UNIFYING EVALUATION THEORY IN WATER RESOURCES PLANNING

Abdul-Mannan Turjoman*

Vice President for Academic Affairs
King Faisal University
Al-Ahsa, Saudi Arabia

الخلاصـــة :

تتعدد اهداف واغراض مشروعات المياه الكبرى ، واهم ما يوليه مخطط هذه المشروعات عنايةً هو تقييم الطرق المختلفة لتخطيط المشروع حتى يحقق الاهداف المرجوة منه ومن ثم يمكن اختيار الطريقة المناسبة للتنفيذ .

وتوجد عدة طرق للدراسات التحليلية للمشروعات ذات الأغراض المتعددة ولكن لم يحاول أحدٌ من الباحثين السابقين أن يربط بين العلاقات المختلفة لهذه الطرق . ويهدف هذا البحث الى توضيح أن جميع طرق التقنية لتحليل المشروعات متعددة الأغراض مبنية على أساس نظري واحد . ولإيضاح ذلك تم اختيار نظامي تقنية هما : Welfare Economics & Surrogate Worth Trade Off الذي ثبت أنهما متشابهان رياضيا .

وهذا يدل على أن المخطط يمكنه استعمال أى نظام تقنية لتحليل المشروعات متعددة الأغراض دون تردد أو احتمال ارتكاب خطأ قانوني أو غيره كمخطط للمشروع .

كما يوضح أن أفضلية استخدام اي نظام تقنية في هذا الغرض عن الآخر لا يرجع الى الأساس النظري ، ولكن يعتمد على مدى مناسبة واختيار نظام التقنية المناسب للمشروع المتعدد الاغراض الواجب دراسته .

*Address for correspondence :
King Faisal University
P. O. Box 380
Al-Ahsa 31982
Saudi Arabia

ABSTRACT

Complex water projects normally have several objectives. One of the most important planning tasks is to evaluate the various alternatives in the light of these objectives in order to select the best one to be implemented. There are a number of techniques used in multi-objective analysis and no one has attempted to show the interrelationships among these techniques. The thesis of this paper is that these techniques of multi-objective analysis can all be derived from a common theoretical base. To demonstrate this, two representative techniques, Welfare Economics and the Surrogate Worth Trade-Off Technique, are shown to be mathematically similar. This implies, that a planner may use any of the valid techniques, without fear of violating some fundamental precept of planning theory. Likewise, claims that one technique is superior to another cannot be based on theoretical grounds but only on practical utility.

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INTRODUCTION

The evaluation of water and agricultural project alternatives has often been considered purely from an economic perspective. Various criteria have been utilized, including maximum net benefits, cost/benefit ratio, in the overall process known as "cost-benefit analysis". Since the mid 1960's, concern over the increasing degradation of both environment and social well-being prompted planners, notably those in the public works sector, to increase the emphasis on these more subjective factors in their evaluations. However, problems in quantifying social and environmental features in terms of monetary units has led to apparently less precise evaluation techniques.

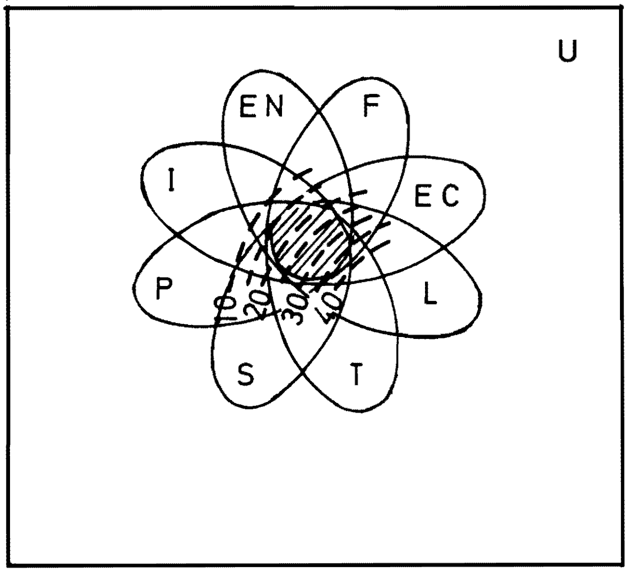
According to Lichfield *et al.* [1], evaluation is "the process of analyzing a number of plans or projects with a view to searching out their comparative advantages and disadvantages and the act of setting down the findings of such analysis in a logical framework". Drawing on Helweg [2], evaluation presupposes three items of information: (1) a knowledge of *what is valuable,* such as objectives; (2) some *way to measure* it, not necessarily quantitatively; and (3) *something to measure,* such as an alternative. In other words, evaluation is the process of determining which of the alternatives is the "best fit" with regard to some objective function.

