# EFFICIENT DESIGN OF COMPOSITE STEEL-CONCRETE I-BEAMS FOR SHORED CONSTRUCTION

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## INTRODUCTION

Composite steel–concrete construction in which a portion of the reinforced concrete slab acts as an integral part of the steel beam is used widely in building construction, owing primarily to its structural efficiency and economy. The design of a composite steel–concrete beam can be furnished following the guidelines of a building code such as, for example, the AISC specification [1] and a procedure similar to the one described in [2, 3]. I-shaped steel beams, either rolled or built-up, are commonly used. In recent years the growing interest in structural optimization has led designers to seek practical methods that can be used to find an economical design. While considerable work has been done in the area of optimization of non-composite built-up I-beams, of which [4–6] can be cited as a representative sample, design oriented work in the area of composite steel–concrete I-beams is limited. In [7], geometric programming is used to arrive at an optimum composite I-section.

The objective of this paper is to present an iterative search technique and to provide design aids that can be used to determine rapidly an economical built-up composite I-section for shored construction. The beam is considered homogeneous, having an unstiffened web. It is further assumed that deflection of the beam under a live load would not control the design. Design charts on the basis of the design method of the AISC specification [1] are presented to proportion readily only unsymmetrical sections, since for a built-up composite I-section such a design would minimize material.

## THEORETICAL CONSIDERATIONS

Referring to a typical unsymmetrical I-section as shown in Figure 1, the total cross-sectional area $A$ is

$$A = A_{ft} + A_{fb} + A_{w}. \tag{1}$$

The flange areas and the web area can be related to the total area by

$$A_{ft} = C_1 A \tag{2a}$$

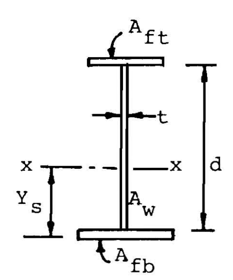


*Figure 1. Typical Unsymmetrical I-Section*

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$$A_w = C_2 A \tag{2b}$$

$$A_{fb} = C_3 A \tag{2c}$$

where

$$C_1 + C_2 + C_3 = 1.0. \tag{3}$$

As the thickness of the flange plates is small compared with the web depth, $d$, the moment of inertia about the $x$-axis, $I_x$, and the distance of the neutral axis from the bottom, $Y_s$, can be expressed as

$$I_x = \frac{Ad^2}{4}[C_1(C_2+2C_3)^2 + C_2/3 + C_2(C_3-C_1)^2 + C_3(2C_1+C_3)^2] \tag{4}$$

and

$$Y_s = \frac{d}{2}(1 + C_1 - C_3). \tag{5}$$

For a typical composite section as shown in Figure 2, the equivalent steel area to replace the concrete slab in the transformed all steel section is $b_e h/n$, where $b_e$ is the effective width of the slab and $n$ is the modular

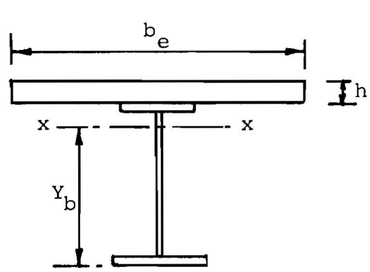


*Figure 2. Composite I-Section*

ratio. Expressing this equivalent area $A_c$ as $A_c = \alpha A (\alpha > 0)$, the location of the neutral axis, $Y_b$, and the moment of inertia of the transformed section, $I_{xc}$, can be written as

$$Y_b = \frac{d(2C_1 + C_2 + 2\alpha) + \alpha h}{2(1+\alpha)} \tag{6}$$

and

$$I_{xc} = A[d^2/4\ \{C_1(C_2+2C_3)^2 + C_2/3 + C_2(C_3-C_1)^2 + C_3(2C_1+C_2)^2\} + \frac{\alpha}{4(1+\alpha)}\{d(C_2+2C_3)+h\}^2 + \alpha h^2/12]. \tag{7}$$

### Bending Requirement

The elastic section modulus, $S_{tr}$, of the composite beam with respect to the bottom fiber of the section is

$$S_{tr} = I_{xc}/Y_b \tag{8}$$

which, in accordance with the design provision of [1], must at least equal $M/F_b$, $M$ being the design moment and $F_b$ the allowable tensile stress in bending.

