A NON-LINEAR REGRESSION METHOD FOR INTERFERENCE TEST ANALYSIS

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الخلاصـــة :

لقد طبقت طريقة التربيع الأقل لتعيين قيم معاملات الناقلية والتخزين التى تنقص مجموع مربعات الفروق بين الضغط الملاحظ والمحسوب باستخدام حل المصدر الخطى . وقد استخدم نهج تكراري يعتمد على طريقة نيوتن – رافسون لحل المعادلتين الآنيتين غير الخطيتين الممثلتين لشرط التدنى . ووجـــد أنَّ هذه الطريقة فعالة وتحقق تقاربا سريعا .

ويمكن استخدام الطريقة المطورة فى حالات المعدل الثابت والاختبارات ذات المعدلين بما فيها حالات التحاشد (الزيادة) وقد استخدمت هذه الطريقة لحالات عديدة من الهبوط والتحاشد كل على حدة أو مجتمعين وقورنت النتائج بنتائج طريقة تطابق المنحنيات النوعية . وقد لوحظ توافق ممتاز فى جميع الحالات .

ABSTRACT

The method of least squares is applied to obtain the values of transmissibility kh/μ and storage $h\phi C_t$ that minimize the sum of the squares of the differences between observed pressures and pressures calculated by the line-source solution. An iterative procedure based on the Newton–Raphson method is used to solve the two simultaneous nonlinear equations representing the minimization conditions. This method is found to be efficient and converges rapidly.

The method developed can be used for cases for constant rate and two-rate tests (including buildup tests). The method was applied to various cases of drawdown, buildup and combinations of both and results were compared to those obtained by type-curve matching. Excellent agreement was observed in all cases.

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NOMENCLATURE

b = constant, Equation (12), h
C_t = total compressibility, psi^{-1} [kPa^{-1}]
h = formation thickness, ft [m]
k = absolute permeability, md
m = constant, Equation (11), psi [kPa]
p = pressure, psi [kPa]
p_D = dimensionless pressure
Δp = pressure drop, psi [kPa]
q = flow rate, bbl d^{-1} [$m^3 d^{-1}$]
r = radial distance, ft [m]
s = storage = $h\phi C_t$, ft psi^{-1} [m kPa^{-1}]
t = time, h
t_{DW} = dimensionless time
T = transmissibility = $kh\ \mu^{-1}$, md ft cp^{-1}
B = formation volume factor, res vol/std vol
γ = Euler constant = 1.78
μ = viscosity, cp [Pa s]
ϕ = porosity, fraction

Subscripts

D = dimensionless
M = match point
t = total
w = well

INTRODUCTION

In an interference test, a change in the production rate in an active well creates a pressure interference in an observation well that can be analyzed for reservoir properties [1]. In interference tests where the skin factor of the producing well does not influence the pressure response at the observation well, test data is analyzed to obtain the reservoir transmissibility kh/μ and storage $h\phi C_t$.

Most well-test analysis methods are based on the line-source solution of the diffusivity equation for infinite acting systems. In this case and for a single well producing at constant rate [2].

$$\Delta p = -\frac{70.6\ q\mu B}{kh}\mathrm{Ei}\left(-\frac{948\ \mu c\phi r^2}{kt}\right), \quad (1)$$

where, Ei is the exponential integral function defined as

$$\mathrm{Ei}(-x) = -\int_x^\infty \frac{e^{-u}}{u}du\ . \quad (2)$$

If the exponential term e^{-u} in Equation (2) is expanded using Taylor's series and the integration is performed term by term we get [3]

$$\mathrm{Ei}(-x) = \ln(\gamma x) - x + \frac{x^2}{2\times 2!} - \frac{x^3}{3\times 3!} + \ \dots \quad (3)$$

It is clear from Equation (3) that for small values of x, the exponential integral function may be approximated by

$$\mathrm{Ei}(-x) = \ln(\gamma x) = \ln x + 0.5772 \quad (4)$$

The substitution of Equation (4) into Equation (1) gives

$$\Delta p = \frac{162.6\ q\mu B}{kh}\left[\log t + \log\frac{k}{\mu c\phi r^2} - 3.23\right]. \quad (5)$$

Equation (5) is the basis for the semi-log plotting techniques that are widely used in well-test analysis. The principle of superposition is applied for the case of multiple rates including buildup and a steady-state skin pressure drop is used to account for formation damage at the wellbore.

