DYNAMIC SIMULATION, OPTIMIZATION AND CONTROL OF MULTI-STAGE, COUNTER- CURRENT WASHING SYSTEMS IN ALUMINA-PRODUCING PLANTS

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الخلاصـة :

يتم استخلاص الصودا الكاوية المذابة مع الطمي الأحمر - وهو منتج جانبي في عملية باير المستخدمة في تصنيع أكسيد الألومنيوم من خامات البوكسيت . وذلك في سلسلة من قطارات الغسيل المتعددة المراحل والتي يتم فيها تغذية سائل الغسيل والطمي الأحمر في إتجاهين متضادين . إن التحكم في عملية الغسيل هذه صعبة نتيجة لوجود فارق زمني بين الفعل التصحيحي والتأثيرات التي تنتج عنه وأيضاً بسبب عدم وجود علاقات رياضية دقيقة تربط بين الاضطرابات في تركيز الصودا الكاوية وسرعات مرور سائل الغسيل والطمي الأحمر في اتجاهما المتضادين .

ولقد تم تصميم برنامج محاكاة ديناميكي لعملية الغسيل هذه باستخدام برامج SLAM II للمحاكاة وذلك بغرض فهم التصرف الديناميكي لعملية الغسيل . وعن طريق تطبيق طرق إيجاد الحلول الـمُـثلى على برنامج المحاكاة ثم التعرف على ظروف تشغيل قطارات الغسيل التي ينتج عنها زيادة المستخلص من الصودا الكاوية نتيجة عملية الغسيل إلى أقصى حد ممكن له . وبناءً على النتائج التي تم الحصول عليها من تطبيق طرق إيجاد الحلول المثلى ، تم تصميم وتطبيق نظام للتحكم - وذلك باستخدام الحاسوب الشخصي - بغرض التحكم في مستويات كلٍ من السائل والطمي الأحمر في أحواض الغسيل المختلفة وكذلك سرعة تغذية البوليمر الـمُـثلى إليها ، هذا بالإضافة إلى أمثل حوض للغسيل يتم إليه تغذية مُرشح البذور الدقيقة . ولقد زادت كفاءة الغسيل بنسبة ٢٠٪ تقريباً نتيجة لاستخدام نظام التحكم .

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ABSTRACT

Entrapped caustic liquor in the red mud residue, a by-product of the Bayer Process used to produce alumina from Bauxite ores, is recovered in a series of counter-current, multi-stage washer trains using the process diluting liquor. Co-ordinated control of the operation of the washer trains is difficult owing to both the long-time constants of elements of the plant, which cause delays between corrective actions and observable effects, and the imprecisely-known interaction of disturbances in composition and rate of the counter-current flows.

A dynamic simulation model has been developed for a multi-stage washer train using SLAM II simulation package in order to gain understanding of the dynamic behavior of the washing system. Optimization of the simulation model has identified sets of operating conditions under which the caustic soda gain from the washing operation would be maximized. Based on optimization results, a PC-based feedforward feedback control system has been designed and implemented to control liquor level and mud level in a washer tank, polymer flow rate to a washer tank and the 'optimal' washer in the train to return the fine seed filtrate. About 20% increase in caustic soda gain from the counter-current washing has been achieved as a result of controlling the operation.

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1. INTRODUCTION

1.1. The Bayer Process

The 100 year old Bayer Process continues to be the most economic means of producing alumina from bauxite ores [1]. The broad outline of the process is shown diagrammatically in Figure 1 [1]. Here, in brief, is the sequence of operations:

1. The bauxite is crushed and ground, then digested with caustic liquor under suitable conditions of temperature and caustic concentration.
2. The slurry from digestion consisting of sodium aluminate liquor and the insoluble residue is cooled and diluted. The residue, in the form of coarse silica sand and red mud, is separated and washed in counter-current decantation, then discarded. A filtration stage ensures complete removal of any remaining residue.
3. The super-saturated sodium aluminate liquor is then cooled further and passed to the precipitation stage. Here controlled precipitation of alumina trihydrate is achieved by seeding the agitated liquor with recycled hydrate crystals.
4. The deposited crystals are then classified according to size. The large crystals form the product hydrate, which is calcined to remove the water of hydration and stored. The medium-sized crystals are recycled to precipitation stage as seed hydrate. The extremely-fine crystals are dissolved in the process liquor and recycled to the precipitation stage.
5. The 'spent' process liquor is then recycled, *via* heat exchangers, to commence the next production cycle. The 'spent' liquor is concentrated by evaporation to allow the intake of wash water at the residue-removal stage of the process.