Introducing $\beta = h/d$, Equations (7) and (8) lead to the following requirement for a safe design

$$\frac{Ad^2}{4}\left[C_1(C_2+2C_3)^2 + C_2/3 + C_2(C_3-C_1)^2 + C_3(2C_1+C_2)^2 + \frac{\alpha}{(1+\alpha)}\{C_2+2C_3+\beta\}^2 + \frac{\alpha\beta^2}{3}\right] \geqslant MY_b/F_b. \tag{9}$$

### Shear Requirement

Following the work of [5], the allowable shear capacity of an unstiffened web in accordance with the AISC specification can be plotted against $d/t$ values as shown in Figure 3. As observed from Figure 3, for a given design shear $V$, an appropriate web thickness can be selected which would carry the shear force within a range of $d/t$ ratios bounded by an upper and a lower limit on the values of $d$. In view of the fact that the compressive stress in the top flange is much less than the bottom flange tensile stress, the value of $d/t$ can be limited to a maximum of 260, provided that the furnished web area is adequate to sustain the design shear.

## SEARCH PROCEDURE

Using a simple iterative search technique, it is possible to identify the minimum weight design within the feasible design space. For this purpose, an acceptable minimum web thickness and a small value of $C_1$ are selected first. As the concrete flange provides a relatively large area, only a small top steel flange area that is feasible and acceptable from a practical viewpoint should be considered. A minimum value of $C_1 = 0.15$ is adopted in this work.

For the selected web thickness, $t$, the minimum and

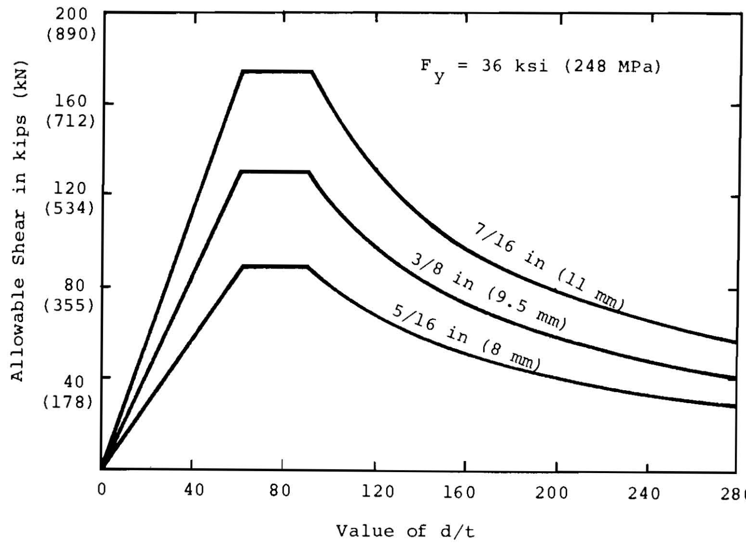


*Figure 3. Allowable Shear for Unstiffened Web*

maximum values of $d/t$, $\varepsilon_1$ and $\varepsilon_2$ respectively, can be determined from Figure 3 (or from formulas prescribed in [5]) to comply with the shear requirement. The objective is then to seek an optimum proportioning on the basis of least area that would satisfy the following constraints in addition to the fulfillment of the condition given in Equation (9);

$$\varepsilon_1 \leqslant d/t \leqslant \varepsilon_2 \tag{10a}$$

$$C_1 = 0.15 \tag{10b}$$

$$C_3 \geqslant 0.15. \tag{10c}$$

It is implicitly assumed here that a beam that would satisfy Equation (9) would not yield an unacceptable concrete stress in excess of $0.45f'_c$, $f'_c$ being the ultimate compressive strength of concrete.