The advantage of using Equation (5) or similar equations in well-test analysis is that a straight line is obtained when Δp is plotted versus log (t) [or p *vs* log ($t+\Delta t/\Delta t$) for buildup]. The slope and intercept of the resulting straight line are directly related to the reservoir properties such as permeability k, porosity ϕ and skin factor S. The two parameters of the straight line can be determined either graphically (slope and intercept) or by simple linear regression using the method of least squares.

It is to be noted that early-time data is usually influenced by wellbore storage and cannot be analyzed by the semi-log method. Only data points lying on the straight line portion are usually considered. A systematic procedure to exclude wellbore-storage influenced data points is to plot Δp *vs* Δt on a log−log graph. Data points lying on a straight line with slope of 1 cycle/cycle are dominated by wellbore storage effects. These points and an additional 1 to 1½ cycles in time should be excluded.

It is obvious that for the conditions for the semi-log plotting technique to be valid, the argument of Ei-function must be small (usually less than 0.01). From Equation (1) it is clear that this condition amounts to

$$t > 94800 \frac{\mu c \phi r^2}{k} \tag{6}$$

For single well tests, where r in Equation (6) is replaced by the well radius r_w, the condition given by Equation (6) is satisfied almost immediately. For a typical case of $\phi = 0.2$, $\mu = 4$ cp, $c = 2 \times 10^{-5}$ psi^{-1}, $k = 100$ md and $r_w = 4$ in, condition (6) is satisfied if $t > 6$ s.

On the other hand, for the case of well interference tests, the pressure is measured in an observation well at a distance r from the active well. In this case, the time needed to allow for the logarithmic approximation of the Ei-function may be extremely large, depending on the value of r. For the previous example with $r = 200$ ft, a value of $t > 25$ days is needed for the approximation to be valid. For greater distances, larger values of t are needed that are usually not reached during the test due to technical and economical considerations. It follows that in most cases, the semi-log plots are not applicable to the analysis of well interference tests.

The method that is being widely used for interference test analysis is the type-curve matching technique [4]. In this method the test data is plotted as Δp *vs* Δt on a log–log graph and is overlayed over the type-curve represented by Equation (1) in dimensionless form

$$P_D = -\tfrac{1}{2}\,\mathrm{Ei}\left(-\frac{r_D^2}{4t_{DW}}\right) \tag{7}$$

The horizontal and vertical axes are moved keeping the grids parallel until the data points match the curve. Match points are selected on the pressure and time scales. Applying the definitions of dimensionless pressure and dimensionless time at the match points we obtain:

$$T = \frac{kh}{\mu} = \frac{141.3\, qB\, (P_D)_M}{(\Delta p)_M} \tag{8}$$

and

$$S = h\phi C_t = \frac{0.000264\, T}{r^2} \frac{(\Delta t)_M}{\left(\frac{t_{DW}}{r_D^2}\right)_M}. \tag{9}$$

Different type-curves are usually used for reservoirs with different shapes and different boundary conditions.

Despite the simplicity of the type-curve matching technique in well-test analysis, it is a manual method lacking the convenience of systematic analysis methods that are amenable to computer processing. Interactive graphics utilizing type curves may be an effective method of analyzing transient tests.

Many authors used reservoir simulation and history matching techniques to determine reservoir properties from transient pressure tests. Jahns [5] presented a method for obtaining a two-dimensional reservoir description from pressure response data in a multi-well system. His method is based on minimizing the sum of squared errors coupled with the numerical solution of the diffusivity equation. Coats *et al.* [6] introduced a technique that couples the method of least squares with linear programming for automatically determining reservoir description from performance data. This method requires a number of runs using a reservoir simulator. Hernandez and Swift [7] applied a least-squares differential algorithm for automatic determination of reservoir description parameters. Their optimization method also requires the numerical solution of the partial differential equation describing the flow problem. In cases where the reservoir is assumed homogenous, such optimization techniques seems to be unnecessary. A more reasonable approach in this case would be to apply the least-squares technique to the exponential integral solution of the diffusivity equation. A somewhat similar approach was used by Earlougher and Kersh [8].

In this work, regression analysis will be applied to determine the reservoir characteristics that minimize the difference (sum of squares) between observed pressures and pressures calculated using the line-source solution. It is to be noticed that the method is applied for data points not affected by wellbore storage in the transient period. Wellbore storage-influenced data points are characterized by a slope d log Δp/d log Δt of unity and thus can be excluded either graphically before the data is entered into the program or numerically within the program itself.

