; (10)$$

where

$$u_1 = F_1\cosh kS,\quad u_2 = F_2\cosh 2kS,\quad u_3 = F_3\cosh 3kS$$
$$w_1 = F_1\sinh kS,\quad w_2 = F_2\sinh 2kS,\quad w_3 = F_3\sinh 3kS \quad (11)$$

In the derivations of the Lagrangian equations of horizontal and vertical components of water particle velocity of the second- and third-order by the authors, (see reference [12]), the third-order derivations were found to be in full agreement with Skjelbreia [11], but in the second-order, the equations for the horizontal component of Lagrangian velocity and the vertical component of displacement of the particle from its still water position given by Wiegel [13] were found to contain some different terms which may be due to a misprint.

In comparing the experimental data with the cnoidal wave theory, the Eulerian equations for velocity and acceleration were derived starting with the equations given by Keulegan and Patterson [7]. The following are the equations for the horizontal and vertical components of velocity respectively.

$$u = \sqrt{(gh)}\Big[-5/4 + 3/2\,\frac{z_t}{2} - \frac{z_t^2}{4h^2} + \frac{H}{2h}\Big(3 - \frac{z_t}{h}\Big)cn^2(\;)$$
$$- \frac{H^2}{4h^2}cn^4(\;) - \frac{8HK^2(k)}{L^2}\Big(\frac{h}{3} - \frac{z^2}{2h}\Big)\{-k^2sn^2(\;)cn^2(\;)$$
$$+ cn^2(\;)dn^2(\;) - sn^2(\;)dn^2(\;)\}\Big] \quad (12)$$

$$w = z\sqrt{(gh)}\,\frac{2HK(k)}{Lh}\Big[3 - \frac{z_t}{h} - \frac{H}{h}cn^2(\;) +$$
$$\frac{32K^2(k)}{3L^2}\Big(h^2 - \frac{z^2}{2}\Big).$$
$$\{k^2sn^2(\;) - k^2cn^2(\;) - dn^2(\;)\}\Big]sn(\;)dn(\;)cn(\;). \quad (13)$$

The components of local acceleration are obtained by differentiating u and w above with respect to time and they are

$$\frac{\partial u}{\partial t} = \sqrt{(gh)}\frac{4HK(k)}{Th}\Big[\Big(\frac{3}{2} - \frac{z_t}{2h}\Big) - \frac{H}{2h}cn^2(\;)$$
$$+ \frac{16K^2(k)}{L^2}\Big(\frac{h^2}{3} - \frac{z^2}{2}\Big)$$
$$\times\{k^2sn^2(\;) - k^2cn^2(\;) - dn^2(\;)\}\Big]sn(\;)cn(\;)dn(\;) \quad (14)$$

and

$$\frac{\partial w}{\partial t} = -z\sqrt{(gh)}\,\frac{4HK^2(k)}{LTh}\Big[\Big(\frac{z_t}{h} - 3\Big)\{sn^2(\;)dn^2(\;)$$
$$- cn^2(\;)dn^2(\;) + k^2sn^2(\;)cn^2(\;)\}$$
$$+ \frac{H}{h}\{3sn^2(\;)dn^2(\;) - cn^2(\;)dn^2(\;)$$
$$+ k^2cn^2(\;)sn^2(\;)\}cn^2(\;) + \frac{32K^2(k)}{3L^2}\Big(h^2 - \frac{z^2}{2}\Big)$$
$$\times[9k^2sn^2(\;)cn^2(\;)dn^2(\;) - k^2sn^4(\;)\{dn^2(\;)$$
$$+ k^2cn^2(\;)\} + k^2cn^4(\;)\{k^2sn^2(\;) - dn^2(\;)\}$$
$$+ dn^4(\;)\{sn^2(\;) - cn^2(\;)\}]\Big] \quad (15)$$

Comparing the equations of velocity and acceleration with the corresponding equations found by Wiegel [14], it is observed that except for the equation of horizontal velocity all the remaining three equations derived by Wiegel [14] contain different terms.

