MAGNETOHYDRODYNAMIC FREE CONVECTION FLOW OF A VISCOUS INCOMPRESSIBLE FLUID DUE TO UNSTEADY MOTION OF A HOT VERTICAL PLATE WITH VARIABLE SUCTION

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الخلاصـــة :

هذا البحث يتعلق بدراسة التدفق الحر الغير مستقر لسائل لزج موصل للكهرباء وغير قابل للانضغاط من خلال صفيحة رأسية ساخنة متغيرة المص وفي وجود مجال مغناطيسي مستعرض حيث تتغير سرعة المص تغيراً رأسيا . وتتميز ظواهر السائل بالأعداد الغير بعدية الآتية :

عدد براندل (P) وعدد غراشوف (G) والعدد المغناطيسي (M) والوسيط الأسسي (n) . هذا وقد توصلنا إلى حلول تقريبية للسرعة وتوزيع الحرارة والاحتكاك السطحي ومثلنا السرعة وتوزيع الحرارة تمثيلاً بيانيا . تبعت ذلك مناقشة تفصيلية تقارن تأثير الوسيط المغناطيسي ووسيط المص على مجالات السرعة والحرارة . وفي النهاية قارنا نتائج هذا البحث مع نتائج بحوث سابقة حول نفس الموضوع .

ABSTRACT

This paper studies the unsteady free convection flow of an incompressible electrically conducting viscous fluid past a hot vertical plate with variable suction in the presence of a transverse magnetic field. The suction velocity is assumed to vary exponentially with time. The flow phenomena are characterized by the non-dimensional numbers, P (Prandtl number), G (Grashoff number), M (Magnetic number), and n (exponential parameter). Approximate solutions for velocity, temperature distributions, and skin friction are obtained. The velocity and temperature distributions are represented graphically. There follows a detailed comparative discussion of the effects of magnetic and suction parameters on the velocity and temperature fields and a comparison of the present work with the previous works on the subject.

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1. INTRODUCTION

Following work by Lighthill [1] and Stuart [2], the oscillatory flow of an incompressible viscous fluid past an infinite plate with variable suction has been studied by Messiha [3]. In their analyses, Stuart and Messiha assumed that the plate is stationary and the free stream oscillates in magnitude about a nonzero constant. Soundalgekar [4, 5] has studied oscillatory flow with and without a magnetic field past an infinite vertical plate with variable suction. He observed a reverse type of flow when the plate moves in a direction opposite to that of the flow. The study of exponential flow with suction was initiated by Pandey [6, 7]. He observed that the velocity field is increased by increasing the suction velocity and that there is no back flow near the wall either in exponentially increasing or in decreasing small perturbation cases. Hydromagnetic flow of a viscous incompressible fluid due to unsteady motion of a plate with suction was investigated by Pandey [8]. He observed that the velocity decreases with increase in the Hartmann number, and that the velocity profile also decreases with increase in the suction parameter; there is no back flow near the wall.

The aim of the present paper is to study the unsteady free convection flow with variable suction of an incompressible, electrically conducting viscous fluid over a hot vertical plate moving exponentially with time in its own plane. The temperature is assumed to vary exponentially with time.

The assumption that the suction velocity varies exponentially with time is taken because we wish to remove the retarded fluid from the boundary layer as quickly as possible, thereby reducing the separation of the flow. The effect of the transverse magnetic field on the velocity profile and that of the suction parameter on the temperature field has been studied in both exponentially increasing and decreasing cases. It has been observed that the velocity profile becomes lower with the increase in magnetic parameter M — and there is no back flow near the wall. The conclusions have been discussed in conjunction with previous works on the subject.

The study of exponential flow could be relevant to the treatment of cardiovascular diseases since cardiovascular flow in the human body often follows exponentially decreasing or increasing profiles. Exponential flow of a Newtonian fluid will first be studied, in order to arrive at an understanding of the proper mechanism, although blood flow itself shows some non-Newtonian properties.

