EFFECT OF FINITE BANDWIDTH ON THE TWO-PLASMON DECAY

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الخلاصـة :

قمنا بدراسة ظاهرة اضمحلال أشعة الليزر الساقطة والتي تحوى مجموعة متدرجة التردد (حزمة) إلى موجتيْن بلازميتيْن. وقد استخدمنا في دراستنا نظرية الموائع وحصلنا على معادلات تحليلية لمعدلات النمو وعتبات الشدة في حالات بلازما متجانسة وبلازما غير متجانسة ذات كثافة متزايدة خطياً. أظهرت هذه المعادلات أنّ عرض حزمة أشعة الليزر يؤثر تأثيرا واضحا على معدلات النمو وعتبات الشدة لهذه الظاهرة.

ABSTRACT

A study is made of the two-plasmon decay when the pump laser beam has a finite bandwidth. Using the fluid theory, analytical expressions for the growth rates and threshold powers for homogeneous plasma and inhomogeneous plasma with linear ramp density were obtained; it has been shown that the bandwidth in the laser beam can significantly affect the growth rates and threshold powers for the two-plasmon decay process.

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1. INTRODUCTION

The decay of an electromagnetic wave into two electron plasma waves (plasmons) has been observed both experimentally [1–3] and in simulations [4]. Recently this instability has been widely investigated theoretically [5–8].

The two-plasmon decay (TPD) is a member of a family of parametric instabilities that occur in an undercritical plasma. They all satisfy the frequency and wave number matching conditions:

$$\omega_0 = \omega_1 + \omega_2 \ , \qquad \mathbf{k}_0 = \mathbf{k}_1 + \mathbf{k}_2$$

where the incident electromagnetic wave $(\omega_0, \mathbf{k}_0)$ decays into two plasmons (ω_1, $\mathbf{k}_1$ and (ω_2, $\mathbf{k}_2$). Since both plasmons are generated at the same location, they are nearly equal in frequency [9] and $\omega_0 = \omega_1 + \omega_2 = 2\omega_{1,2}$; hence this process occurs at densities near quarter critical density $(n_c/4)$, where $n_c = m\omega_0^2/4\pi e^2$, and it is inherently a two (or three) dimensional process.

The TPD instability is absorptive in nature and therefore it is of great significance in high intensity laser-pellet interaction because the electron plasma waves associated with this instability may have phase velocities of the order of the speed of light and therefore produce very energetic electrons when they damp [3, 10, 11]. These electrons cause preheating of the pellet core and thus prevent efficient compression which is necessary for the production of useful amounts of fusion energy [12]. Thus it is natural to try to control or modify the TPD instability. As the power of a laser with bandwidth $\Delta\omega_0$ is dispersed, it is expected that the use of a laser with finite bandwidth would lower the growth rates. Any laser has a finite natural bandwidth due to the finite time of pulse duration, but this is usually neglected because the fractional frequency width is small compared to the laser frequency. An increase in the frequency bandwidth can be achieved artificially by disturbing the coherence of the driving laser [13,14].

Thomson [15] considered the effect of a finite bandwidth on the parametric instability in an inhomogeneous plasma. He concluded that the bandwidth affects the threshold for convective instabilities. His results showed that for large bandwidth, for which the interaction length is much greater than the correlation length for the pump wave, the intensity threshold is proportional to $(\Delta\omega_0)^2$.

Berger [16] solved the linearized coupled mode equations in two dimensions numerically. His results showed that, for plasma lengths only a few times threshold, a surprisingly small temporal bandwidth that is substantially less than the growth rate is effective in suppressing absolute growth. Only if the plasma length greatly exceeds that threshold is the necessary temporal bandwidth the larger value obtained from homogeneous plasma theory.

Guzdar *et al.* [17] studied the effect of bandwidth on the convective amplification of Raman instability in the underdense, inhomogeneous plasma. They concluded that for the case when the homogeneous growth rate $\gamma_0 \ll \Delta\omega_0$ there is no effect of bandwidth on the convective amplification. Their results showed that for $\gamma_0 \gg \Delta\omega_0$ there is a statistical enhancement in the amplification factor.

Recently, we investigated the effect of bandwidth on Raman backscattering and an expression for convective amplification was derived, where the effects of collision and phase mismatch were taken into consideration [18]. We found that if we neglect the collision frequency, the dependence of the amplification factor on the bandwidth disappears. The growth rate was also investigated and found to decrease with bandwidth while the threshold increases with the bandwidth.

