DENSITY OF FROST LAYER DURING THE CRYSTAL GROWTH PERIOD

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1. INTRODUCTION

Frost formation phenomenon, encountered in the field of refrigeration and air-conditioning, has a significant adverse effect upon the heat transfer and pressure drop. For instance, the frost formation on heat exchanger surfaces can be extremely detrimental to their efficient operation since the frost will act as a thermal insulator, thus reducing the ability of the surface to transfer heat. Also, accumulations of the frost often become thick enough to restrict and block the air flow. There is a need for a fundamental understanding of the nature of frost formation including the vapor-condensation process to assist in predicting rate of frost formation.

Most of the previous investigations were concerned with the frosting phenomena after a porous layer of frost had grown to a certain thickness. In this study, however, the crystal growth period of frost formation is investigated and the attention is focused on the density variation during this period.

In the crystal growth period, a thin frost layer covers the cryosurface initially. Next, frost crystals, which are relatively far apart from each other, appear on the thin frost layer, and grow in the direction perpendicular to the surface at about the same rate. Thus, the frost formation in this period is best characterized by crystal growth in one linear dimension, such that the frost becomes like a forest of trees, without the growth of a homogeneous layer [1, 2].

It has been well established that vapor diffusion does occur within the frost layer due to the vapor pressure gradient which is a consequence of the temperature gradient. Diffusion is a relatively slow process and Chung and Algren [3] concluded that the diffusion process has little effect on changing the local frost density or thermal conductivity.

Several variables have been suggested in the literature as having an apparent effect on the density of frost. These include plate temperature, air temperature, air velocity, and humidity ratio [4]. However, it is noted that none of these are properties of the frost layer in that they are all variables external to the frost layer and do not uniquely define properties within the frost layer itself. If a useful analytical tool is to be developed, it was felt that the frost model must include the correlation of the frost layer density with some other property (or properties) which could be calculated directly. A likely candidate for this correlation appears to be the surface temperature of the frost (at the time of formation).

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From experimental evidence, it is anticipated that frost crystal density at the moment of formation under a given set of conditions (such as the surface temperature of the frost) is very critical and crucial in the process of frost nucleation and growth, which is a crystallization process. In other words, if the role that is played by crystal density under certain conditions during frost formation is determined correctly, the development of a model could be easy and accurate.

There have been many attempts to simulate crystal growth numerically by cloud physicists [5], since ice crystal growth is such an important part of the precipitation process. In this study, it is felt appropriate to utilize the results of cloud physicists Hallett and Mason [5] and Fukuta [6] on ice crystals' shape and density. The results are found to be quite accurate, and explain many aspects of frost nucleation and growth.

2. ICE CRYSTAL STRUCTURE

Many workers have noted that the habit (shape) of ice crystals exhibits a striking dependence on temperature and, to a lesser degree, on supersaturation.

Mason and Hallet [5] were able to grow crystals over the temperature range 0°C to −50°C and under supersaturations ranging from a few per cent to about 300 per cent, using a thin nylon or glass fibre running vertically through the center of a water vapor diffusion chamber. The crystal habit varied along the length of the fiber as shown in Table 1.

Table 1. Variation of Crystal Habit with Temperature (Hallet and Mason, 1958).

Temperature (°C)	Form of ice crystal
0 to −3°C	Thin hexagonal plates
−3 to −5°C	Needles
−5 to −8°C	Hollow prisms
−8 to −12°C	Hexagonal plates
−12 to −16°C	Dendritic crystals
−16 to −25°C	Plates
−25 to −50°C	Hollow prisms

Transitions from plate growth to prism growth, back to plates, and then to prisms again may be noted as the temperature decreases. The classification of laboratory-produced crystals according to the temperature of formation bears a marked similarity to that of natural snow crystals, showing that it is possible to simulate quite well the early stages of growth of snow crystals in the laboratory and, at the same time, determine the transition temperatures for the different crystal forms more precisely than can be done in the atmosphere.