2. FORMULATION OF THE PROBLEM

For the present investigation, the equation of motion, the energy equation, and the equation of continuity [4] are taken as:

$$\frac{\partial u^*}{\partial t^*}+u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=\nu^*\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right)+g_x^*\beta(T^*-T_\infty^*)-\frac{\sigma^*}{\rho^*}B_0^* u^*, \tag{1}$$

$$\frac{\partial v^*}{\partial x^*}+u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=-\frac{1}{\rho^*}\frac{\partial p^*}{\partial y^*}+\nu^*\left(\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{\partial^2 v^*}{\partial y^{*2}}\right) \tag{2}$$

$$\frac{\partial T^*}{\partial t^*}+u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}\frac{K^*}{\rho^* c_p^*}\left(\frac{\partial^2 T^*}{\partial x^{*2}}+\frac{\partial^2 T^*}{\partial y^{*2}}\right) \tag{3}$$

and

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0 \tag{4}$$

where the x^*-axis is assumed to be along the vertical infinite plate in the upward direction, and the y^*-axis is taken perpendicular to the plate and the origin of the coordinate system is assumed to be at the lowest point of the plate.

In these equations, u^* is the velocity in the x^*-direction, v^* the velocity normal to the plate in the y^*-direction, t^* the time variable, ν^* the kinematic viscosity, ρ^* the density, B_0^* the external magnetic field, σ^* the electrical conductivity of the field, P^* the static pressure, g_x^* the acceleration due to gravity, β the coefficient of volume expansion, $\perp_p^*$ the specific heat at constant pressure, K^* the thermal conductivity, T^* the temperature in the boundary layer, and T_∞^* the temperature far away from the plate. Assuming all the physical quantities to be independent of x^*, the

Equations (1)–(4) reduce to

$$\frac{\partial u^*}{\partial t^*}+v^*\frac{\partial u^*}{\partial y^*}=v^*\frac{\partial^2 u^*}{\partial y^{*2}}+g_x^*\beta(T^*-T_\infty^*)-\frac{\sigma^*}{\rho^*}B_0^{*2}u^*, \quad (5)$$

$$\frac{\partial v^*}{\partial t^*}+v^*\frac{\partial v^*}{\partial y^*}=-\frac{P^*}{y^*}+v^*\frac{\partial^2 v^*}{\partial y^{*2}}, \quad (6)$$

$$\frac{\partial T^*}{\partial t^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{K}{\rho^* c_p^*}\frac{\partial^2 T^*}{\partial y^{*2}} \quad (7)$$

and

$$\frac{\partial v^*}{\partial y^*}=0, \quad (8)$$

subject to the boundary conditions

$$u=v_0\left(a_0+\varepsilon\sum_{n^*=1}^{\infty} C_{n^*}\exp(\pm n^*t^*)\right),$$

$$T=T_m^*\left(a_0+\varepsilon\sum_{n^*=1}^{\infty} C_{n^*}\exp(\pm n^*t^*)\right)$$

$$-\varepsilon T_\infty^*\sum_{n^*=1}^{\infty} C_{n^*}\exp(\pm n^*t^*) \text{ at } y^*=0$$

and

$$u^*\to 0,\ T^*\to T_\infty^* \text{ as } y^*\to\infty, \quad (9)$$

where T_m^* is the mean value about which the temperature fluctuates, $\varepsilon\leq 1$, n^* is the exponential parameter, and a_0 is a positive real constant. The series under summation is taken such that C_{n^*} decreases as n^* increases, so that the convergence of the series is maintained.

We assume that the suction velocity varies exponentially with time, so that:

$$v^*=-v_0^*\left(a_0+\varepsilon A\sum_{n^*=1}^{\infty} C_{n^*}\exp(n^*t^*)\right) \quad (10)$$

where v_0^* is a constant mean suction velocity and a_0 and A are positive, real constants.