In this paper we discuss in detail the effects of pump bandwidth on the TPD instability both in a homogeneous plasma and in an inhomogeneous plasma. In Section 2, we obtain the slow coupling equations for the waves involved in the TPD. In Section 3 we consider the effect of pump bandwidth on the homogeneous growth rate and threshold intensity of the TPD. In Section 4 we consider the effect of pump bandwidth on the convective TPD in an inhomogeneous plasma with linear density profile. Finally in Section 5 we present our conclusions.

2. THE SLOW COUPLING EQUATIONS

Let the electron total density n_e be composed of two parts such that:

$$n_e = n_e(x) + \tilde{n}_e(x,t), \tag{1}$$

where $n_e(x)$ is the slowly varying density due to the inhomogeneity and is given by:

$$n_e(x) = n_0\left(1 + \frac{x}{L}\right), \tag{2}$$

where L is the inhomogeneity scale length and $\tilde{n}_e(x,t)$ is the fast harmonically varying density due to the electrons' fast response to the high frequency fields. Because of the ions' inertia, they will be considered immobile and their density is denoted by $n_i(x) = n_e(x)$; the motion of the electrons is then governed by the following equations:

(*a*) The electron equation of motion:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}.\nabla)\mathbf{v} = -\frac{e}{m}\mathbf{E} - \frac{e}{mc}\mathbf{v}\times\mathbf{B} - \frac{3T}{mn_e}\nabla n_e - v\mathbf{v}. \tag{3}$$

(*b*) Poisson's equation:

$$\nabla.\mathbf{E} = -4\pi e \sim n_e(x,t). \tag{4}$$

(*c*) Faraday's law:

$$\nabla\times\mathbf{E} = -\frac{\mathbf{1}}{\mathbf{c}}\frac{\partial\mathbf{B}}{\partial\mathbf{t}}. \tag{5}$$

(*d*) Ampere's law:

$$\nabla\times\mathbf{B} = -\frac{4\pi}{c}n_e e\mathbf{v} + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}, \tag{6}$$

where $\mathbf{v}$ is the electron velocity, T is the electron temperature in energy units, v is the electron ion collision frequency, and $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields of the waves involved in the three wave process.

Taking the curl of Equation (5), and using Equations (1), (4), and (5), we obtain the following equation:

$$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} - \nabla(\nabla.\mathbf{E}) - \frac{4\pi n_e(x)}{mc^2}\mathbf{E} - \frac{4\pi n_e(x)}{c^2}ev\mathbf{v} + \frac{3T}{mc^2}\nabla(\nabla.\mathbf{E})$$
$$= \frac{1}{c^2}\frac{\partial}{\partial\tau}[\mathbf{v}(\nabla.\mathbf{E})] + \frac{4\pi n_e(x)e}{c^2}\left[\frac{1}{2}\nabla(\mathbf{v}.\mathbf{v}) - \mathbf{v}\times\left(\nabla\times\mathbf{v} - \frac{\mathbf{eB}}{\mathbf{mc}}\right)\right]. \tag{7}$$

This equation is the generalized inhomogeneous wave equation. From the linear theory of waves we have $\nabla \times \mathbf{v} = e\mathbf{B}/mc$, then Equation (7) transforms to:

$$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} - \nabla(\nabla.\mathbf{E}) - \frac{4\pi n_e(x)e^2}{mc^2}\mathbf{E} - \frac{4\pi n_e(x)e}{c^2}\,v\mathbf{v} + \frac{eT}{mc^2}\nabla(\nabla.\mathbf{E})$$

$$= \frac{1}{c^2}\frac{\partial}{\partial t}\left[\mathbf{v}(\nabla.E)\right] + \frac{4\pi n_e(x)e}{c^2}\nabla(\mathbf{v}.\mathbf{v}). \tag{8}$$

We represent the total electric field as a superposition of the fields associated with the three interacting waves:

$$\mathbf{E}(x,t) = \widehat{\mathbf{y}}E_0(x,t)\exp\{i(k_0x - \omega_0 t)\}$$

$$+(\widehat{\mathbf{x}}+\widehat{\mathbf{y}})E_1(x,t)\exp\{i(k_{1x}x + k_{1y}y - \omega_1 t)\}$$

$$+(\widehat{\mathbf{x}}+\widehat{\mathbf{y}})E_2(x,t)\exp\{i(k_{2x}x + k_{2y}y - \omega_2 t)\} + c.c. \tag{9}$$

