A COMPARISON OF URBAN RUNOFF LUMPED-PARAMETER MODELS USING SPARSE DATA

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الخلاصة :

مع إزدياد الإهتمام بإمكانية توقيع كمية الصرف السطحي التي يمكن أن تتعرض لها الأحواض المائية الواقعة في المناطق المعمرة من البلاد القاحلة والتي تتعرض لعواصف مطرية غزيرة بصفة غير منتظمة . وقد قام الباحثون بمقارنة النتائج التي يتم التوصل إليها من النماذج بنتائج الطبيعة التي يمكن قياسها . . . فحاول بعض الباحثين عبثاً حساب قيمة العوامل الرياضية التي يشملها النموذج من خصائص الحوض المائي والبعض الآخر إهتموا ببعض النماذج ووصفوها بالدقة بدون إيضاح أسباب لذلك .

وهذا البحث يقوم بمقارنة ثلاث نماذج : نموذج هيدروجراف الوحدة HEC-1 نموذج براند ستتر الخطي مع نموذج براند ستتر غير الخطي وذلك في ١٤ حوض مائي ، وذلك بمعايرة كل نموذج أولاً ثم مطابقته مع نتائج معلومة .

ولقد كانت المعاير سهلة لجميع النماذج ألا أن عملية المطابقة أظهرت عدم الدقة المتأصلة في جميع الطرق والأسس التي بنيت عليها النماذج ، وقد أظهر نموذج براند ستتر أحسن النتائج .

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ABSTRACT

The ability to predict runoff from urban watersheds has become increasingly more important in arid regions which experience infrequent high intensity storms. Work has been done to compare the various lumped-parameter models with data. Some investigators have vainly attempted to calculate the model parameters from watershed characteristics. Others have attributed more accuracy to a particular model than warranted. This paper compares a unit hydrograph model, HEC-l, the linear versions of the Brandstetter model and the non-linear version of the Brandstetter model, for 14 watersheds, by first calibrating each model and then verifying it with another known event. All models were easily calibrated, but the performance in the verification phase demonstrated the inherent inaccuracy of all approaches; however, the linear version of the Brandstetter model performed the best.

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1. INTRODUCTION

The rapid growth of urban centers in Saudi Arabia has caused an increasing need for the planning and design of urban storm drainage for flood control, water harvesting, etc. In arid areas like Saudi Arabia, high-intensity winter storms cause extensive flooding. The U.S. Army Corps of Engineers, among others, has done extensive research into the impact of urbanization on flood hydrograph characteristics [1]. Urbanization increases the magnitude of the runoff and reduces the time to peak of the runoff hydrograph.

To compound the problem, many arid regions lack adequate data to calibrate runoff models. Planners and decision makers must either postpone flood control projects until more data are collected or use what data are available, accepting the inherent inaccuracies that will result.

There are many runoff models, such as the EPA Storm Water Management Model (SWMM) [2, 3], the Dynamic Model for Urban Hydrologic Systems (HYDRA) [4, 5], the Illinois Urban Drainage Area Simulator (ILLUOAS) [5], the University of Cincinnati Urban Runoff Model [6], etc. Readers interested in a review of 18 current models may refer to Ishaq [7]. Other comparison studies may be found in reference [2].

Since the objective of the project was merely to compare the unit hydrograph based models with a nonlinear model, the HEC-1 model and the Brandstetter model were chosen. Though HEC-1 contains many complex routing, infiltration, and optimization methods. It is possible to use only the unit hydrograph portion to simulate runoff for a small catchment. The Brandstetter model likewise can be run with only the runoff generating portion. Moreover, the nonlinear part can be 'turned off' so the linear 'curve fitting' part may be run separately.

There are two general classes of runoff models: lumped parameter and distributed parameter. The former 'lumps' all the characteristics of a catchment into one or two parameters. Examples of these are the unit hydrograph techniques and curve (function) fitting methods. Distributed parameter models attempt to measure each significant factor that effects runoff, such as catchment slopes, amount of impervious area, rate of infiltration, etc. The first and most famous example of this type is the Standford watershed model. The distributed parameter models will not be covered in this paper because they have already been treated by Ishaq [7] and the amount of data needed to run them is often not available.