The difficulty with incommensurable objectives forces planners to ask the question, "How does one measure value when dealing with multiobjective analysis?" One present practical approach is simply to consider each objective separately and hope that the tradeoffs are clear enough so that the decision-maker can choose the best alternative. There are, however, other approaches (methods) that make evaluation more rigorous.

The existence of many methods confuses some people and suggests a lack of a unified theory of evaluation. The aim of this paper is to examine two evaluation methods from different disciplines and show that there is a consistent theory underlying them both.

EVALUATION THEORY

It may be useful to view evaluation as a two step process: one step being to define the feasible region for a given problem by considering all of the constraints, and the second step being that of finding the best alternative within that feasible region. Again, following Helweg [2], Figure 1 is a Venn or



U = The Universe of all Possible Alternatives
EN = The Set of Environmentally Feasible Alternatives
F = The Set of Financially Feasible Alternatives
I = The Set of Institutionally Feasible Alternatives
Shaded Area = EN∩F∩I = The Set of Feasible Alternatives
-------- = The Net Benefit of Alternatives (Objective Function)

Figure 1. The Objective Function and Set of Feasible Alternatives.

Euler Diagram showing this concept. Here eight constraints: Environmental (EN), Financial (F), and Institutional (I), etc. are assumed. The alternatives that lie outside the intersection of these sets are infeasible and should be eliminated. The second step, then, is to select the best feasible alternative. Assuming that benefit, if multiobjective, is made commensurable where all of the objectives have been grouped under a common measure, then the decision maker chooses the alternative that lies on the greatest net benefit curve. Notice that the unconstrained best alternative lies outside the feasible region as it is infeasible environmentally. Notice also the assumption that feasibility can be determined *a priori.* Though this is not always true, Figure 1 clarifies the concept of feasibility over and against optimality.

Various disciplines are interested in the theory of evaluation. Perhaps the two most relevant disciplines are Welfare Economics and Operations Research (OR). An Operations Researcher would approach evaluation as a constrained, multi-objective, optimization problem, remembering the assumed commensurability in Figure 1. Welfare economists consider evaluation in the light of Pareto Optimality. Both of these approaches will be briefly examined.

Welfare Economics

The theoretical basis used by welfare economists to find the best alternative is like finding the point where all of the available resources are used optimally to satisfy the desires of society [3]. There are four simplifying assumptions that help illustrate the theory. First, assume only two homogeneous and perfectly divisible inputs or labor (L) and capital (K). Second, there are only two homogeneous goods: water supply (W) such as might be provided by building a reservoir and a scenic recreational area (R). Third, assume society consists of only two individuals, A and B. Fourth, there exists a social welfare function, $W = W(U_A, U_B)$, whereby society can order all possible combinations of A's and B's utilities. This last assumption is highly controversial, because constructing it demands a decision as to the relative distribution of goods between A and B.

First construct an Edgeworth box, see Figure 2,

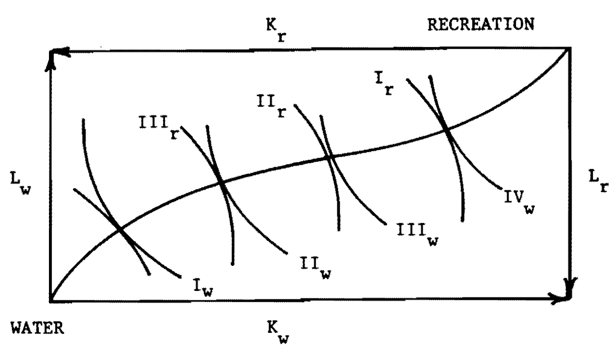


Figure 2. Edgeworth Box Showing the Pareto Optimal Contract Curve or Distribution of Inputs Between Water and Recreation.

showing the isoquants of the production functions, i.e. curves showing the different combinations of inputs that produces the same number of output units. The amounts of K and L available determine the dimensions of the box, and the Pareto optimal contract curve is determined by the locus of points where the slope of the isoquants for W and R are equal ($MRS_W = MRS_R$). (I_W through IV_W show isoquants of increasing output of water.) Any point not on the optimal contract curve can be moved to the curve yielding an increase in production of both W and R.

