MACROECONOMIC DYNAMICS: A CYBERNETIC COMPUTER-ORIENTED APPROACH

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الخلاصة

ان أغلب النماذج الاقتصادية الكبيرة النظرية المستخدمة هي بسيطة ،ولذا تكون خاطئة وزائفة حيث أن الأساليب التقليدية المتعلقة بعلوم الاقتصاد النظرية ، والتي لا بد وأن تقرر الحل الشامل لمجموعة من المعادلات، تتصل اتصالا وثيقاً بحدود ممارسة الوسائل الحسابية . وستبقى هذه المشكلة قائمة ملحة لأن رجال الاقتصاد غالباً ما يتجاهلون تطور المعارف والطرق التقنية في مختلف المجالات التي قد يكون اسهامها في تعزيز التقدم جوهرياً .

ويستهدف هذا البحث عرض طريقة لتشكيل وصياغة الاقتصاد القائم على علم التحكم الأوتوماتي وتجارب الآلات الحاسبة (الكمبيوتر) التي تتيح الفرصة لتصور امتداد واسع وجدير بالاهتمام يتناول تحسين وضع انشاء النماذج والتدريب الاقتصادي .

ABSTRACT

Most theoretical macroeconomic models are kept simple and are consequently false because the traditional approach in theoretical economics, which is to determine the general solution of a system of equations, is closely related to the limits of manipulation of the mathematical tool. This problem persists because economists generally ignore the development of knowledge and techniques in various fields whose contribution to further progress might be substantial.

The purpose of this paper is to present a method of economic modeling based on cybernetics and computer simulation experiments which permits to envisage considerable extension of both models building and economic training.

INTRODUCTION

Dynamic analysis in macroeconomics generally consists in:

1. expressing macroeconomic processes by dynamic models - which are simplified mathematical representations of economic systems written as a set of either differential or finite recursive difference equations,**
2. Studying the solutions of these sets of equations with respect to a given variable for example income or output,
3. giving an economic interpretation of each possible solution.

This type of approach is called positive economics. Generally, in such analysis, theoretical models are presented according to a logical progression in which each of them focusses on some particular aspect of the economic problem. However, models are neither complicated nor comprehensive since their construction is closely related to the limits of manipulation of the mathematical tool. As a result, certain economic relationships are neglected within each model because they are not considered as essential with respect to the particular focus of the model. Consequently the conclusions of the analysis are affected by this assumption and it is difficult to understand both the real impact of a given economic policy or relationship on the whole economic process and the relationships between the different models.

A solution to this problem is to supplement the usual mathematical tools by computer simulation techniques. They first eliminate the problem of model complexity and, also, they are related to a certain methodological approach which is most useful in economic analysis. Any economic model indeed may be presented as a cybernetic system to which a diagram and a simulation program are associated. Furthermore, a given cybernetic system may act as a subsystem within a more elaborated system. It is consequently

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** it is worth noting that these equations rarely exceed the second order although the simplest model which can be developed to simulate a total economy to any degree of realism must be a fourth order one.

possible to present the models of the macroeconomic theory as a system of models starting with a very simple cybernetic system and being progressively complicated by the introduction of complementary subsystems. In doing so, one may easily introduce relationships or feedback loops which are not considered as essential with respect to the particular focus of a given model but which may have an action on the system behavior. In other words, it should always be possible to plug into a given economic model, considered as a cybernetic system, one or several subsystems or modules in order to study economic functions or policies which are not usually introduced in that particular model. An immediate advantage of this approach is a considerable extension of the realism of models. It is furthermore worth noting the important pedagogical aspect of this procedure in economic training since the computer may be used as a laboratory for either models testing and improving or economic case studies.

The purpose of this paper is to present dynamic macroeconomics as a computer oriented analysis of cybernetic systems. The computer background is based upon the DYNAMO (DYNAmic MOdeling) simulation language developed at M.I.T. and particularly designed to treat socio-economic systems[1]. The main advantages of DYNAMO are related to its conceptual construction: the concepts of the language are identical to the concepts of dynamic macroeconomics. Furthermore, the language is very simple and does not require computer knowledge.

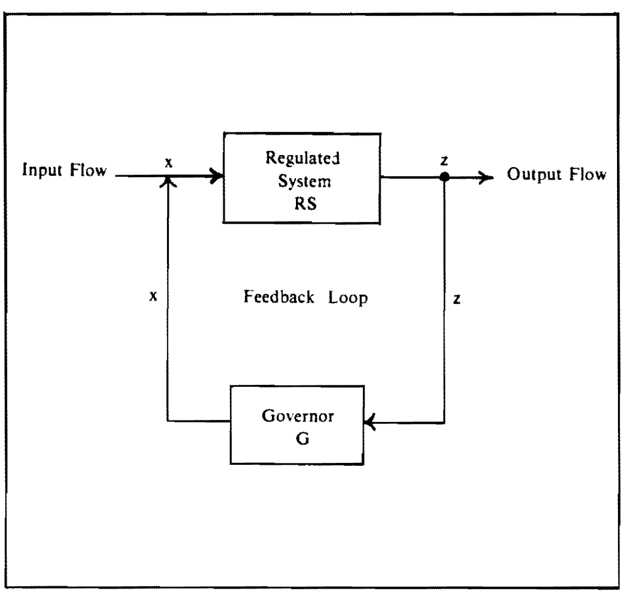


Figure 1. Representation of a Cybernetic System

THE CYBERNETIC BACKGROUND [2]

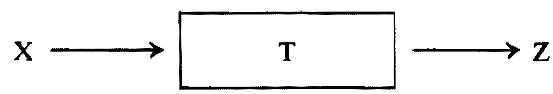
A cybernetic system is a device which accepts one or several inputs, generates one or several outputs and is characterized by one or several feedback loops including a governor whose effect is to modify the variations or the level of the input flow.

A simple representation of this definition is shown in Figure 1.

Most economic models have this common structure defined by the existence of one or several feedback loops. They are regulated systems. In such systems a certain transformation T takes place so that

$$z = Tx,$$

or



This transformation T depends on what happens in RS and G (Figure 1). We may assume that, in RS, the transformation

$$z = RS(x)$$

is proportional, so that the state of input x is multiplied by the real number K:

$$z = Kx.$$

If an analog transformation takes place in G,

$$x = Lz.$$

The whole system is such that

$$z = K(x + \triangle x) = K(x + Lz) = Kx + KLz$$

or

$$z = \frac{K}{1 - KL} x \qquad (1)$$

This is the basic formula of the theory of regulation. The expression $\frac{K}{1-KL}$ is called the transmittance of the regulation system and the factor $\frac{1}{1-KL}$ the feedback operator. So Equation (1) may be written

$$z = \frac{1}{1-KL} Kx,$$

or

z = (operation of the governor) (operation of the regulated system).

In dynamics, the feedback operator may be regarded as the sum of the infinite geometric series:

$$\frac{1}{1-KL} = 1 + KL + (KL)^2 + \ldots + (KL)^p + \ldots$$

which is convergent if $|KL| < 1$. Then

$$z = (1 + KL + (KL)^2 + \ldots + (KL)^p + \ldots) Kx \quad (2)$$

Consequently, regulation systems may be considered dynamically as infinite processes of continuous weakening influences whose sum gives a finite effect (the operation of the governor consists in producing consecutive increments in the value of z which get smaller as $t \to \infty$).

A full picture of the dynamics of the regulation process is obtained in setting up a mathematical dynamic model of the cybernetic system and in computing the time path of each variable of the system as a function of its initial values and coefficients. The output is then compared either to experimental data or to a given norm. If the model is not a good representation of the analyzed process, the values of the coefficients or/and the model itself are modified up to the obtainment of a satisfactory result. This operation is called simulation.