The terminology of elliptic functions and integrals used above are the same as was used by Milne-Thomson [15]. For finding the values of the functions *sn*, *cn* and *dn* and integrals $E(k)$, and $K(k)$ the five-figure tables by Milne-Thomson [15] were used. Before finding the values of the above functions and integrals, the following equation was used to find k by trial and error, using the known data of wave period, wave height, and water depth.

$$T\Big(\frac{g}{h}\Big)^{1/2} = \Big(\frac{16h}{3H}\Big)^{1/2}\left[\frac{kK(k)}{\Big\{1 + \frac{H}{hk^2}\Big(1/2 - \frac{E(k)}{K(k)}\Big)\Big\}}\right] \quad (16)$$

The value of the wave-length to be used in the above equations was calculated using the following equation:

$$\frac{L^2H}{h^3} = 16/3\;[kK(k)]^2\,. \quad (17)$$

The limiting conditions for applying cnoidal and

Stokes' wave theories for predictive behavior of a permanent finite amplitude wave may be mentioned here. Laitone [16] stated that cnoidal wave theory should be applied for waves having $L > 5h$ and Stokes' wave theory was suitable for waves having $L < 8h$. Thus there is a overlapping range, $5h < L < 8h$, where both the theories can be applied.

4. EXPERIMENTAL PROGRAMME

Experiments were performed in the large wave tank in the Michell laboratory, Department of Civil Engineering at the University of Melbourne. The principal dimensions of the tank are (approximately) - 65 m long, 2.03 m deep, and 1.83 m wide. The generator of the type known as the 'nodding duck' was used for generating waves.

Large amplitude waves were generated using the method of Le Mehaute and others [1]. A horizontal platform was constructed at the test area with sloping bottoms at each end to concentrate the wave energy in high amplitude intermediate depth waves. Waves were generally accompanied by second harmonics which is an usual phenomenon for laboratory waves as reported by Morison and Crooke [4], Le Mehaute and others [1], Iwagaki and Sakai [2], and Hensen and Svendsen [17]. This is different from the secondary wave crest at the trough predicted by the second-order wave theory for some combinations of H/L and h/L and multiple crests for shallow water waves as revealed from numerical computations of Madsen and Mei [18]. Although most of the waves were observed to have second harmonics, only the better few waves with minimum disturbances are presented in this paper.

A two component suspension wire type velocity probe, developed by Sharp [19], was used for the Eulerian velocity measurements. The instrument detects the drag experienced by a cotton suspension wire and the water particle velocity is derived from a calibration relation. In this application the suspension wires were about 55 mm long and the component of velocity direction and magnitude was determined by the orientation of the enclosed cantilever at one end as described in Sharp [19–21]. Before and after each test each probe was calibrated for two directional responses by towing it backward and forward using a towing carriage over the wave tank in the range 2 to 20 cm s^{-1}.

The data was sampled directly online to the department's ID-70 mini computer storing in a buffer of 2048 points at a sampling rate of 40 milli-seconds. A control unit in the laboratory in the form of a Tektronix Alpha-numeric graphics display terminal was connected to a general purpose interface box which incorporated an eight channel analogue to digital converter for various inputs. Figure 1 shows the line diagram of the online data acquisition system. Three channels were used for reading simultaneously wave height and two velocity components. The system allowed a preliminary estimate of the quality of the data to be made. The acceptable

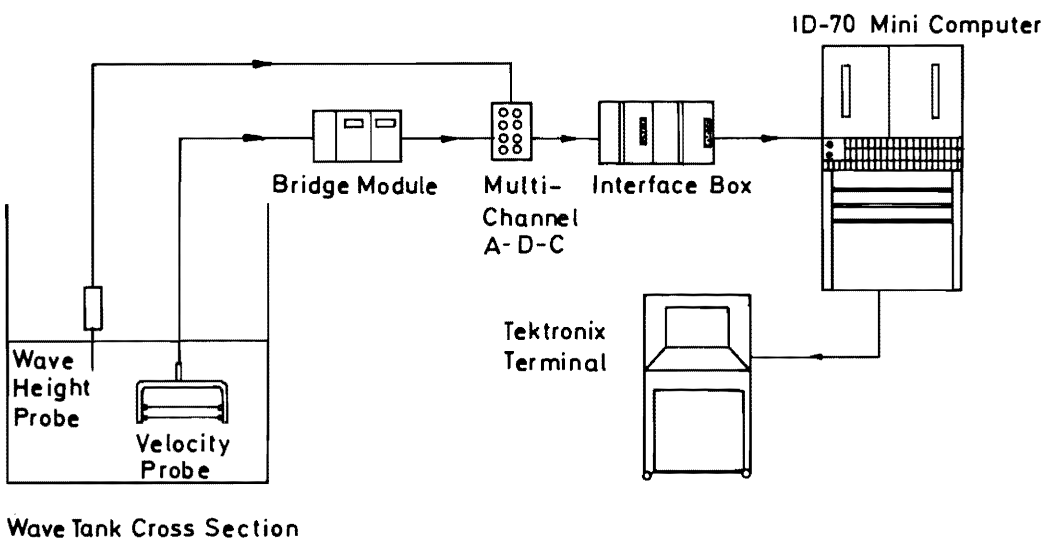

Figure 1. Line Diagram of Online Acquisition of Data.

data were stored on floppy discs for processing later on.