Second, using the preceding results, construct the product transformation curve, $T-T'$, in Figure 3.

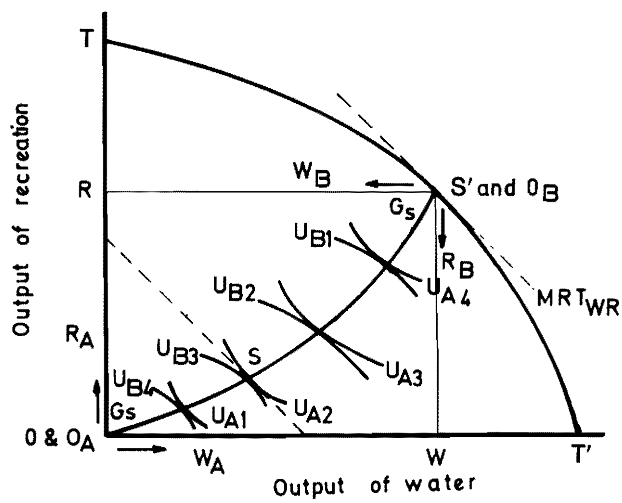


Figure 3. Product Transformation Curve. T−T', and Edgeworth Box Showing the Pareto Optimal Contract Curve or the Distribution of Water and Recreation Between A and B.

This curve shows the combinations of W and R that can be produced with the limited resources of L and K. Remember that the slope of the curve, $T-T'$, at any point equals the marginal rate of transformation (MRT) (i.e., the number of units of W needed to be given up for an additional unit of R). One can pick any point on this curve and construct another Edgeworth box which will define the Pareto optimal contract curve for A and B showing the distribution of W and R between them. This is done by plotting the utility functions of each person and constructing a line over the locus of points where the slopes of the respective utility functions are equal ($MRS_A = MRS_B$).

It can be shown mathematically that the optimal division of the two goods between A and B is the point, S' (i.e., that point on the pareto optimal contract curve where the slope of the utility functions (MRS) equal the Marginal Rate of Transformation

(MRT)). Note that this automatically divides up the available utility between A and B but it does not say that this is the correct point because there are an infinite number of points and Edgeworth boxes that can be drawn on the product transformation curve. These cover the spectrum of possibilities along the contract curve, so this one point gives just one curve on the utility possibilities envelope shown in Figure 4.

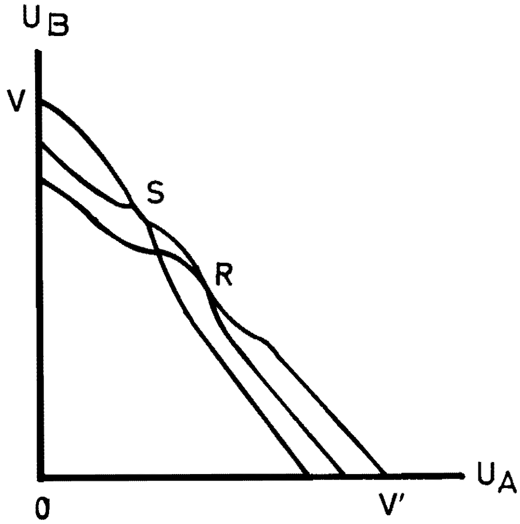


Figure 4. Utility–Possibility Frontier Where V–V' is the Envelope of All Possible Curves.

This brings the reader to the third step. Each point on the product transformation curve generates a utility possibility curve in Figure 4. The result of all of the infinite curves is an envelope of curves, or the utility–possibility frontier. This curve $V-V'$, is the locus of points farthest from the origin or those points that give the maximum attainable utility for any product mix. This curve is analogous to the product possibility curve common in microeconomic theory. Later it will be shown that the utility possibilities frontier equals the noninferior set. Notice that the possibilities range from maximum utility for A and zero utility for B to vice versa. And, the point that should theoretically be chosen is the one that is tangent to the maximum social welfare function as seen in Figure 5.