THEORY

The line source solution given by Equation (1) can be written in the form

$$\Delta p = -m\,\mathrm{Ei}\left(-\frac{b}{t}\right) \tag{10}$$

where

$$m = \frac{70.6\, qB}{T} \tag{11}$$

and

$$b = 948\, \frac{sr^2}{T} \tag{12}$$

Given n values of Δp_i corresponding to the n values of t_i ($i = 1, 2, \ldots n$), it is required to determine the values m and b such that the given data best fits Equation (10). The method of least squares is based on minimizing the sum of the squares of the differences between observed and calculated pressures. In mathematical terms, it is required to minimize the function $F(m, b)$ where

$$F(m, b) = \sum_{i=1}^{n} \left[\Delta p_i + m \, \mathrm{Ei}\left(-\frac{b}{t_i} \right) \right]^2 \tag{13}$$

The condition for $F(m, b)$ to be minimum is

$$\frac{\partial F(m, b)}{\partial m} = \frac{\partial F(m, b)}{\partial b} = 0 \tag{14}$$

From the definition of the Ei-function, Equation (2) it follows that,

$$\frac{\partial}{\partial b}\mathrm{Ei}(-x) = \frac{\partial}{\partial x}\mathrm{Ei}(-x)\,\frac{\partial x}{\partial b} = \frac{e^{-x}}{b}\,\frac{\partial x}{\partial b}$$

Setting $x = \frac{b}{t_i}$ and noting that $\frac{\partial x}{\partial b} = \frac{1}{t_i}$ we get

$$\frac{\partial}{\partial b}\mathrm{Ei}\left(-\frac{b}{t_i} \right) = \frac{\exp\left(-\frac{b}{t_i} \right)}{b} \tag{15}$$

Differentiating Equation (13) with respect to m and b respectively and making use of Equation (15) we get

$$\sum_{i=1}^{n} \Delta p_i \, \mathrm{Ei}\left(-\frac{b}{t_i} \right) + m \sum_{i=1}^{n} \left[\mathrm{Ei}\left(-\frac{b}{t_i} \right) \right]^2 = 0 \tag{16}$$

and

$$\sum_{i=1}^{n} \Delta p_i \, \exp\left(-\frac{b}{t_i} \right) + m \sum_{i=1}^{n} \mathrm{Ei}\left(-\frac{b}{t_i} \right) \exp\left(-\frac{b}{t_i} \right) = 0\,. \tag{17}$$

Equations (16) and (17) are two equations in the two unknowns m and b. The difficulty arises from the implicitness of the unknown b in the argument of the Ei-function. If b is assumed either of the Equations (16) or (17) can be directly solved for m. Using Equation (16), we obtain

$$m = -\left[\sum_{i=1}^{n} \Delta p_i \, \mathrm{Ei}\left(-\frac{b}{t_i} \right) \right] \Big/ \sum_{i=1}^{n} \left[\mathrm{Ei}\left(-\frac{b}{t_i} \right) \right]^2 . \tag{18}$$

The second equation, Equation (17), is then checked and corrected for b. The iteration is continued until Equation (17) is satisfied within a prescribed tolerance.

To accelerate the convergence of the iteration process, the Newton–Raphson method is used. In this case

$$b^{k+1} = b^k - \frac{f(b_k)}{f'(b_k)} \tag{19}$$

where,

$$f(b) = \sum_{i=1}^{n} \Delta p_i \, \exp\left(-\frac{b}{t_i} \right) + m \sum_{i=1}^{n} \mathrm{Ei}\left(-\frac{b}{t_i} \right) \exp\left(-\frac{b}{t_i} \right). \tag{20}$$

In evaluating the derivative of f with respect to b it must be noted that m is also a function of b according to Equation (18), so

$$f'(b) = \frac{\partial f}{\partial b} + \frac{\partial f}{\partial m}\,\frac{\partial m}{\partial b} \tag{21}$$

From Equation (20) and (18)

$$\begin{aligned} \frac{\partial f}{\partial b} = {} & \sum \Delta p_i \, \exp\left(-\frac{b}{t_i} \right)\left(-\frac{1}{t_i} \right) \\ & + m\left\{ \sum \mathrm{Ei}\left(-\frac{b}{t_i} \right) \exp\left(-\frac{b}{t_i} \right)\left(-\frac{1}{t_i} \right) \right. \\ & \left. + \sum \exp\left(-\frac{b}{t_i} \right) \exp\left(-\frac{b}{t_i} \right)\left(\frac{1}{b} \right) \right\} \end{aligned} \tag{22}$$

$$\frac{\partial f}{\partial m} = \sum \mathrm{Ei}\left(-\frac{b}{t_i} \right) \exp\left(-\frac{b}{t_i} \right) \tag{23}$$

$$\frac{\partial m}{\partial b} = -\frac{\sum \Delta p_i \, \exp\left(-\frac{b}{t_i} \right) + 2m \sum \mathrm{Ei}\left(-\frac{b}{t_i} \right) \exp\left(-\frac{b}{t_i} \right)}{b \sum \left[\mathrm{Ei}\left(\frac{b}{t_i} \right) \right]^2} \tag{24}$$