According to the study of Fukuta [6], the sublimation density of ice crystals as a function of crystal temperature is shown in Figure 1. The least squares approximation of the experimental data is

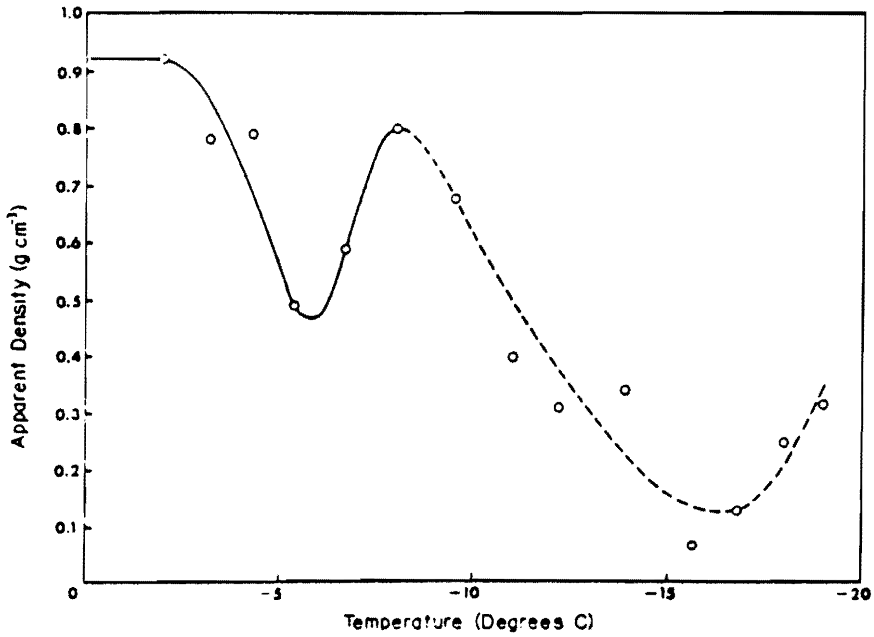

Figure 1. The Sublimation Density as a Function of Crystal Temperature. [6]

$$\rho_c = \begin{cases} 926.376 + 41.574\,T_c & -18°C < T_c < 0°C \\ 180 & T_c < -18°C \end{cases} \qquad (1)$$

where T_c is the crystal temperature in °C. The sublimation density of ice crystals for the temperature below −18°C is assumed to be constant since there is no experimental data available for these temperatures as seen in Figure 1. The least squares approximation of the experimental data is considered to be appropriate for the analysis because the local temperature in the frost layer and the temperature of the frost layer surface do not stay constant but change continuously with time. Because of this timewise variation in temperature, the crystals formed during frost formation are the complex type of crystals mentioned in [5].

3. CRYSTAL GROWTH MODEL OF THIN FROST LAYER

The frost formation during the crystal growth period can be represented by frost columns with moist air surrounding them, with the columns characterized by linear crystal growth.

A simple model, shown in Figure 2, has been selected for this work in which the frost layer is assumed to consist of several frost columns.

The following assumptions are made for this model:

- frost columns consist of ice crystals
- n frost columns exist on the heat transfer plate per unit area,
- the radius of each column is identical and uniform in $r(t)$, a function of time,
- the density of frost columns, $\rho_{fc}(t)$, is uniform,
- the temperature of the frost column at a cross-section y has the same temperature as that of the void fraction at this position,
- the humidity of the air at this cross-section corresponds to saturation humidity at that temperature, and
- the conductivity of frost columns, k_{fc}, is a function of the frost column density only (as in Sander's Equation [7]).

Furthermore, the process of adhesion and growth of frost on a heat-transfer surface is a very slow but unsteady process. However, because this process is slow, it is treated in a quasi-steady state manner.

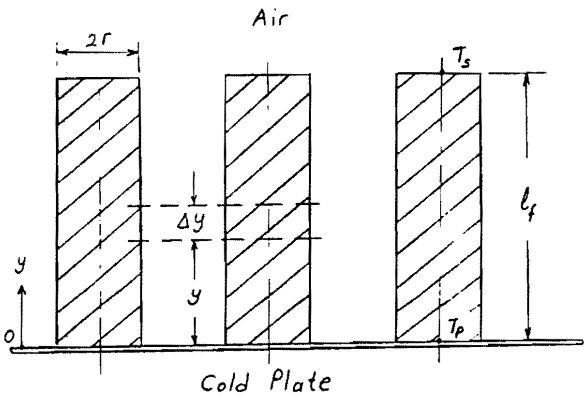

Figure 2. Frost Model Consisting of Cylindrical Frost Columns.