The social welfare function is difficult, if not impossible, to construct, because it decides the relative deservedness of A and B, which was assumption four above. As interesting as this is, it

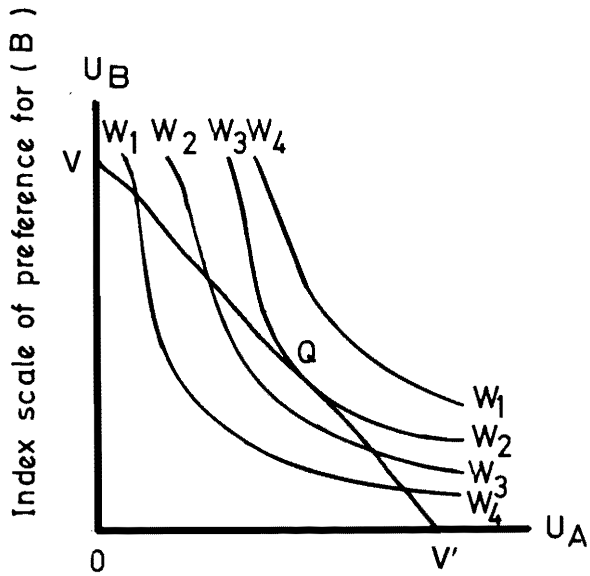


Figure 5. Point Q Shows the Point on W_3-W_3' Where the Social Welfare Function is Maximized and is Tangent to the Utility–Possibility Frontier V–V'

has limited practical application to the economist; however, the theoretical value is important as will be shown later.

Operations Research

Multiobjective optimization, also called vector optimization and multiobjective programming in the literature is the Operations Research approach to optimizing incommensurables.

Following the discussion by Cohen and Marks [4], multi-objective optimization can be described mathematically as:

$$\max \ \mathbf{Z}(\mathbf{x}) = Z_1(\mathbf{x}),\ Z_2(\mathbf{x}), \ldots\ldots, Z_p(\mathbf{x}) \qquad (1)$$

subject to

$$g_i(\mathbf{x}) \le 0 \qquad i = 1, 2, \ldots\ldots, m$$
$$\mathbf{x} \ge 0$$

in which $\mathbf{Z}(\mathbf{x})$ is a p-dimensional objective function, $\mathbf{x}$ is an n dimensional vector of decision variables, and $g_i(\mathbf{x})$ is the constraint set. See references [4] and [5] for a classification of the main multi-objective optimization techniques. Table 1 is adopted from [4].

Most multi-objective techniques seek to identify the noninferior set, $\mathbf{Z}(\mathbf{x}^*)$. This can be done without preference information from the decision-maker. It is then up to the decision-maker to choose the best

Table 1. Classification of Multiobjective Solution Techniques †

1. Generating Techniques
Weighting method Constraint method Derviation of a functional relationship for the noninferior set Adaptive search
2. Techniques Which Rely on Prior Articulation of Preferences
Goal programming Assessing utility functions Estimation of optimal weights Electre method Surrogate worth trade-off method
3. Techniques Which Rely on Progressive Articulation of Preferences
Step method Iterative weighting method Sequential multiobjective problem solving (Semopes)

† After Cohen & Marks [4]

alternative from the non-inferior set. The best alternative is sometimes called "the best compromise solution" reminding the decision-maker that a tradeoff is involved. To illustrate the concept of the noninferior set, consider the multiobjective problem of:

$$\max\ 5x_1-2x_2 \text{ and } -x_1+4x_2 \tag{2}$$

subject to

$$\begin{aligned} -x_1+x_2 &\le 3 \\ x_1+x_2 &\le 8 \\ x_1 &\le 6 \\ x_2 &\le 4 \\ x_1,\ x_2 &\ge 0 \end{aligned}$$

This is shown in Figure 6. Notice that if there were only one objective function, this would be a classic linear programming problem; nevertheless, even with two objective functions, the Fundamental Theory of Linear Programming applies. This states that any optimal feasible solution will be at an external point or on the boundary.

The feasible region is the one that has satisfied all of the constraints which is the shaded area in Figure 6 and comparable to the intersection in Figure 1. It is described mathematically as:

$$X=\{x|g_i(\mathbf{x}) \le 0,\ \forall\, i,\ x_j \ge 0\ \forall\, j\} \tag{3}$$

Within this feasible region there is a subset of X which is called the set of non-inferior solutions. That is $\mathbf{x} \in X$ in which $\mathbf{Z}(\mathbf{x})$ has no values that will become better for both objective functions. This set of points is shown by the dark line on the boundary of this feasible region between points (1, 4) and (6, 0). This is derived by calculating all of the values of $Z_1(\mathbf{x})$ and $Z_2(\mathbf{x})$ for external points and applying the definitions for noninferiority. Figure 7 shows these values. The two scales for the two objective functions are in different units.