Equations (22) to (24) are used to evaluate $f'(b_k)$ which is then used in Equation (19) to obtain the new value b^{k+1} for the $k+1$ iteration.

It can be seen that at any iteration with the known value of b, the following summations must be computed to be used in estimating m, f, and f':

$\Sigma \Delta p\, \mathrm{Ei}$, $\Sigma \Delta p \exp$, $\Sigma \mathrm{Ei} \exp$, $\Sigma \mathrm{Ei}^2$, $\sum \frac{\Delta p \exp}{t}$, $\sum \frac{\mathrm{Ei} \exp}{t}$

and Σ exp^2, with the argument $\left(-\frac{b}{t}\right)$ of the Ei and exp functions dropped.

TWO-RATE TESTS

The principle of superposition is applied to the two rate case. If the flow rate is q_1 for $0 < t < t_1$ and q_2 for $t > t_1$ then

$$\Delta p = -m \text{ Ei}\left(-\frac{b}{t}\right) \qquad 0 < t < t_1$$

$$\Delta p = -m\left[\text{Ei}\left(-\frac{b}{t}\right) + \left(\frac{q_2-q_1}{q_1}\right)\text{Ei}\left(-\frac{b}{t-t_1}\right)\right] \qquad t > t_1 \quad (25)$$

In general if n_1 points are in the range $0 < t < t_1$ and $(n-n_1)$ points in the range $t > t_1$, the function to be minimized is

$$F(b,m) = \sum_{i=1}^{n}\left[\Delta p_i + m \text{ Ei}\left(-\frac{b}{t_i}\right)\right]^2 + \sum_{i=n_1+1}^{n}\left\{\Delta p_i + m\left[\text{Ei}\left(-\frac{b}{t_i}\right) + \frac{q_2-q_1}{q_1}\text{Ei}\left(-\frac{b}{t_i-t_1}\right)\right]\right\}^2 \quad (26)$$

Differentiating F w.r.t. b and m and equating to zero we get the condition for F to be minimum.

Following the same procedure as for the constant rate case and dropping subscripts, we get

$$m = -\frac{\sum_1^n \Delta p \text{ Ei} + \Delta q \sum_{n+1}^{n} \Delta p \ \overline{\text{Ei}}}{\sum_1^n \text{Ei}^2 + 2\Delta q \sum_{n_1+1}^{n} \text{Ei}\ \overline{\text{Ei}} + \Delta q^2 \sum_{n_1+1}^{n} \overline{\text{Ei}}^2} \quad (27)$$

and

$$b^{k+1} = b^k - \frac{f}{f'} \quad (28)$$

with

$$f = \sum_1^n \Delta p \text{ exp} + \Delta q \sum_1^n \Delta P \text{ exp} + m\left\{\sum_1^n \text{Ei exp} + \Delta q\left(\sum_{n_1+1}^{n} \text{Ei}\ \overline{\text{exp}} + \sum_{n_1+1}^{n} \overline{\text{Ei}} \text{ exp}\right) + \Delta q^2 \sum_{n_1+1}^{n} \overline{\text{Ei}}\ \overline{\text{exp}}\right\} \quad (29)$$

$$f' = \frac{\partial f}{\partial b} + \frac{\partial f}{\partial m}\frac{\partial m}{\partial b} \quad (30)$$

$$\frac{\partial f}{\partial b} = -\sum_1^n \frac{\Delta p \text{ exp}}{t} - \Delta q \sum_{n_1+1} \frac{\Delta p\ \overline{\text{exp}}}{t-t_1} + m\left\{-\sum_1^n \frac{\text{Ei exp}}{t} + \frac{1}{b}\sum_1^n \text{exp}^2 + \Delta q\left(\frac{2}{b}\sum_{n_1+1}^{n} \text{exp}\ .\ \overline{\text{exp}} + \sum_{n_1+1}^{n} \frac{\overline{\text{Ei}}\text{exp}}{t} - \sum_{n_1+1}^{n} \frac{\text{Ei}\ \overline{\text{exp}}}{t-t_1}\right) + \Delta q^2\left(\frac{1}{b}\sum_{n_1+1}^{n} \overline{\text{exp}}^2 - \sum_{n_1+1}^{n} \frac{\overline{\text{Ei}}\ \overline{\text{exp}}}{t-t_1}\right)\right\} \quad (31)$$

