DYNAMICS OF THE GENERALIZED MULTIPHOTON JAYNES–CUMMINGS MODEL IN THE LOGARITHMIC STATE

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الخلاصـــة :

تمّت دراسة ديناميكية نموذج (جينز وكمينجز) المتعدد الفوتونات وذلك عندما يكون المجال المشارك في التفاعل عند البداية حيث له توزيع إحصائي لوغاريتمي . وشملت الدراسة كلاً من المؤثرات التي تمثل المجال والذرة . وعلى وجه الخصوص تمت مناقشة ظاهرة التداعي والبعث بالاضافة إلى دراسة التأثيرات الكلاسيكية وغير الكلاسيكية للمجال مثل تعنقد وانفراط الفوتونات وكذلك إمكانية الحصول على موجات ذات تشوش منضغط من التفاعل .

ABSTRACT

The dynamics of the generalized multiphoton Jaynes-Cummings model (J.C.M.) when the mode has initially a logarithmic distribution is investigated. The atomic and the field dynamics are both studied. In particular, the phenomena of collapses and revivals as well as bunching and antibunching and squeezing effects are shown.

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1. INTRODUCTION

The studying of the quantum effects in a system comprising of two-level atom interacting with a single non-decaying electromagnetic field mode inside a cavity lies at the heart of quantum optics. Some of these quantum effects have been investigated experimentally [1, 2] and theoretically [3–7]. The effects of finite cavity damping have also been discussed [8]. In particular, the Jaynes–Cummings model [9] of a two-level atom interacting with a quantized single mode electromagnetic field is at the core of many problems in quantum optics and quantum electronics. The importance of this model lies in that it is perhaps the simplest solvable model that describes the essential physics of radiation–matter interaction. The dynamics of this model is very sensitive to the statistical properties of the field. For example, if the atom is prepared in its excited state and the field is initially prepared in a number state $|n\rangle$, the population oscillates sinusoidally between the ground and the excited state at Rabi frequency, which is proportional to $\sqrt{(n+1)}$ [9]. However, if the field is a superposition of the number states, the oscillations in the population inversion collapse and revive repeatedly, exhibiting apparently chaotic behavior over a long period of time [4–8]. These revival features are due to the quantum nature of the cavity field which manifests itself in the discreteness of the photon number in statistical averages [6]. Field quantization is essential to predict collapse–revival phenomena in a system in a cavity. The nature of this phenomenon has been studied in detail, when the field is in a coherent state [4, 5] and chaotic state [6, 8] as well as when there is a superposition of coherent and chaotic fields, by Puri and Agarwal in 1987 [8]. The effects of squeezing have been discussed by considering the initial state of the field to be squeezed state [7]. The effects of squeezing and a sub-Poissonian photon number distribution on the phenomenon of collapse and revival have been investigated when the field is in a binomial state [10]. The dynamics of the JCM in the logarithmic state has been studied in [11]. Normal squeezing of JCM has been investigated for large mean photon number ($\bar{n} > 10$) [12]. Recently, the time evolution of squeezing in a single-mode single-atom JCM has been examined [13]. The phenomena of collapses and revivals for JCM have recently been observed using Rydberg atom masers in ultra-high-Q cavities [2].

Sukumar and Buck [14] proposed two exactly solvable generalizations of the Jaynes–Cummings model, one involving intensity dependent coupling and the other involving multiphoton interaction between the field and the atom. These models also exhibit periodic decay and revival of atomic coherence. The emphasis, however, has been on the atomic dynamics. On the other hand, Singh [15] has studied the effect of interaction on the field statistics and has shown that the mean photon number may also exhibit periodic decay and revival. The concept of higher order squeezing [16, 17] has also been applied to the generalized multiphoton Jaynes–Cummings model of a single two-level atom interacting with an initially coherent cavity field mode [18, 19]. Very recently, we have investigated JCM and the present model for amplitude-squared squeezing [20, 21].

Recently, the logarithmic state has been introduced and investigated [22]. The dynamics of the JCM when the mode initially has a logarithmic distribution was studied by Mahran and Obada [11]. The temporal behavior of the atomic inversion and dipole moment have been shown. Photon statistical averages such as, mean photon number, the second-order correlation function, and finally squeezing effects have also been studied. In conclusion, we found that the dynamical effects of the logarithmic state initially seem to be similar to the single photon effect (as $|q| \to 0$ with fixed $|c|$, where $|q|$ and $|c|$ are the parameters of the logarithmic state), but, as $|q|$ increases chaotic behavior begins to appear.

In this paper, we continue our investigation for the effects of the logarithmic state [22]. We investigate the dynamics of the generalized multiphoton Jaynes–Cummings model of a single two-level atom interacting with an initially logarithmic cavity field mode. The previous results of JCM in the logarithmic state [11] will be taken into consideration when we discuss the results of the present model. It should be noted that for certain values of the parameters, the logarithmic state becomes a linear combination of the vacuum and the one-photon state [22]. Such a superposition state has been studied recently [23], in the context of squeezing. Moreover, the logarithmic state exhibits squeezing as well as photon antibunching, but this squeezed state is not a minimum-uncertainty product state [22]. The effect of squeezing in this case should therefore be contrasted with those discussed in [7]. In fact, logarithmic state can be viewed as a interpolation between the generalized Bose–Einstein state and coherent state. The latter interpolation is true only as far as the counting distribution, *i.e.* the diagonal elements of the density matrix in the occupation number representation, is concerned [22]. It is important to note that logarithmic states are pure states whereas Bose–Einstein states are mixed states. Finally, logarithmic states have attracted interest because they have the properties of both classical and nonclassical light [22]. It is therefore of interest to investigate the dynamics of the generalized multiphoton Jaynes–Cummings model of a single two-level atom interacting with an initially logarithmic cavity field-mode, because of its rich characteristics.