1.2. The Red Mud Washing Process

The washing circuit recovers caustic soda from the thickener red mud using counter-current decantation. Mud from thickener under-flows is distributed as evenly as possible to two trains of six washers on each stage. Configuration of elements in a washer in the train is shown in Figure 2. The mud at each washing stage is combined with the overflow from the preceding washer in a mixing tank from which it flows by gravity into the washer feed well. Polymer solution is added to the mixing tank to flocculate the mud particles to enhance settling rate and improve mud under flow densities. The settled mud, now lower in caustic, is raked to the tank perimeter where it is removed and pumped to the next washer down in the series. The raking action also has the effect of deliquoring the settled mud. The clear washer overflow, now higher in caustic, is pumped to the next washer up in the series. The mud decreases in caustic content as it moves down the washing train, while the washing liquor increases in caustic as it moves up the train. Each washing train has four input and two output streams as shown in Figure 1. Each of these are determined by factors outside of the washing area:

1. Mud exit thickener (input): The liquor content of the mud is determined by the thickener area performance and bauxite charge.
2. Fine seed filtrate FSF (input): This stream is directed to either the 3rd, 4th, or 5th washer depending on the plant oxalate purging requirements and is determined by the fine seed washing requirements.
3. Weak liquor dilution to digester, WCL (output): This stream is determined by the dilution demand of digestion which is linked to the evaporation plant performance and the strong feed liquor concentration.
4. Washed mud to disposal (output): The bauxite throughout determines the mud flow rate down the washing train.

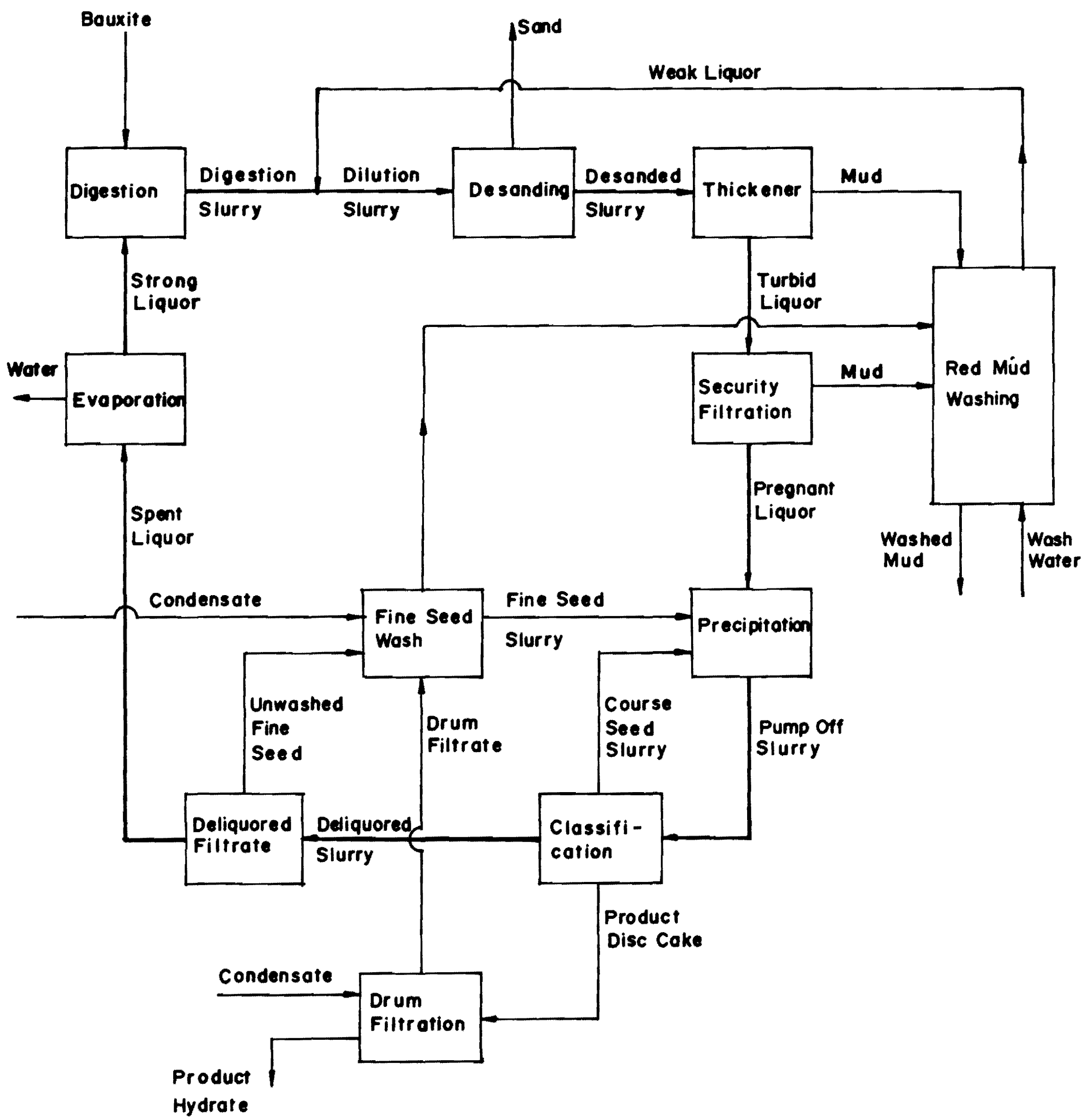


Figure 1. Bayer Process Flow Sheet.