Lumped parameter models may be further classified as linear or nonlinear. Linear models assume the principles of super position and proportionality [8] while nonlinear models do not. The limitations of linear models are:

1. The model may give only a peak discharge and does not reproduce the entire runoff hydrograph;
2. The model may not reflect the nonlinearity that can exist in many watersheds.

The major advantages of the Brandstetter model [9] to be used in this study are that the model does produce a complete runoff hydrograph, and that it can take into account the nonlinearities that might be inherent in a catchment. Consequently, this study compares the ability of the nonlinear version of the Brandstetter model both to reconstitute known storm events and to predict runoff from future storm events with the results obtained by the linear version of the Brandstetter model and a commonly-used unit hydrograph model, HEC-1.

To conduct this study, storm event information was collected by literature search for several different small, urban watersheds, most in semi-arid regions. These data were then used to calibrate and verify the three runoff models. The model performances were then compared in terms of their ability to predict hydrograph peak, shape, and volume. This is called 'split sampling'.

2. INTRODUCTION TO THE BRANDSTETTER MODEL AND HEC-1

The basic purpose behind rainfall–runoff models is to transform a given rainfall into runoff. The nonlinear (functional series) model used in this study is the model developed by Brandstetter and Amorocho [9] and modified by Finch [2]. This model can be classified as a nonlinear lumped parameter rainfall–

runoof model. The model has the following characteristics:

1. Since runoff is directly related by a mathematical equation to the rainfall, there is no need to separate the base flow component from the runoff hydrograph before beginning the analysis;
2. The model assumes that there is no external input of surface water and that groundwater flow components are small;
3. All components of the hydrologic cycle are lumped and are independent of spatial distribution and therefore they are assumed to be uniformly distributed over the watershed. This assumption best suits frontal storms.

The Brandstetter model uses a functional series to represent the input/output response of the watershed. This sum of integral functions can be considered a nonlinear generalization of the convolution integral [8, 9]:

$$y(t)=h_0+\int_0^\infty h_1(s_1)x(t-s_1)ds_1 \quad (1)$$
$$+\int_0^\infty\int_0^\infty h_2(s_1s_2)x(t-s_1)x(t-s_2)ds_1ds_2$$
$$+\cdots$$
$$+\int_0^\infty\cdots\int_0^\infty h_n(s_1,\ldots,s_n)x(t-s_1)\ldots x(t-s_n)ds_1\ldots ds_n$$
$$+\cdots$$

where:

$h(s_1,\ldots,s_n)$ = continuous n-th order response function,
s = continuous variable of lag time,
t = continuous variable of time,
$y(t)$ = continuous system output,
$x(t)$ = continuous system input.

The first term in Equation (1) represents an internal source or sink and for most natural watersheds this term is zero. For modeling purposes, experience has shown that this series can be truncated at the third term with no loss of accuracy.

For practical applications, a system can be assumed to have a finite memory. The system output then will depend only on the input between some past time and the present. Equation (1) then becomes:

$$y(t)=\int_0^u h_1(s_1)x(t-s_1)ds_1+\int_0^u\int_0^u h_2(s_1,s_2)x(t-s_1)x(t-s_2)ds_1ds_2 \quad (2)$$

where:

u = length of memory for a continuous system, and all other terms are as previously defined.

Since the rainfall input data is a discrete series of points, Equation (2) is transformed to its discrete form:

$$Y\{T\}=\sum_{S_1=0}^{U} H_1(S_1)X(T-S_1)+\sum_{S_1=0}^{U}\sum_{S_2=0}^{U} H_2(S_1,S_2)X(T-S_1)X(T-S_2) \quad (3)$$

where,

$H_1(S_1)$ = discrete linear system response function,
$H_2(S_1,S_2)$ = discrete nonlinear system response function,
T = discrete variable of time
U = system memory (a positive integer)
$X\{T\}$ = discrete system input,
$Y\{T\}$ = discrete system output.