Output voltage of the strain gauges in water corresponding to no relative motion between the gauges and the surrounding water, termed here as 'Zero', was recorded by clamping the velocity probe near the bed in such a position that its cantilever plate remained parallel to the longitudinal axis of the channel. While doing tests, every alternate data file was used for recording the 'zero' and the time of recording every data file was noted down. The 'zeros' of the intermediate data files were obtained from this time variation.

An interactive Fortran programme was developed for calculating orbital velocity by trial and error for two different conditions — balanced mass transport profile over the whole depth and zero mass transport at each point. For vertical component of velocity, however, only the second condition was satisfied.

The Lagrangian tests were perfomed by filming fluoresceine dye traces of the water particle motions, through the grid-marked window glass of the wave tank, with the help of a variable speed movie camera. To minimize the parallax error, the camera was placed at the maximum possible distance of 5 meters and a telephoto lens was used to magnify the field of observation.

The setup for collecting data from the processed film, as shown in Figure 2, consists of a movie projector having a wide range of variable speed suitable for photo-optical data analysis, an optical digitizing device (PCD digital data reader), a SOLARTRON data logger and a FACIT paper tape punch. Calibration relations between voltage reading of the data logger and actual distance were determined by reading the grid of the focused film. For determining orbital velocity, readings were taken following a particular particle in every frame over one wave cycle. These data in the paper tape were processed with the help of the ID-70 mini computer, using interactive Fortran programme.

5. PRESENTATION AND DISCUSSION OF RESULTS

For both the Eulerian and Lagrangian results, data have been plotted over one wave cycle, starting from the crest. The levels of the points of measurement with respect to the bottom or the surface are indicated by the mean line of oscillation of velocity. A large number of data have been processed but only a representative few are presented here. Table 1 lists particulars of the waves presented here.

From Table 1 it is observed that all the waves fall in the intermediate depth range having high relative heights. It is also observed from Laitone's [16] finding (see section 3) that the cnoidal wave theory is suitable for waves in Figures 5, 8, and 9 and both the cnoidal and Stokes' wave theories could be used for the rest four waves in Table 1. But as the objective of this study is to check the suitability of wave theories against experimental data, the limiting conditions by Laitone [16] are not strictly applied here.

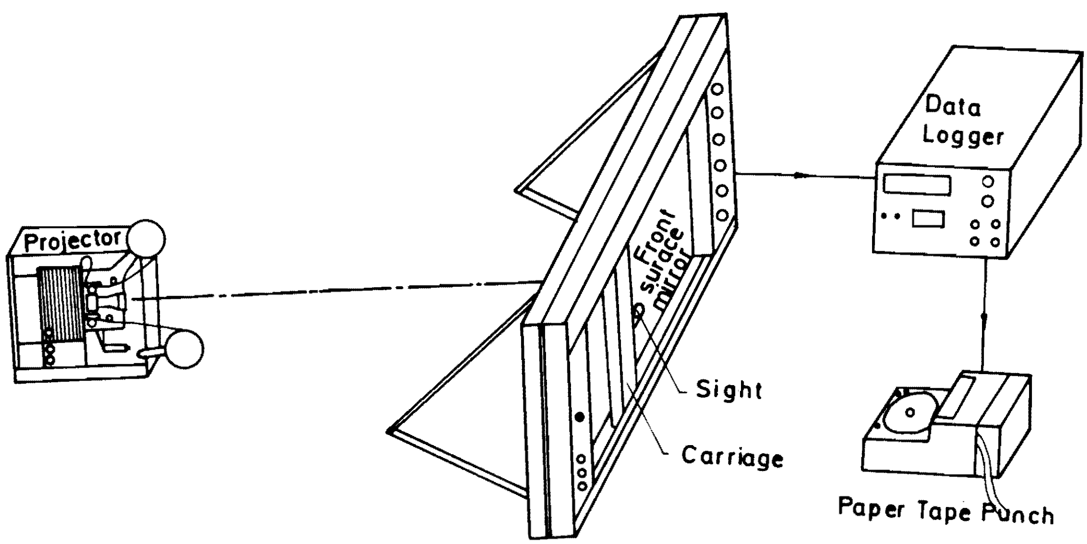

Figure 2. Optical Digital Data Reader Setup.