5. Security filtration wash effluent (input): This stream, which is small compared with the main washer flows, is dependent on filter washing operations in security filtration and thickener performance.
6. Wash water (input): This stream is controlled by volume balance after allowing for all other input and output streams.

Losses of caustic soda, which are calculated by soda balance, due to inefficient washing of the red mud represents about 50% of the net caustic losses from an alumina plant (about 110 kg NaOH/ton of calcined alumina produced). This figure takes into account the caustic soda which returns to the plant in the form of supernatant liquor, SNL.

This paper presents a dynamic simulation model, constructed using SLAM II simulation package [2], that describes the dynamic behavior of the counter-current washing process and one which is valid over a wide range of process operations. With maximizing caustic soda gain from the washing operation as the objective function, a number of 'significant' process load and manipulated variables have been identified through off-line optimization and hence a PC-based control system has been designed and implemented on the process. Detailed description of the washers train control will be avoided because of its commercial value.

2. THE COUNTER-CURRENT WASHING SIMULATION MODEL

2.1. Assumptions Used in the Model

The following assumptions were used in the model development:

1. In the settling tank, there are two separate phases, namely, a liquor phase and a slurry phase. The slurry phase consists of solid mud particles and entrapped liquor. There is no solids transfer between the two phases [3].
2. Height of the liquor phase in the settling tank is determined by flow rates of input and output liquors to, or from, that tank.
3. Height of the slurry phase in the settling tank is determined by flow rates of input and output slurries to, or from, that tank and by slurry solids concentration (density).

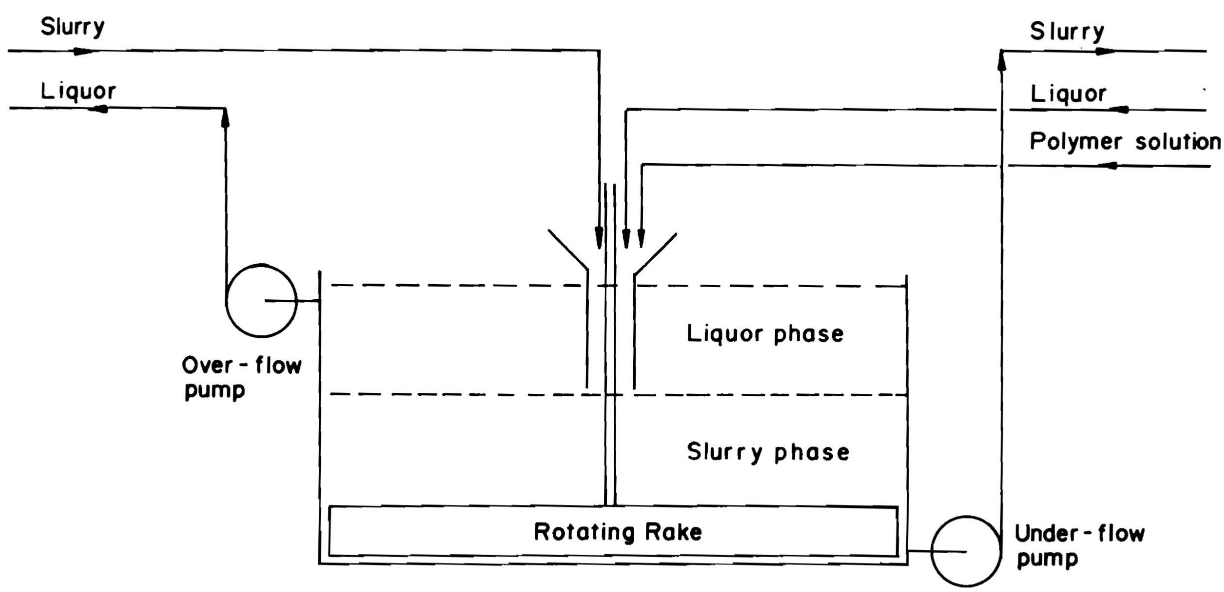


Figure 2. Configuration of Elements in Mixing–Settling Stage N.

4. Density of slurry leaving the settling tank is a function of polymer to slurry flow rates ratio to the mixing tank (experimental unpublished work).
5. There is a variable order exponential delay for the effect of polymer to slurry flow rates ratio on density of slurry leaving the settling tank (experimental unpublished work).
6. Liquor density is dependent on both liquor caustic concentration and liquor temperature.
7. There is no liquor transfer between slurry phase and liquor phase.
8. The only caustic soda in the slurry phase is that which exists in the entrapped liquor in that phase (*i.e.*, caustic soda adsorbed on red mud particles are neglected).
9. The concentration of caustic soda is uniform in both liquor phase and entrapped liquor in the slurry phase.
10. In both mixing and settling tanks, caustic soda concentration can be expressed as a function of both rate of diffusion of caustic soda from slurry phase to liquor phase and convection flow.
11. Rate of diffusion of caustic soda from slurry phase to liquor phase is a function of temperature.