From Equation (10), Equations (5) and (7) become:

$$\frac{\partial u^*}{\partial t^*}-v_0\left(a_0+\varepsilon A\sum_{n=1}^{\infty} C_{n^*}\exp(n^*t^*)\right)\frac{\partial u^*}{\partial y^*}$$

$$=v^*\frac{\partial^2 u^*}{\partial y^{*2}}+g_x^*\beta(T^*-T_\infty^*)-\frac{\sigma^*}{\rho^*}B_0^{*2}u^*, \quad (11)$$

and

$$\frac{\partial T^*}{\partial t^*}-v_0^*\left(a_0=\varepsilon A\sum_{n^*=1}^{\infty} C_{n^*}\exp(n^*t^*)\right)\frac{\partial T^*}{\partial y^*}$$

$$=\frac{K^*}{\rho^* C_p^*}\frac{\partial^2 T^*}{\partial y^{*2}}. \quad (12)$$

Introducing the nondimensional variables,

$$y=\frac{y^*v_0^*}{v^*},\ t=\frac{v_0^{*2}t^*}{4v^*},\ n=\frac{4v^*n^*}{v_0^{*2}},\ u=\frac{u^*}{v_0^*},$$

$$a_n=\frac{C_n^*v_0^{*2}}{4v^*},\ \theta=\frac{T^*-T_\infty^*}{T_m^*-T_\infty^*},\ P=\frac{\eta C_p^*}{K^*},$$

$$G=vg_x^*\beta\left(\frac{T_m^*-T_\infty^*}{v^{*3}}\right),\ M=\frac{4v^*\sigma^*B_0^{*2}}{v_0^{*2}\rho^*}. \quad (13)$$

Equations (11) and (12) become:

$$\frac{\partial^2 u}{\partial y^2}+\left(a_0+\varepsilon A\sum_{n=1}^{\infty} a_n\exp(nt)\right)\frac{\partial u}{\partial y}-\frac{1}{4}\frac{\partial u}{\partial y}-\frac{1}{4}M=-G\theta \quad (14)$$

and

$$\frac{\partial^2\theta}{\partial y^2}+P\left(a_0+\varepsilon A\sum_{n=1}^{\infty} a_n\exp(nt)\right)\frac{\partial\theta}{\partial y}-\frac{1}{4}P\frac{\partial\theta}{\partial t}=0. \quad (15)$$

Boundary conditions reduce to:

$$u=a_0+\varepsilon\sum_{n=1}^{\infty} a_n\exp(nt),$$

$$\theta=a_0+\varepsilon\sum_{n=1}^{\infty} a_n\exp(\pm nt), \text{ at } y=0 \quad (16)$$

and

$u\to 0,\ \theta\to 0$ as $y\to\infty$.

3. SOLUTION OF THE PROBLEM

We solve Equations (14) and (15) subject to the boundary conditions (16).

Suppose

$$u(y,t)=u_1(y)+\varepsilon\sum_{n=1}^{\infty} a_n\exp(\pm nt)u_2(y), \quad (17)$$

and

$$(y,t)=a_0+\varepsilon\sum_{n=1}^{\infty} a_n\exp(\pm nt)-\theta_1(y)$$

$$-\varepsilon\sum^{\infty} a_n\exp(\pm nt)\theta_2(y) \quad (18)$$

Then, substituting Equations (17) and (18) into Equations (14) and (15), comparing the harmonic terms, and neglecting the coefficient of ε^2, we obtain:

$$u_1''+a_0u_1'-\tfrac{1}{4}Mu_1=-Ga_0+G\theta_1, \quad (19)$$

$$u_2''+a_0u_2'-\tfrac{1}{4}(M\pm n)u_2+Au_1'=G\theta_2-G, \quad (20)$$

$$\theta_1''+Pa_0\theta_1'=0 \quad (21)$$

and

$$\theta_2'' + Pa_0\theta_2' \pm \tfrac{1}{4}nP\theta_2 + PA\theta_1' \pm \tfrac{1}{4}nP = 0 \quad (22)$$

where primes denote differentiation with respect to y. The boundary conditions on u_1, u_2, θ_1, and θ_2 are:

$$u_1 = a_0,\ u_2 = 1,\ \theta_1 = \theta_2 = 0 \quad \text{at } y = 0$$
$$u_1 \to 0,\ \theta_1 \to 1,\ \theta_2 \to 1 \quad \text{as } y \to \infty. \quad (23)$$

Solving Equations (21) and (22) with the conditions of (23), we obtain for exponentially increasing (decreasing) cases:

$$\theta_1 = 1 - \exp(-Pa_0y) \quad (24)$$

and

$$\theta_2 = 1 \pm \frac{4Pa_0y}{n}\exp(-Pa_0y) - \left(1 \pm \frac{4Pa_0A}{n}\right)\exp(-Pa_0dy) \quad (25)$$