The search begins with initial values of $C_1 = C_3 = 0.15$ (minimum permissible) and $d/t = \varepsilon_2$ (maximum). As $C_2 = 0.7$, the area $A$ is known ($A = dt/C_2$). This proportioning is acceptable if it satisfies Equation (9) and would represent the least area corresponding to a trial value of $d/t$. If Equation (9) is not satisfied, the value of $C_2$ is gradually decreased in small steps thus increasing $A$ in small increments until Equation (9) is satisfied. Thus the optimum $C_2$ corresponding to the chosen $d/t$ ratio is determined. Next, the ratio $d/t$ is reduced in small steps, and at each step the corresponding minimum area is determined by varying $C_2$ from the maximum value of 0.7 as before. Thus, for various feasible values of $d/t$ in the design space, the minimum area proportions are generated. From all these admissible designs, the one with the least area (global minimum) is accepted as the optimum design.

It should be noted that the area $A$ is very insensitive to the variation in $d$ in the neighborhood of the minimum area, as depicted in Figure 4. Thus for all practical purposes, the depth $d$ can be varied by a small range in the vicinity of the optimum value, keeping the area essentially unchanged.

## RESULTS

Based on a generalized computer program incorporating the proposed search procedure, the results of optimum proportioning for a wide range of $S_{tr}$ values are presented graphically in Figures 5 and 6 for slab thicknesses of 4 in (100 mm) and 5 in (125 mm). For a required $S_{tr}$, the minimum area $A$ and the web depth $d$ can be obtained directly from these plots for a chosen web thickness. As $C_1 = 0.15$ in all cases, the values of $A$ and $d$ for a selected $t$ would enable the determination of the entire cross section.

In presenting the results in Figures 5 and 6, it has tacitly been assumed that the effective width of the concrete slab, $b_e$, is controlled by the AISC provision of $b_e$ being equal to 16 times the thickness of the slab plus the width of the top flange of the steel beam. A nominal value of 6 in (150 mm) is assumed as the width of the top flange. This is acceptable due to the

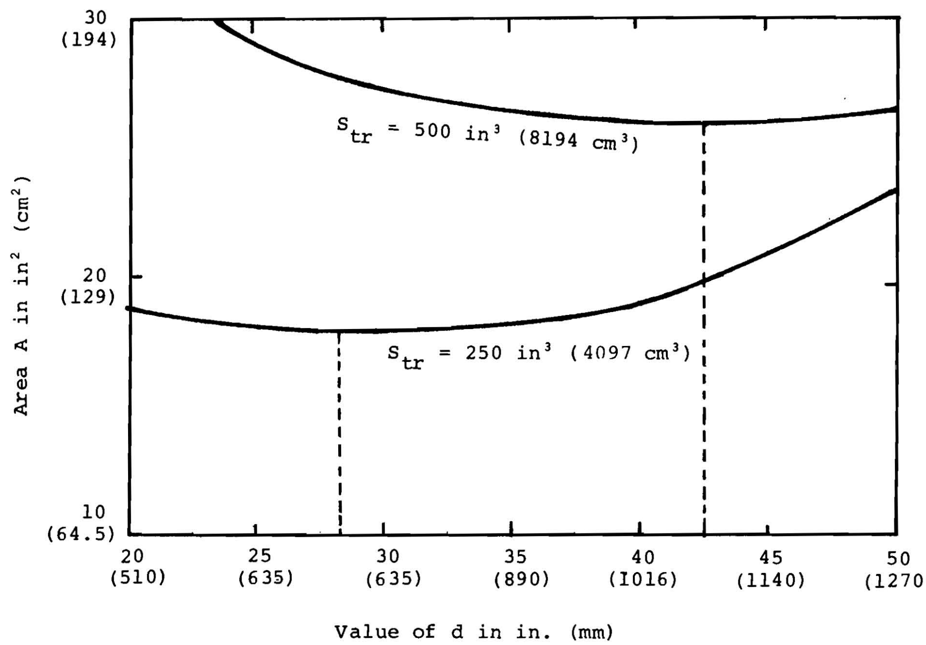


*Figure 4. Sensitivity of A with Respect to d*

fact that a small variation in $b_e$ does not significantly alter the value of $S_{tr}$.