2.2. Equations Used in the Model

Figure 3 shows stream line sequence for the counter-current decantation model. It must be noted that fine seed filtrate stream, FSF, is directed to either the 3rd, 4th, or 5th washer. For each mixing-settling stage, the following equations were used:

1. Rates of diffusion of caustic soda from slurry phase to liquor phase have been calculated according to McKibbins [4] as follows:

$$R_m = P_1\, T + Q_1 \tag{1}$$

$$R_s = P_2\, T + Q_2 \tag{2}$$

where P_1, Q_1, P_2, and Q_2 are constants.

2. Density of liquor input to the mixing tank,

$$\rho_\ell = K_1 + K_2 C_{11} + K_3 T \tag{3}$$

where K_1, K_2, and K_3 are constants.

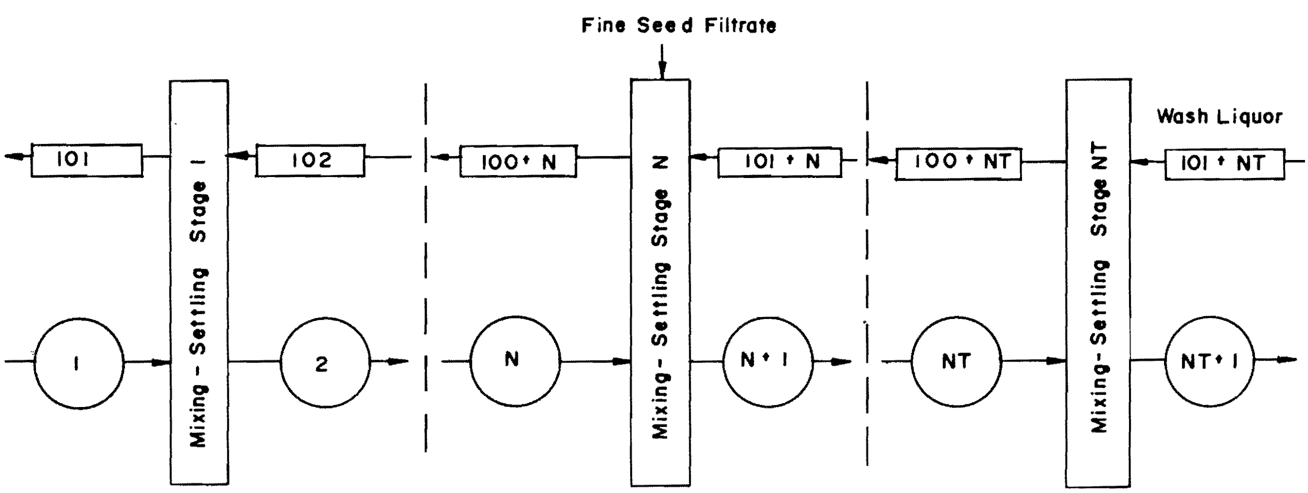


Figure 3. Stream Number Sequence for Counter-Current Decantation Model.

3. Mass balance on the caustic soda in the liquor phase in mixing tank is written as the sum of gain by mass transfer from the entrapped liquor in slurry and by convection flow, respectively:

$$d(C_{21})/\mathrm{d}t = -R_m(C_{22} - C_{21})H_m\left[\{\rho_{si}[1-(\rho_l/\rho_m)] + \rho_l - \rho_{si}\}/\rho_l\right]/H_m$$
$$+\left[Q_{ti}/(H_m\ A_m)\right](C_{11} - C_{21})\,. \tag{4}$$

4. Mass balance on total caustic in mixing tank gives:

$$d(C_{22})/\mathrm{d}t = -R_m(C_{22} - C_{21}) + Q_{si}\left[\{\rho_{si}[1-(\rho_l/\rho_{si})] + \rho_l - \rho_{si}\}/\rho_l\right]/$$
$$\left[H_m\,A_m\{[\rho_{si}(1-(\rho_\ell/\rho_m)) + \rho_\ell - \rho_{si}]/\rho_\ell\}\right](C_{12} - C_{22})\,. \tag{5}$$

5. Overall volume balance on liquor phase in settling tank gives:

$$d(h_\ell)/\mathrm{d}t = (Q_{\ell i} - Q_{\ell o})/A_s\,. \tag{6}$$

6. Overall volume balance on slurry phase in settling tank gives:

$$d(h_s)/\mathrm{d}t = (Q_{si} - Q_{so})/A_s\,. \tag{7}$$

7. Mass balance on the caustic soda in the liquor phase in settling tank is written as the sum of gain by mass transfer from the entrapped liquor in slurry phase and by convection flow, respectively, as follows:

$$d(C_{31})/\mathrm{d}t = R_s(C_{22} - C_{31})h_s\left[\{\rho_{so}[1-(\rho_l/\rho_m)] + \rho_l - \rho_{so}\}/\rho_l\right]/h_\ell$$
$$+\left[Q_{ti}/(h_\ell\,A_s)\right](C_{21} - C_{31})\,. \tag{8}$$