The indices 0, 1, 2 refer to the pump wave (usually laser) and the two plasmons respectively. We assume that the variation in the amplitudes of the interacting waves is along the plasma density gradient (in the x-direction), and that their variation is slow on the time scale $\frac{1}{\omega_j}$, $i = 0, 1, 2$. This approximation is known as the weak coupling limit

$$\frac{\partial^2}{\partial x^2}(E_0, E_1, E_2) = 0 \tag{10a}$$

$$\frac{\partial^2}{\partial t^2}(E_0, E_1, E_2) = 0. \tag{10b}$$

Inserting Equation (9) into Equation (8) and making use of Equations (10a, 10b) then selecting terms of equal exponentials on both sides, we obtain the slow coupling equations:

$$\left[\frac{\partial}{\partial t} + \frac{1}{2\omega_1}\left\{3v_{th}^2(2k_{1x}+k_{1y}) - k_{1y}c^2\right\}\frac{\partial}{\partial x} + \frac{1}{2i\omega_1}\left\{k_{1y}(c^2 - 3v_{th}^2)(k_{1x}-k_{1y})\cdot\right.\right.$$

$$\left.\left. -\omega_p^2(o)\frac{x}{L} + i\frac{\omega_p^2(o)}{\omega_1}\nu\left(1+\frac{x}{L}\right)\right\}\right]E_1 = ik_{1x}\omega_p^2(o)v_0E_2^* \tag{11}$$

$$\left[\frac{\partial}{\partial t} + \frac{1}{2\omega_2}\left\{3v_{th}^2(2k_{2x}+k_{2y}) - k_{2y}c^2\right\}\frac{\partial}{\partial x} - \frac{1}{2i\omega_2}\left\{k_{2y}(c^2 - 3v_{th}^2)(k_{2x}-k_{2y})\cdot\right.\right.$$

$$\left.\left. -\omega_p^2(o)\frac{x}{L} - i\frac{\omega_p^2(o)}{\omega_2}v\left(1+\frac{x}{L}\right)\right\}\right]E_2^* = -i\frac{k_{2x}\omega_p^2(o)v_0^*}{2\omega_1\omega_2}E_1, \tag{12}$$

where $v_{th}^2 = \sqrt{\frac{2T}{m}}$ and $v_0 = \mid \frac{eE_0}{im\omega_0} \mid$ are the thermal and quiver velocities of the electron fluid, respectively.

3. HOMOGENEOUS PLASMA ANALYSIS

In the limit where the inhomogeneity scale length $(L) \to \infty$, the slow coupling equations reduce to that of homogeneous plasma:

$$\left[\frac{\partial}{\partial t} - \frac{i}{2\omega_1}\left\{k_{1y}\left(k_{1x} - k_{1y}\right)\left(c^2 - 3v_{th}^2\right) + \frac{i\omega_p^2(o)}{\omega_1}\nu\right\}\right] E_1 = \frac{ik_{1x}\omega_p^2(o)v_0}{2\omega_1\omega_2} E_2^* \tag{13}$$

$$\left[\frac{\partial}{\partial t} + \frac{i}{2\omega_1}\left\{k_{2y}\left(k_{1x} - k_{2y}\right)\left(c^2 - 3v_{th}^2\right) - \frac{i\omega_p^2(o)}{\omega_2}\nu\right\}\right] E_2^* = -\frac{ik_{2x}\omega_p^2(o)v_0^*}{2\omega_1\omega_2} E_2. \tag{14}$$

Defining $E_i = \sqrt{\omega_p(o)}a_i (i = 1, 2)$, and substituting for $k_{1x} = k_{2x} = \frac{1}{2}k_0$, $k_{1y} = k_{2y}$, $\omega_1 = \omega_2 = \omega_p(0)$ and using the abbreviation $\alpha = \frac{1}{2\omega_i}\left[k_{iy}\left(c^2 - 3v_{th}^2\right)\left(k_{iy}\right)\right]$, $i = 1, 2$, Equations (13) and (14) transform into the following forms:

$$\left[\frac{\partial}{\partial T} + \left(\frac{\nu}{2} - i\alpha\right)\right] a_1 = \frac{ik_0v_0}{4} a_2^* \tag{15}$$

$$\left[\frac{\partial}{\partial t} + \left(\frac{\nu}{2} + i\alpha\right)\right] a_2^* = -\frac{ik_0v_0^*}{4} a_1, \tag{16}$$

where we have made use of the fact that $\omega_1 \approx \omega_2 \approx \omega_p(0)$.