TYPES OF ECONOMIC VARIABLES; CONCEPT OF ECONOMIC SYSTEM

In economics as in any systems study, the mathematical basis of a dynamic analysis is integration which is a process to relate level variables and flow variables according to the following relationship:

level now = level earlier + (elapsed time) (rate of change)

If "now" and "earlier" are defined as indicated in Figure 2, then

$$\text{level .K} = \text{level .J} + (\text{DT}) (\text{rate of change .JK})$$

$$\frac{\text{level .K} - \text{level .J}}{\text{DT}} = \text{rate of change .JK.} \quad (3)$$

Within dynamic economic models, the time is introduced either in continuous or in period terms.

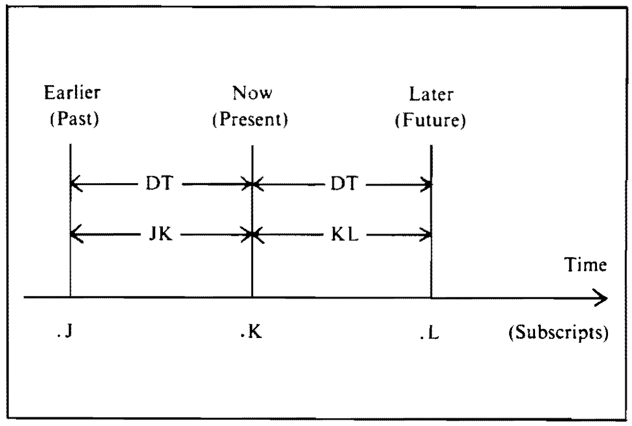


Figure 2. Definitions of Time Variables

In period analysis, variables are written V_t for periods t = 0, 1, 2, 3, ... and the conditions of the model are finite recursive difference equations. In continuous analysis, variables are written V(t) at time t as a rate per unit of time and the model conditions are differential equations.

Consequently Equation (3) is either

$$D(\text{level}) = \frac{d\ \text{level}}{dt} = \frac{\text{level .K} - \text{level .J}}{\text{DT}} = \text{rate of change .JK}$$

in continuous analysis, or

$$\Delta(\text{level}) = \text{level .K} - \text{level .J} = \text{rate of change .JK}$$

in discrete analysis (DT = 1).

When the rate of change is variable, the value of the level at time T is

$$\text{level .T} = \text{level .0} + \int_{t=0}^{t=T} (\text{rate of change})(dt) = \text{level .0} + \int_{t=0}^{t=T} f(t)\, dt$$

A simple period term or digital approximation for

$$\int_{t=0}^{t=T} f(t)\, dt \quad \text{is} \quad \sum_{t=0}^{t=T} f(t)\, \text{DT and}$$

depends on the size of DT (see Figure 3).

This is the way DYNAMO integrates.

A level is consequently a quantity which exists during several DT. A level is independent of the time flow and has no time dimension. In contrast, a rate of

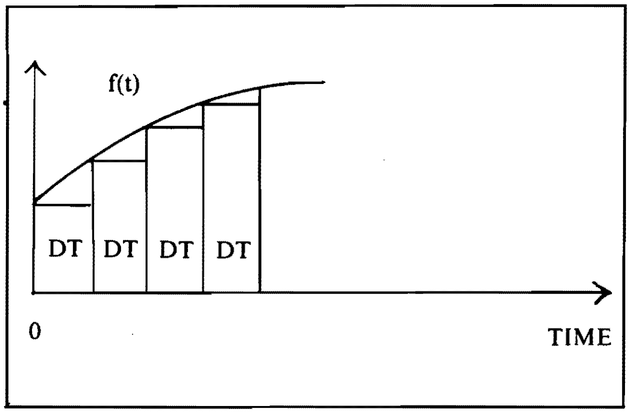


Figure 3. *Approximation for* $\int_{T=0}^{t=T} f(t)\,dt$

change or flow takes a specific value each DT, is created and destroyed each time period, and has a time dimension.

Levels and rates are two of the three types of economic variables. The third one is the auxiliary type. An auxiliary is a variable which is neither a level nor a flow.*

A logic assemblage of level, rate and auxiliary equations is called a model of a system or, by extension, a system. An economic system is defined as being composed of

a. one or several levels,
b. flows that transport the contents of levels,
c. valves that control the rates of flows, and
d. information channels that define the functionnal relationships within the system.

THE MACROECONOMIC CIRCUIT: CONCEPT OF DELAY

The simplest macroeconomic circuit of a closed economy is represented by the diagram of Figure 4.

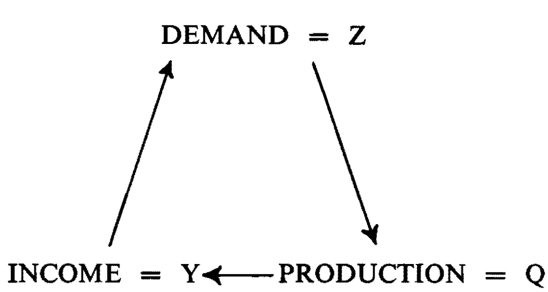


Figure 4. *Simple Macroeconomic circuit*

This diagram is called the circular flow of income and shows how the income is circulated and conserved within the economy.

The static equilibrium of the circuit is

$$Y = Z = Q.$$

In dynamic analysis, delays are introduced and a specific lag applies to each flow. Figure 5 illustrates this.

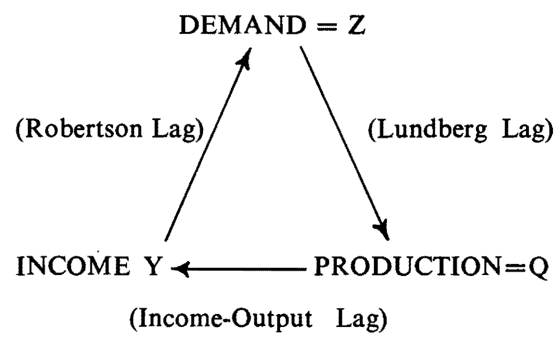


Figure 5. *Circuit with Delays*

Consequently a variation (e.g., positive) of a variable (e.g., Z) generates a deficit (e.g., Z-Q) and a specific behavior of the system to achieve equilibrium again.

A general procedure to lift Q to the new level of demand Z is to fix a time interval T and to increase Q and consequently Y to fill regularly the Z − Q gap during the specified time interval. Thus, during the first period, Q must grow an amount given by

$$\frac{Z - Q}{T}$$

In differential analysis, we have

$$DQ = dQ/dt = (1/T)(Z - Q)$$
$$TDQ = Z - Q$$
$$Q(TD + 1) = Z$$
$$Q = \frac{1}{TD + 1} Z \qquad (4)$$

or, with $\lambda = 1/T$ = reaction velocity of the delay,

$$DQ = \lambda(Z - Q)$$
$$DQ + \lambda Q = \lambda Z$$
$$Q(D + \lambda) = \lambda Z$$
$$Q = \frac{\lambda}{D + \lambda} Z \qquad (5)$$

* The DYNAMO Subscripts conventions and the simulation procedure are explained in Reference [1].

Equations (4) and (5) define a simple or first order exponential delay between Z and Q. This may be extended to double, triple, etc., exponential delays according to the relationship

$$Q = \left[\frac{\lambda n}{D + \lambda n} \right]^n Z$$

n = 1,2,3,... =order of the delay.

The DYNAMO equations for the first order material delay are

Level : A.K = A.J + (DT)(Z.JK − B.JK)

Rate : B.KL = A.K/T (7)

These constitute a stored macro-instruction which means that the computation sequence is automatically implemented when a given keyword is read by the compiler. Thus the computation of Equations (7) is ordered by

Q = DELAY 1 (Z, T) (8)

The corresponding diagram is shown in Figure 6.