8. Mass balance on total caustic in settling tank gives:

$$d(C_{32})/\mathrm{d}t = R_s(C_{32} - C_{31}) + \left[Q_{si}\{[\rho_{si}(1-(\rho_l/\rho_m)) + \rho_l - \rho_{si}]/\rho_l\}\right]/$$
$$\left[h_s\,A_s\{[\rho_{so}(1-(\rho_l/\rho_m)) + \rho_l - \rho_{so}]/\rho_l\}\right](C_{22}/C_{32})\,. \tag{9}$$

2.3. Validation of the Model

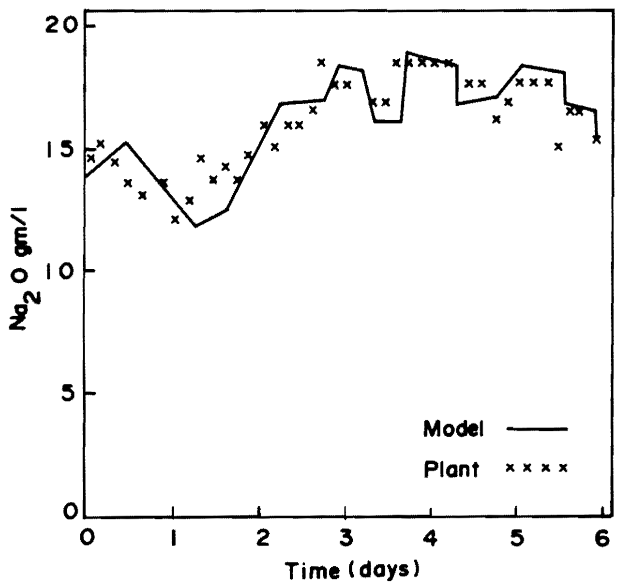
The counter-current washing train considered in this investigation is composed of 5 serially-connected mixing-settling stages (1 through to 5). Data from stage 3, shown in Figure 4, were used to tune the model parameters. The tuned model was, then, validated using data from stage 5 as shown in Figure 5. Data were collected from a major alumina-producing plant on a 4-hourly basis over a period of 6 operational days. Figure 5 indicates a good agreement between the model and the plant measurements.

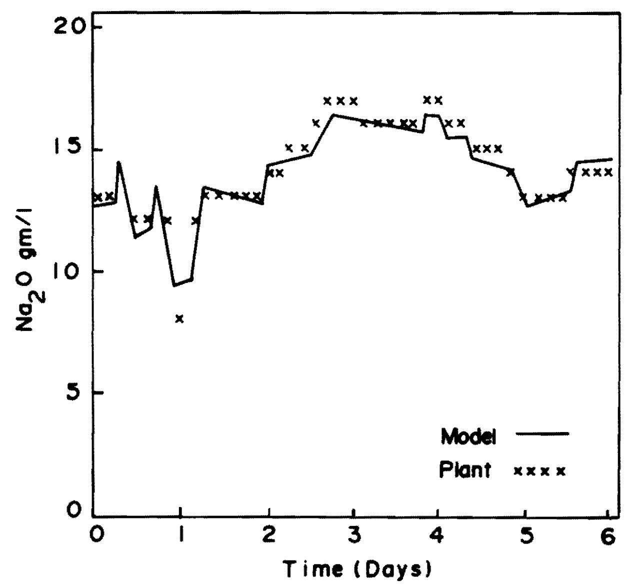
Considering that the model has shown a reasonable agreement with the plant in terms of absolute caustic concentration and dynamic form, the model is considered to be sufficiently accurate for the purpose of a control investigation.

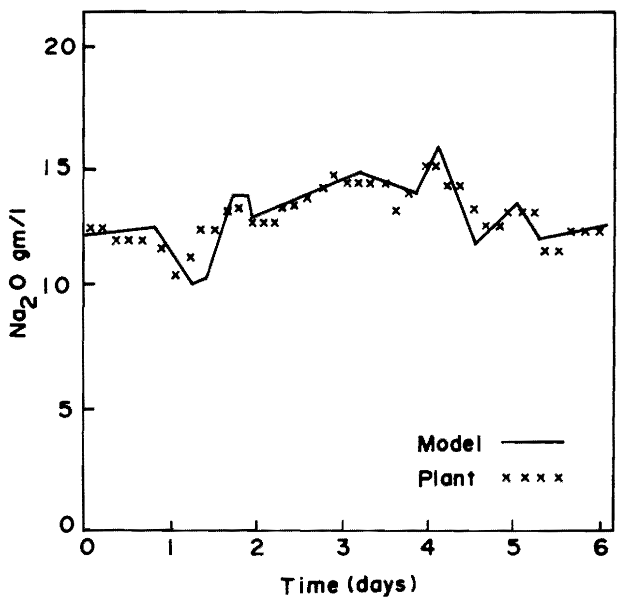
3. APPLICATIONS OF THE SIMULATION MODEL

3.1. Optimization of the Simulation Model

The multi-stage, counter-current washing simulation model has been coupled with an optimum-search algorithm, DYNAM [5] in a single computer program.