Other multi-objective techniques are described in the literature but one of the most popular has been the Surrogate Worth Trade-off Method (SWT) [5]. The advantage of this technique is that it takes into account the difference in marginal utility because of quantity available. That is, an additional $1000 has a different utility to a millionaire than an average citizen.

Using the SWT method a Surrogate Worth Trade-off function (W_{ij}) may be set up by displaying tradeoffs among the objective functions of the linear programming problem. That is, the shadow costs (similar to the Lagrangian multipliers) are displayed and the decision maker orders these lexicographically from -5 if the tradeoff is strongly not desired to $+5$ if the tradeoff is highly desired. If the tradeoff is slightly desired, the decision maker would assign a $+1$, $+2$, and so on. If the decision maker is neutral, he would assign a zero to the overall objective function. The zero value (indifference by the decision maker) determines the "best compromise solution." Such an objective function may be described mathematically as:

$$\text{Min } |W_{ij}| \tag{4}$$

CORRESPONDENCE BETWEEN THE SWT METHOD AND WELFARE ECONOMICS

The "proof" of correspondence will take two steps, first the welfare economics approach will be formulated as a general optimization problem and second, the constraints and objective functions of both methods will be converted into "well behaved" generalized functions. The latter step is sufficient to show that the two methods are interchangeable and the demonstration is complete.