The plan of this paper is as follows. In Section 2, we derive an expression for the density matrix of the system in the logarithmic state of the field. In Section 3, we use these results to study the evolution of the atomic population inversion and dipole moment. We discuss the statistical properties of the field in Section 4. A summary is contained in Section 5.

2. DESCRIPTION OF THE SYSTEM

We consider a two-level atom interacting with a single-mode radiation field in a lossless resonant cavity *via* a k-photon transition mechanism. The Hamiltonian for this system in rotating wave approximation (RWA) [14] is

$$H = \omega \hat{a}^{+} \hat{a} + \omega_{o} \hat{\sigma}_{z} + \lambda(\hat{\sigma}_{+} \hat{a}^{k} + \hat{a}^{+k} \hat{\sigma}_{-}) \ , \qquad (1)$$

where ω and ω_o are the frequencies of the field and the atom, respectively, λ is the multiphoton atom-radiation coupling constant, k is the photon multiple, $\hat{\sigma}_z$, $\hat{\sigma}_+$, and $\hat{\sigma}_-$ are the atomic pseudospin operators, and $\hat{a}^+$ and $\hat{a}$ are the creation and annihilation operators of the field.

When we take the detuning parameter

$$\Delta = \omega_o - k\omega \ ,$$

and then use the following constants of motion

$$\hat{N} = \hat{a}^{+} \hat{a} + (½) k \hat{\sigma}_{z}, \qquad \hat{C} = \Delta \hat{\sigma}_{z} + \lambda(\hat{\sigma}_{+} \hat{a}^{k} + \hat{a}^{+k} \sigma_{-}),$$

the Hamiltonian becomes solvable [15].

Initially, let the field be in the logarithmic state $|q;c\rangle$ where [22]

$$|q;c\rangle = c|0\rangle + \beta \sum_{n=1}^{\infty} [q^{n}/\sqrt{n}]|n\rangle \ , \qquad (2)$$

with q, c complex numbers with $|c| \leq 1$ and β is defined as

$$\beta^{2} = -(1-|c|^{2})/\log(1-|q|^{2}) \ . \qquad (3)$$

The field density operator $\hat{\rho}_f(t)$ is given by

$$\hat{\rho}_{f}(t) = Tr_{A}\, \hat{\rho}(t) \ , \qquad (4)$$

where $\hat{\rho}(t)$ is the density operator of the system. In the resonance case (*i.e.* $\Delta = 0$) and for the atom initially taken in its excited state, $\hat{\rho}_f(t)$ is found to be [15]

$$\begin{aligned}\hat{\rho}_{f}(t) = \sum_{m,n=o}^{\infty} & [\cos(\lambda t \sqrt{[(n+k)!/n!]}) \\ & \times \cos(\lambda t \sqrt{[(m+k)!/m!]})\ \hat{\rho}_{n,m}(0) \\ & + \sin(\lambda t \sqrt{[n!/(n-k)!]}) \\ & \times \sin(\lambda t \sqrt{[m!/(m-k)!]})\ \hat{\rho}_{n-k,m-k}(0)], \qquad (5)\end{aligned}$$

and $\hat{\rho}_{n,m}$ the initial density operator, is given by

$$\hat{\rho}_{n,m}(0) = P_{n,m}|n\rangle\langle m| \ , \qquad (6)$$

where [11]

$$\begin{aligned}P_{n,m} = & |c|^{2} \delta_{n,o} \delta_{m,o} + c\beta [q^{*m}/\sqrt{m}]\, \delta_{n,o}(1-\delta_{m,o}) \\ & + \beta c^{*}[q^{n}/\sqrt{n}]\, \delta_{m,o}(1-\delta_{n,o}) \\ & + \beta^{2}[q^{n} q^{*m}/\sqrt{(nm)}]\,(1-\delta_{n,o})(1-\delta_{m,o}). \qquad (7)\end{aligned}$$

It is clear from the above equation that logarithmic states have the counting distribution:

$$P_{n,n} = \begin{cases} |c|^{2} & \text{for } n = 0 \\ \beta^{2}[|q|^{2n}/n] & \text{for } n = 1, 2 \ldots. \ , \end{cases} \qquad (8)$$

that is, the vacuum has probability $|c|^2$ and n-photon probability for $n \geq 1$ follows a logarithmic counting distribution. Thus with fixed value of c we have a two parameter family of normalized states and it is this family that is termed the logarithmic state [22]. Further discussion for the behavior of the probability as well as the dispersion and mean photon number for different values of parameters q and c is found in the appendix.

3. ATOMIC DYNAMICS

The expectation value for the inversion operator $\hat{\sigma}_z(t)$ when the atom starts from its excited state is given by:

$$P_e(t) = Tr\,\hat{\sigma}_z\,\rho(t)\ , \tag{9}$$

where the trace is taken over the atomic and field state.

Thus by using Equation (5), we obtain:

$$P_e(t) = \langle\hat{\sigma}_z(t)\rangle = \sum_{n=0}^{\infty} P_{n,n}\cos(2\lambda t\sqrt{[(n+k)/n!)]})\ , \tag{10}$$

where $P_{n,n}$ is given by Equation (7).