Table 1. Wave Particulars

Figure Number	h m	T s	H m	L m	h/gT^2	H/gT^2	kh	L/h
3	0.25	1.176	0.0615	1.58	0.0184	0.00453	0.994	6.32
4	0.25	1.428	0.0698	1.96	0.0125	0.00349	0.801	7.84
5	0.219	2.222	0.06	3.12	0.0045	0.00124	0.441	14.24
6	0.25	1.25	0.082	1.76	0.0163	0.00535	0.892	7.04
7	0.2305	1.111	0.0543	1.46	0.019	0.00448	0.992	6.33
8	0.232	2.857	0.092	4.15	0.0029	0.00115	0.351	17.88
9	0.219	2.222	0.06	3.12	0.0045	0.00124	0.441	14.24
10	0.25	1.175	0.0546	1.57	0.0185	0.00403	1.0	6.28

Figures 3 to 5 show the variation of the horizontal component of the Eulerian velocity. Observation of all the plots show that at high values of kh, agreement between the theories and the data is better and in a few cases the data and the theoretical plots surprisingly coincide over all the wave cycle. But the theories depart more and more from the data as the values of kh decreases. Disagreement of data at low values of kh is partly due to the presence of the second harmonics at these values of kh. It should be noted here that the velocities in Figures 3 and 4 have been processed for the condition of approximately balanced mass transport profile over the whole depth and thus they represent actual velocity distributions in a closed channel. In Figure 5, however, the data have been processed for the condition of zero mass transport at each point and thus these data should compare better with the theories yielding no mass transport.

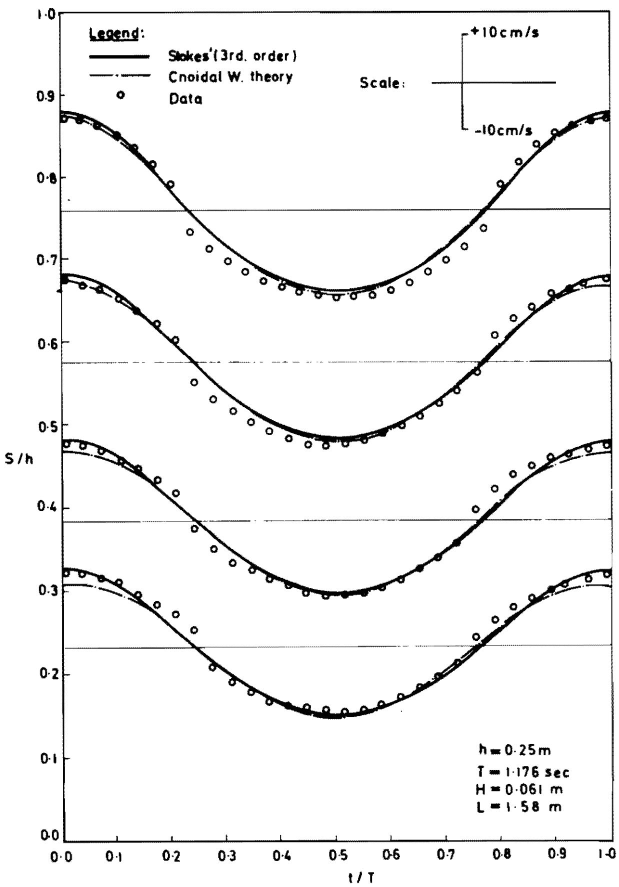

Figure 3. Horizontal Component of Eulerian Velocity, kh = 0.994.