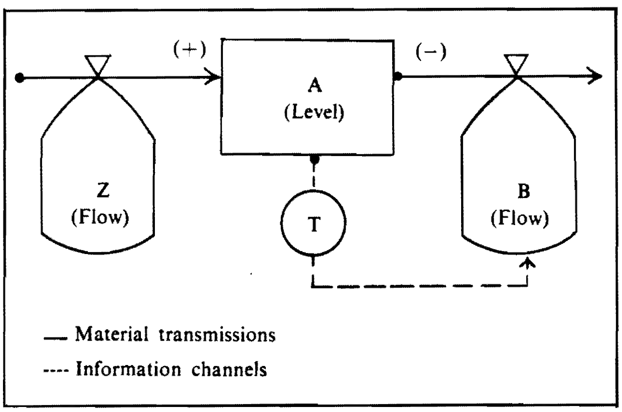


Figure 6. First Order Material Delay.

Equation (7) may be written

A.K = T*B.KL

and

$$\frac{T\,(B.KL - B.JK)}{DT} = Z.JK - B.JK$$

or:

TDB = Z − B

B (TD + 1) = Z

$$B = \frac{1}{TD + 1} Z$$

Consequently Equations (4) and (7) are identical with Q = B.

Simple exponential delays are usually sufficient to express the lag between demand of consumption goods and production or between production and income. However, the lag between demand of investment goods and production is more complicated and triple exponential delays are generally introduced.

A triple exponential delay is written

$$Q = \left(\frac{3\lambda}{D + 3\lambda}\right)^3 Z \qquad (9)$$

and the corresponding macro-instruction is

Q = DELAY3 (Z, T) (10)

An important part of the dynamic analysis in macroeconomics is carried on in terms of finite periods with finite recursive difference equations. In this case, the adjustment mechanism

$$Q = \frac{\lambda}{D+\lambda} Z$$

is replaced by

$$Q_t = Z_t - \theta \qquad (11)$$

θ being a real number and t = 0,1,2,3,...

DYNAMO does not have buit-in macro-instructions corresponding to this kind of lag. However macro-instructions may be written by the user and added to the simulation program.

A one-period lag, (K + DT) − K, will be indicated by the keyword LAG1:

Q.K = LAG1(Z.K); (12)

this means

Q.K = Z.J

The 2 or 3 periods lags are noted LAG2 or LAG3, as follows:

Q.K = LAG2 (Z.K) (13)

Q.K = LAG3 (Z.K). (14)

It is worth noting that these macro-instructions are not required when rates are involved since the subscript conventions permit writing directly any lag between two rates; thus

Rate : A.KL = B.JK

Rate : B.KL = C.JK

Rate : C.KL = D.JK

is equivalent to

$$A_t = D_t - 3DT .$$

ECONOMIC MODELING : DISEQUILIBRIUM DYNAMICS

The basic macroeconomic model is directly deduced from the circular flow of income (Figure 4) and is expressed by Figure 7.

The previous circuit is balanced if and only if the distribution of income between C and S corresponds exactly to the distribution of demand between C and I:

$$Y = Z$$
$$C + S = C + I$$
$$S = I$$

The fact of assuming that this balanced situation exists *a priori* is called ex-post analysis. An ex-post approach is the *a priori* assumption that global demand generated by income Y is equal to global supply (C+1) or that investment I is totally financed by saving S :

$$Z = Y = Q = C + I \ . \qquad (15)$$

This is the basic assumption of model building in economics.

The basic macroeconomic model is mathematically expressed in supplementing Equation (15) by a relationship indicating the distribution of Y between C and S. This is the consumption function,* the simplest expression of which is

$$C = cY \ , \qquad 0 < c < 1 \qquad (16)$$

where c is called marginal propensity to consume (MPC).

Consequently

$$Z = Y = Q = C + I \qquad (17)$$
$$C = cY, \ \ 0 < c < 1$$

System (17) is static. A system is dynamic when the system variables are dated. In dynamics the main distinction is between equilibrium dynamics and disequilibrium dynamics.

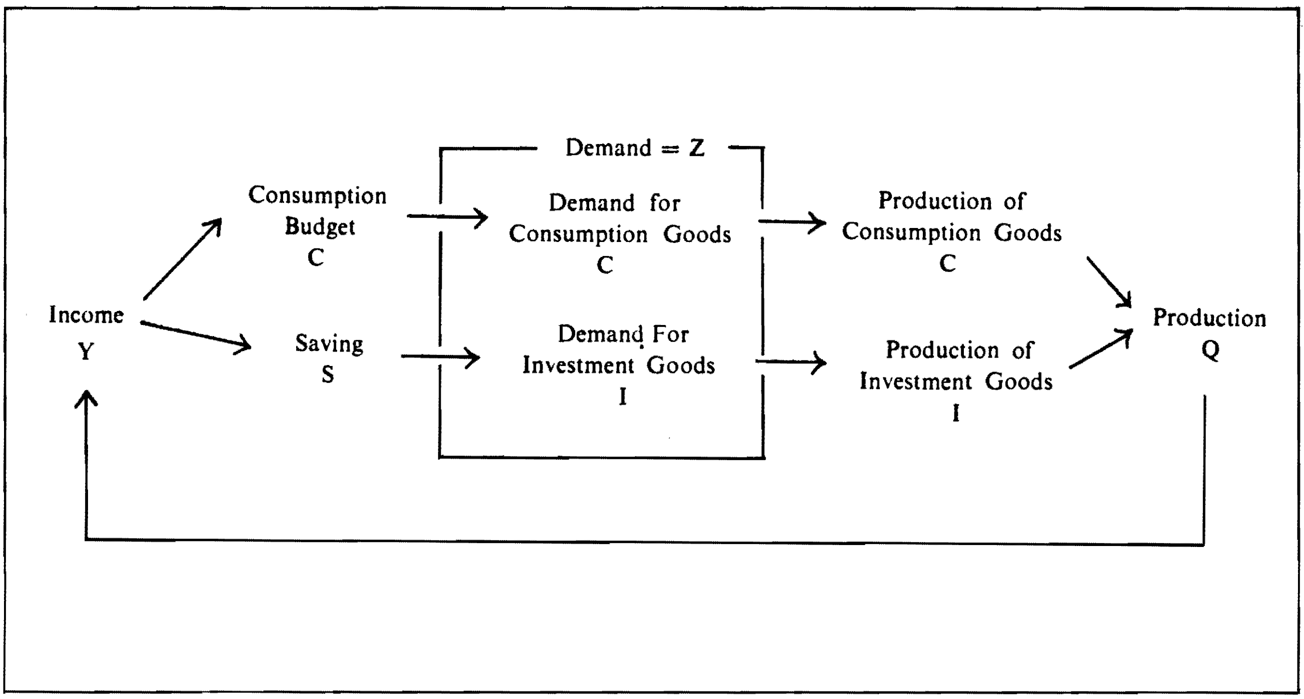


Figure 7. Basic Macroeconomic Model.

* It is worth noting that the consumption function is totally independent of the occurence of equilibrium. It is simply a way to relate planned consumption and income. This type of relationship is called an *ex-ante* relationship.

Equilibrium dynamics refers to a dynamic balanced system without time lags or delays. Disequilibrium dynamics refers to a system in which at least one lag or delay is introduced.

Our objective is to deal with disequilibrium dynamics systems according to the following procedure: from the simplest dynamic state of a system, the statio nary state**, an external or exogenous impulse generates an initial disequilibrium situation and the adjust ment procedure. The problem is then to define the paths of each variable of the system from the initialstationary state to the new one if it exists.

THE MULTIPLIER EFFECT

System (17)

$$Z = Y = Q = C + I$$
$$C = cY , \quad 0 < c < 1$$

is equivalent to

$$Z = Q = Y = cY + I$$
$$Y(1 - c) = I$$
$$Y = \frac{1}{1-c} I \quad \text{or} \quad \triangle Y = \frac{1}{1-c} \triangle I \quad (18)$$

As $0 < c < 1$, a $|\triangle I| = 1$ generates a $|\triangle Y| > 1$.

Consequently the quantity $\frac{1}{1\text{-}c}$ is called the multiplier.

System (17) or Equations (18) describe a system in which all the reactions have acted. They constitute the basic model of a static macroeconomic equilibrium or the stationary state of the corresponding dynamic model.