The dipole moment $D(t)$ can also be calculated by using the same technique. In this case $D(t)$ is given by:

$$D(t) = \langle\sigma_+(t)\rangle = Tr\,\sigma_+\,\rho(t)\ ,$$

$$D(t) = \mathrm{i}\sum_{n=k} P_{n-k,n}\sin(\lambda t\sqrt{[n!/(n-k)!]}) \times \cos(\lambda t\sqrt{[(n+k)!/n!]})\exp(-ik\omega t). \tag{11}$$

Note, for a pure number state $P_{n-k,n}=0$

In Figures 1, 2 we have plotted $P_e(t)$ of equation (10) as a function of λt for different values of $|q|$, $|c|$ and k with $\theta=\phi=0$, where θ and ϕ are the phases of q and c respectively. We note that for the small values of q and c, which is almost the state of one-photon, the oscillation of $P_e(t)$ is much similar to the case of number state discussed in [9]. The oscillations are almost sinusoidal in this case. As the value of k increases the oscillations become very rapid. It is also clear from the figure that the population oscillates sinusoidally between the ground state and the excited state at Rabi frequency, proportional to $\sqrt{[(n+1)]}$, $\sqrt{[(n+1)(n+2)]}$ and $\sqrt{[(n+1)(n+2)(n+3)]}$ for $k=1$, $k=2$, and $k=3$ respectively. Thus as k increases the Rabi frequency

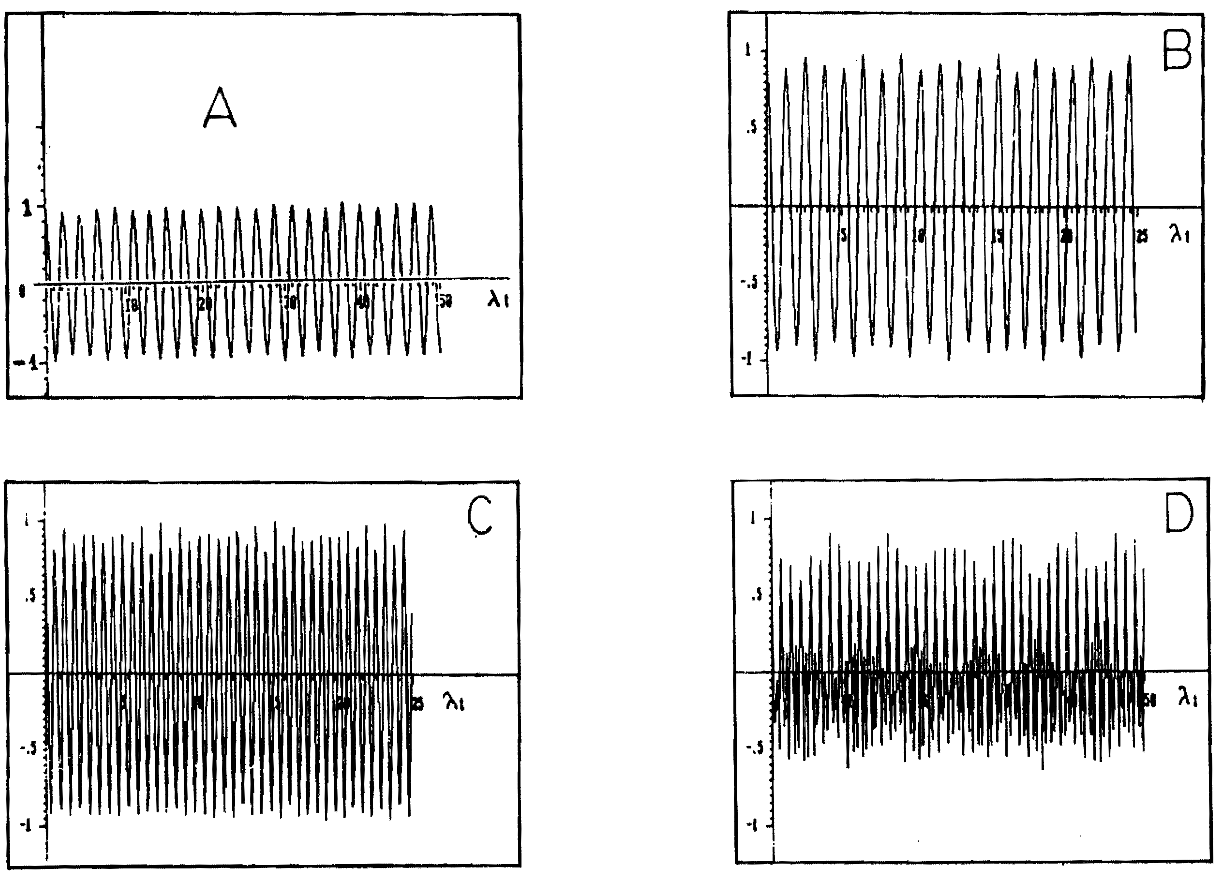

Figure 1. The Temporal Behavior of the Atomic Inversion (Equation 10) for $\theta=\phi=0$: A $-$ q = c = 0.2 & k = 1; B $-$ q = c = 0.2 & k = 2; C $-$ q = c = 0.2 & k = 3; D $-$ q = 0.8, c = 0.6, & k = 3.