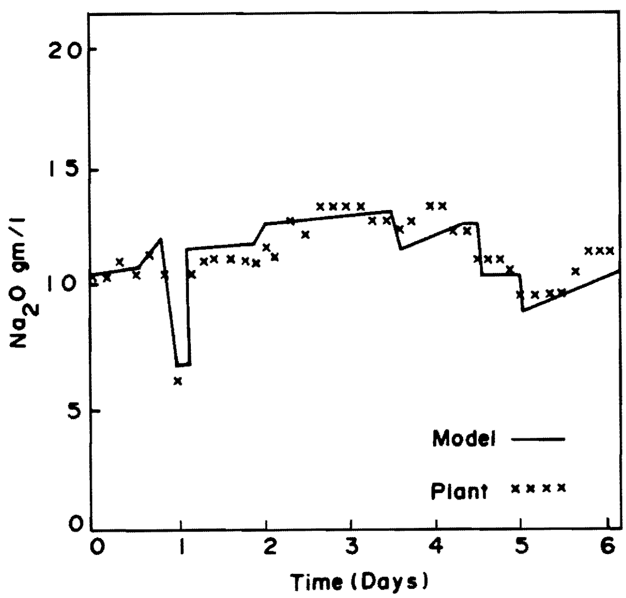


Figure 4. A Graphical Comparison Between the Model and Plant Data for Caustic Concentration.
(a) In slurry output from mixing-settling stage 3;
(b) in liquor output from mixing-settling stage 3.

Figure 5. A Graphical Comparison Between the Model and Plant Data for Caustic Concentration.
(a) In slurry output from mixing-settling stage 5;
(b) in liquor output from mixing-settling stage 5.

The DYNAM procedure is based on the "principle of optimality" [6]. This principle relates the properties of decisions made in sequence. Whatever the initial decisions are, the remaining decisions must be optimal for the overall set of decisions to be optimal.

The DYNAM algorithm and the counter-current washing simulator work together as follows: A base case of a set of washing conditions, which closely approximates those of commercial importance, is chosen. Given these washing conditions, the simulator then calculates the value of the "objective function," *i.e.*, the goal of the optimization procedure. This is a well-defined quantity, such as washing efficiency. Once the initial value of the objective function has been determined, the DYNAM algorithm tries new washing conditions, gradually moving toward the desired maximum, or minimum, in the objective function.

According to Figure 3, and for mixing-settling stage N, washing efficiency is defined as:

$$\text{Washing Efficiency} = \frac{C_u(N) - C_u(N+1)}{C_u(N) - C_o(100+N)}, \tag{10}$$

where

$C_u(N)$ = Caustic soda concentration in slurry to stage N

$C_u(N+1)$ = Caustic soda concentration in slurry from stage N

$C_o(100+N)$ = Caustic soda concentration in liquor from stage N.

Optimization results have shown the following parameters to be of utmost significance on washing efficiency:

1. Settled mud is kept as dense (thick) as possible without bogging the washer rake. This is controlled by manipulating the ratio of polymer-to-mud flow rates to a mixing tank. Increasing the polymer dosage to a washer increases the rate of flocculation and density up to a maximum. Beyond this maximum, the polymer molecules occupy all the available active sites. The mud will become stable and mud densities will fall. In the washing circuit, the washer rakes tend to bog well before this turnover point is reached.
2. Flows of all streams between wash trains and stages are to be balanced.
3. Mud level in the last washes is kept as high as possible to maximize mud compaction. In the first washes, however, mud level is kept low as liquor clarity is required for digestion dilution.
4. Fine seed filtrate, FSF, addition point is optimized to ensure the pregnant liquor oxalate is maintained at its optimal commercial value. There is, apparently, a conflict between the requirement of the washers circuit to maximize the caustic recovery but maintain an oxalate purge which in itself is a soda loss. As caustic losses from the washers are minimized, so too are the losses of the other components, including oxalate. Further, oxalate returned to the plant in the supernatant liquor, SNL, is seasonal due to evaporation and rainfall at the ponds. Therefore, the oxalate purged from the plant *via* the red mud must also be varied to maintain the oxalate balance. To achieve this, it has been found that the fine seed filtrate steam, FSF, which is oxalate rich, should be moved between the third and the fifth washers. This apparently alters the oxalate/total soda ratio in the red mud of the last washers. Directing fine seed filtrate stream, FSF, to the fifth washer increases this ratio and a higher oxalate purge will result. If an even higher purge rate is required than can be achieved using the fifth washers, facilities should exist to dump fine seed filtrate stream directly to the ponds.