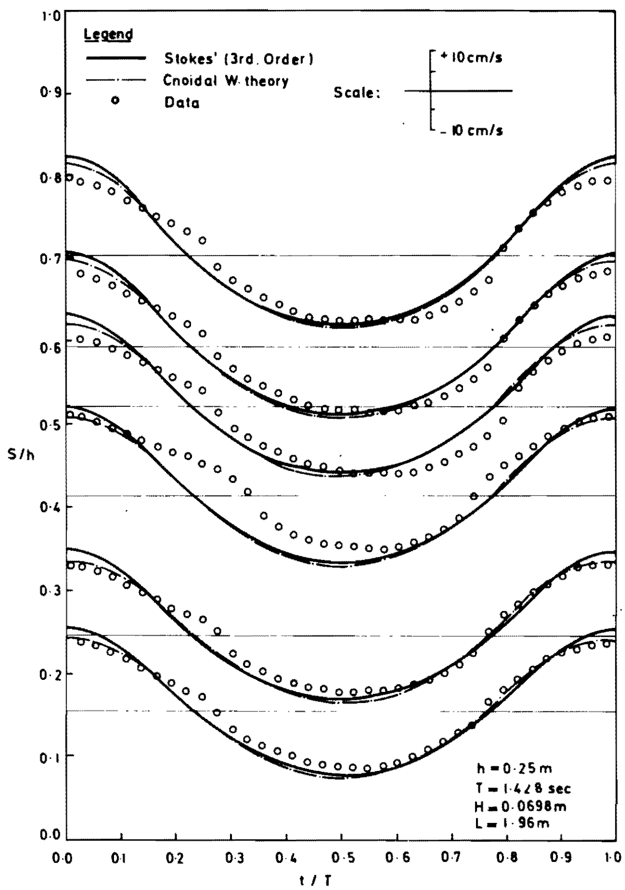

Figure 4. Horizontal Component of Eulerian Velocity, kh = 0.801.

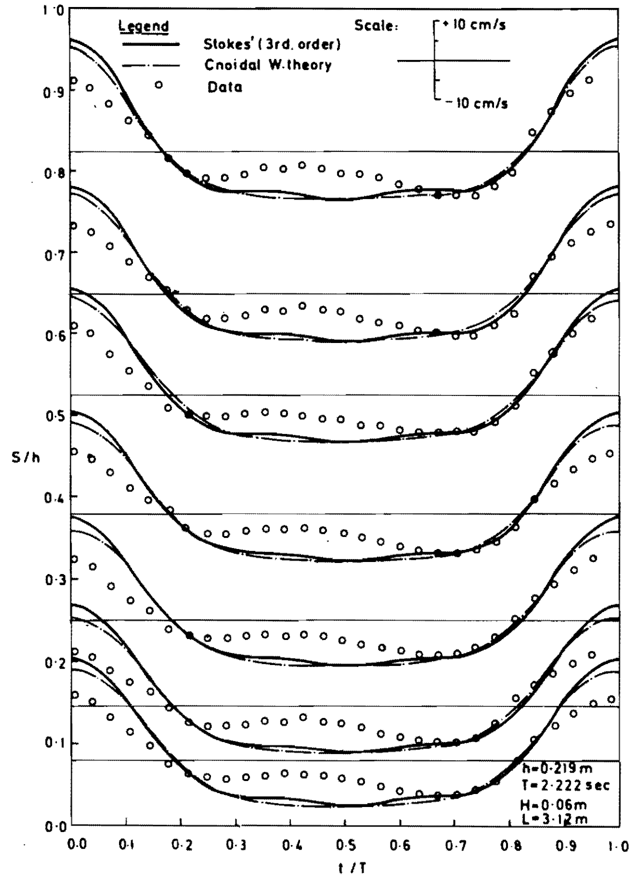

Figure 5. Horizontal Component of Eulerian Velocity, kh = 0.441.

The Figures plotted over the whole wave period clearly show the difference between the experiments concentrating at particular phases of the wave and those dealing with the waves over the full wave cycle. Present results clearly show how the data is affected by the presence of the second harmonics which if otherwise plotted for points in phase at the crest could claim much better agreement with the theories.

Ullah [12] has shown that the theories overestimate the increase of velocity with the increase of wave height which is in agreement with a similar observation by Tsuchiya and Yamaguchi [3].

Comparison between the two theories reveal that, for the horizontal component of velocity, at the phase of the crest, Stokes' (third-order) wave theory always produces higher values than the cnoidal wave theory. At the phase of the trough, the reverse is true — for the horizontal component of velocity, the Stokes' (third-order) wave theory always produces lower values than the cnoidal wave theory. The difference between the two theoretical plots at the crest is always higher than the corresponding difference at the trough. Observation of all the data show that the Stokes' (third-order) wave theory always overestimate the horizontal water particle velocity at the phase of the crest. The pattern of difference is similar to the difference in the wave profiles. This is because both the formulas of wave profile and horizontal component of velocity consist of the same cosine functions. It has been mentioned that the difference between the two theories is greatest at the crest and observation of the pattern of data around the vicinity of the crest lead to the conclusion that, in general, the cnoidal wave theory is better than the Stokes' (third-order) wave theory for the description of the waves tested.