The objective of this study is to investigate the density variation of frost during the crystal growth period of frost formation using the above model and utilizing the sublimation density of ice crystals in Equation (1). Since the variation of the frost density is closely related to the surface temperature of the frost layer, the temperature variation inside the frost layer and, thus, the surface temperature of the frost layer will be determined first in the following. The model is designed to be accurate during approximately the first one hour period. Therefore, the accuracy of the model after one hour is not assured, since a different phenomenon, vapor diffusion through the frost layer and densification, becomes a significant part of the frost layer development.

Vapor diffusion through the void portions of frost layer, which is not a dominating factor during the crystal growth period of frost formation, causes the radius of a column to grow with time. The diffusion rate of mass of water vapor through the frost layer is given by

$$\dot{m}_d = -D\,\rho_a \frac{d\omega_s}{dy} = -D\,\rho_a \frac{d\omega_s}{dP_g}\frac{dP_g}{dT}\frac{dT}{dy}\,. \qquad (2)$$

The partial pressure under frost formation conditions ($T < 0°C$) is such that

$$P_g \ll 100\,\text{kPa}$$

so, the humidity ratio in Equation (2) can be approximated by

$$\omega_s \approx 0.622 \times 10^{-2} P_g\,. \qquad (3)$$

Thus, the diffusion rate of mass of water vapor can be expressed in terms of the temperature by using Equation (3)

$$\dot{m}_d \approx -0.622 \times 10^{-2}\, D\, \rho_a \frac{h_{ig}}{RT^2} P_g \frac{dT}{dy}, \quad (4)$$

where the partial pressure of water vapor, P_g, is given as

$$P_g = P_o \exp\left[\frac{h_{ig}}{R}\left(\frac{1}{T_o} - \frac{1}{T}\right)\right] \quad (5)$$

and P_o is the partial pressure of water vapor at T_o= 273.15 K.

The amount of mass which crosses the frost surface and diffuses through the void parts of the frost, $\dot{m}_d$, solidifies around the frost columns, causing a horizontal growth with time. During this horizontal growth the radius of a frost column is assumed to remain the same everywhere in the vertical direction. The density of frost columns is also assumed to be uniform along the frost columns, that is, equal to the average sublimation density of ice crystals at the surface temperature of the frost layer. The overall density, however, is an average density, including the void portions of the frost layer, as well.

Defining the volumetric ratio of frost columns, β, as

$$\beta = n\pi r^2 = \frac{\rho_f}{\rho_{fc}}, \quad (6)$$

where ρ_f and ρ_{fc} are the densities of frost layer and frost column respectively, an energy balance on the differential control volume in the frost layer can be written as,

$$-(1-\beta)\, h_{ig} \frac{d\dot{m}_d}{dy} - \beta \frac{dq_c}{dy} - (1-\beta) \frac{dq_a}{dy} = 0 \quad (7)$$

where q_c and q_a are the heat transfer through the frost columns and the void portions of the frost layer respectively. Using Equations (2) and (3) the energy balance becomes

$$\frac{d}{dy}\left\{\left[0.622 \times 10^{-2}(1-\beta)\, h_{ig}\, D\, \rho_a \frac{dP_g}{dT} + \beta k_{fc} + (1-\beta) k_a\right] \frac{dT}{dy}\right\} = 0 \quad (8)$$

with the bounday conditions:

$$y = 0, \qquad T = T_p \quad (9)$$

$$y = l_f, \qquad T = T_s. \quad (10)$$

k_{fc} in Equation (8) is the conductivity of the frost column and k_a is the conductivity of moist air in the void portions of the frost layer.

The temperature variation in the frost layer is obtained in terms of the frost surface temperature by integrating Equation (8), subject to boundary conditions, Equations (9) and (10);

$$A \exp\left(\frac{h_{ig}}{RT_o}\right)\left[\exp\left(-\frac{h_{ig}}{RT}\right) - \exp\left(-\frac{h_{ig}}{RT_p}\right)\right] + k_f(T - T_p) = \frac{y}{l_f}\left\{A \exp\left(\frac{h_{ig}}{RT_o}\right)\left[\exp\left(-\frac{h_{ig}}{RT_s}\right) - \exp\left(-\frac{h_{ig}}{RT_p}\right)\right] + k_f(T_s - T_p)\right\} \quad (11)$$

where

$$A = 0.622 \times 10^{-2}\,(1-\beta)\, h_{ig}\, D\, \rho_a\, P_o \quad (12)$$

and k_f is the frost thermal conductivity;

$$k_f = \beta\, k_{fc} + (1-\beta)\, k_a\,. \quad (13)$$

y in Equation (11) is the distance from the cold plate surface.