Integrating Equation (16), we obtain

$$a_2^*(t) = -i\int_{-\infty}^{t} \gamma_0^*(t')a_1(t')\exp\left\{-\left(\frac{\nu}{2} + i\alpha\right)(t - t')\right\} dt' \tag{17}$$

where $\gamma_0 = \frac{k_0v_0}{4}$ is the maximum growth rate for TPD in a homogeneous plasma.

Substituting Equation (17) into Equation (15), we have

$$\left[\frac{\partial}{\partial t} + \left(\frac{\nu}{2} - i\alpha\right)\right] a_1 = \int_{-\infty}^{t} \gamma_0(t)\gamma_0^*(t')a_1(t')\exp\left\{-\left(\frac{\nu}{2} + i\alpha\right)(t - t')\right\} dt'. \tag{18}$$

Using Equation (15) we obtain an equation for the intensity $\mid a_1 \mid^2$:

$$\left[\frac{\partial}{\partial t} + v\right] \mid a_1 \mid^2 = \int_{-\infty}^{t} \left[\gamma_0(t)\gamma_0^*(t')a_1(t)a_1^*(t')\exp\left\{-\left(\frac{v}{2} + i\alpha\right)(t - t')\right\}\right.$$
$$\left. + \gamma_0(t')\gamma_0^*(t)a_1(t)a_1^*(t')\exp\left\{-\left(\frac{\nu}{2} - i\alpha\right)(t - t')\right\}\right] dt'. \tag{19}$$

Assuming that the growth time for the two plasmons to be much greater than the correlation time of the pump wave, this assumption implies that the bandwidth $\Delta\omega_0 \ll \gamma_0$. The integrand on the right is finite only over the correlation time, during this time the amplitude $a_1(t')$ will not change much, so it can be set to $a_1(t)$

and taken outside the integral. Now, averaging Equation (19) over a period shorter than the growth time but larger than the correlation time, we obtain:

$$\left[\frac{\partial}{\partial t} + \nu\right] \langle | a_1 |^2 \rangle = \langle | a_1 |^2 \rangle \int_{-\infty}^{t} \gamma_0(t)\gamma_0^*(t') \exp\left\{-\left(\frac{\nu}{2} + i\alpha\right)(t - t')\right\} dt'$$

$$+\langle | a_1 |^2 \rangle \int_{-\infty}^{t} \gamma_0(t)\gamma_0^*(t') \exp\left\{-\left(\frac{\nu}{2} + i\alpha\right)(t - t')\right\} dt', \quad (20)$$

where the angular brackets denote the averaging process described. The procedure followed to arrive at Equation (20) from Equation (19) is known as the Bourret approximation [19].

Choosing

$$\langle \gamma_0^*(t')\gamma_0(t) \rangle = | \gamma_0 |^2 \exp\{-\Delta\omega_0(t - t')\},$$

Equation 20 becomes

$$\left[\frac{\partial}{\partial t} + \nu - \frac{2 | \gamma_0 |^2 \Delta\omega_0}{(\Delta\omega_0 + \nu/2)^2 + \alpha^2}\right] \langle | a_1 |^2 \rangle = 0. \quad (21)$$

We can easily solve this equation to obtain the intensity as a function of time:

$$\langle | a_1(t) |^2 \rangle = \langle | a_1(o) |^2 \rangle \exp\left[\frac{2 | \gamma_0 |^2 \Delta\omega_0}{(\Delta\omega_0 + v/2)^2 + \alpha^2} - \nu\right] t. \quad (22)$$

The effective growth rate (γ_{eff}) is readily obtained from this equation:

$$\gamma_{eff} = \frac{2 | \gamma_0 |^2 \Delta\omega_0}{(\Delta\omega_0 + \nu/2)^2 + \alpha^2} - v. \quad (23)$$

For collisionless plasma ($v \to 0$), the effective growth rate becomes:

$$\gamma_{eff} = \frac{2 | \gamma_0 |^2 \Delta\omega_0}{(\Delta\omega_0)^2 + \alpha^2}. \quad (24)$$

Furthermore, if we allow the y-components of the propagation vectors of the plasmons to approach zero, then we have:

$$\gamma_{eff} = \frac{2 | \gamma_0 |^2}{\Delta\omega_0} \quad (25)$$

From this result, we conclude that the bandwidth has reduced the growth rate by a factor of $\frac{2 | \gamma_0 |}{\Delta\omega_0}$ provided that $\Delta\omega_0 \gg \gamma_0$.