Figure 6 is a sample representation of a range of test results for the vertical component of Eulerian velocity which shows good agreement between the data and the theories. In a few cases, at some phases of the wave period, theories are found to underesti-

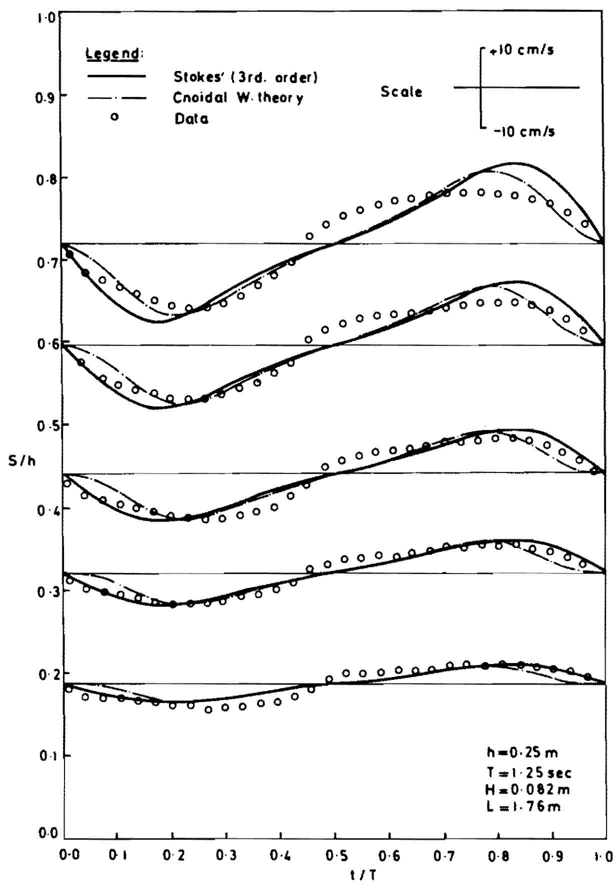

Figure 6. Vertical Component of Eulerian Velocity.

mate the vertical velocity. This may be due to the presence of second harmonics in those tests. A similar observation was reported by Morison and Crooke [4].

The influence of second harmonics on the water particle velocity is found to be more in the case of the vertical component than the horizontal component. The reason may be explained this way. Generally, the second harmonics travel at slower speed than the main wave and thus may occur at different phase lags with the main wave; this becomes distinctly noticeable, specially when the crest of the second harmonic appears near the trough of the main wave. The appearance of the secondary wave crest near the trough of the main wave can highly influence the very low vertical velocity but cannot greatly influence the high horizontal velocity. The influence of the second harmonics on the vertical component of velocity is found to be most near the surface.

A range of data for the horizontal and vertical components of velocity by Lagrangian measurement were given by Ullah [12] and the data were found to confirm the conclusions reached from Eulerian experiments. Figures 7 and 8 are sample representations of the horizontal and vertical components of Lagrangian velocity respectively.

It has been mentioned previously that the reason for the very few experimental studies on water particle acceleration is the assumption that the checking of the velocity field is sufficient for assuming the acceleration field to be correct. But it was realized in this study that the determination of the acceleration accurately from Lagrangian photography was extremely difficult. Acceleration is a time differential of velocity and it is extremely sensitive to even the most minor error in calculating the velocity; and estimates from the Lagrangian acceleration data are too scattered for presentation. However the local acceleration from the Eulerian data was found to be very consistent, mainly because of the use of the computer for recording and analyzing the data, which reduces human error. Figures 9 and 10 represent horizontal and vertical components of local acceleration.