$$\frac{\partial f}{\partial m} = \sum_1^n \text{Ei exp} + \Delta q\left(\sum_{n_1+1}^{n} \text{Ei}\ \overline{\text{exp}} + \sum_{n_1+1}^{n} \overline{\text{Ei}}\text{exp} + \Delta q^2 \sum_{n_1+1}^{n} \overline{\text{Ei}}\ \overline{\text{exp}}\right) \quad (32)$$

and

$$\frac{\partial m}{\partial b} = \left\{\sum_1 \Delta p \text{ exp} + \Delta q \sum_{n_1+1} \Delta p\ \overline{\text{exp}} + 2m\left[\sum_1 \text{Ei exp} + \Delta q\left(\sum_{n_1+1} \text{Ei}\ \overline{\text{exp}} + \sum_{n_1+1} \overline{\text{Ei}} \text{ exp}\right) + \Delta q^2 \sum_{n_1+1} \overline{\text{Ei}}\ \overline{\text{exp}}\right]\right\} / \text{D} \quad (33)$$

where,

$$D = -b\left[\sum_1 \text{Ei}^2 + 2\Delta q \sum_{n_1+1} \text{Ei}\ \overline{\text{Ei}} + \Delta q^2 \sum_{n_1+1} \overline{\text{Ei}}^2\right]; \quad (34)$$

the argument for Ei and exp is $(-b/t_i)$, and for $\overline{\text{Ei}}$ and $\overline{\text{exp}}$ is $[-b/(t_i-t_1)]$, and $\Delta q = (q_2-q_1)/q_1$.

In this case 20 summations must be evaluated, 7 of which are for all data points while the rest of the 13 summations are for points in the second range $(t > t_1)$. It can easily be seen that if $\Delta q = 0$ the case is reduced to the constant rate case, Equations (18) to (24). It is also clear that the buildup case is a special case of the two-rate case with $\Delta q = -1$.

These facts were utilized to develop a general computer program that can be used in the analysis of cases of constant rate, two-rate, including buildup, and a combination of both cases.

The Ei-function is evaluated using equations presented in the IBM Scientific Subroutine Package [11]. These equations are:

For $x > 1$

$$-x\, e^{x}\, \mathrm{Ei}(-x) = (x^4+8.5733287401\, x^3+18.0590169730\, x^2 +8.6347608925\, x+0.2677737343) / (x^4 +9.573322454\, x^3+25.6329561486\, x^2 +21.0996530827\, x+3.9584969228) \quad (35)$$

For $x < 1$

$$[-\mathrm{Ei}(-x)+\ln|x|+0.5772] / x = 9.999999\text{ E}-1 -2.500001\text{ E}-1\, x+5.555682\text{ E}-2\, x^2 -1.041576\text{ E}-2\, x^3+1.664156\text{ E}-3\, x^4 -2.335379\text{ E}-4\, x^5+2.928433\text{ E}-5\, x^6 -1.766345\text{ E}-6\, x^7 +7.122452\text{ E}-7\, x^8 \quad (36)$$

ILLUSTRATIVE EXAMPLES

Example 1: Constant Rate Interference Test

The data of this example was reported by Ramey *et al.* [9] for a gas well interference test. A total of 8 data points are used. Results of regression analysis are shown in Table 1 and a plot of the data points and the generated curve are shown in Figure 1. Using type-curve matching technique Ramey *et al.* Obtained values of 1.77 md for the permeability and 4.03×10^{-5} psi^{-1} for the ϕC_t. The results obtained by the semi-log method are given at the top of Table 1. Values of transmissibility and storage are 2072.3 md ft/cp and 3.065 ft/psi respectively. There values correspond to values of 3.5 md and 2.55×10^{-5} psi^{-1} for permeability and porosity−compressibility product respectively. Comparison of these results with those obtained by type-curve matching indicates the inadequacy of the semi-log method. This can also be verified by calculating the argument of the Ei-function at the largest time of the test. Since the value of b is about 110 h, the smallest argument of the Ei-function at the largest time of the test (504 h) is 0.22. This value is too large to allow for the logarithmic approximation of the Ei-function.