The collisional threshold is simply given [20, 21] by the condition $\gamma = \frac{\nu}{2}$, hence we obtain:

$$k_0^2 v_0^2 = \frac{3\nu}{\Delta\omega_0}\left[\left(\Delta\omega_0 + \frac{\nu}{2}\right) + \alpha^2\right]. \quad (26)$$

If we allow the y-components of the propagation vectors of the plasmons to approach zero and assume that $\Delta\omega_0 \gg \nu$, a condition which is readily met by hot plasmas or lasers with large bandwidth, we obtain the threshold expression $(k_0 v_0)^2_{th} = 3\nu\Delta\omega_0$, which when compared with the usual threshold condition $(k_0 v_0)^2_{th} = \nu^2$, shows that the bandwidth has increased the threshold by a factor of $3\Delta\omega_0/\nu$.

4. CONVECTIVE TPD IN AN INHOMOGENEOUS PLASMA

In an inhomogeneous plasma the presence of density gradient significantly modifies the features of the TPD instability and adds to the complexity of its analytical treatment. To investigate the effect of the pump wave bandwidth on the TPD, we set $k_{1x} \approx k_{2x} \approx \dfrac{k_0}{2}$, $\omega_1 \approx \omega_2 \approx \omega_p(o)$, and $k_{1y} \approx k_{2y} \approx k_y$, we also assume that $k_0 \gg k_y$, a situation which is typical in experiments of laser wavelength below $1\mu m$ [22], hence Equations (11) and (12) transform to:

$$\left[\frac{\partial}{\partial t} + V\frac{\partial}{\partial x} + \left\{\frac{\nu}{2}\left(1+\frac{x}{L}\right) + \frac{i\omega_p(o)}{2}\frac{x}{L}\right\}\right] E_1 = i\gamma_0 E_2^* \tag{27}$$

$$\left[\frac{\partial}{\partial t} + V\frac{\partial}{\partial x} + \left\{\frac{\nu}{2}\left(1+\frac{x}{L}\right) - \frac{i\omega_p(o)}{2}\frac{x}{L}\right\}\right] E_2^* = i\gamma_0^* E_1 \tag{28}$$

where

$V = \dfrac{3v_{th}^2}{\omega_p(o)}k_1$ is the group velocity of the two plasmons.

Defining $E_i = \sqrt{\omega_p(o)}a_i$, $i = 1, 2$, and normalizing space and time using the transformation

$$x_n = \frac{x}{x_0}, \qquad t_n = \frac{t}{t_0}$$

where

$$x_0 = \sqrt{\frac{2VL}{\omega_p(o)}}, \qquad t_0 = \sqrt{\frac{2L}{V\omega_p(o)}}.$$

Substituting into Equation (27) and (28) we obtain:

$$\left[\frac{\partial}{\partial t_n} + \frac{\partial}{\partial x_n} + \left\{\frac{\nu}{2}\left(t_0 + \frac{2x_n}{\omega_p(o)}\right) + ix_n\right\}\right] a_1 = i\gamma_0 t_0 a_2^* \tag{29}$$

$$\left[\frac{\partial}{\partial t_n} + \frac{\partial}{\partial x_n} + \left\{\frac{\nu}{2}\left(t_0 + \frac{2x_n}{\omega_p(o)}\right) + ix_n\right\}\right] a_2^* = i\gamma_0^* t_0 a_1. \tag{30}$$

Since the amplitudes are slowly varying in space and time, then Equation (30) can be integrated to yield:

$$a_2^* = -it_0 \int_{-\infty}^{t} \gamma_0^*(t') \exp\left[-\left\{\frac{\nu}{2}\left(t_0 + \frac{2x}{\omega_p(o)}\right) - ix'\right\}(t-t')\right] dt'. \tag{31}$$

