INFLUENCE OF RANDOM MEDIA ON THE POLARIZATION OF ELECTROMAGNETIC WAVES

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الخلاصة

تم تقديم بعض الصيغ البسيطة التي تصف كيف أن حالة استقطاب موجة مستوية تنتقل عبر الوسط العشوائي تتغير بفعل تقلبات الوسط نفسه .

ABSTRACT

Simple formulas are given which describe how the state of polarization of a plane wave traveling through a random medium is changed by the random fluctuations of the medium.

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INTRODUCTION

When an electromagnetic wave travels through a random medium the wave is depolarized progressively by the random fluctuations in the medium's refractive index. These random fluctuations cause energy to be transferred from the wave's polarized part to the wave's unpolarized part. Accordingly, the wave's degree of polarization decreases with distance and asymptotically approaches zero at infinity.

In this paper we shall review this phenomenon from a quantitative viewpoint.

DESCRIPTION OF RANDOM MEDIUM

Let us consider a partially polarized plane wave that is normally incident on a slab of dielectric whose index of refraction n is given by

$$n = n_o(1+\varepsilon) \qquad (1)$$

where n_o is the average value of the refractive index and ε is its fluctuating part [1]. We assume that ε is small, i.e., $|\varepsilon| \ll 1$, and its average value is zero, i.e.,

$$\langle \varepsilon \rangle = 0 \qquad (2)$$

We use Cartesian coordinates x,y,z. The slab occupies the region $0 \leqq z \leqq d$ where d is the thickness of the slab. The wave travels in the z-direction and is independent of the transverse coordinates x and y.

The variance of the fluctuations is $\langle \varepsilon^2 \rangle$. The correlation length of the fluctuations is l. The incident wave is quasi-monochromatic, i.e. narrow-band, and the magnitude of its mid-band wave vector is k. The autocorrelation function of ε is taken to be Gaussian. The depolarization length is $1/\alpha$ where

$$\alpha = \frac{\langle \varepsilon^2 \rangle}{(k\, l)^2 l} \qquad (3)$$

For our present purpose the random medium is described by the parameter α.

COHERENCY MATRIX

The components of the electric vector of the quasi-monochromatic wave are

$$E_x = \mathrm{Re}\{A_x \exp(-i\omega t\},\ E_y = \mathrm{Re}\{A_y \exp(-i\omega t)\}),$$
$$E_z = 0 \qquad (4)$$

where ω is the mid-band frequency and where A_x and A_y are the phasor components of the electric vector. In keeping with the quasi-monochromatic nature of the wave, A_x and A_y are slowly varying functions of time, slow compared with $\exp(-i\omega t)$. In terms of these phasor components the coherency matrix of the wave is given by [2]:

$$J = \begin{bmatrix} J_{xx} & J_{xy} \\ J_{xy} & J_{yy} \end{bmatrix} = \begin{bmatrix} \langle A_x A_x^* \rangle & \langle A_x A_y^* \rangle \\ \langle A_y A_x^* \rangle & \langle A A_y^* \rangle \end{bmatrix} \qquad (5)$$

The matrix is hermitian, i.e. $J_{xy} = J_{yx}^*$. If A_x and A_y are physically independent, then $\langle A_x A_y^* \rangle = 0$.

TRANSMISSION EQUATIONS

How does each of the components of the coherency matrix vary with z as the wave penetrates the random medium? We answer this question by assuming that the random medium occupies the entire half-space $z \leqq 0$ and then examining the transmission equations for the coherency matrix in the half-space.

From the supposition that the depolarization is a diffusion process with α as its diffusion constant, and from the limit condition that the wave must be completely unpolarized at $z=\infty$ it can be reasoned that the transmission equations are:

$$J_{xx}(z) = \tfrac{1}{2}\{1+\exp(-2\alpha z)\}\, J_{xx}(0) + \tfrac{1}{2}\{(1-\exp(-2\alpha z)\} J_{yy}\,(0)$$

$$J_{xy}(z) = \exp(-2\alpha z)[\tfrac{1}{2}\{1+\exp(-2\alpha z)\} J_{xy}(0) + \tfrac{1}{2}\{1- \exp\,(-2\alpha z)\}\, J_{yx}\,(0)] \qquad (6)$$

$$J_{yx}(z) = \exp(-2\alpha z)[\tfrac{1}{2}\{1-\exp(-2\alpha z)\}\, J_{xy}(0) + \tfrac{1}{2}\{1+ \exp(-2\alpha z)\}\, J_{yx}\,(0)]$$

$$J_{yy}(z) = \tfrac{1}{2}\{1-\exp(-2\alpha z)\}\, J_{xx}(0) + \tfrac{1}{2}\{1+\exp(-2\alpha x)\} J_{yy}(0)$$

These equations express, in terms of the components of the coherency matrix at $z=0$, the components of the coherency matrix at any depth z in the random medium. When $z \to \infty$, we see that J_{xx} and J_{yy} become equal and J_{xy} and J_{yx} approach zero, which means that at $z=\infty$ the wave, as required, is completely unpolarized.