Equation (3) can now be directly used to find the kernel values. This is done by solving a set of linear equations defined by [9]:

$$[Y]=[H][X] \quad (4)$$

where,

$[Y]$ = an $\{N\}$ row matrix of N observations of stream flow values,
$[X]$ = an $\{L\}$ row matrix of L unknown values of the system response functions,
$[X]$ = an $\{L\times N\}$ square matrix of rainfall input $S_1(T-S_1)$ and its products $X(T-S_1)X(T-S_2)$.

It is now assumed that the kernels can be expanded into a finite series with M_1+1 and M_2+1 terms respectively [9]:

$$H_1(T_1)=\sum_{i=0}^{M_1} a_iP_i(T_1) \quad (5)$$

and,

$$H_2(T_1,T_2)=\sum_{i=0}^{M_2}\sum_{j=0}^{M_2} a_{ij}P_i(T_1)P_j(T_2) \quad (6)$$

where,

M_1 = the order of expansion for the linear term of the kernel function,
M_2 = the order of expansion for the nonlinear term of the kernel function,
$P(T)$ = function of series expansion of a discrete function,
a_i = coefficient of series expansion of discrete linear response function.

In these terms, a_i and a_{ij} are coefficients to be determined and $P_i(T)$ is a known function of T chosen for the expansion. The function $P_i(T)$ used by Amorocho and Branstetter in this model is the Meixner function, which was found to give good hydrograph reproduction during model development.

If we let:

$$a_i(T) = \sum_{S_1=0}^{U} P_i(S_1)X(T-S_1) \quad (7)$$

for the linear term, and;

$$a_j(T) = \sum_{S_1=0}^{U} P_i(S_1)X(T-S_2) \quad (8)$$

for the nonlinear term, Equation (3) can be written as:

$$Y\ T = \sum_{i=0}^{M1} \alpha_i a_i\ (T) + \sum_{j=0}^{M2} \alpha_j a_j^2\ (T)$$

$$+2\sum_{i=i}^{M2}\sum_{j=0}^{i=1} \alpha_{ij} a_i\ (T) a_j(T) \quad (9)$$

This equation is linear with respect to the α coefficients of both orders of the discrete form of the convolution integral shown here. These α values are computed by performing a linear multiple least-squares regression analysis on the rainfall–runoff input to the model. This minimizes the error of the output prediction. The matrix equation shown as Equation (4) can now be transformed into [2]:

$$[Y] = [\alpha][a] \quad (10)$$

where,

$[Y]$ = as defined in Equation (4),
$[\alpha]$ = a matrix of α coefficients,
$[a]$ = a matrix of $a(T)$ coefficients.

The response function can then be calculated from the expansions of in terms of the α coefficients, which can now be found from:

$$[\alpha] = [Y]\ [a]^{-2} \quad (11)$$

The Brandstetter model can also be used to predict the response of systems that are predominantly linear. This is done by using only the first order or linear response function in the series shown in Equation (9). The mathematical reduction of the series to the matrix form is the same as in the nonlinear case. The response function is still calculated in terms of α coefficients using the Meixner function in expansion.

The second hydrologic model used in this study to analyze the response of urban watersheds to rainfall is the computer model HEC-1 [10–12]. HEC-1 is one of a series of hydrologic models developed by the U.S. Army Corps of Engineers Hydrologic Engineering Center. The model can be characterized as a linear, lumped parameter, single event rainfall–runoff model. HEC-1 can be used on either rural or urban catchments with or without snow melt. It can be used to:

1. Optimize the routing parameters in a river reach, given inflow and outflow hydrographs and reach parameters,
2. Optimize the rainfall loss rate equation and find the unit hydrograph for a basin, given the rainfall–runoff data from a historical event,
3. Perform a generalized streamflow network analysis given local runoff to subareas within the basin and a specified routing method through the basin,
4. Develop a set of depth–area storm hydrographs, given a depth–area precipitation relationship for a watershed and a precipitation pattern.
5. Perform multiplan economic analysis given storm, runoff, and routing characteristics for the stream reaches in a basin.