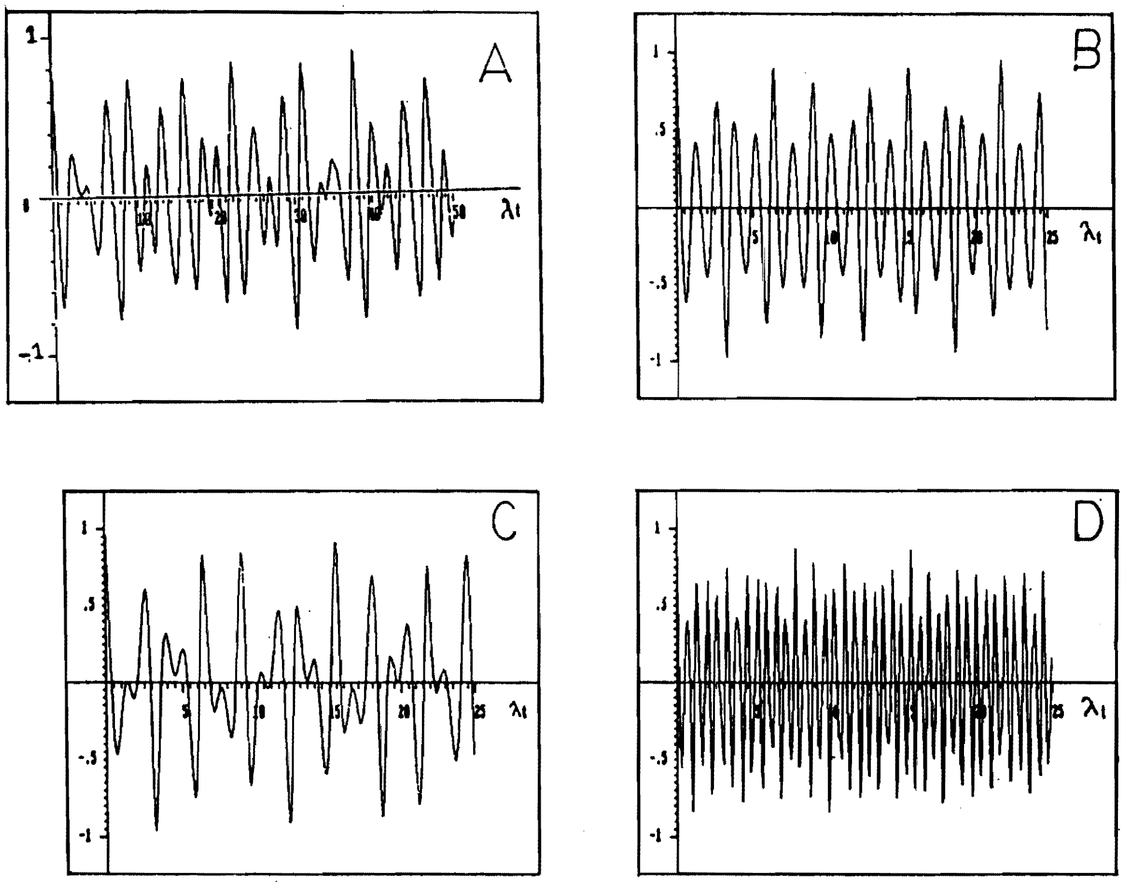

Figure 2. The Temporal Behavior of the Atomic Inversion for $\theta = \phi = 0$ *and:* A $-$ $q = 0.8$, $c = 0.6$, & $k = 1$; B $-$ $q = 0.8$, $c = 0.2$, & $k = 2$; C $-$ $q = 0.8$, $c = 0.6$, & $k = 2$; D $-$ $q = 0.8$, $c = 0.2$, & $k = 3$.

increases and consequently the oscillations of the inversion become very rapid. This explains why the oscillations of the population become very fast as the value of k increases.

In Figure 1(a, b, c) we take $c = q = 0.2$. We find for this case $\bar{n} = 0.98$ and $var(n) \simeq 0.06$. This means that the distribution is narrowly peaked around $\bar{n}$. Thus the one-photon state is predominant.

We observe also that as the values of q and c increase the states of more than one photon start to contribute as well as the vacuum state. This means that the photon number as well as the dispersion increase. This is almost clear in Figure 2a, b, c, d (where as an example, we find $\bar{n} \simeq 1.7$ and $\Delta(n) = 1.4$ for $q = 0.8$ and $c = 0.2$). The behavior of the population is different from that noted for the small value of q and c. The sinusoidal oscillations start to disappear and the picture starts to look like the collapse and revival (see Figure 2a, b, c). The disappearance of the sinusoidal oscillations is mainly attributed to the interference between the different frequencies which destroys the sinusoidal oscillations. It may be noted also that as k increases we note a decrease in the collapse time (Figure 2c, d). This is due to the collapse time being inversely proportional to the spread in Rabi frequencies. Therefore as k increases the spread in the Rabi frequencies increases and consequently the collapse time decreases. In fact the dependence of the collapse time here on the strength of the field is completely different from that noted for initially coherent field ($t_c = 1/\lambda$) [5] independent of the field strength. The dependence of the collapse time on field strength was noted for the atom initially in its ground state interacting with chaotic field for large value of $\bar{n}$ in [6]. The figure shows also that the revivals in the initially-logarithmic generalized multiphoton Jaynes–Cummings are similar to those noticed in [6]. The revivals are not regularly spaced out but appear to be generated more rapidly as time progresses (Figure 1d).