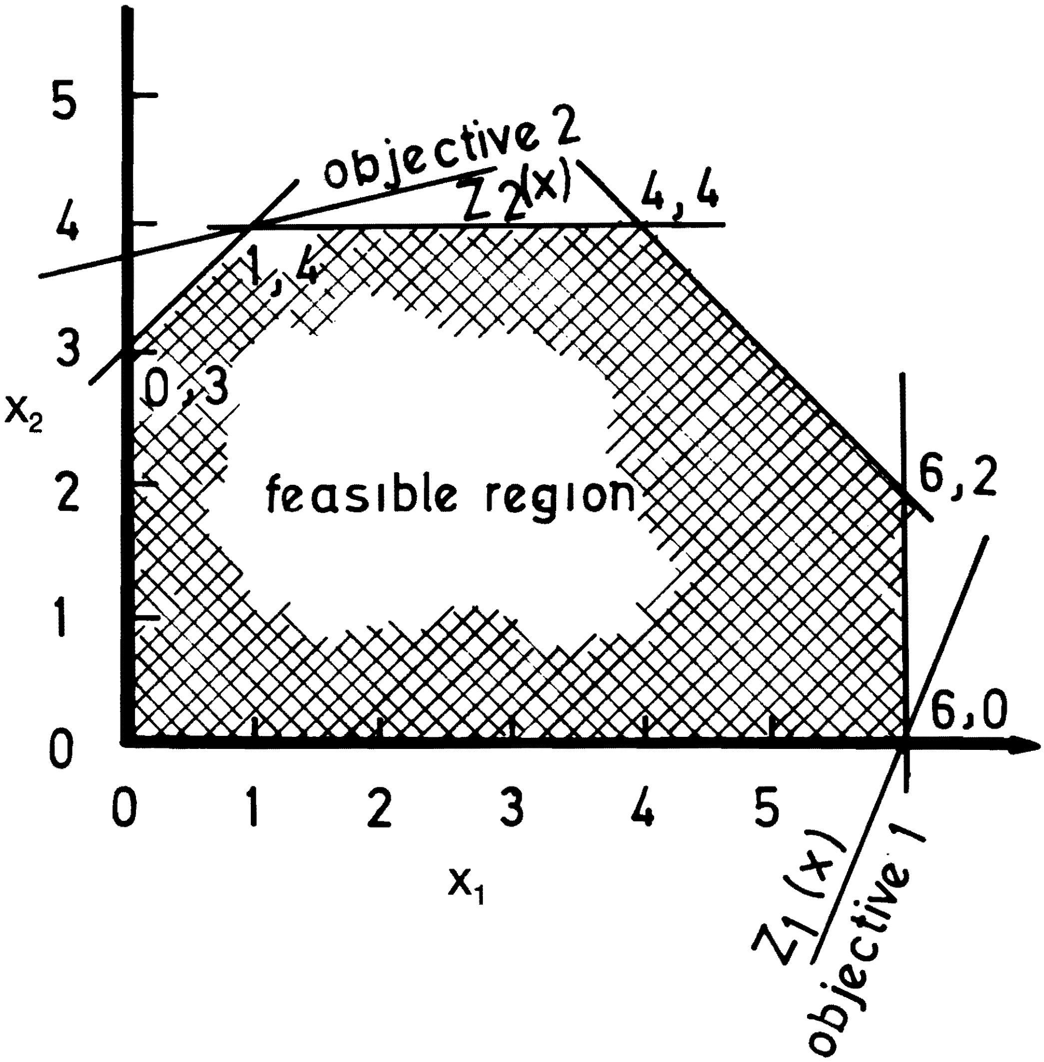


Figure 6. Multiobjective Optimization Problem with Two Objectives.

Correspondence of the Constraint Sets

A. Welfare Economics

Showing the non-inferior sets to be similar will show, *a fortiori*, that the constraints are similar. Using the same notation as above, define $\mathbf{x} \in X$, $X \simeq \{A, B, K, L, R, W\}$, and $\mathbf{x} \geq 0$. The non-inferior set for welfare economics may be defined as:

$$k_1(\mathbf{x}) = MRS_A - MRS_B = 0, \ \forall \ A, \ B$$

$$k_2(\mathbf{x}) = MRS_K - MRS_L = 0, \ \forall \ K, \ L$$

$$k_3(\mathbf{x}) = MRT_{RW} - MRS_{RW} = 0, \ \forall \ R, \ W \qquad (5)$$

in which all variables are as previously defined. Since each constraint is based on the slope of a function they can be rewritten as:

$$k_1(\mathbf{x}) \simeq f'_A(\mathbf{x}) - f'_B(\mathbf{x}) = 0 \qquad (6)$$

in which $f_A(\mathbf{x}) = U_A$ in Figure 3 and so on. This yields a general non-linear constraint set:

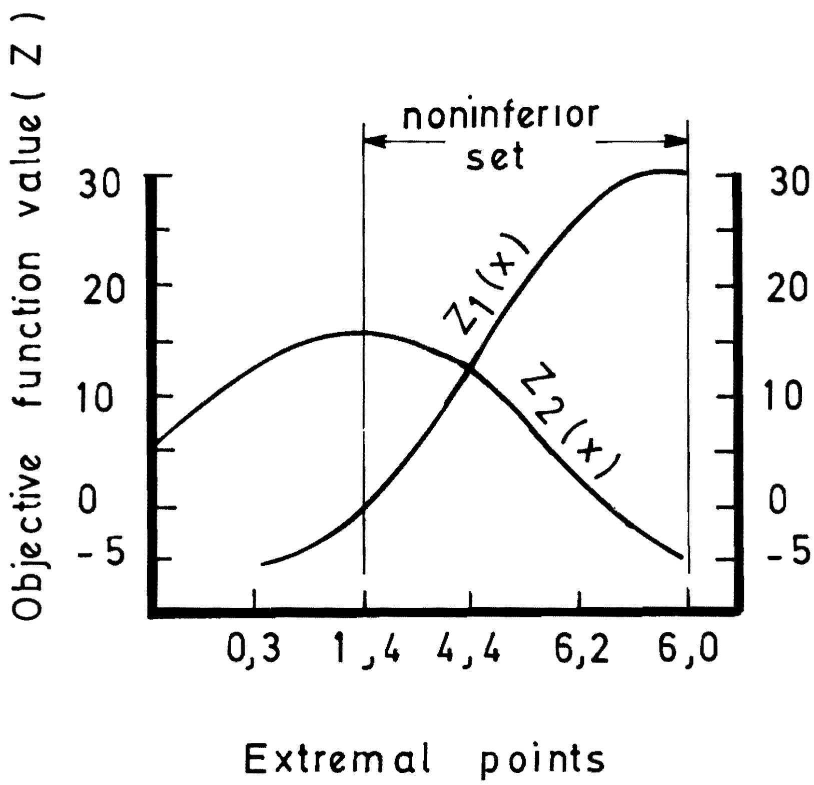
$$\mathbf{k}(\mathbf{x}) \leq 0 \,. \qquad (7)$$