The solution of Equation (11) can be obtained by Newton's method.

An energy balance across the frost layer surface in Figure 3 is established to calculate the frost surface temperature

$$\dot{q}_s = k_f \left.\frac{dT}{dy}\right|_{T=T_s} \quad (14)$$

where

$$\dot{q}_s = h_c(T_a - T_s) + L_H\left[\dot{m} - \dot{m}_d\Big|_{T=T_s}(1-\beta)\right]. \quad (15)$$

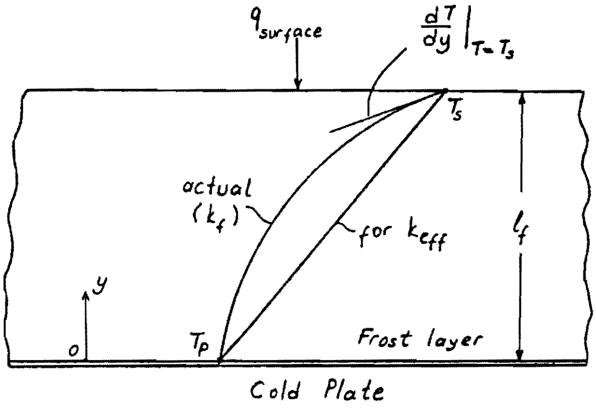

Figure 3. Energy Balance on the Frost Layer Surface.

All the terms in both sides of Equation (14) are functions of the frost surface temperature which is the only unknown. The solution is obtained by a trial and error procedure with the bisection method being used in this study. Simultaneous solution of Equations (11) and (14) give the actual temperature profile in the frost layer during the crystal growth period of frost formation.

Once the temperature profile in the frost layer is obtained, mass of vapor diffusion rate can be calculated. Thus, the density variation and all other frost layer growth properties can be predicted by an iterative procedure with the assumption of quasi-steady state mentioned above. Since ice crystals grow at the temperature of the frost layer surface, the overall frost density is obtained by averaging the ice crystal density values at the frost surface temperature for every step in the numerical procedure and also considering the void portion of the frost layer.

The initial volumetric ratio of frost columns, β_o, which is the only empirical constant used in the analysis, is obtained from experimental results [1] and depends on the experimental conditions such as T_p, T_s, and ω_a. A regression analysis in [1] based on the experimental results gives the following linear correlation of β_o with the experimental conditions;

$$\beta_o = -11.8916 + 0.01371\ T_p + 0.03269\ T_a - 112.677\ \omega_a. \quad (16)$$

Regression analysis shows that the effect of the Reynolds number on β_o is negligible.

The variation of the volumetric ratio of frost columns with time is calculated using the rate of mass of vapor diffusing through the frost layer, $\dot{m}_d$, and the frost column density, ρ_{fc}.

4. RESULTS AND DISCUSSION

Since the development of a theoretical model for ice crystal density and growth is out of the scope of this study, it is deemed proper to utilize the experimental results of cloud physicists [5, 6] regarding crystal density, rather than using a constant or linear crystal formation density, which are the assumptions made by several researchers in frost formation, or a very complicated empirical correlation based on experimental observations of frost formation and having no basis as regards ice crystal formation and growth.

One of the most important properties of frost formation is the frost surface temperature, since this is the temperature at which the ice crystals develop and it is the temperature which, as a prime factor, determines the shape and therefore the density of ice crystals. During the crystal growth period of frost formation, the frost surface temperature change is quite extensive, rather than being constant (melting point) as some researchers assume, thus affecting the ice crystal density directly. Therefore, frost height and other relevant frost properties such as density, mass, heat flux, and effective thermal conductivity of frost are also affected by frost surface temperature.

In the present model, frost surface temperature tends to rapidly increase, especially for high air humidity ratios (Figure 6), high Reynolds numbers (Figure 7), and low plate temperatures (Figure 4), and then approaches the melting temperature asymptotically. Air temperature (Figure 5) does not seem to affect the surface temperature much, while higher humidity ratio (Figure 6) and Reynolds number (Figure 7) yield higher frost surface temperature since the boundary layer over the frost layer surface gets thinner and heat transfer coefficient gets higher.