While a typical plot of $A$ is a smooth curve through all data points, that of $d$, however, requires some smoothing. This is possible without an error in $A$, as the value of $A$ remains essentially unchanged with a small variation in $d$ in the neighborhood of the optimum $d$ (Figure 4). Only three web thicknesses, namely $\frac{5}{16}$ in (8 mm), $\frac{3}{8}$ in (9.5 mm), and $\frac{7}{16}$ in (11 mm) are considered in this study, as they cover most designs with built-up sections.

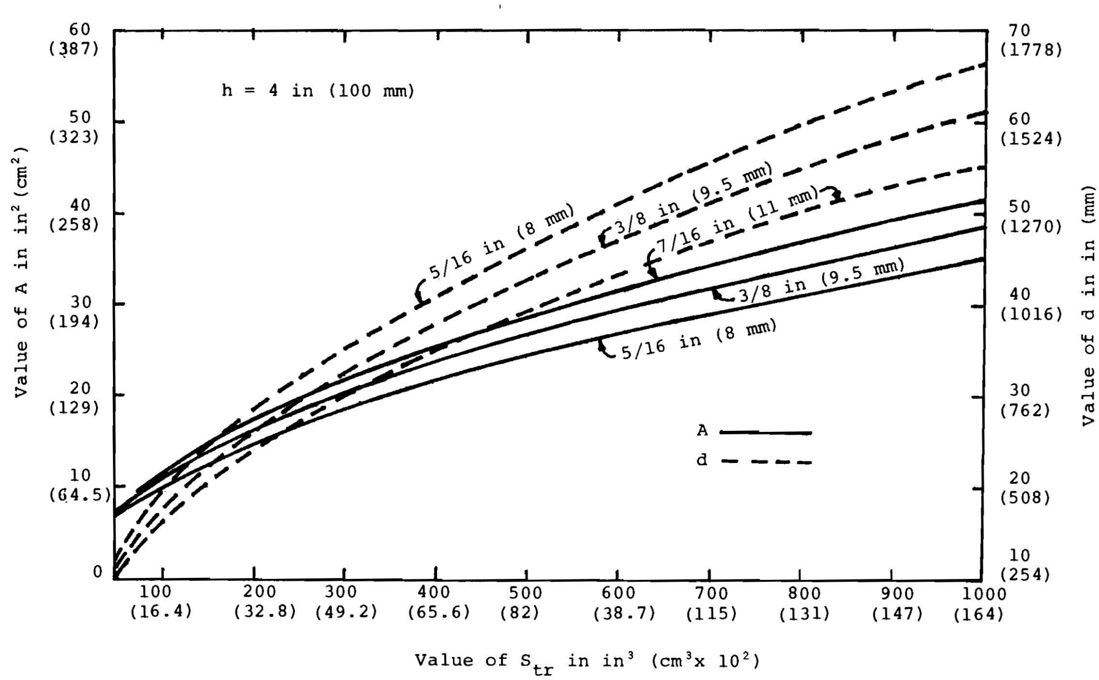


*Figure 5. Optimum Values of A ahd d versus $S_{tr}$ for a Slab Thickness of 4 in (100 mm)*