Also solving Equations (12) and (20), using Equations (23), (24), and (25), we obtain for both the cases:

$$u_1 = 4G\left(\frac{1-a_0}{M}\right)\exp(-a_0Ky) + \left(a_0 + \frac{G}{A_1}\right)\exp(-a_0K_1y) - 4G\left(\frac{1-a_0}{M}\right) - \frac{G}{A_1}\exp(-Pa_0y) \quad (26)$$

and

$$u_2 = \exp(-Sa_0y) \pm \frac{B_1}{B_2}[\exp(-Pa_0y) - \exp(-Sa_0y)] - \frac{G}{B_3}[\exp(-Pa_0dy) - \exp(-Sa_0y)] \pm \frac{B_1}{B_3}[\exp(-Pa_0dy) - \exp(-Sa_0y)] + B_4[\exp(-a_0Ky) - \exp(-Sa_0y)] + B_5[\exp(-a_0K_1y) - \exp(-Sa_0y)] - \frac{B_6}{A_1B_2}[\exp(-Pa_0y) - \exp(-Sa_0y)] \quad (27)$$

where

$$d = \tfrac{1}{2}\left\{1 + \left(1 \pm \frac{n}{Pa_0^2}\right)^{1/2}\right\},\quad K = \tfrac{1}{2}\left\{1 - \left(1 - \frac{M}{a_0^2}\right)^{1/2}\right\}$$

$$K_1 = \tfrac{1}{2}\left\{1 + \left(1 + \frac{M}{a_0^2}\right)^{1/2}\right\},\quad A_1 = \left(P^2a_0^2 - Pa_0^2 - \frac{M}{4}\right),$$

$$S = \tfrac{1}{2}\left\{1 + \left(1 + \frac{M+n}{a_0^2}\right)^{1/2}\right\},\quad B_1 = \frac{4Pa_0GA}{n},$$

$$B_2 = \left(P^2a_0^2 - Pa_0^2 - \frac{M \pm n}{4}\right),$$

$$B_3 = \left(P^2a_0^2d^2 - Pa_0^2d - \frac{M+n}{4}\right),$$

$$B_4 = \frac{4GAa_0K\left(1 - \frac{a_0}{M}\right)}{a_0^2K^2 - a_0^2K - \left(\frac{M+n}{4}\right)},$$

$$B_5 = \frac{Aa_0K_1\left(a_0 + \frac{G}{A}\right)}{a_0^2K_1^2 - a_0^2K_1 - \left(\frac{M+n}{a_0^2}\right)}$$

and

$$B_6 = Pa_0AG.$$

Hence from Equations (17) and (18), the velocity field $u(y, t)$ and temperature distribution $\theta(y, t)$ are respectively given by:

$$u(y, t) = 4G\left(\frac{1-a_0}{M}\right)\exp(-a_0Ky) + \left(a_0 + \frac{G}{A_1}\right)\exp(-a_0K_1y) - 4G\left(\frac{1-a_0}{M}\right) - \frac{G}{A_1}\exp(-Pa_0y) + \varepsilon\sum_{n=1}^{\infty} a_n\exp(\pm nt)\{\exp(-Sa_0y) \pm \frac{B_1}{B_2}\{\exp(-Pa_0y) - \exp(-Sa_0y)\} = \frac{G}{B_3}\{\exp(-Pa_0dy) - \exp(Sa_0y)\} \pm \frac{B_1}{B_3}\{\exp(-Pa_0dy) - \exp(-Sa_0y)\} + B_4\{\exp(-a_0Ky) - \exp(-Sa_0y)\} + B_5\{\exp(-a_0K_1y) - \exp(-Sa_0y)\} - \frac{B_6}{A_1B_2}\{\exp(-Pa_0y) - \exp(-Sa_0y)\}\} \quad (28)$$

and

$$\theta(y, t) = a_0 + \varepsilon\sum_{n=1}^{\infty} a_n\exp(\pm nt)$$

$$-\{1-\exp(-Pa_0A)\}-\varepsilon\sum_{n=1}^{\infty} a_n\exp(\pm nt)$$
$$\left\{1\pm\frac{4Pa_0A}{n}\exp(-Pa_0 y)\right.$$
$$\left.-\left(1\pm\frac{4Pa_0A}{n}\right)\exp(-Pa_0dy)\right\} \tag{29}$$