3.2. PC-Based Control of the Counter-Current Washing Operation

The objectives of the PC-based control of the washing operation are:

1. Maintain both liquor and slurry levels within all five stages of the washers train as close as possible to their respective "desired" levels as determined by the model optimization results.
2. Manipulate the ratios of polymer-to-mud flow rates to mixing tanks in order to obtain an optimal value for settled mud density.

3. Manipulate the fine seed filtrate addition point to the washing train so that washing efficiency is maximized while maintaining a 'desired' oxalate purge in the circuit.

A feedforward feedback control strategy has been designed and implemented using a personal computer for process control purpose.

Underflow of the red mud slurry from each tank was controlled by the application of an electrical signal to a variable-speed pump, whilst upstream flow of liquor was maintained *via* another variable speed pump.

3.2.1. Selection of Feedback Term

In the non-equilibrium situation, the actual height, h_{si}, of slurry in settling tank i differed from the desired slurry height, h_{di}, by an amount x_i, where:

$$x_i = h_{si} - h_{di} . \qquad (11)$$

Feedback control to restore x_i to zero was therefore possible *via* the introduction of a term $k.x_i$ to the control equation, where k, which should be greater than zero, was a constant of proportionality relating the demanded slurry volumetric underflow rate to unit slurry height error. Such a term could have been applied equally, apart from a sign change, to the pump feeding mixing-settling stage i or the pump draining it. However, the plant configuration was such as to make the latter application preferable.

The initial estimate of the constant of proportionality in the feedback term was made as follows. Given a nominal slurry height error, x_N (greater than zero), then a corresponding nominal slurry volume error can be calculated as $A_s\, x_N$. Given a nominal time, t_N, during which recovery from such an error is desired, then a nominal slurry volumetric underflow rate, u_N, to facilitate recovery can be calculated as:

$$u_N = A_s\, x_N / t_N . \qquad (12)$$

Given a nominal equilibrium slurry volumetric underflow rate, u_E, then provided $(u_E - u_N, u_E + u_N)$ is within the operating range of the underflow pumps, the constant of proportionality, k, is given by:

$$\begin{aligned} k &= u_N / x_N \\ &= \left(A_s x_N / t_N\right) x_N \\ &= A_s / t_N . \end{aligned} \qquad (13)$$

3.2.2. Selection of Feed-Forward Term

In classical control strategy, the feed-forward term was to be that element of each control equation, which, under equilibrium conditions, facilitated the steady-state underflow of slurry from the mixing-settling stage in question. Furthermore, this term was to ensure that a disturbance in the input slurry flow to the train, was immediately transmitted to all tanks of the train. This happens in preference to being heralded by the gradual departure of successive tanks from their desired states, as the disturbance propagated down the train. Accordingly, the solids mass underflow rate from each settling tank in the train was set equal to the solids mass inflow to the train which led to the feed-forward equation:

$$u_i = u_o , \quad \text{for} \quad i = 1, \ldots\ldots, 5 \qquad (14)$$

where,

u_i = volumetric underflow rate of slurry from settling tank i

u_o = volumetric flowrate of slurry at the start of the train (a primary measurable input to the system).

Simulation studies revealed that although the choice of feed-forward term, u_o, for all control equations advised all tanks of variations in slurry inflow rate, such a term was not the best choice to minimize the recovery time following slurry height errors. Analysis of the situation revealed that there are 3 separable components which constituted the slurry underflow:

(*i*) a component to accommodate the inflow to the train;

(*ii*) a component to remove the slurry height error in the tank being serviced by the pump in question, and,

(*iii*) a component, not previously accounted for, to accommodate the disturbances in the inflow rate resulting from variations in the underflow from the tank-pump combination feeding the tank in question.

Components (*i*) and (*iii*) above could be accommodated together by replacing u_o by u_{i-1} as the feed-forward term in the control equation for settling tank i. The choice of u_{i-1} as the feed-forward term, rather than u_o, was based on the following:

(*i*) If new control settings were calculated upon the basis of parameters from the last control interval, then the delay in propagating an inflow disturbance down the train, would be, at most, five control intervals, otherwise,

(*ii*) If new control settings were calculated for each settling tank down the train, using the last calculated underflow rate as the inflow rate to the settling tank in question, then no propagation delay would result.

Accordingly, the control equations, based upon underflow rates, become:

$$u_i(t) = \left[1 + \xi_i(t)\right]u_{i-1}(t) + k.x_i(t) \tag{15}$$

where $\xi_i(t)$ is an adaptive term in the form of an integral of the error term. The adaptive term was introduced to compensate for the errors in measurement instruments, initial poor estimates of pump characteristics, and variations of pump characteristics with time which is associated with constrictions from deposits of scale inside the pump mechanisms.