Substituting into Equation (29) we obtain

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\left\{\frac{\nu}{2}\left(t_0+\frac{2x_n}{\omega_p(o)}\right)+ix\right\}\right]a_1$$
$$=t_0^2\int_{-\infty}^{t}\gamma_0(t)\gamma_0^*(t')a_1^*(t')a_1(t)\exp\left[-\left\{\frac{\nu}{2}\left(t_0+\frac{2x}{\omega_p(o)}\right)-ix\right\}(t-t')\right]dt'. \qquad (32)$$

From this equation we can obtain for the intensity $|a_1|^2$, the following equation:

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\nu\left(t_0+\frac{2x}{\omega_p(o)}\right)\right]|a_1|^2$$
$$t_0^2\int_{-\infty}^{t}dt'\gamma_0(t)\gamma_0^*(t')a_1^*(t')a_1(t')\exp\left[-\left\{\frac{\nu}{2}\left(t_0+\frac{2x}{\omega_p(o)}\right)-ix\right\}(t-t')\right]$$
$$+t_0^2\int_{-\infty}^{t}dt'\gamma_0^*(t)\gamma_0(t')a_1^*(t')a_1(t')\exp\left[-\left\{\frac{\nu}{2}\left(t_0\frac{2x}{\omega_p(o)}\right)+ix\right\}(t-t')\right]. \qquad (33)$$

Using Bourret approximation we obtain for the average intensity

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\nu\left(t_0+\frac{2x}{\omega_p(o)}\right)\right]\langle|a_1|^2\rangle$$
$$=t_0^2\langle|a|^2\rangle\langle\gamma_0(t)\gamma_0^*(t')\rangle\frac{[\nu(t_0+2x/\omega_p(o)+2\Delta\omega_u)]}{\left[\frac{\nu}{2}[t_0+2x/\omega_p(o)+2\Delta\omega_0)\right]^2+x^2}. \qquad (34)$$

Choosing:

$t_0^2\langle\gamma_0(t)\gamma_0^*(t')\rangle=|\bar{\gamma_0}|^2\,e^{-\Delta\omega_0(t-t')}$, then Equation (34) transforms to:

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\nu\left(t_0+\frac{2x}{\omega_p(o)}\right)\right]\langle|a_1|^2\rangle$$
$$=\frac{|\bar{\gamma_0}|^2\left[\nu\left(t_0+2x/\omega_p(o)\right)+2\Delta\omega_0\right]}{\left[\frac{\nu}{2}\left(t_0+2x/\omega_p(o)\right)+2\Delta\omega_0\right]^2+x^2}\langle|a_1|^2\rangle. \qquad (35)$$

Now, let us use the variable $y=t_0+2x/\omega_p(o)$, then Equation (35) becomes:

$$\left[\frac{\partial}{\partial t}+\frac{2}{\omega_p(o)}\frac{\partial}{\partial y}+\nu y\right]\langle<|a_1|^2\rangle$$
$$=\frac{|\bar{\gamma_0}^2\left[\nu y+2\Delta\omega_0\right]}{\left[\frac{\nu}{2}y+2\Delta\omega_0\right]^2+\frac{1}{4}\left(\omega_p(o)y-t_0\right)^2}\langle|a_1|^2\rangle. \qquad (36)$$

Using the abbreviations:

$$\alpha=\frac{2\left(\nu\Delta\omega_0-\omega_p(o)t_0/2\right)}{\nu^2+\omega_p^2(o)},\qquad \beta^2=\frac{4\left(\Delta\omega_0\right)^2+t_0^2}{\nu^2+\omega_p^2(o)}-\alpha^2,$$

Equation (36) takes the form

$$\left[\frac{\partial}{\partial t} + \frac{2}{\omega_p(o)}\frac{\partial}{\partial y} + \nu y\right] \langle | a_1 | \rangle$$

$$= \frac{4\, |\bar{\gamma_0}|^2 \, [\nu y + 2\Delta\omega_0]}{[\nu^2 + \omega_p^2(o)]\, [(y+\alpha)^2 + \beta^2]} \langle | a_1 |^2 \rangle. \tag{37}$$