The nondimensional skin-friction τ_0 is given by:

$$\tau_0=\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$\tau_0=B_9-B_8-B_7+\varepsilon\sum_{n=1}^{\infty} a_n\exp(\pm nt)$$
$$\left(\pm\frac{B_1}{B_2}(Sa_0-Pa_0)-\frac{G}{B_3}(Sa_0-Pa_0d)\pm\right.$$
$$\pm\frac{B_1}{B_3}(Sa_0-Pa_0d)+B_4(Sa_0-a_0K)$$
$$\left.+B_5(Sa_0-a_0K_1)-\frac{B_6}{A_1B_2}(Sa_0-Pa_0)-Sa_0\right) \tag{30}$$

where

$$B_9=G\left(\frac{Pa_0}{A_1}-\frac{4}{M}(1-a_0)\right),\ B_8=4G\left(\frac{1-a_0}{M}\right)a_0K$$

and

$$B_7=a_0K\left(a_0+\frac{G}{A_1}\right)$$

where the upper and lower signs in $\mp$ or $\pm$ refer to exponentially increasing and decreasing cases respectively.

4. GRAPHICAL INTERPRETATIONS AND CONCLUSION

Figures 1 and 2 are obtained from Equation (28), and show the relation of the velocity distribution u against y. Here, we have taken $a_0=1$, $\varepsilon=0.1$, $n=1$, $t=1$, $a_n=0.5$, $A=1$, $P=5$, and $G=10$. We have plotted the velocity distributions u against y for $M=0$, 2, and 4 for exponentially increasing and decreasing cases in Figures 1 and 2, respectively. It is noted that the velocity u decreases with increase in magnetic parameter M in both exponentially increasing and decreasing cases. A transverse magnetic field reduces the boundary layer thickness and there is no back flow near the wall in either case.

Figures 3 and 4 have been obtained by using Equation (29) and show the temperature distribution plotted against y. Here we have taken $a_0=1$, $\varepsilon=0.1$, $n=1$, $t=1$, $a_n=0.5$, $P=5$, and $G=10$. We have plotted the temperature distribution θ against y for $A=0$, 1, and 2 for exponentially increasing and decreasing cases, respectively. It is noted that the temperature θ decreases with increase in suction parameter A in both cases. Also there is no effect of the magnetic parameter M on the temperature field, probably due to neglecting the induced magnetic field. It is obvious from the figures that the boundary layer contracts with increasing suction velocity and that there is no back flow near the wall.

Equation (30) determines the skin friction amplitude for exponential flow and the authors propose to discuss the effects of the transverse magnetic field and suction parameter on skin friction in a subsequent paper.