The static multiplier process consists in deducing the balanced situation B

$$Y^B = C^B + I^B$$
$$C^B = cY^B$$

from the balanced situation A

$$Y^A = C^A + I^A$$
$$C^A = cY^A$$

when

$$Y^B = Y^A + \triangle Y = Y^A + \frac{1}{1-c}\triangle I = Y^A + \frac{1}{1-c}$$
$$(I^B - I^A) = \frac{1}{1-c} I^B$$

In other words, static equilibrium B is immediatly obtained from static equilibrium A when $\triangle I = I^B - I^A$ is injected into system A. This is the basic process of economic growth.

The cybernetic interpretation of the static multiplier is obvious when Equations (18) and (1) are compared. Equations (18) are equation (1) with K=1 and L=c. Consequently the multiplier effect is a feedback whose diagram is:

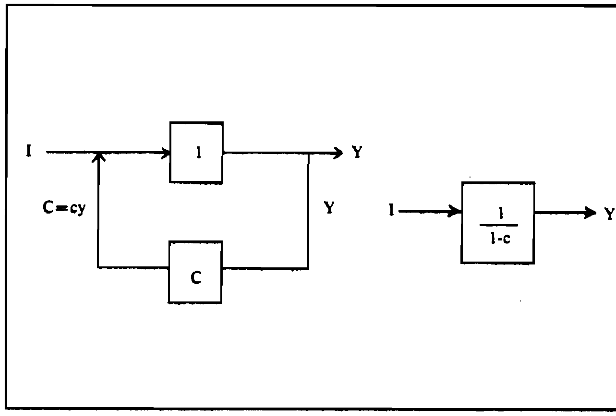


Figure 8. Diagram of Multiplier.

The multiplier $\frac{1}{1-c}$ is simply the feedback operator of system (17).

In dynamic period analysis system (17) is written either

$$Z_t = Q_t = Y_t = C_t + \overline{I}$$
$$C_t = cY_{t-1} \quad (19)$$

if a Robertson lag is considered, or

$$Y_t = Q_t = Z_{t-1}$$
$$Z_t = C_t + \overline{I}$$
$$C_t = cY^t \quad (20)$$

with a Lundberg lag.

** The stationary state is defined as a position of static equilibrium in which the variables are dated but do not change from a given period to another one. System (17) is the stationary state of a dynamic model. A natural extension of the stationary state is called steady-state growth. In a stationary state, the variables are kept constant through time whereas a steady-state growth is characterized by constant rates of growth of the model variables. So, the stationary state is a particular case in which variables' rates of growth are nul.

Both systems may be written

$$Y_t - cY_{t-1} = \overline{I} = \text{constant} \; ; \qquad (21)$$

and the solution of Equation (21) is :

$$Y_t = \left(\frac{\overline{I}}{1-c}\right) + \left(Y_0 - \frac{\overline{I}}{1-c}\right)\left(1 - (1-c)\right)^t \qquad (22)$$

which is the path of Y from the initial level Y^0. The stationary state is

$$Y = \frac{\overline{I}}{1-c}, \quad \text{when } t \to \infty . \qquad (23)$$

The solution of Equation (21) may also be written as

$$Yt = (1 + c + c^2 + \dots + c^{n-1})\,(\overline{I}) + b^t y_0 \qquad (24)$$

Equation (24) is consequently the dynamic basic formula of the theory of regulation.

Equation (24) is Equation (2) with K=1, L=c, Y_0=0.

In continuous analysis, the simplest adjustment mechanism is a Lundberg lag generated by a variation of I:

$$Q = Y = \frac{\lambda}{D+\lambda} Z = \frac{\lambda}{D+\lambda}(C+1) = \frac{\lambda}{D+\lambda}(cY+1). \qquad (25)$$

The solution of equation (25) is

$$Y = \left(\frac{I}{1-c}\right) + \left(Y_0 - \frac{I}{1\text{-}c}\right) e^{-\lambda(1-c)^t}, \qquad (26)$$

which is the continuous version of Equation (22). The stationary state is:

$$Y = \frac{I}{1-c} \qquad \text{when } t \to \infty . \qquad (27)$$

We may now use the previously developed computer background to associate a diagram and a simulation program to each of the multiplier models.

According to the DYNAMO coding, Model (19) may be written

Auxiliary : Z.K = Q.K
Auxiliary : Q.K = Y.K (28)
Auxiliary : Y.K = C.K + I
Auxiliary : C.K = MPC*LAG1(Y.K) ,

and corresponds to the diagram of Figure 9.

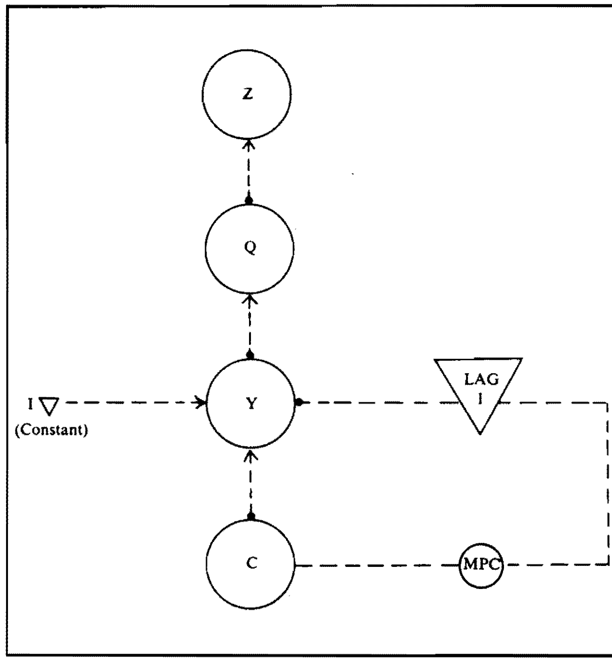


Figure 9. Diagram for Model (19)

Identically, Model (20) is written

Auxiliary : Z.K = C.K + I
Auxiliary : C.K = MPC*Y.K (29)
Auxiliary : Q.K = Y.K
Auxiliary : Y.K = LAG1 (Z.K) ,

and the diagram for this model is as shown in Figure 10.

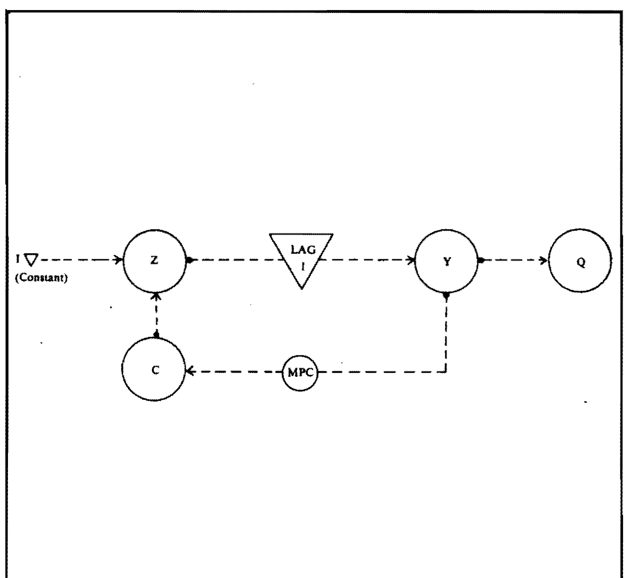


Figure 10. Diagram for Model (20)

At last Model (25) is

$$\begin{aligned} &\text{Auxiliary} &:\quad & Z.K = C.K + I \\ &\text{Auxiliary} &:\quad & C.K = MPC*Y.K \\ &\text{Auxiliary} &:\quad & Q.K = Y.K \\ &\text{Auxiliary} &:\quad & Y.K = DELAY1(Z.K,T) \; ; \end{aligned} \qquad (30)$$

the diagram is shown in Figure 11.