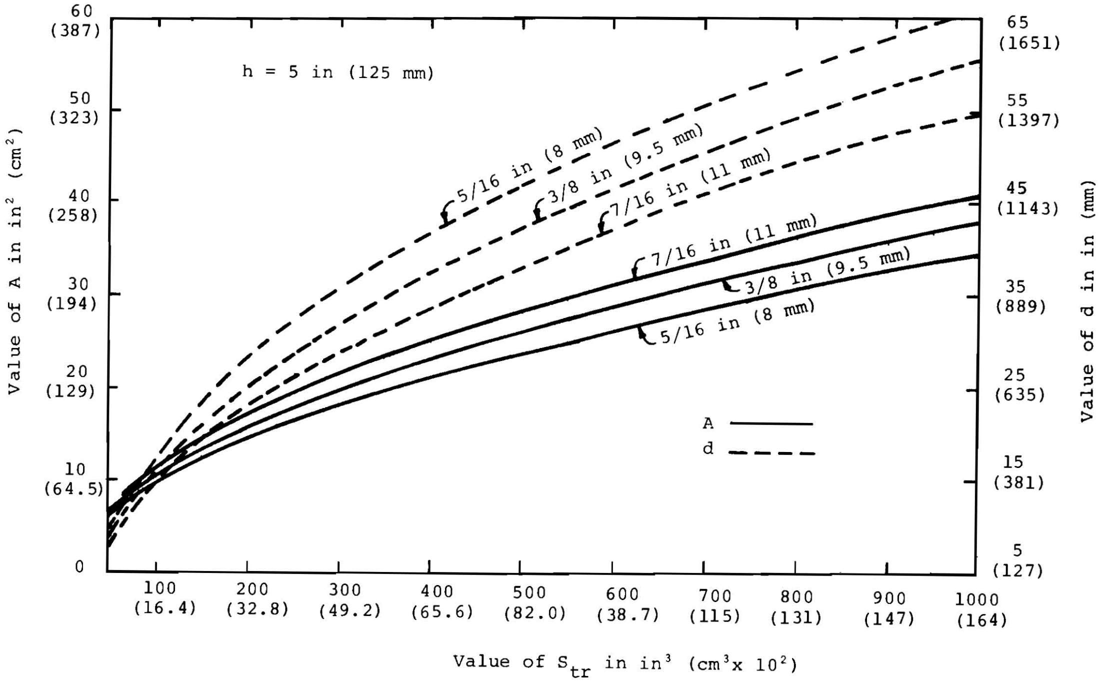


*Figure 6. Optimum Values of A and d versus $S_{tr}$ for a Slab Thickness of 5* in (*125* mm)

## Example

Proportion a built-up steel I-section for a composite beam of 20 ft (6.1 m) simple span. The beam is subjected to a uniformly distributed dead load of 2.0 K/ft (29.2 kN/m) inclusive of self weight and a live load of 1.6 K/ft (23.3 kN/m). Concrete slab thickness = 5 in (125 mm) and the modular ratio = 9. Yield stress of steel, $F_y = 36$ ksi (248 MPa).

Design $M = 180$ ft-K (244 kN·m) and $V = 36$ kips (160 kN). Assuming $\frac{5}{16}$ in (8 mm) is the acceptable minimum web thickness $t$, from Figure 3 $\varepsilon_1 = 25$ and $\varepsilon_2 = 220$. With an allowable $F_b$ of $0.6F_y = 21.6$ ksi (149 MPa) for a noncompact section, the required $S_{tr} = 180 \times 12/21.6 = 100$ in$^3$ (1639 cm$^3$). Entering Figure 6 with this value of $S_{tr}$, $d = 18.0$ in (457 mm) and $A = 9.8$ in$^2$ (63.2 cm$^2$) for $t = \frac{5}{16}$ in (8 mm). The corresponding proportioning gives $A_{ft} = 0.15 \times 9.8 = 1.47$ in$^2$ (9.5 cm$^2$), $A_w = 5.62$ in$^2$ (36.3 cm$^2$) and hence $A_{fb} = 2.71$ in$^2$ (17.5 cm$^2$). Select the top flange plate as $\frac{5}{16} \times 5$ in ($8 \times 127$ mm), the bottom flange plate as $\frac{3}{8}$ in $\times 7\frac{1}{2}$ in ($9.5 \times 191$ mm), and the web plate as $\frac{5}{16}$ in $\times 18$ in ($8 \times 457$ mm).

## CONCLUSIONS

An iterative search procedure has been described to find an economical design for composite steel–concrete I-beams for shored construction based on the allowable stress design method of the AISC specification. The steel beam is assumed to be homogeneous with an unstiffened web. It has been observed that the minimum steel area within the specified constraints is not sensitive to a small variation in web depth in the neighborhood of the optimum web. Results have also shown that a $\frac{5}{16}$ in (8 mm) web thickness would yield the minimum weight design provided that there is no practical limitation on the web height and that the design shear does not critically control the web depth.

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