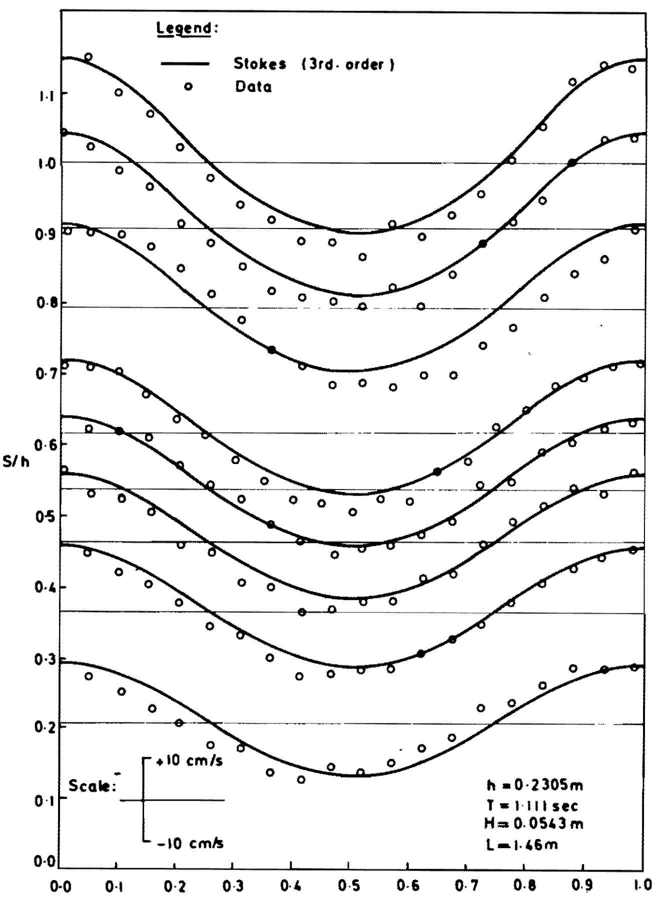

Figure 7. Horizontal Component of Lagrangian Velocity.

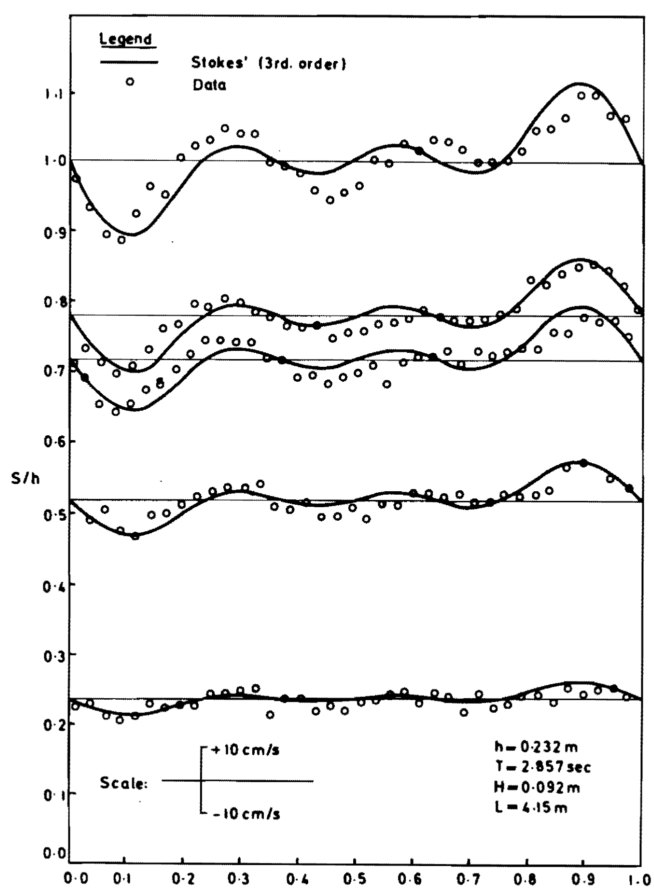

Figure 8. Vertical Component of Lagrangian Velocity.

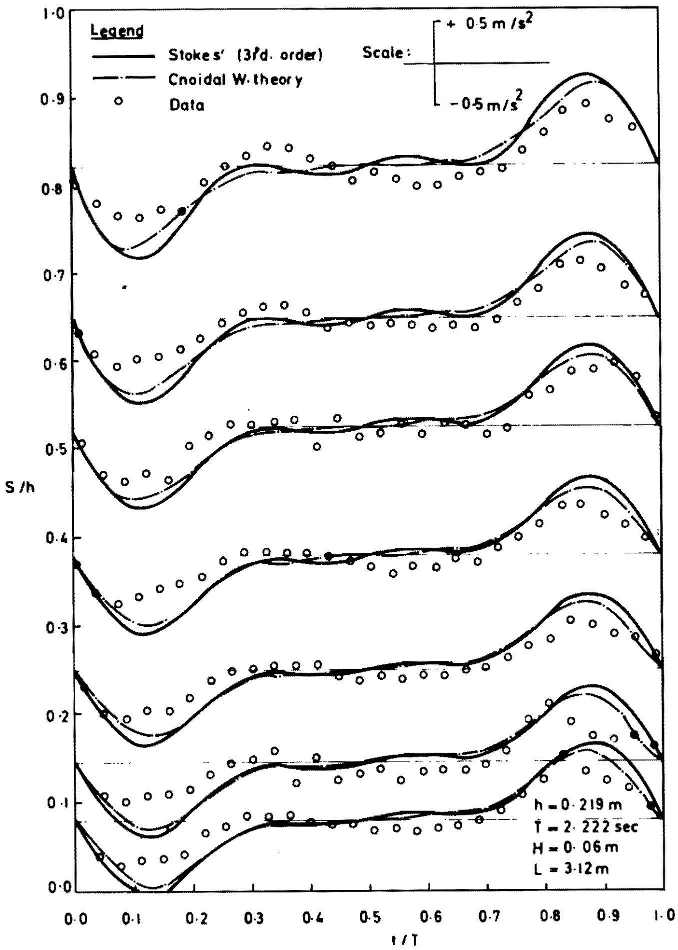

Figure 9. Horizontal Component of Local Acceleration.