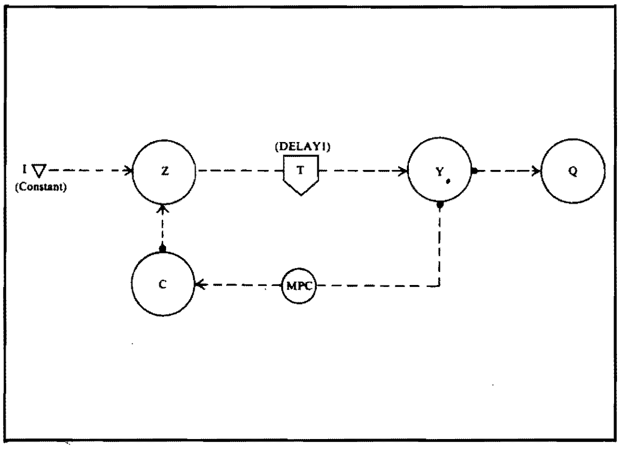


***Figure* 11. *Diagram for Model* (25).**

Model (25) is, of course, the continuous version of Model (20) since both models consider a Lundberg lag between Y and Z.

It is naturally possible to develop the previous presentation considering the different models corresponding to the introduction of various delays in the circular flow of income. For example, the continuous version of the multiplier effect with both a Lundberg lag and an output-income lag corresponds to the diagram shown in Figure 12.

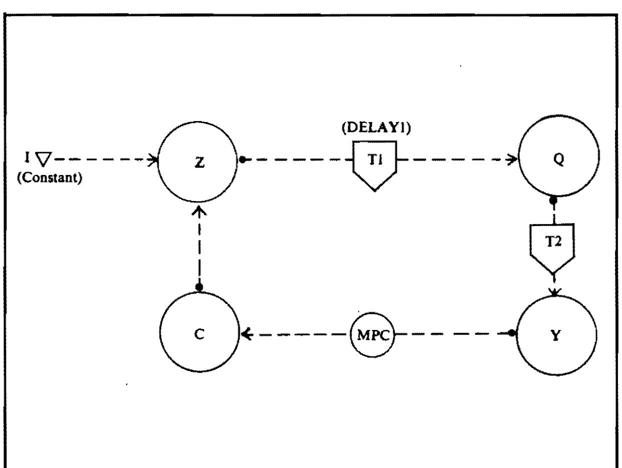


Figure* 12. *Diagram for Continuous Version of Multiplier Effect with Lundberg lag and out-put-income lag.

The equations for the model diagrammed in Figure 12 are as follows:

$$\begin{aligned} &\text{Auxiliary} &:\quad & Z.K = C.K + 1 \\ &\text{Auxiliary} &:\quad & C.K = MPC*Y.K \\ &\text{Auxiliary} &:\quad & Y.K = DELAY1(Q.K,T2) \\ &\text{Auxiliary} &:\quad & Q.K = DELAY1(Z.K,T1) \end{aligned} \qquad (31)$$

Consequently the most general circuit includes three delays or lags. This is shown on Figure 13 for the continuous approach and on Figure 14 for the time-period approach.

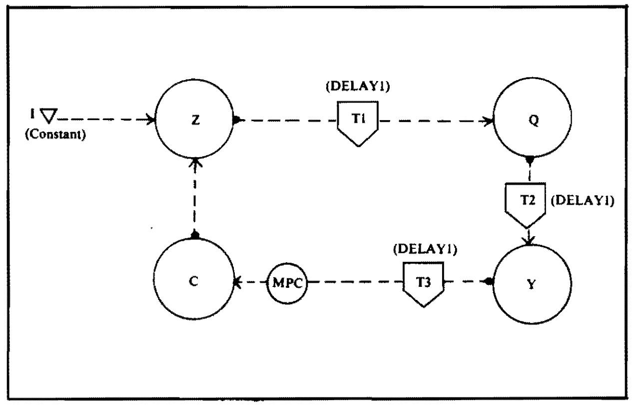


Figure* 13. *General Circuit with Three Delays, Continuous Approach.

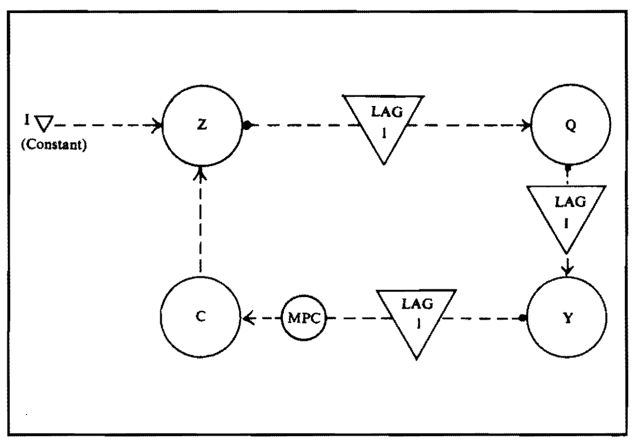


Figure* 14. *General Circuit with Three lags, Time-period Approach.

The previous development suggests two methodological remarks. First, there is no explicit level in the previous models; however level variables are present in the delay macro-instruction, and indeed any simulation program must include at least one integrator.

Secondly, it is worth noting that any economic variable is actually either a flow or a level. However, the DYNAMO subscript conventions are such that it is more convenient to use auxiliary equations and variables to program the previous systems. Consequently, as far as convenience is involved, the replacement of economic flow variables by DYNAMO auxiliary variables will be implemented whenever it is not required that specific consideration be given to a variable as a flow.

An immediate extension of the multiplier models illustrates the previous remarks. It consists in assuming that entrepreneurs have adequate inventories of consumption goods so that any discrepancy between Z and Q may be met by inventory fluctuations [3]. This assumption introduces in the previous models an explicit level variable (inventory) and two rate variables (demand Z and output Q).

In this case the path of Q or Y is no longer exponential but is cyclical . The cyclical behavior of the model is the result of the constant adjustment between desired inventory KBAR and actual inventory K : if K < KBAR, Q increases to compensate the gap; if K > KBAR, Q decreases to reduce the excess.

The simplest assumption is the existence of a constant desired inventory KBAR. It is naturally possible to introduce this new set of assumptions in any of the previous multiplier models, for example in the model of Figure 13. The equations are

$$
\begin{aligned}
Z &= C + I \\
C &= c \, \frac{\lambda_1}{D+\lambda_1} Y \\
Y &= \frac{\lambda_2}{D+\lambda_2} Q \\
Q &= (KBAR - K) + \frac{\lambda_3}{D+\lambda_3} Z \, ,
\end{aligned}
\tag{32}
$$

or, in time-period analysis,

$$
\begin{aligned}
Z_t &= C_t + I_t \\
C_t &= cY_{t-1} \\
Y_t &= Q_{t-1} \\
Q_t &= (KBAR - K_t) + Z_{t-1} \, .
\end{aligned}
\tag{33}
$$

However both model (32) and model (33) must be completed by the identity explaining inventory fluctuations. In (32) it is a differential equation:

$$dK/dt = DK = Q - Z \, . \tag{34}$$

In model (33), it is a difference equation:

$$\Delta K = Q_t - Z_t \, . \tag{35}$$

The corresponding diagram (continuous version) is that shown in Figure 15.

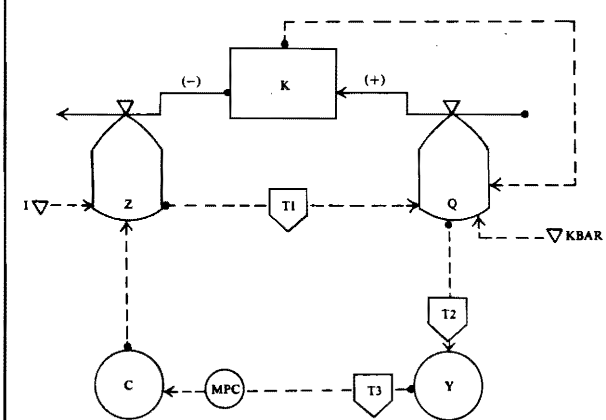


Figure 15. Diagram (Continuous Version) for General Circuit with Constant Inventory Assumption Included.