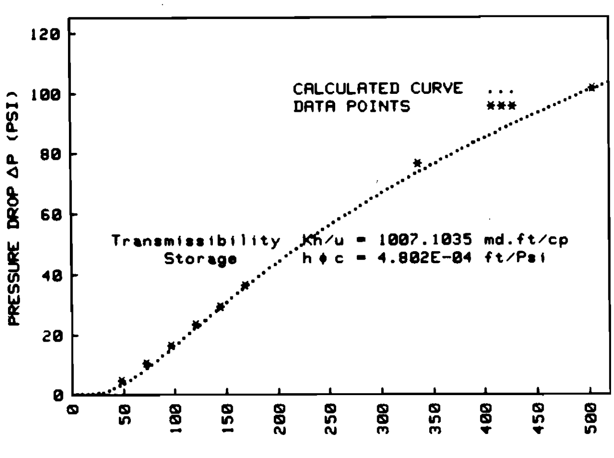

Figure 1. Regression Analysis Results: Example 1.

Table 1. Data and Regression Analysis of Example 1

Results Using Semi-log Analysis

Slope m/2.303		= 42.6993 psi
P(1h)		= −175.6634 psi
Transmissibility	Kh/u	= 2072.3017 md ft cp^{-1}
Storage	$h\phi c$	= 3.0650×10^{-4} ft psi^{-1}

Results Using Non-Linear Regression

Iteration Number	b(h)	m(psi)	f(b)
1	34.3557	36.9054	−23.1380203
2	79.3566	66.7303	−6.0948121
3	103.2402	82.7406	−1.1510298
4	110.2398	87.5087	−0.0737681
5	110.7530	87.8599	−0.0003663
6	110.7556	87.8616	−0.0000000

Final Results

Transmissibility	Kh/u	= 1007.1035 md ft cp^{-1}
Storage	$h\phi c$	= 4.8020×10^{-4} ft psi^{-1}

Point Number	Time h	P_{obs} psi	P_{calc} psi	% Error
1	48.0	3.0	2.8275	−5.750672
2	72.0	9.0	8.3032	−7.741705
3	96.0	15.0	14.9838	−0.107763
4	120.0	22.0	21.9704	−0.134727
5	144.0	28.0	28.8671	3.096874
6	168.0	35.0	35.5107	1.459284
7	336.0	75.0	73.5322	−1.957077
8	504.0	100.0	100.7142	0.714191

Standard Deviation = 0.8360

The values of transmissibility kh/μ and storage $h\phi C$ calculated using non-linear regression are 1007.1 md ft/cp and 4.802×10^{-4} ft psi^{-1} respectively. These values correspond to a permeability k of 1.704 md and $a\phi C$ of 4.0×10^{-5} psi^{-1}. These results compare very well with the results of Ramey *et al.* of 1.77 md for the permeability and 4.03×10^{-5} psi^{-1} for the porosity−compressibility product.

Comparison between the calculated and observed values of pressure drop reveals a very good agreement between the two values. The iterative procedure converges in 7 iterations with an error of less than 1×10^{-6}.

Example 2: Interference Tests During Injectivity and Falloff Periods

The data of this example was reported by Earlougher [10] for the pressure in an observation well 119 ft away from an injection well in which water was injected at a rate of 170 BPD for 48 hours. The pressure in the observation well was recorded for a total of 148 h during both the injection and shut-in periods. Results of type curve matching based on the injection were 5.1 md for the permeability and 0.11 for the porosity. The data for the shut-in period after subtraction from the expected trend was found to match well with the injectivity curve but no specific values were given.

The method of nonlinear regression was applied to the data of this example in 3 different ways. First the data for the injection period alone is analyzed, then the shut-in data is analyzed alone and finally the combined data for both injection and shut-in periods is analyzed. Results of analysis are shown in Table 2 and Figures 2–4. The analysis of the injection data alone results in a value of 228.55 md ft cp^{-1} for transmissibility and 4.829×10^{-5}ft psi^{-1} for the storage. This corresponds to values of 5.079 md for k and 0.119 for ϕ. The analysis of the shut-in data alone results in values of 5.815 md for k and 0.0794 for ϕ, while the combined data for injection and

Table 2. Data and Regression Analysis of Example 2

Results Using Semi-log Analysis

Slope m/2.303		= 32.0725 psi
P(1h)		= −2.5020 psi
Transmissibility	Kh/u	= 374.2151 md ft cp^{-1}
Storage	$h\phi c$	= 1.6921×10^{-5} ft psi^{-1}