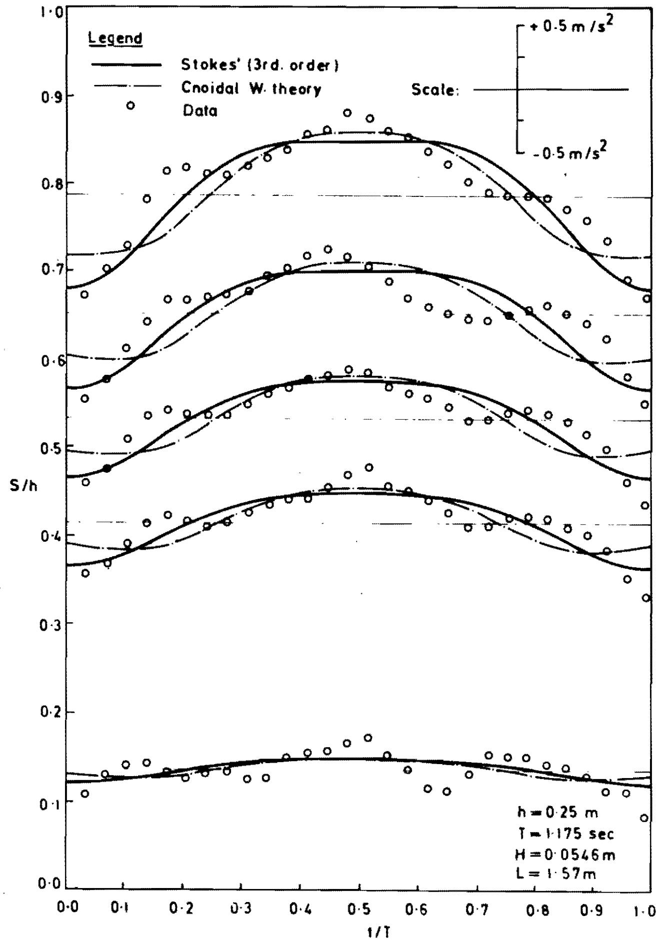

Figure 10. Vertical Component of Local Acceleration.

Most of the observations for velocity mentioned above are also appropriate for acceleration. For example at high values of *kh*, the acceleration data are in close agreement with the theoretical values which depart more and more from the experimental data with the increase of the relative wave-length. Also, with the increase of the relative height, the theories are found to overestimate the acceleration values by bigger margins.

The maximum values of acceleration occur at the instant of flow reversal and this instant varies for experimental data and theories and also from wave to wave. In general, after passing the wave crest, flow reversal happens first for the Stokes' (third-order) wave theory followed by the cnoidal wave theory and the experimental data respectively. Because of the symmetry of the wave system, the flow reversal before the next wave crest happens first for the experimental data followed by the cnoidal wave theory and Stokes' (third-order) wave theory respectively. This time lag between the data and the theories was found to increase with the increase of the relative wave-length and relative wave height.

In general the magnitude of acceleration was lower than those predicted by the theories and the cnoidal wave theory values were closer to the data than the Stokes' (third-order) wave theory prediction.

For the vertical component of local acceleration, with the increase of the relative height of the wave, the theories were found to overestimate the experimental data. At these high values of the wave height, near the wave crest, the theoretical curves become complicated in appearance with sharp changes and the difference between the two theoretical curves become very large, both in magnitude and form. Around the wave crest, the theoretical prediction by Stokes' (third-order) wave theory is closer to data than the cnoidal wave theory and near the wave trough the theoretical lines are almost coincident.

6. CONCLUSIONS

At high values of kh and low values of relative wave height (H/h), agreement between the data and the theories, for both the horizontal and vertical components of Eulerian water particle velocity, was found to be good. The theories were found to overestimate the horizontal and vertical velocity by increasing margins with the decrease of kh and increase of H/h. The conclusions were found to be equally true for the components of water particle acceleration. The plots of the Lagrangian velocity confirmed the conclusions drawn from the Eulerian experiments.

Measurements and plotting of data over the whole cycle of wave showed how the components of velocity are affected by the presence of second harmonics.

Between the two theories used, the cnoidal wave theory was comparatively a better description of the experimental findings.

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