The simulation program for this case is expressed by the following set of equations:

Level	: K.K = K.J + (DT) (Q.JK – Z.JK)	
Rate	: Q.KL = (KBAR – K.K) + DELAY1(Z.JK,T1)	
Rate	: Z.KL = C.K + I	(36)
Auxiliary	: C.K = MPC*DELAY1(Y.K,T3)	
Auxiliary	: Y.K = DELAY1(Q. JK,T2)	

The previous diagram illustrates the difference between the information channel (dashed lines) showing the functional relationships within the system and the material flows (solid lines) indicating the movement of products through inventory.

THE ACCELERATOR EFFECT

Model (36) is easily improved by assuming that KBAR is no longer a constant but rather a constant proportion of sales of consumption goods:

Auxiliary : KBAR.K = ALPHA*C.K (37)

The effect of the assumption is to introduce a second feedback loop into the system; the diagram becomes that shown in Figure 16.

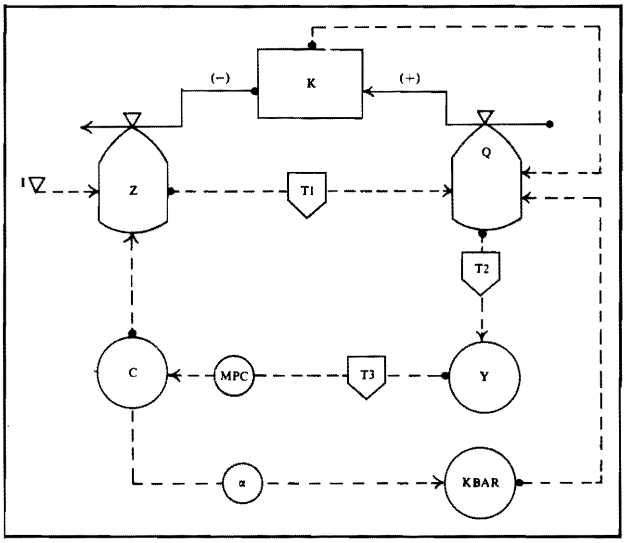


Figure 16. General Circuit with Variable Inventory Assumption.

The system shown in Figure 16 is called inventory accelerator. The autonomous impulse I not only generates a multiplier effect on Y, given by

$$Y = \frac{I}{1-c},$$

but also produces an accelerator effect on K, since at equilibrium

$$K = KBAR = \alpha C = \alpha cY.$$

Thus, the reaction chain is :

$$\Delta I \Longrightarrow \Delta Y = \frac{\Delta I}{1-c} \Longrightarrow \Delta K = \alpha c \Delta Y \quad (38)$$

The model just discussed is incomplete, however, since it does not link the net growth of K to Z. In other words, DK is not an investment since it does not appear as a component of Z, and I is not an investment either since it is not related to the capital stock.* As a result the second feedback loop is incomplete and the feedback operator cannot bedefined. Consequently, it is necessary to generalize this approach.

A general version of the accelerator effect consists in considering demand Z as divided in 3 parts: consumption C, investment I, and autonomous expenditures A. The acceleration principle is then written

$$I = \Delta K = f(\Delta Q) = v\,\Delta Q,$$

or (39)**

$$I = DK = vDQ$$

When the appropriate delays are introduced, including a delay between desired investment vDQ and actual investment I, the model is definable by the following equations :

$$Z = C + I + A$$

$$C = \frac{\lambda_1}{D+\lambda_1}\, cY$$

$$Y = \frac{\lambda_2}{D+\lambda_2}\, Q$$

$$I = \frac{\lambda_3}{D+\lambda_3}\, vDQ = DK \quad (40)$$

$$Q = \frac{\lambda_4}{D+\lambda_4}\, Z$$

This model then corresponds to the diagram show in Figure 17.

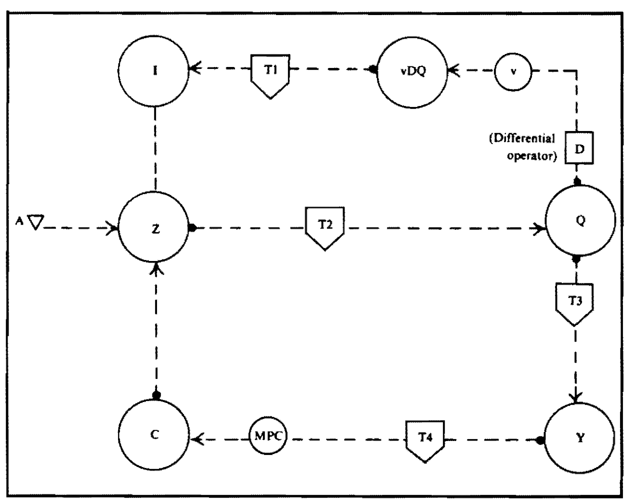


Figure 17. General Model of Accelerator Effect.

* This approach is economically justified by the fact that it constitutes a short period analysis in which the capital stock is assumed to be constant so that I is not an investment but any exogenous or autonomous injection.

** These equations hold, because as long as Q is constant, the net capital stock is sufficient and no investment is required. However, a ΔQ requires an increase in the capital stock and I is positive.

The simulation program is defined by the following equations:

$$
\begin{aligned}
&\text{Auxiliary} : Z.K = C.K + I.K + A \\
&\text{Auxiliary} : C.K = MPC{*}DELAY1\ (Y.K,T4) \\
&\text{Auxiliary} : Y.K = DELAY1\ (Q.K,T3) \\
&\text{Auxiliary} : Q.K = DELAY1\ (Z.K,T2) \qquad (41) \\
&\text{Auxiliary} : I.K = DELAY1\ (VDQ.K,T1) \\
&\text{Auxiliary} : VDQ.K = (VQ.JK\text{-}VQ1.JK)/DT \\
&\text{Rate} \quad : VQ.KL = V{*}Q.K \\
&\text{Rate} \quad : VQ1.KL = VQ.JK
\end{aligned}
$$

In terms of cybernetics, Figure 17 represents a parallel coupling. It may be represented (cf. Figure 8) by a simplified diagram, where delays have not been taken into consideration in order to reduce the complexity of the diagram, shown in Figure 18.

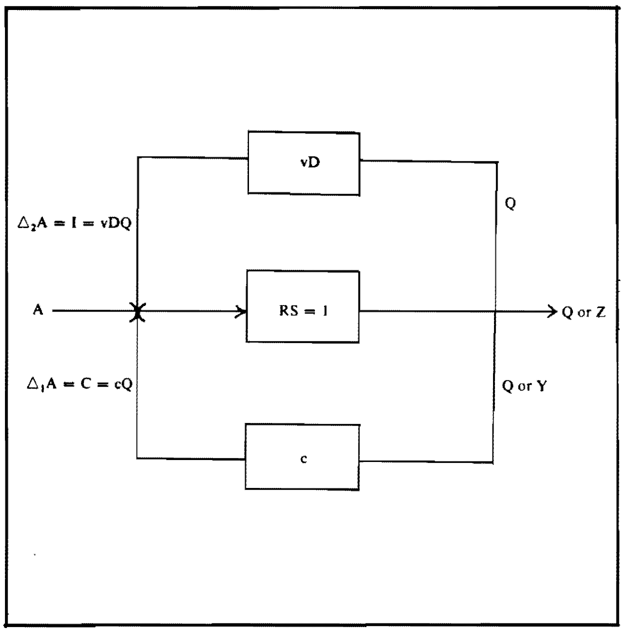


Figure 18. Simplified Diagram of Accelerator Model.