For comparison with the results of the Brandstetter model, the HEC-1 subroutine to optimize loss rate and unitgraph parameters was used to reconstitute storm events from urbanized catchments. These optimal parameters were then used to predict runoff from other storm events in the basins. The results were again compared with the Brandstetter model predictions.

The unit hydrograph used in the basin can be either user supplied or computed using Clark method by specifying the appropriate Clark or Snyder coefficients. A more detailed explanation of the Clark unitgraph method can be found in reference [11].

The runoff hydrograph recession and base flow parameters are empirically determined functions related to basin characteristics. The total runoff hydrograph is then the sum of the unitgraph convolution and the computed base flow.

3. DATA COLLECTION

In order to compare the models of small urban watersheds, it was felt that any data found in the literature should meet the following two criteria: (1) the drainage area of the watershed should be less than 20 square miles; and (2) the watershed should be at least 10 percent urbanized as defined by the amount of impervious cover. Using these criteria, the watersheds given in Table 1 were chosen for further analysis [13, 14].

To analyze accurately the rainfall–runoff response functions of a watershed, it is necessary to gather storm data that would reflect all possible combinations of antecedent moisture conditions, areal distribution, seasonal variations, and storm trackings that could occur in the basin [15]. Since this type of data is seldom (probably never) available, the storms chosen for the analysis should reflect the general types of storms that can occur in the basin, in this case, frontal storms.

While the optimal approach to determining the response function of a basin is to use a large number of storms in the analysis, in practice, the usual procedure is to use what data is available. Since one of the purposes of this paper is to compare models with minimum data, at least two storm events are required for each basin. This allows for the two-step modeling process of calibration and verification.

The time interval of data was a much more important consideration in the data reduction process. The watershed storm data had sampling intervals varying from 2 to 15 minutes. The choice of time interval had to be such that there was not excessive 'noise' or oscillations in either the linear or the non-linear kernels of the functional series model. Experiments by Hossain [16] on similar types of functional series models have shown that there is no excessive noise in the kernel response for time intervals as short as 4 minutes. Thus, for the purpose of formatting consistency and so that there would be additional data points in the shorter duration storm events, a time interval of 5 minutes was chosen as the standard time step for all model runs and, all data were converted to that standard.

4. RESULTS AND CONCLUSIONS

The rainfall–runoff data from the fourteen urbanized watersheds listed in Table 1 were used in the two-step

Table 1. Basin Physiographic Characteristics

No.	Basin	Data source	Drainage area (square miles)	I 96	L (miles)	L (miles^{ca})	S (ft/mile)	Γ
1	Waller Creek at 38th Street	USCE	2.31	33	4.37	1.75	49	1.09
2	Waller Creek at 23rd Street	USCE	4.13	35	5.24	1.90	48	1.44
3	Turtle Creek	USCE	7.98	47 (37)**	5.91	2.78	28	3.10
4	Hunting Bayou	USCE	3.92	20	2.41	1.58	6	1.55
5	Boneyard Creek	USCE	4.46	37	2.84	1.32	10	1.19
6	Big Dry Creek	USGS	0.95	25 (15)	2.16	1.02	90	0.23
7	36th Street Storm Drain	USGS	3.50	65 (40)	*	*	*	*
8	Spring Harbor	USGS	3.29	21 (16)	3.74	1.43	40	0.84
9	Willow Creek	USGS	3.16	29 (24)	2.96	1.50	34	0.76
10	Olbrich Park	USGS	2.36	17 (9)	2.79	1.38	50	0.54
11	Warner Park	USGS	0.58	28 (25)	1.15	0.11	78	0.01
12	El Modena-Irvine Storm Channel	Pedersen	11.9	40	6.35	4.51	52	3.97
13	Victoria Street Storm Drain	Pedersen	0.61	22	2.18	1.10	3.21	0.13
14	Castro Valley	USCE	5.00	70	4.51	1.83	1.79	0.61

**() USGS effective impervious
*Data not available

calibration/verification process. The first storm was used to define the improtant model parameters and the second storm was used to test the accuracy of the original parameter optimization. The results of both the calibration and verification runs were then compared wtih similar results from the linear version of the Brandstetter model, the nonlinear version of the Brandstetter model and HEC-1.