The Stokes parameters S_o, $S_1, S_2 S_3$, are related to the components of the coherency matrix as follows:

$$S_o = J_{xx} + J_{yy},\ S_1 = J_{xx} - J_{yy},$$
$$S_2 = J_{xy} + J_{yx},\ S_3 = i(J_{yx} - J_{xy}) \qquad (7)$$

Hence, from (6) and (7) it is clear that the transmission equations for the Stokes parameters are

$$S_o(z) = S_o(0), \qquad S_1(z) = S_1(0)\exp(-2\alpha z) \qquad (8)$$
$$S_2(z) = S_2(0)\exp(-2\alpha z),\ \ S_3(z) = S_3(0)\exp(-4\alpha z).$$

POLARIZATION

The incident wave can be decomposed into a polarized part and an unpolarized part. The degree of polarization P is defined as the ratio of the intensity of the polarized part to the intensity of the composite wave. In terms of the Stokes parameters, P is given by

$$P = \frac{(S_1^2 + S_2^2 + S_3^2)^{\frac{1}{2}}}{S_o} \quad \text{(degree of polarization)} \quad (9)$$

The range of P is $0 \leqq P \leqq 1$. When $P=0$ the wave is completely unpolarized . When $P=1$ the wave is completely polarized. When $0<P<1$ the wave is partially polarized.

The degree of ellipticity E of the polarized part of the wave is given by:

$$E = \frac{S_3}{(S_1^2 + S_2^2 + S_3^2)^{\frac{1}{2}}} \qquad (10)$$

The range of E is $-1 \leqq E \leqq 1$. When $E=0$ the polarization is linear. When $E=1$ the polarization is L.H. circular. When $E = -1$ the polarization is R. H. circular. When $0<E<1$ the polarization is L.H. elliptical When $-1<E<0$ the polarization is R.H. elliptical.

The degree of linear polarization L of the polarized part of the wave is given by

$$L = \frac{(S_1 + S_2^2)^{\frac{1}{2}}}{(S_1^2 + S_2^2 S_3^2)^{\frac{1}{2}}} \qquad (11)$$

The range of L is $0 \leqq L \leqq 1$. When $L=1$ the polarization is linear. When $L=0$ the polarization is circular. When $0<L<1$ the polarization is elliptical.

Substituting (8) into (9), (10), and (11) we get

$$P(z) = \exp(-2\alpha z)\,[L^2(0) + E^2(0)\exp(-4\alpha z)]^{\frac{1}{2}} \quad (12)$$

$$E(z) = \frac{E(0)}{[E^2(0) + L^2(0)\exp(4\alpha z)]^{\frac{1}{2}}} \qquad (13)$$

$$L(z) = \frac{L(0)}{(L^2(0) + E^2(0)\exp(-4\alpha z)^{\frac{1}{2}}} \qquad (14)$$

If the incident wave is completely unpolarized, the wave has no polarized part and hence these equations are not applicable. In this case we, of course, have $P=0$. However this result does not follow from (12); it follows from (9) and from the fact that for a completely unpolarized wave $S_1 = S_2 = S_3 = 0$ and $S_o \neq 0$.

If the incident wave is partially polarized and the polarized part is circularly polarized, i.e. $L(0)=0$ and $E(0)=1$ or $E(0)=-1$, then we see from (12), (13) and (14) that

$$P(z) = \exp(-4\alpha z),\ E(z) = \pm 1,\ L(z) = 0 \qquad (15)$$

As the wave penetrates into the random medium, the circularly polarized part of the wave remains circularly polarized but its intensity decreases to zero. If the polarized part of the incident wave is elliptically polarized, then $L(0)=0$ and $E(0)=0$. We see from (12), (13), and (14) that

$$P(z) \rightarrow L(0)\exp(-2\alpha z)$$
$$E(z) \rightarrow \frac{E(0)}{L(0)}\exp(-2\alpha z) \qquad (16)$$
$$L(z) \rightarrow 1$$

as $z \rightarrow \infty$. The polarization ellipse becomes thinner with distance and eventually turns into a straight line along the major axis of the ellipse. Also the polarized part of the wave becomes weaker and finally disappears completely.

CONCLUSION

From the discussion above, it is clear that a random medium increases the unpolarized part of a wave and decreases the polarized part. This result agrees with the idea the random medium increases the entropy of waves traveling through it.

Moreover, as the wave penetrates the medium, the state of polarization of the polarized part becomes linear. Recalling that the polarized part of a wave is the superposition of two circularly polarized waves, one left-handed and the other right-handed, one can reason that the random medium tends to equalize the two circularly polarized waves so that they will become of equal amplitude and thus will yield a linearly polarized wave.

REFERENCES

[1] V.I. Tatarski, *Wave Propagation in a Turbulent Medium.* New York: McGraw-Hill, 1961.

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