Accordingly, the central equations from which underflow pump signals, $P_i(t)$, were calculated become:

$$P_i(t) = k_{P_i}\left\{\left[1 + \xi_i(t)\right]\frac{P_{i-1}(t)}{k_{P_{i-1}}} + k.x_i(t)\right\}, \quad i = 2,\ldots\ldots,5 \tag{16}$$

and

$$P_1(t) = k_{P_i}\left\{\left[1 + \xi_i(t)\right]u_o(t) + k.x_1(t)\right\} \tag{17}$$

where K_{p_i} is the initial estimate of the signal-to-flowrate constant for the pump servicing settling tank i.

In the computer control strategy, it is assumed that the total fine seed filtrate stream to each stage of washers is split equally between the two trains in each stage. Half of the measured fine seed filtrate flow signal is added as a feedforward component to the signal to the liquor feed central valve of the tank to which the fine seed filtrate stream is being added.

The problem of unequal fine seed filtrate split is overcome by the use of a liquor mass balance technique, which calculates the fine seed filtrate flow to each train and thus ensures that the correct fine seed filtrate feedforward signals are used.

Using Equation (10), Table 1 shows values of calculated washing efficiencies for the washing train over 6 operational days at 8 hours intervals for both cases when washing train is under no control and when control system is implemented.

Table 1. Calculated Washing Efficiencies for the Washing Train.

Control System→ Day/Hour↓	No	Yes
1/00	0.58	0.73
1/08	0.58	0.72
1/16	0.60	0.73
1/24	0.61	0.71
2/08	0.59	0.73
2/16	0.60	0.73
2/24	0.57	0.70
3/08	0.62	0.70
3/16	0.62	0.73
3/24	0.62	0.73
4/08	0.59	0.72
4/16	0.58	0.70
4/24	0.58	0.71
5/08	0.60	0.71
5/16	0.60	0.70
5/24	0.59	0.72
6/08	0.57	0.71
6/16	0.60	0.73
6/24	0.62	0.70
Average	0.5957	0.7163

$$\text{Average increase in washing efficiency} = \frac{0.7163 - 0.5957}{0.5957} \times 100$$

$$= 20.24.$$

So, about 20% increase in caustic soda gain from the counter current washing has been achieved as a result of controlling the operation.

4. SUMMARY AND CONCLUSIONS

A simulation model for the red mud counter-current washing operation in alumina-producing plants has been developed and validated. This model has been used to optimize the red mud washing operation with maximizing washing efficiency as the objective function. Based on optimization results, a PC-based control system has been designed and implemented on a washing train. About 20% increase in caustic soda gain has been achieved as a result of control system implementation. As it has been mentioned earlier, however, all input and output streams to a mixing-settling stage are determined by factors outside the washing area. Therefore, it is important that the washing model be integrated into a larger simulation model of the Bayer Process to achieve a more effective control of the washing process.

NOMENCLATURE

Q_{li} = Flow rate of liquor input to the mixing tank (m^3/hr)
Q_{lo} = Flow rate of liquor leaving the settling tank (m^3/hr)
Q_{si} = Flow rate of slurry input to the mixing tank (m^3/hr)
Q_{so} = Flow rate of slurry leaving the settling tank (m^3/hr)
C_{11} = Concentration of caustic soda in liquor input to mixing tank (expressed in grams Na_2O/liter)

C_{21} = Concentration of caustic soda in liquor leaving, or in, the mixing tank (expressed in grams Na_2O/liter)
C_{12} = Concentration of caustic soda in entrapped liquor in slurry input to the mixing tank (expressed in grams Na_2O/liter)
C_{22} = Concentration of caustic soda in entrapped liquor in slurry leaving, or in, the mixing tank (expressed in grams Na_2O/liter)
C_{31} = Concentration of caustic soda in liquor leaving, or in, the settling tank (expressed in grams Na_2O/liter)
C_{32} = Concentration of caustic soda in entrapped liquor in slurry leaving, or in, the settling tank (expressed in grams Na_2O/liter)
T = Liquor Temperature (in degrees Celsius)
ρ_{si} = Density of slurry input to mixing tank (kg/m^3)
ρ_{so} = Density of slurry leaving, or in, the settling tank (kg/m^3)
R = Ratio of polymer to mud flow rates to mixing tank
h_l = Height of liquor in the settling tank (meters)
h_s = Height of slurry in the settling tank (meters)
ρ_ℓ = Density of liquor input to the mixing tank (kg/m^3)
H_m = Height of the mixing tank (meters)
A_m = Cross-sectional area of the mixing tank (m^2)
ρ_m = Dry mud density (kg/m^3)
A_s = Cross-sectional area of the settling tank (m^2)
R_m = Rate of mass transfer in the mixing tank (1/second)
R_s = Rate of mass transfer in the settling tank (1/second).

ACKNOWLEDGMENT

It is upon the request of the alumina-producing plant from which data were collected that the author has not identified its name or names of personnel within the plant who have contributed to this work. The author, however, wishes to acknowledge the contributions made without which this work could not have been completed.

The author gratefully acknowledges the assistance of King Fahd University of Petroleum and Minerals in the performance of this work.

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Paper Received 11 October 1993; Revised 14 February 1994; Accepted 28 June 1994.