It should be noted that Buck and Sukumar [14] proved that the atomic inversion of the multiphoton JCM has a periodic dynamics with careful choice of the detuning, Δ, (for 2-photon JCM) for arbitrary initial conditions of the atom and field. On the other hand, and for the case of two-photon JCM, we can obtain a quasi-periodic dynamics for sufficient high initial mean photon number. We believe that this is not observed in our case for two reasons. First, since we investigate the inversion in the case of resonance ($\Delta = 0$) the linear dependence of the arguments of the trigonometric functions on the oscillator eigenvalues disappears. Therefore we cannot find any commensurable frequencies for terms in the series representation of the inversion for the case of resonance. Second, because of the behavior of the logarithmic state we observe as q increases with small value of c the dispersion increases very rapidly (Figure 6*b*) and the behavior of the probability is almost Gaussian in this case (Figure 6*a* & *c*). Therefore the chaotic effect is dominant for this case. For example, when we take $c = 0.05$ and $q = 0.98$ we find $\bar{n} = 12.54156$ and $\Delta(n) = 21.74622$. Increasing q and decreasing c further gives larger values for $\bar{n}$ and even larger amounts for the dispersion. Hence we cannot expect any quasi-periodic behavior for this state.

Figure 3, shows Im $D(t)$ of Equation (11) against λt in a rotating frame for different values of $|q|$, $|c|$, and k. We observe a regular chaotic behavior for the case of $k = 2$ (Figure 3*a* and *b*). This behavior disappears for $k = 3$ and rapid oscillations take place (Figure 3*b* and *d*). This is because the frequency here depends on the square root of $(n+1)(n+2)(n+3)$. It is interesting to note that for a fixed value of k the observed amount of dipole increases as the value of q approaches unity (see Figure 3*a, c* for $k = 2$ and Figure 3*b, d* for $k = 3$). On the other hand for fixed value of q the observed amount of the imaginary part of the dipole moment decreases as the value of k increases (Figure 3*a, b* for $q = 0.2$ and Figure 3*c, d* for $q = 0.8$). This behavior could be attributed to

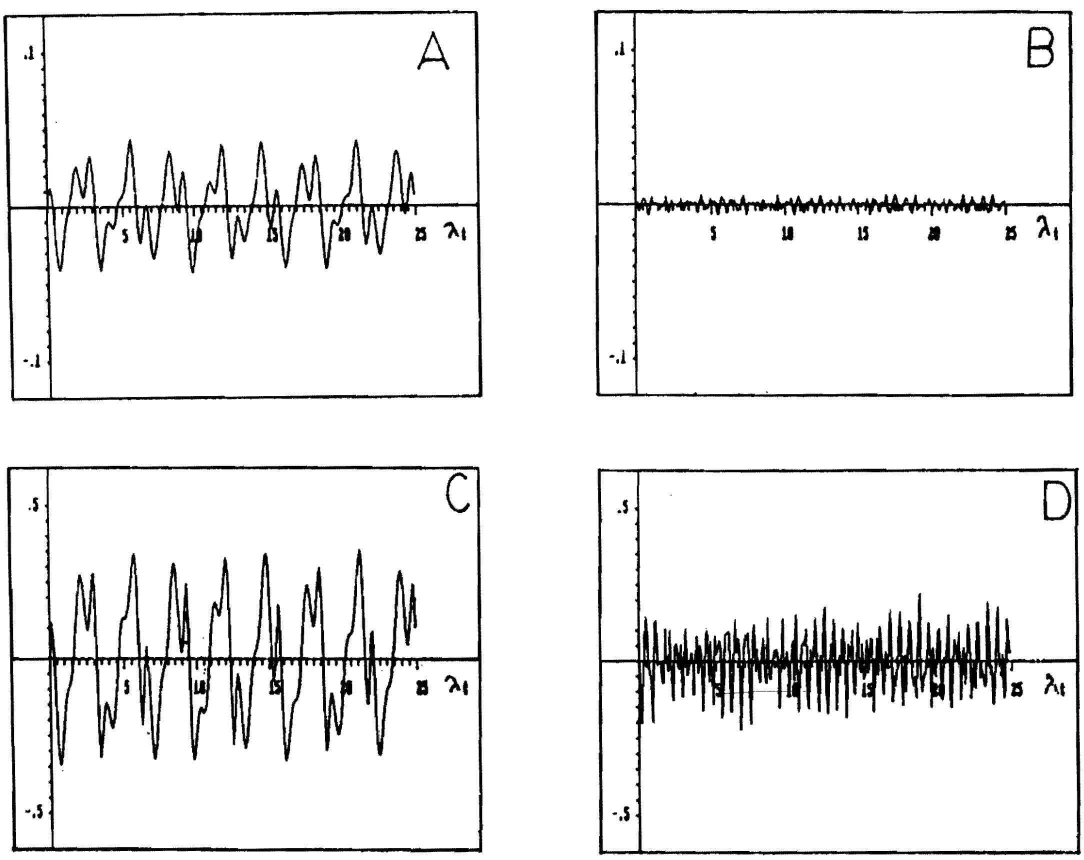

Figure 3. The Temporal Behavior of the Imaginary Part of the Dipole Moment (Equation 11) for $\theta = \phi = 0$ and: A – q = c = *0.2* & k = *2;* B – q = c = *0.2* & k = *3;* C – q = *0.8,* c = *0.2,* & k = *2;* D – q = *0.8,* c = *0.2,* & k = *3.*

the appearance of the factor $|q|^k/\sqrt{(n+k)}$ in Equation (11). For example, for fixed value of q this factor minimizes the value of the dipole as the value of k increases. Thus the inclusion of multiples of photons decreases the observed amount of dipole moment for the case of a field having initially a logarithmic distribution.