The general procedure for calibrating the nonlinear Branstetter model involved the optimization of three model parameters:

U – the system memory,
M1 – truncation of the first order Meixner function,
M2 – truncation of the second order Meixner function.

This optimization can be done by using either a trial-and-error technique or by an internal subroutine developed by Finch [17].

When the linear Brandstetter model is used, the procedure for selecting model parameters is somewhat simplified in that only system memory (U) and the first-order Meixner truncation (M1) need to be optimized. At present, the only way to achieve the 'best fit' U and M1 for the linear model is through the use of a trial-and-error method. The model optimization subroutine will always choose the nonlinear model as the best solution in calibration runs. However, as will be seen later, the nonlinear model does not always yield the best verification/prediction results with limited data. This investigation used both measures to evaluate which model most nearly reproduced the watershed response function.

The HEC-1 calibration procedure is well documented in reference [11].

The results of the calibration are interesting for several reasons:

1. All models gave extremely good reproduction of storm volume during the calibration runs.
2. The peak discharge error for all models is also very low but the nonlinear model gave the best result with an average error of 1.75%; the linear model average error was 5.77%. The HEC-1 model was the worst with an average error of 10.95%. The good fit of the nonlinear model is not surprising because it has more parameters with which to fit any given curve. A note of caution: just considering the error of peak discharge and volume may be misleading because the shapes of the computed and observed hydrograph may be dissimilar; consequently, these should be examined visually.
3. For the nonlinear model, the optimization subroutine was able to choose reasonable values for M1, M2, and U in only 6 of the 14 test storms. Thus, it is imperative that optimization routine results be carefully examined.
4. For the nonlinear model, several of the watersheds (Turtle Creek, Hunting Bayou, El Modena-Irvine Storm Channel, and Castro valley) gave unusually high values for α_0. This was the result of inadequate data on the recession characteristics of basin storm hydrographs. By checking the flow conditions on these basins during period of no precipitation, it was found that the recession limbs of the runoff hydrographs in the literature had been truncated before returning to base flow levels. Therefore α_0's were higher then if the hydrograph data had been complete.
5. The nonlinear model showed a tendency to miss flash peaks that occurred early in the storm, (see Figure 1).
6. The most noticeable result in calibrating the nonlinear Brandstetter model is the occurrence of several negative α_0 values. This caused the model to predict negative flow values during low flow periods. This phenomenon was most evident in the Waller Creek at 38th Street and Waller Creek at 23rd Street calibration results.