According to the computations carried on to obtain Equation (1), we may write:

$$Q = RS\,(A + \Delta_1 A + \Delta_2 A) = A + cQ + vDQ$$

$$Q\,(1 - c - vD) = A$$

$$Q = \frac{1}{1-c-vD} A = \frac{1}{1-(c+vD)} A \qquad (42)$$

The result consequently the same as if parallel coupling was replaced by a single feedback whose operator would be the sum of the feedback transmittances of the parallel coupling system*. This result is quite general. It may be generalized to any number of parallel feedbacks and holds for all transformations which occur within systems whose operators are linear.

The general expression of Equation (42), cf. Equation (1), is written as follows:

$$Q = \frac{K}{1-K(L+L')} A\,. \qquad (43)$$

So, if the delays of system (41) are introduced, the multiplier-accelerator system may be represented as shown in Figure 19, since it is easily demonstrated that the delay operator is linear.

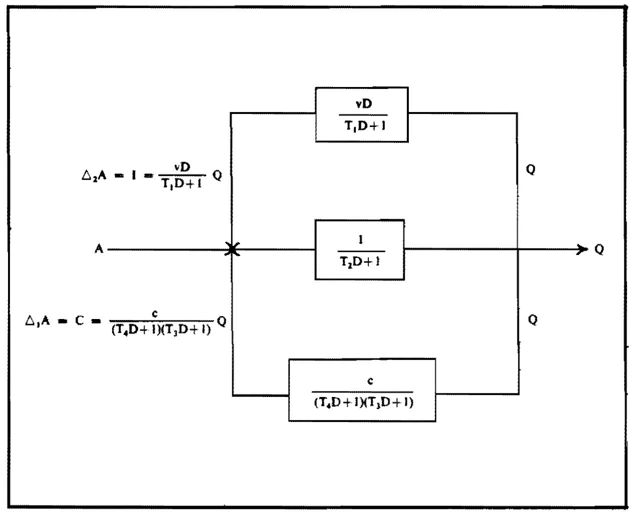


Figure 19. Model of Accelerator Effect with Delays.

The feedback operator of the system shown in Figure 19 is equivalent to the expression

$$\frac{\dfrac{1}{T_2D+1}}{1-\left(\dfrac{1}{T_2D+1}\right)\left(\dfrac{c}{(T_4D+1)\,(T_3D+1)}+\dfrac{vD}{T_1D+1}\right)} \qquad (44)$$

In discrete analysis, Model (40) is written as

$$
\begin{aligned}
Z_t &= C_t + I_t + A \\
C_t &= cY_{t-1} \\
Y_t &= Q_{t-1} \qquad (45) \\
I_t &= v(Q_{t-1} - Q_{t-2}) \\
Q_t &= Z_{t-1}
\end{aligned}
$$

* It is however necessary to assume: $0 < c < 1$, $0 < v < 1$, and $c + v < 1$.

or, using the linear backing operator E^{-1} defined by $E^{-1} x_t = x_{t-1}$,

$$
\begin{aligned}
Z_t &= C_t + I_t + A \\
C_t &= cE^{-1}Y_t \\
Y_t &= E^{-1}Q_t \qquad (46) \\
I_t &= v(E^{-1} Q_t - E^{-2} Q_t) \\
Q_t &= E^{-1}Z_t
\end{aligned}
$$

The structural diagram of the system is then that shown in Figure 20, and the feedback operator is given by the expression

$$\frac{E^{-1}}{1-(E^{-1})\,[cE^{-2} + v(E^{-1} - E^{-2}]} \qquad (47)$$

The previous systems are totally linear. However, the economic reality is rarely and perhaps never linear. It is consequently interesting to introduce non-linear elements in the multiplier-accelerator interaction. One of them is the non linear accelerator and was introduced by R.M. Goodwin [4].

According to Goodwin's approach , the accelerator appears as a relationship between decisions to invest ID_t and the current rate of change of output DQ_t :

$$ID_t = \phi\,(DQ_t) \qquad (48)$$

where ϕ is a non-linear function. Furthermore there exists a periods lag between decisions to invest and actual investments:

$$I_t = ID_{t-\theta} \qquad (49)$$

Function ϕ is such that

$$
\begin{aligned}
DQ &\longrightarrow \infty \Longrightarrow ID \longrightarrow IDMAX \\
DQ &\longrightarrow -\infty \Longrightarrow ID \longrightarrow -IDMIN \\
DQ &= 0 \Longrightarrow ID = 0
\end{aligned}
$$

with

IDMAX = production capacity of the capital good industries,

IDMIN = discard rate for capital goods,

IDMAX > IDMIN

Consequently, function ϕ may be drawn as shown in Figure 21.

A specification of ϕ is given by Allen [5] and is written

$$\phi = IDMIN \left| \frac{IDMAX+IDMIN}{(IDMAX)e^{-vDQ} + IDMIN} - 1 \right| \qquad (50)$$

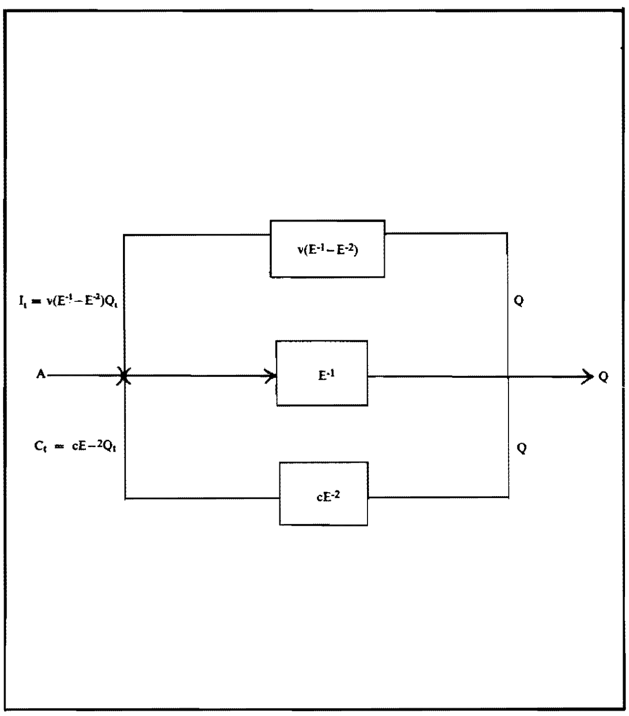


Figure 20. Structural Model of Accelerator Effect.

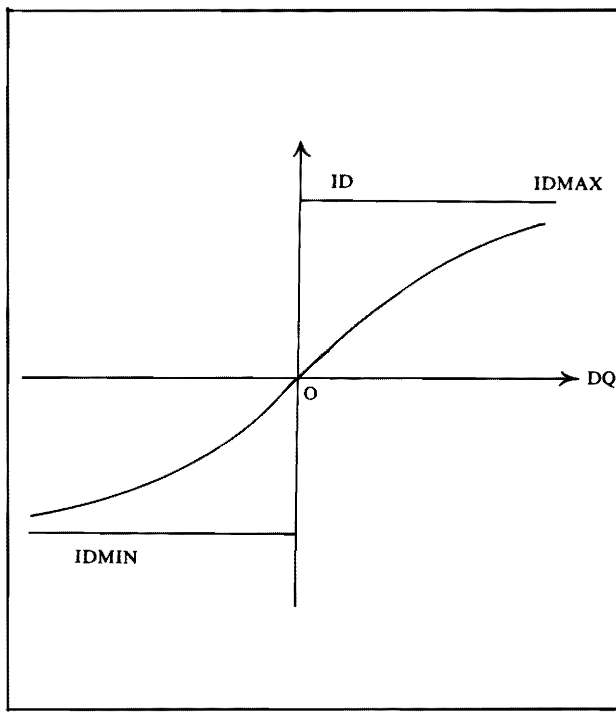


Figure 21. The Function ϕ in Goodwin's Equation.