4. FIELD STATISTICS

In this section, we study squeezing, bunching, and antibunching as an example of the field dynamics.

First, we study squeezing effects [24] by evaluting $S_1(t)$ and $S_2(t)$ where

$$S_1(t) = \tfrac{1}{4}\{2\langle\hat{n}\rangle + 1 + \langle\hat{a}^2\rangle + \langle\hat{a}^{+2}\rangle\} - \tfrac{1}{4}\{\langle\hat{a}\rangle + \langle\hat{a}^+\rangle\}^2 , \qquad (12)$$

$$S_2(t) = \tfrac{1}{4}\{2\langle\hat{n}\rangle + 1 - \langle\hat{a}^2\rangle - \langle\hat{a}^{+2}\rangle\} + \tfrac{1}{4}\{\langle\hat{a}\rangle - \langle\hat{a}^+\rangle\}^2 . \qquad (13)$$

The state of the field is said to be squeezed if:

$$S_{1,2}(t) < \tfrac{1}{4} . \qquad (14)$$

It should be noted that the probability distribution function for finding n photons in the mode at time $t > 0$ is defined as

$$P(n,t) = \langle n|\hat{\rho}_f(t)|n\rangle ,$$

therefore using Equation (5) one finds,

$$P(n,t) = \cos^2(\lambda t\sqrt{[(n+k)!/n!]})\,P(n) + \sin^2(\lambda t\sqrt{[n!/(n-k)!]})\,P(n-k), \quad (15)$$

where $P(n)$ is given by Equation (7). Hence from Equation (15) we can obtain the expectation value for operators $\hat{n}$ and $\hat{n}^2$ at any time $t > 0$. For the rest of operators we use Equation (5).

For example, we can obtain the expectation value for the photon number operator $\hat{n}(t)$ as

$$\langle\hat{n}(t)\rangle = \sum_{n=0}^{\infty} nP(n,t) = \bar{n} + \frac{k}{2} - \frac{k}{2}\sum_{n=0}^{\infty} P(n)\cos(2\lambda t\sqrt{[(n+k)!/n!]}). \qquad (16)$$

It is clear from Equations (10) and (16) that $\langle n(t)\rangle + k/2\,P_e(t)$ is a constant of motion as should be expected. It is therefore expected that the temporal behavior of $\langle n(t)\rangle$ and $P_e(t)$ are the same (except for a phase factor of π). Hence what has been said in the previous section about the features of $P_e(t)$ can be transcripted for $\langle n(t)\rangle$.

In Figure 4, we have plotted $S_1(t)$ of Equation (12) for different values of $|q|$, $|c|$, and k. The effect of multiplicity on the squeezing is shown in the Figure. It is noted that as the value of k increases the squeezing disappears. This could be attributed to the increase in the interferences between the different frequencies which spoil the squeezing. However, it is well known that producing squeezing from a two-level atom in its excited state is very difficult, because squeezing is associated with a higher population in the upper level that produces simultaneous emissions that spoil squeezing [24]. We believe that the lack of the squeezing due to the increase of k is compatible with the temporal behavior of the inversion explained in the previous section. We observe for the inversion that as the value of k increases random evolution starts to appear. In this region the waves are almost decorrelated and dephasing is almost complete; we therefore expect a very high level of noise and squeezing in the field operators is washed out.

Next, we investigate the bunching and antibunching properties of the field [25] by evaluating the second-order correlation function $g^{(2)}(t)$, where

$$g^{(2)}(t) = \frac{\langle\hat{n}^2(t)\rangle - \langle\hat{n}(t)\rangle}{\langle\hat{n}(t)\rangle^2} .$$

A field is said to be antibunched if $g^{(2)}(t) < 1$ and bunched for $g^{(2)}(t) \geq 1$.

In Figure 5, we show $g^{(2)}(t)$ as a function of λt for different values of $|q|$, $|c|$, and k. It is noted that the antibunching decreases and bunching starts to contribute as the values of $|q|$ and k increase. The comparison between the cases of $k = 1, 2$ and 3 shows that the observed amount of bunching increases as the value of k increases. This lack of antibunching is expected for the same reason as mentioned in the case of inversion. Finally, we would mention that although $g^{(2)}(t) < 1$ means a non-classical effects, there is no direct relation between squeezing and this phenomenon.

5. SUMMARY

We have investigated the dynamics of the system of a two-level atom interacting with a single-mode