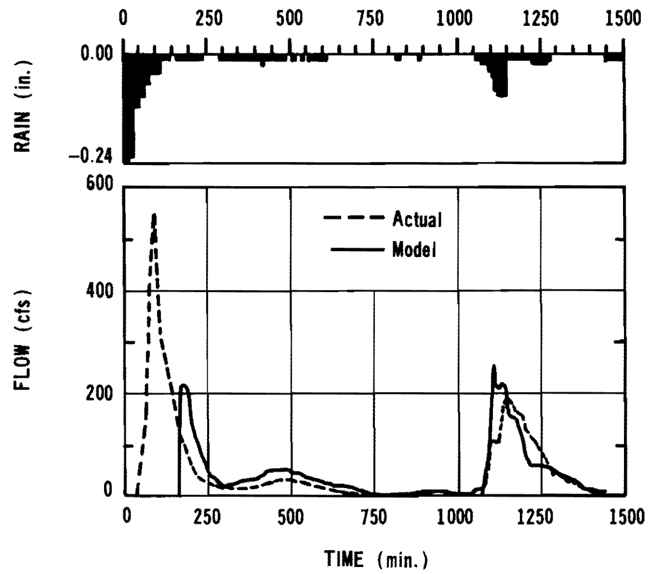

Figure 1. Brandstetter Nonlinear Model Verification of Waller Creek

Table 2. Verification Results

No.	Basin	Storm	Storm	Nonlinear BAFSM			HEC-1			Linear BAFSM		
		Date	Peak cfs cfs	% Error peak	% Error volume	Model peak cfs	% Error peak	% Error volume	Model peak cfs	% Error peak	% Error volume	Model peak cfs
1	Waller Creek 38th St.	8/28–29/74	551	31.9	56.2	254	90.9	105.21	1 052	112.4	32	413
2	Waller Creek 23rd St.	10/21–22/74	1 110	25.4	15.0	1 074	5.57	7.15	1 183	7.67	4.17	1 195
5	Boneyard Creek	8/18–19/66	418	[2]	[2]	[2]	206	266	1 282	24.4	30.8	520
7	36th St. Storm Drain	8/1/76	150	[3]	[3]	[3]	281	198	572	[3]	[3]	[3]
8	Spring Harbor	8/13–14/76	144	[2]	[2]	[2]	319	387	604	18	22	171
9	Willow Creek	8/12–14/76	766	[2]	[2]	[2]	14.1	112	874	−46	−8.77	416
10	Olbrich Park	8/13–14/76	99	47.1	17.3	146	146	178	244	−30	−17.4	124
11	Warner Park	8/13–14/76	175	97.7	182	347	120	176	385	−28.8	−17.4	124
14	Castro Valley	2/5–6/73	496	130	11.8	1 141	18.3	18.6	587	−6.45	−20.9	464

[1] Major peak occured during system memory
[2] Percent Error Peak greater than 500
[3] Storm Peak occured during system memory

Despite the negative flow values, the linear model runs resulted in good hydrograph shapings. (The model plotting routine treats negative flow values as zero.)

7. The HEC-1 model varied more when predicting peak discharges. Moreover, the shape of the HEC-1 predicted hydrographs were less precise than the non-linear model hydrograph. The most apparent difference was on storms with multiple or elongated peaks such as Castro Valley, 36th Street Storm Drain, and El Molina Storm Channel. HEC-1 performed best when the calibration storm had a large single peak.

To test how well the three calibration models would predict runoff from the sample basins, a second set of storm data from 9 of the 14 watersheds was used. The 9 watersheds and the date of the storm events are given in Table 2.

The performance of the nonlinear model in the verification runs was very erratic. In only two of the 9 basins was the storm reproduction within reasonable limits. In several cases, the predicted runoff was orders of magnitude greater than the observed event. The model again had trouble predicting early large peaks in the runoff hydrograph. The results of the nonlinear Brandstetter model verification are shown in Table 2 and actual storm hydrographs along with the model predictions are given in Figures 2a and 3a.

The verification results of the linear Brandstetter model, while still somewhat erratic, were a large improvement over the nonlinear verification hydrographs. In no case were the linear Brandstetter model predicted hydrographs totally unreasonable. However, there were several negative flow predictions in most of the model runs. The test results in terms of percent error peak, percent error volume and actual storm peak versus model prediction are shown in Table 2. Figures 2 and 3 show some selected results.

The third set of values in Table 2 show the results of the HEC-1 verification. Again, the results are very erratic with peak discharge error varying from 6.75% for Waller Creek at 23rd Street to 387% for the Spring Harbor data. The same range of error was observed for volume predictions. While the results of the HEC-1 runs are disappointing, they were not totally unexpected since many of the HEC-1 model parameters are storm-dependent.