B. SWT Method

The non-inferior set for the SWT Method may be defined as:

$$\mathbf{x} = [x \,|\, g_i(\mathbf{x}) \leq 0, \ \forall \ i, \ x_j \geq 0, \ \forall \ j] \text{ AND}$$

$$[x \,|\, Z_{k+1} \ (Z^*_k(\mathbf{x})) \leq Z_k \leq Z^*_k(\mathbf{x}), \ \forall \ k] \qquad (8)$$



Point	$Z_1(\mathbf{x})$	$Z_2(\mathbf{x})$	
0,3	−6	12	
1,4	−3	15*	
4,4	12	12	noninferior set
6,2	26	2	
6,0	30*	−6	
0,0	0	0	

Figure 7. Finding the Noninferior Set.

in which all symbols are as previously defined. The first set is equivalent to the general constraints to a linear programming problem:

$$g_i(\mathbf{x}) \leq 0 \qquad \forall\ i \qquad (9)$$
$$\mathbf{x} \geq 0$$

The second set merely states that the non-inferior set must lie on the hyperplane between the maxima (or minima) of the objective functions as illustrated in Figure 6. This set may be written as:

$$Z_k(\mathbf{x}) - Z^*_k(\mathbf{x}) \leq 0 \qquad \forall\ k$$
$$Z_{k+1}(Z^*_k(\mathbf{x}) - Z_k(\mathbf{x})) \leq 0 \quad \forall\ k \qquad (10)$$

in this form it is obvious that the constraint set of the SWT may also be written as:

$$\mathbf{h}(\mathbf{x}) \leq 0 \qquad (11)$$

which is the same form as Equation (7), though not the same function. The fact that equations 10 are linear does not weaken the similarity because the SWT method may be applied to general non-linear programming problems as well as the more familiar linear programming ones. Using slack and surplus variables, S_m, the correspondence can be set up as:

Welfare Economics		SWT
$\mathbf{k}(\mathbf{x}) = 0$	$\simeq$	$\left\{ \begin{array}{l} \mathbf{g}(\mathbf{x}) + S_m = 0 \\ \mathbf{h}(\mathbf{x}) + S_m = 0 \end{array} \right.$ (12)

Correspondence of the overall objective functions

A. Welfare Economics

The fact that the Social Welfare Function, $W\ (U_A,\ U_B)$ is assumed but no practical method of obtaining it is given in the literature does not effect its theoretical validity. It is required by economic theory to be a convex function and can be written as:

$$\text{Max } W(\mathbf{x}) \tag{13}$$

in which $\mathbf{x} \in U$ and $U=\{U_A, U_B\ldots\}$, the set of all possibility utility dimensions.

B. SWT Method

For practical convenience the SWT function is derived from a table of discrete values [2]; however, the function can be made continuous using the theorems of limits as follows:

$$\lim_{\Delta x \to 0} \frac{\Delta Z_{k+1}(\mathbf{x})}{\Delta Z_k(\mathbf{x})} = h(\mathbf{x}) \tag{14}$$

and Equation 4 can be written as:

$$\text{Max } V(\mathbf{x}) = -|k(h(\mathbf{x}))| \tag{15}$$

in which $k(\cdot)$ is the decision maker's worth mapped on the trade-off space (the negative converts a minimum objective into a maximum). The correspondence can be seen as:

Welfare Economics		SWT	
Max $W(\mathbf{x})$	$\simeq$	Max $V(\mathbf{x})$	(16)

SUMMARY

The preceding discussion has shown the similarity between the SWT method and Welfare Economics by formulating both approaches in the form of a general optimization problem and showing the similarity between the two constraint sets and the two objective functions. While it might be argued that because the derivative of Equation 15 is not continuous, it does not correspond to Equation 13, the form of Equation 15 is arbitrary and it could easily be made differentiable by changing the instructions to the decision maker.

The author, though not having rigorously proven the similarity of all known methods, has looked at them and found that all of those investigated could be treated as the two in this paper. That is, they all could be formulated as a general mathematical programming problem and the constraints and objective functions could be shown to be similar.

CONCLUSIONS

Water resources planners today have before them a wide array of tools from which to choose in the evaluation of alternatives. What this paper has suggested is that the actual evaluation method (assuming it is a valid method) is unimportant theoretically. This leaves the planner free to choose the method most convenient to the particular planning environment. Or, the planner may choose the method with which he is most comfortable.

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