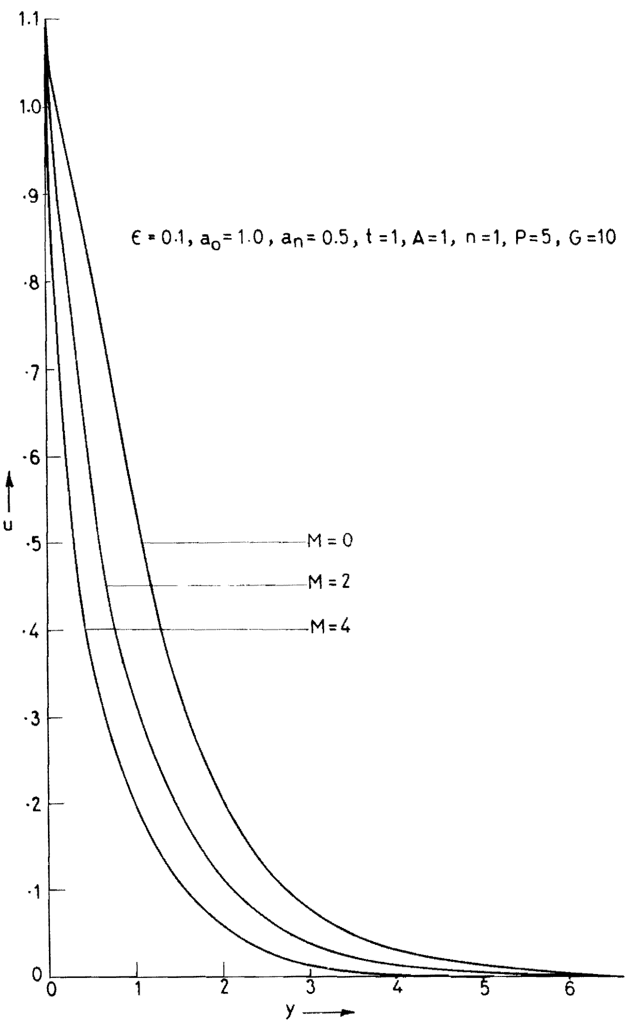

Figure 1. Exponentially Increasing Case: Velocity u against y.

5. COMPARISON WITH RELEVANT PREVIOUS WORK

The expression for the velocity field given by (28) reduces to that of Kishore and Pandey [8] for non-convective flow and the graphic conclusions remain almost the same. The free convection effect was not considered earlier for exponential flow, though it has been considered by Soundalgekar [4] for oscillating MHD flow past an infinite vertical flat plate with variable suction. He has studied the effect of the magnetic field on the velocity profile and concluded that the velocity field decreases with increase in M. The shapes of his velocity profiles are entirely different from our's but the conclusion is of a similar nature.

Comparing our temperature profile with those of Mishra and Mohapatra [9], we observe that the shapes of the temperature profiles are entirely different but the suction parameter has a similar effect on the temperature field. That is, a decrease in temperature profile is observed with increase in suction parameter A for both exponential and oscillatory flow.

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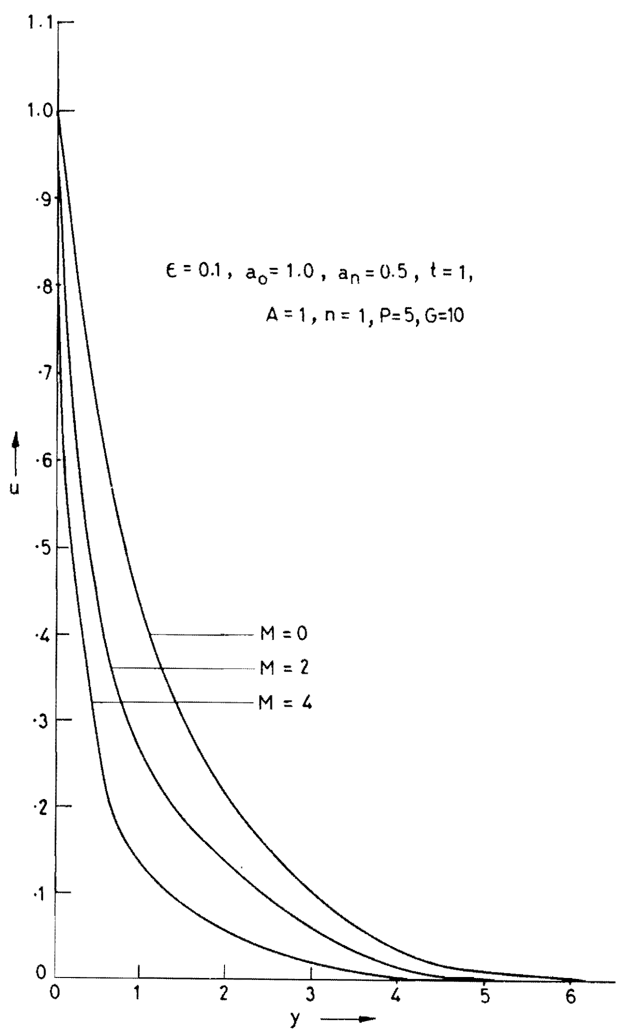

Figure 2. Exponentially Decreasing Case: Velocity u against y.