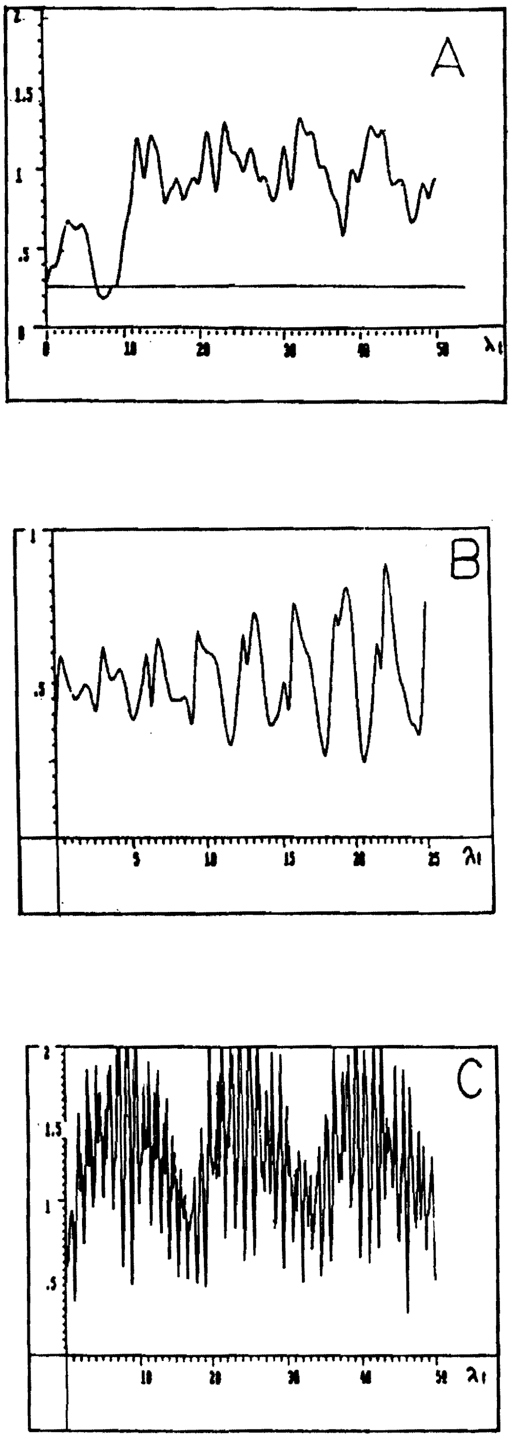

Figure 4. Squeezing in $S_1(t)$ *of Equation (12) for* $\theta = \phi = 0$ *and:* $A - q = 0.8,\ c = 0.6,\ \&\ k = 1;\ B - q = 0.8,\ c = 0.6,\ \&\ k = 2;\ C - q = 0.8,\ c = 0.6,\ \&\ k = 3.$

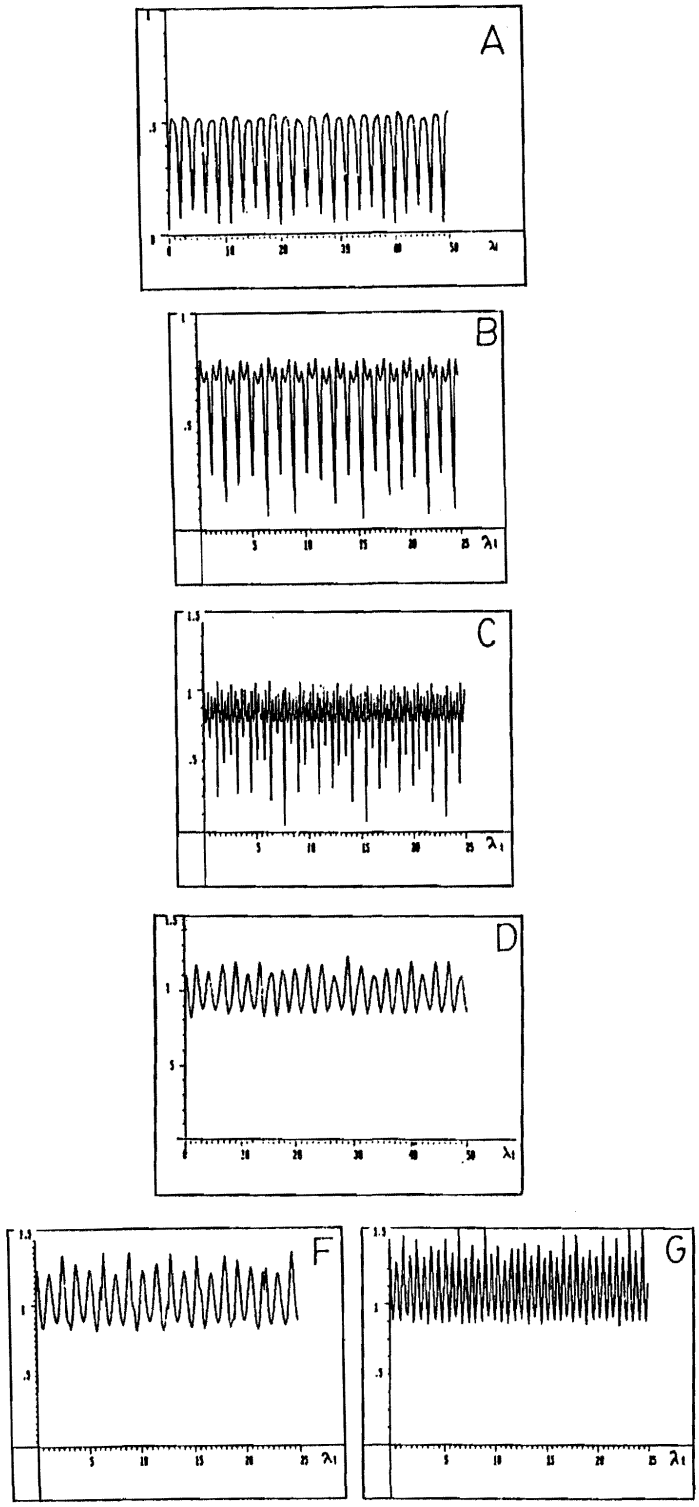

Figure 5. The Second-Order Correlation Function $g^{(2)}(t)$ *Against* λt *for* $\theta = \phi = 0$ *and the Following Values:* $A - q = c = 0.2\ \&\ k = 1;\ B - q = c = 0.2\ \&\ k = 2;\ C - q = c = 0.2\ \&\ k = 3;\ D - q = 0.8,\ c = 0.2,\ \&\ k = 1;\ F - q = 0.8,\ c = 0.2,\ \&\ k = 2;\ G - q = 0.8,\ c = 0.2,\ \&\ k = 3.$