Results Using Non-Linear Regression

Iteration Number	b(h)	m(psi)	f(b)
1	0.6070	31.7116	−39.8079174
2	1.7188	43.3173	−10.8352229
3	2.2991	47.7272	−1.4130441
4	2.3990	48.4308	−0.0304433
5	2.4013	48.4465	−0.0000146
6	2.4013	48.4465	−0.0000000

Final Results

Transmissibility	Kh/u	= 247.7373 md ft cp^{-1}
Storage	$h\phi c$	= 4.4313×10^{-5} ft psi^{-1}

Point Number	Time h	P_{obs} psi	P_{calc} psi	% Error
1	4.3	22.0	23.9625	8.920645
2	21.6	82.0	83.6978	2.070457
3	28.2	95.0	95.4144	0.436178
4	45.0	119.0	116.5674	−2.044231
5	51.0	109.0	107.2987	−1.560808
6	69.0	55.0	53.9173	−1.968469
7	73.0	47.0	48.9511	4.151323
8	93.0	32.0	33.8609	5.815195
9	142.0	16.0	19.5718	22.323801
10	148.0	15.0	18.6195	24.130228

Standard Deviation = 2.5005

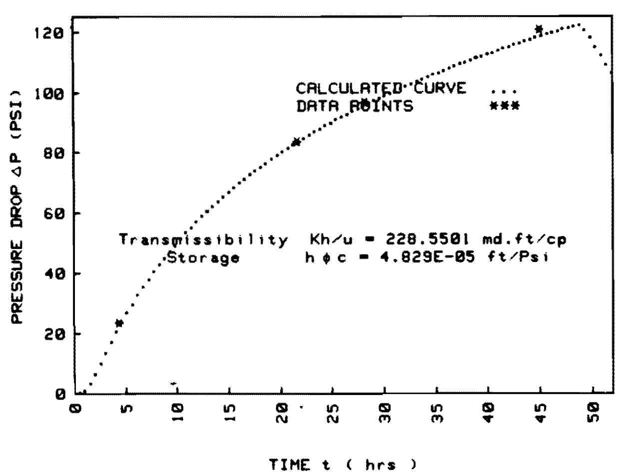

Figure 2. Regression Analysis for Injection Period: Example 2.

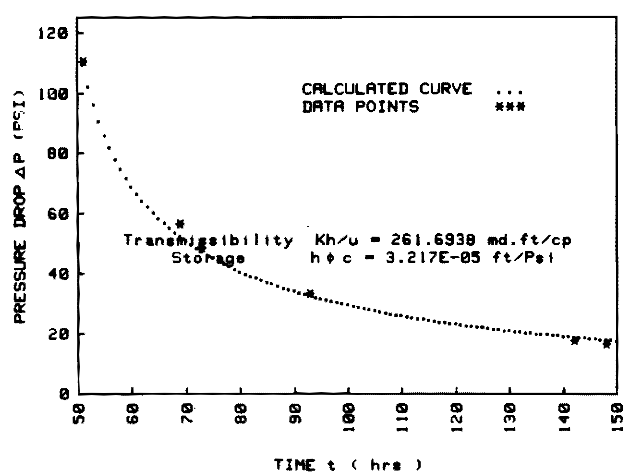

Figure 3. Regression Analysis for Shut-In Period: Example 2.

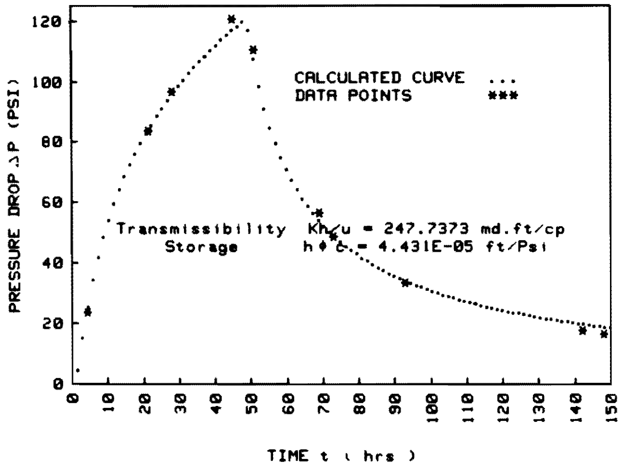

Figure 4. Regression Analysis for Combined Period: Example 2.

shut-in periods gives a value of 5.505 md for k and 0.1094 for ϕ. The closeness of the results of this method to those of type curve matching is clear, except for the value of ϕ for the shut-in data alone. It is apparent that the use of injection and shut-in data combined gives the best representative results.