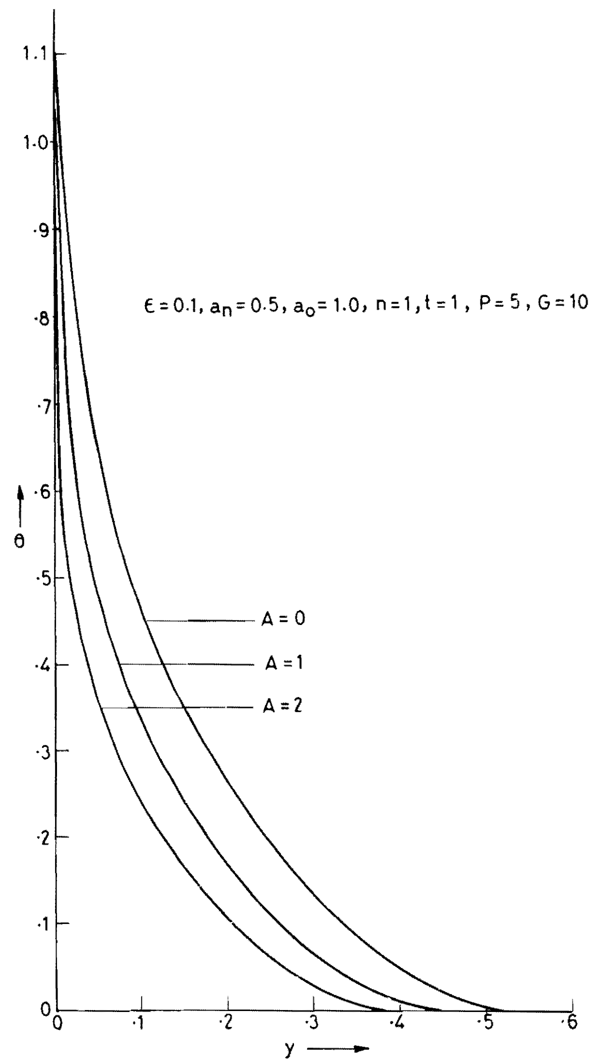

Figure 3. Exponentially Increasing Case: Temperature θ against y.

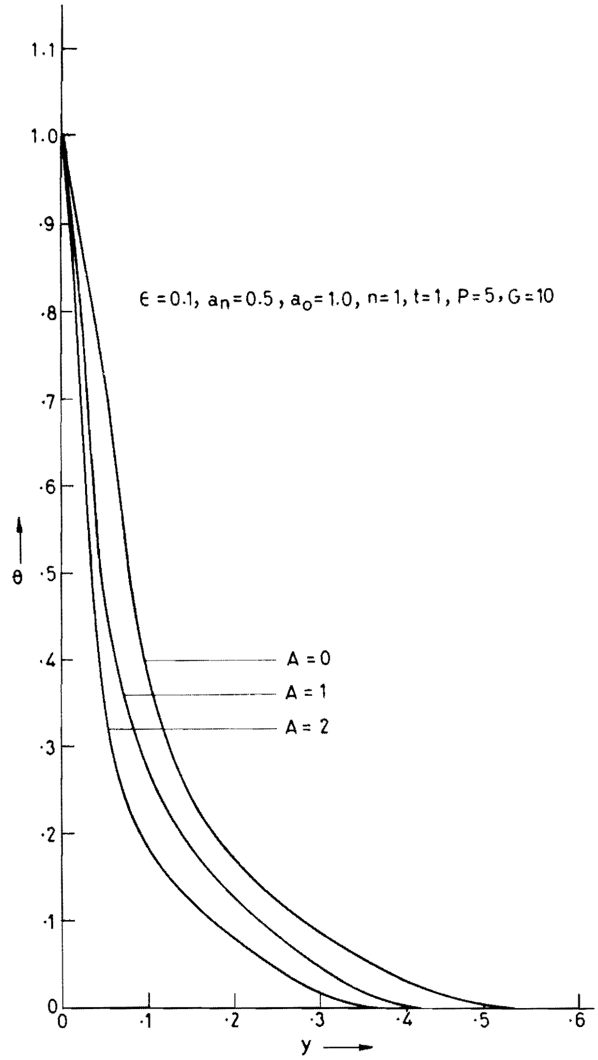

Figure 4. Exponentially Decreasing Case: Temperature θ against y.

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