To illustrate the variability of the model optimization from storm to storm, a second calibration run was done on the four Madison, Wisconsin basins, using the storm of August 13–14. As expected, there was a great deal of variation in the model parameters. The variation is greatest in those variables associated with the loss rate function. Obviously as many storms as available (and reasonable) should be used in calibrations; however, as practitioners know, these kinds of data are frequently unavailable; consequently, the results of any model

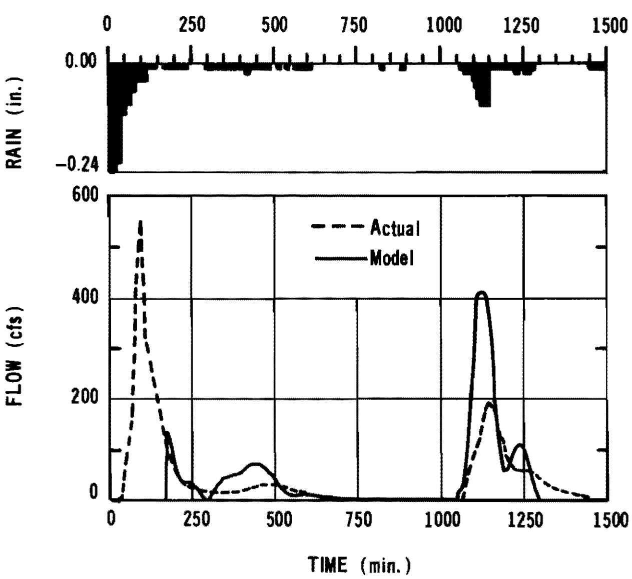

Figure 2. Brandstetter Linear Model Verification of Waller Creek

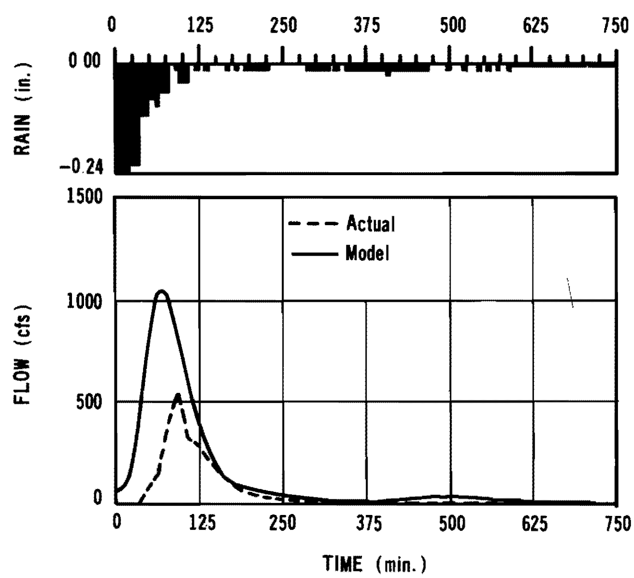

Figure 3. HEC-1 Model Verification of Waller Creek

calibrated with sparse data must be viewed with caution!

From the results of the test on the fourteen basins in this study, the nonlinear Brandstetter model does not seem well suited for predicting runoff from small urban watersheds with sparse data. The model is unable to reproduce the types of flash peaks that can occur in these small basins. However, the model does perform well in predicting more extensive (long duration) storm events (note the results from the Waller Creek at 23rd Street Basin.) This could make the model quite useful for producing time-series data and could be used to fill gaps in runoff records where adequate rainfall data are available.

While the results of none of the models tested could be termed outstanding, the linear Brandstetter model consistently gave better verification results. Outstanding results would not be expected from calibrating on only one storm. However, the linear Brandstetter model exhibited a greater flexibility in its potential applications than the nonlinear model. Again this should not be totally unexpected because small urban watersheds exhibit 'flashy' hydrographs and seem to fit the linear system assumptions better than larger, non-urban watersheds.

The storm dependency of all the models parameters became apparent after the first few calibration/verification runs. This was not totally unexpected. The HEC-1 users manual suggests that extensive regional studies be performed to find suitable values for the model parameters. This is due to the high degree of storm-dependence of these variables.

Finally, attempting to use the Brandstetter model on ungauged watersheds by relating parameters (α coefficients, etc.) to regional equations or watershed characteristics appears unpromising. Even if this does not prove to be true, the variation of the values of memory (U), and the α coefficients for different seasons, land use, soil type, antecedent moisture, and storm caterings would need much more study.

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