Then:

$$DQ \longrightarrow \infty \Longrightarrow \phi \longrightarrow IDMIN \left| \frac{(IDMAX+IDMIN}{IDMIN} - 1 \right| = IDMAX$$

$$DQ \longrightarrow -\infty \Longrightarrow \phi \longrightarrow -IDMIN$$

$$DQ = 0 \Longrightarrow \phi = IDMIN \left| \frac{IDMAX+IDMIN}{IDMAX+IDMIN} - 1 \right| = 0$$

The introduction of the non-linear accelerator in Model (40) gives

$$\begin{aligned} Z_t &= C_t + I_t + A \\ C_t &= \frac{\lambda_1}{D+\lambda_1} cY_t \\ Y_t &= \frac{\lambda 2}{D+\lambda_2} Q_t \\ Q_t &= \frac{\lambda_3}{D+\lambda_3} Z_t \\ I_t &= DK_t = ID_{t-\theta} \\ ID_t &= IDMIN \left| \frac{IDMAX + IDMIN}{(IDMAX)e^{-vDQ^t} + IDMIN} - 1) \right| \end{aligned} \qquad (51)$$

This formulation in equations (51) corresponds to the diagram shown in Figure 22.

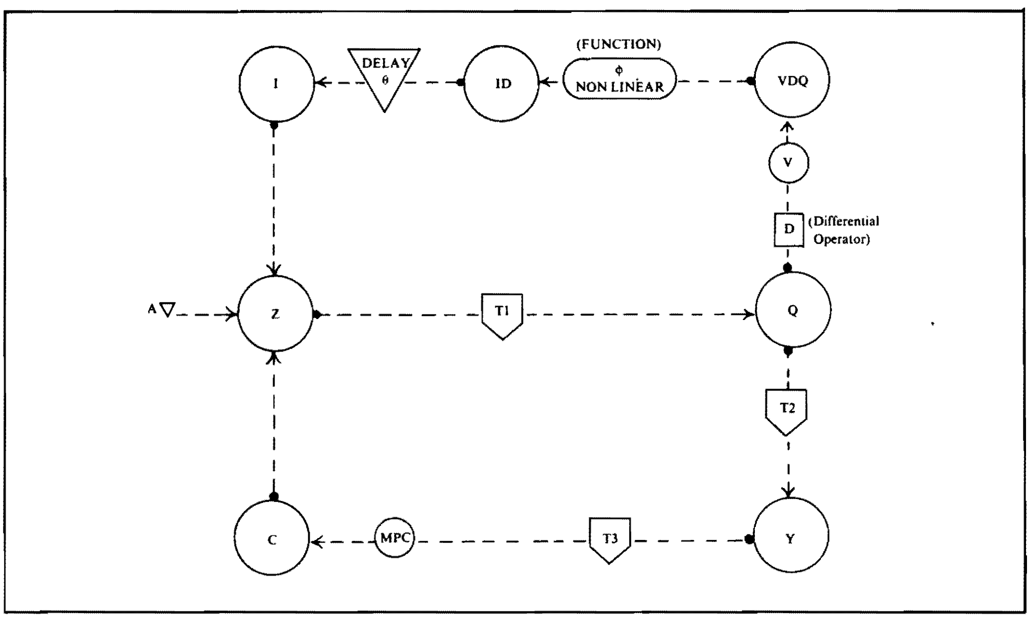


Figure 22. A non-linear Accelerator Model.

The simulation program is definable by the following set of equations:

Auxiliary : Z.K = C.K + I.K + A

Auxiliary : C.K = MPC*DELAY1(Y.K,T3)

Auxiliary : Y.K = DELAY1(Q.K,T2)

Auxiliary : Q.K = DELAY1(Z.K,T1)

Auxiliary : I.K = LAG θ (ID.K)

Auxiliary : ID.K = IMIN*(IMAX+IMIN)/(IMAX*EXP(−VDQ.K)+IMIN)−1)
Auxiliary : VDQ.K = (VQ.JK − VQ1.JK)/DT
Rate : VQ.KL = V*Q.K
Rate : VQ1.KL = VQ.JK

So the improvement of Model (51) consists in replacing (cf. Model (40) the relationship shown in the upper part of Figure 23 by that shown in the lower part of the figure.

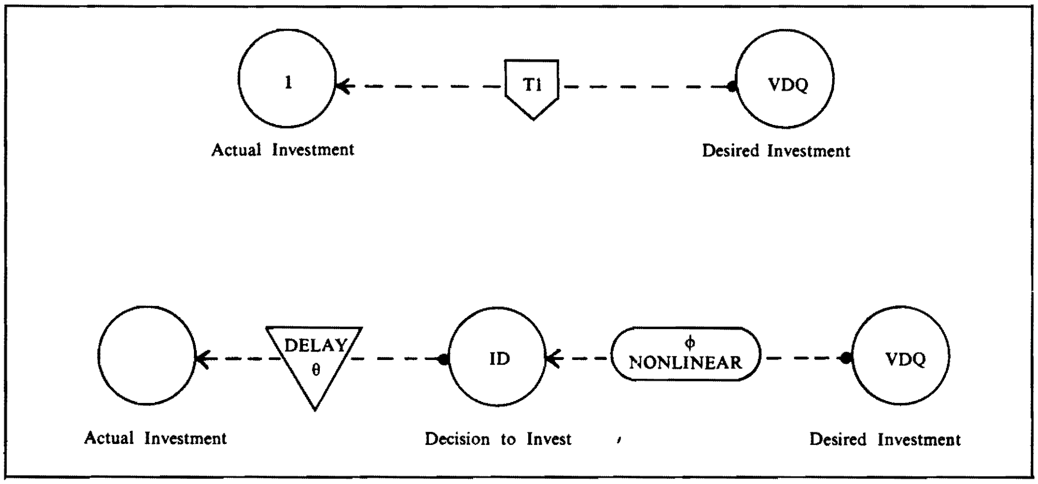


Figure 23. Non-linear Functions that Replace Linear Functions in Economic Models.

A further improvement of Model (51) would be to introduce a two-sectors analysis (production of consumption goods, production of investment goods) in order to include the inventory adjustment mechanism of Figure 16 in Model (51).

The new formulation of the model would then be

$Z_t = C_t + I_t + A$	Total demand
$C_t = \frac{\lambda 1}{D+\lambda_1} cY_t$	consumption demand
$Y_t = \frac{\lambda_2}{D+\lambda_2} Q_t$	total income
$Q_t = QC_t + QI_t$	total production
$QC_t = (INVBAR_t - INV_t) + \frac{\lambda_3}{D+\lambda_3}(C_t + A)$	production of consumption goods
$INVBAR_t = \tau Ct$	desired inventory
$D(INV_t) = QC_t - C_t$	actual inventory
$QI_t = \frac{\lambda_4}{D+\lambda_4} I_t = D(K_t)$	production of investment goods
$I_t = ID_t - \theta$	investment demand
$ID_t = IDMIN \left\| \frac{IDMAX+IDMIN}{(IDMAX)e^{-vDQ_t} + IDMIN} - 1 \right\|$	desired investment

CONCLUSION

This paper has tried to demonstrate that a formal similarity between economic and engineering systems is most useful to economic analysis and that some of the techniques used by engineers apply to economic analysis.

It has also introduced a logic progression in economic models building which consists in setting up a new model from the previous one so that no theoretical information is lost in the process. Naturally this method quickly generates complicated models and a computer approach is then an immense help since one can experiment with complicated systems far beyond the scope of a simple mathematical treat, ment. It is indeed clear that, mathematically speaking-simple multiplier-accelerator models are sufficiently complex before the addition of such factors as government policies, external trade, monetary mass, prices, and the interest rate, all of which, however, must be introduced if a rather comprehensive and usable economic system is to be defined. Furthermore, non-linear elements, which complicate the mathematical treatment of a model, must appear within economic systems.

Consequently, it seems very important for the economist to go over the simplification of macro-economic systems in order to represent them by second order difference or differential models and to become familiar with the language and techniques of engineers in order to generate this cross-fertilization process which is so much needed in social sciences.

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