This differential equation shows the variations of the intensity in space and time, but since we are interested in the convective instability we set $\frac{\partial}{\partial t} = 0$, and then integrating over a symmetric interaction region from $y_-(-y)$ to $y_+(+y)$, we obtain:

$$\ln \frac{\langle | a_1(y_+) |^2 \rangle}{\langle | a_1(y) |^2 \rangle} = \frac{2\omega_p(o)\, |\bar{\gamma_0}|^2}{\nu^2 + \omega_p^2(o)} \left(\frac{2\Delta\omega_0 - \alpha\nu}{\beta}\right) \left[tan^{-1}\frac{y_+ + \alpha}{\beta} - tan^{-1}\frac{y_- + \alpha}{\beta}\right].$$

Extending the region of interaction from $-\infty$ to $+\infty$ we obtain

$$\frac{\langle | a_1(\infty) |^2 \rangle}{\langle | a_1(-\infty) |^2 \rangle} = \exp\left[\frac{2\pi\omega_p(o)\, |\bar{\gamma_0}|^2\, (2\Delta\omega_0 - \alpha\nu)}{\beta\,(\nu^2 + \omega_p^2(o))}\right]. \tag{38}$$

Hence the convective amplification factor in the vicinity of $x \cong 0$, where the interaction is at resonance (*i.e.* no phase mismatch) is given by:

$$A = \frac{2\pi\, |\bar{\gamma_0}|^2\, \omega_p(o)}{\nu^2 + \omega_p(o)} \left(\frac{2\Delta\omega_0 - \alpha\nu}{\beta}\right). \tag{39}$$

This equation gives the amplification factor as a function of the pump bandwidth and the electron–ion collision frequency. It is interesting to note that in the limit when $\nu \rightarrow 0$, the amplification factor reduces to $2\pi t_0^2 \mid \gamma_0 \mid^2$ *i.e.*, independent of the bandwidth. Since the intensity of the decay mode can be written in the form $I = I_0 e^A$, thus $A > 1$ represents the growth in the mode under consideration and the threshold can be estimated [23, 24] by setting $A = \pi$, so substituting for $A = \pi$ in Equation (39) we obtain the threshold condition:

$$\left(\frac{v_0}{v_{th}}\right)^2 k_0 L = \frac{6\beta\, [\omega_p^2(o) + \nu^2]}{\omega_p^2(o)\, [2\Delta\omega_0 - \alpha\nu]}. \tag{40}$$

It is interesting to note that in the limit of collisionless plasma ($\nu \rightarrow 0$), the threshold dependence on the bandwidth disappears and we obtain the following condition for the threshold:

$$\left(\frac{v_0}{v_{th}}\right)^2 k_0 L > 6. \tag{41}$$

This is the same threshold condition for TPD instability that is given in the literature [20, 21]. To determine the growth rate, we focus on the time variation in the vicinity of $y = 0$. At this point Equation (37) becomes:

$$\frac{\partial}{\partial t}\langle | a_1 |^2 = \frac{8\, |\bar{\gamma_0}|^2\, \Delta\omega_0}{4\,(\Delta\omega_0)^2\, t_0^2} \langle | a_1 |^2 \rangle, \tag{42}$$

hence

$$\langle | a_1(o,t) |^2 \rangle = \langle | a_1(0,0) |^2 \rangle \exp\left[\frac{8\, |\bar{\gamma_0}|^2\, \Delta\omega_0}{4\,(\Delta\omega_0)^2 + t_0^2} t\right]. \tag{43}$$

From this equation we obtain the effective growth rate:

$$\gamma_{eff} = \left[\frac{8\,|\bar{\gamma}_0|^2\,\Delta\omega_0}{4\,(\Delta\omega_0)^2 + t_0^2}\right]. \tag{44}$$

5. CONCLUSIONS

The effect of bandwidth on the TPD instability has been investigated when $(\Delta\omega_0 \gg \gamma_0)$. Homogeneous plasma growth rate is reduced by a factor of $\frac{2\gamma_0}{\Delta\omega_0}$ and the threshold is increased by a factor of $\frac{3\Delta\omega_0}{\nu}$. Convective TPD instability in an inhomogeneous plasma of a linear ramp density was also investigated under the same condition $(\Delta\omega_0 \gg \gamma_0)$ and analytical expressions for the amplification factor and threshold were obtained. We noted that when the electron–ion collision frequency approaches zero the dependence of the threshold and amplification factor on the pump bandwidth disappears, and the usual threshold expression given in the literature is recovered. We found that the effective growth rate does not depend on the electron–ion collision frequency but it is reduced when the bandwidth of the pump wave is taken into consideration.

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