electromagnetic field *via* a k-photon process when the mode has initially a logarithmic distribution. The dynamics of both atomic and field operators are shown. As an example for the atomic dynamics, we have investigated the inversion and the dipole moment. Squeezing effects as well as photon bunching and antibunching are also shown as an example for the field dynamics. The effects of multiplicity on the dynamics of both atomic and field operators are pointed out for this model.

APPENDIX

In this appendix we feel it is instructive to investigate some of the characteristics of the logarithmic state [22]. Using Equation (3) one can find the mean photon number $\langle n \rangle = \langle a^+ a \rangle$ and the variance as

$$\bar{n} = \beta^2 |q|^2 / [1 - |q|^2] \,, \qquad (1a)$$

and

$$\begin{aligned} \mathrm{var}(n) &= \langle n^2 \rangle - \langle n \rangle^2 \\ &= \beta^2 |q|^2 [1 - \beta^2 |q|^2] / [1 - |q|^2]^2 \,. \qquad (2a) \end{aligned}$$

In Figure 6*a* we have plotted $P(n)$ (Equation (8)) against n for different values of c and q. We observe as q and c approach unity the probability behaves almost like the Gaussian distribution (see curve e).

Figure 6*b* and 6*c* shows the mean photon number (Equation (1*a*)) and dispersion ($\Delta(n) = \sqrt{[\mathrm{var}(n)]}$) as a function of q for different values of c. We notice that the largest mean photon number corresponds to a large amount of dispersion.

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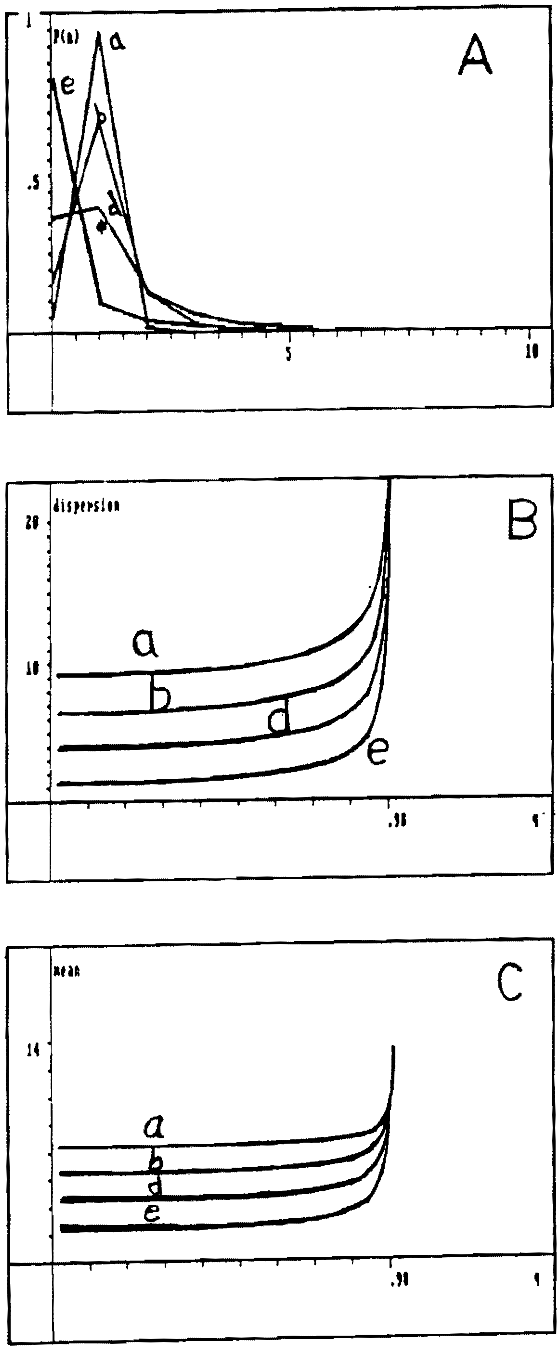

Figure 6a. *Probability,* P(n), *(Equation (8)) for Different Values of* c *and* q. *Curves* a, b, d, *and* e *are for* $c = q = 0.2$, $c = 0.4$, *and* $q = 0.6$, $c = 0.6$, *and* $q = 0.8$, *and* $c = 0.9$, *and* $q = 0.9$ *respectively.*

Figure 6b. *Dispersion,* $\Delta(n)$, ($\sqrt{[Var(n)]}$) *as a function of* q. *Curves* a, b, d, *and* e *are for* $c = 0.8(\Delta(n) + 7)$, $c = 0.6(\Delta(n) + 5)$, $c = 0.4(\Delta(n) + 3)$, *and* $c = 0.2(\Delta(n) + 1)$.

Figure 6c. *Mean Photon Number as a Function of* q. *Curves* a, b, d, *and* e *are for the Same Value and the Same Shifts as in Figure* 6b.

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