In all three cases of the analysis described in this example, the use of the semi-log approach resulted in erroneous values for m and b indicating the inadequacy of the logarithmic approximation.

Example 3: Simulated Drawdown and Buildup Example

In this example data is generated using the line-source solution for an observation well located 250 ft from an active well producing at a rate of 500 res bbls/day for 500 h after which it was shut-in. The pressure at the well was calculated during the drawdown and buildup periods using values of 15 psi and 5 h for m and b respectively. The data for the test and the results of the analysis are shown in Table 3. A plot of observed and calculated pressures are shown in Figure 5. The excellent agreement between the two pressures is clear.

The value of m obtained from the analysis of the drawdown period, the buildup period and the combined period were 14.9353, 15.053, and 15.036 psi respectively. Values of b were 5.0497, 5.1541, and 5.1032 h for the three cases respectively.

It is also noted that the convergence of the iterative scheme is fast, requiring between 5 and 7 iterations for an allowable error of $<1\times10^{-6}$. If this allowable error is relaxed to <0.001 the number of iterations decreases by 2–3 iterations without any significant loss in accuracy.

Table 3. Data and Regression Analysis of Example 3

Results Using Semi-log Analysis

Slope m/2.303		= 9.7234 psi
P(1h)		= 1.1720 psi
Transmissibility	Kh/u	= 3630.4045 md ft cp^{-1}
Storage	$h\phi c$	= 3.0496 ft psi^{-1}

Results Using Non-Linear Regression

Iteration Number	b(h)	m(psi)	f(b)
1	0.4977	9.7837	−68.6124581
2	2.2528	12.9884	−26.0760163
3	3.9921	14.3862	−7.2656368
4	4.9293	14.9427	−0.9917248
5	5.1012	15.0360	−0.0249243
6	5.1057	15.0385	−0.0000166
7	5.1057	15.0385	−0.0000000

Final Results

Transmissibility	Kh/u	= 2347.3144 md ft cp^{-1}
Storage	$h\phi c$	= 2.0227×10^{-4} ft psi^{-1}

Point Number	Time h	P_{obs} psi	P_{calc} psi	% Error
1	5.0	3.4	3.1847	−6.333300
2	10.0	7.9	8.2283	4.155929
3	20.0	15.6	15.4600	−0.897186
4	30.0	20.6	20.4048	−0.947755
5	50.0	27.0	27.1296	0.480042
6	500.0	60.4	60.4128	0.021218
7	505.0	57.2	57.3763	0.308169
8	510.0	52.8	52.4793	−0.607382
9	520.0	45.4	45.5367	0.301143
10	530.0	40.7	40.8757	0.431642
11	550.0	34.9	34.7026	−0.565530

Standard Deviation = 0.2239

CONCLUSIONS

A technique based on nonlinear regression using the method of least squares is introduced for interference well test analysis. The resulting equations are solved efficiently by an iterative procedure based on the Newton–Raphson method.

The developed method is applicable to analysis of interference tests in cases of constant rate and two-rate tests, including buildup and in combinations of the two cases.

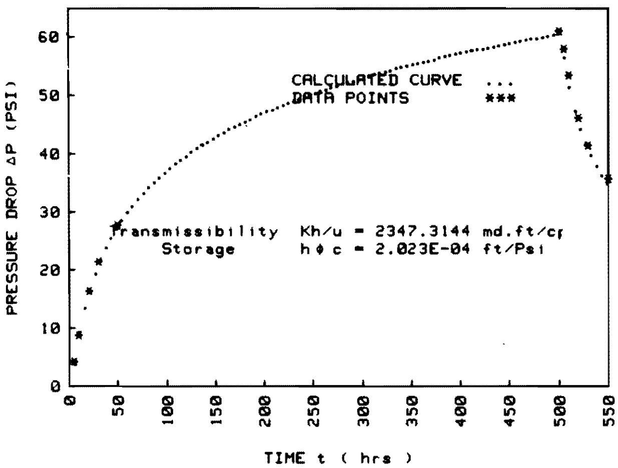

Figure 5. Regression Analysis for Combined Period: Example 3.

Application of the method to field and simulated examples showed a good agreement with results obtained by type-curve matching in all cases. The method has the advantage of being suitable for systematic computer-based analysis.

ACKNOWLEDGEMENT

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