Frost density primarily depends on frost surface temperature, besides other parameters, because frost formation indeed develops at that particular temperature with the principles of ice crystal growth explained above. This means that the type of the crystals developing at the frost surface temperature determine all the properties of frost formation.

Density of frost layer variations with respect to four external variables, namely plate temperature, air temperature, air humidity ratio, and Reynolds number, are investigated (Figure 8). Plate temperature is one of the most significant variables affecting frost density. Frosts formed at high plate temperatures have high densities, and those formed at very low temperatures are light and fluffy. Frost will grow at a rate, and with a density, such that the latent heat released is conducted away. The higher the plate temperature, the smaller the temperature gradient available in the frost layer to carry away the latent heat released, and, hence, the frost grows more densely so that the thermal conductivity is larger.

Higher air temperature yield denser frost layer as it is seen in Figure 8. As pointed out above, the temperature gradient in the boundary layer for higher air temperatures is greater, and therefore the

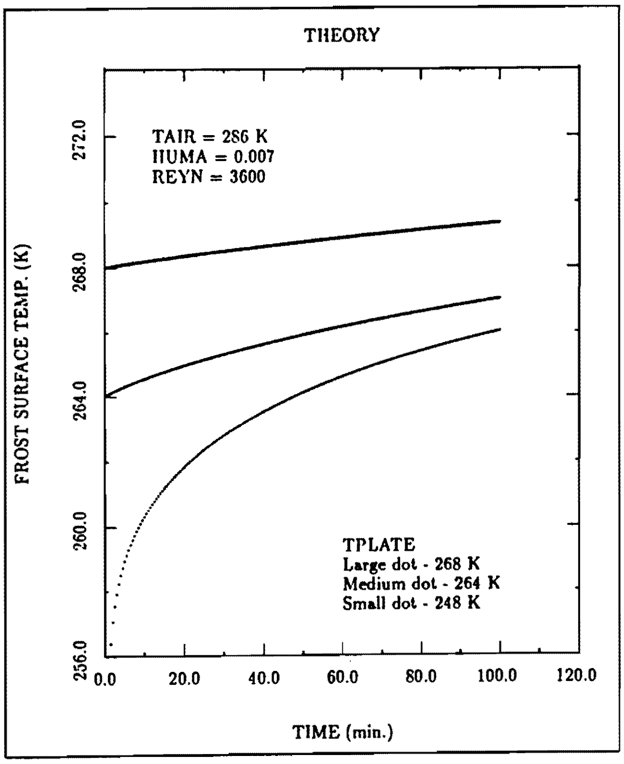

Figure 4. The Effect of Plate Temperature on Frost Surface Temperature.

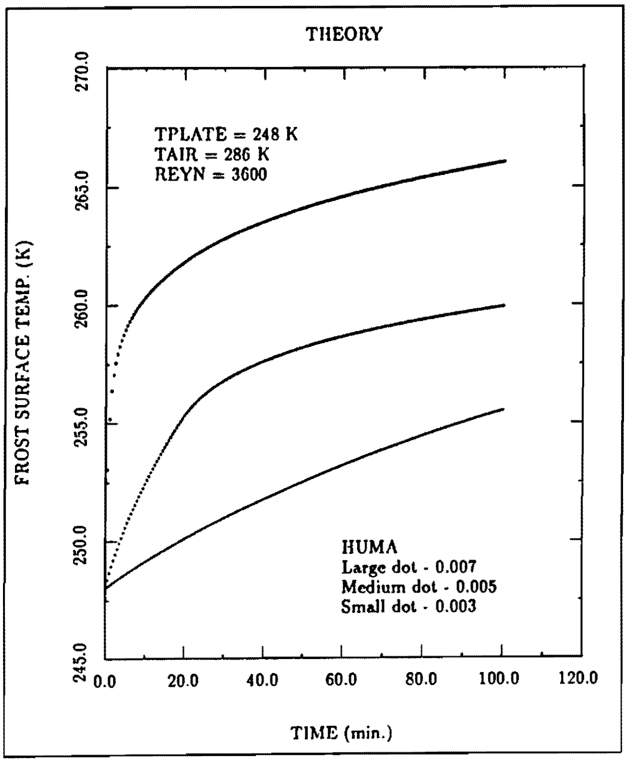

Figure 6. The Effect of Air Humidity Ratio on Frost Surface Temperature.

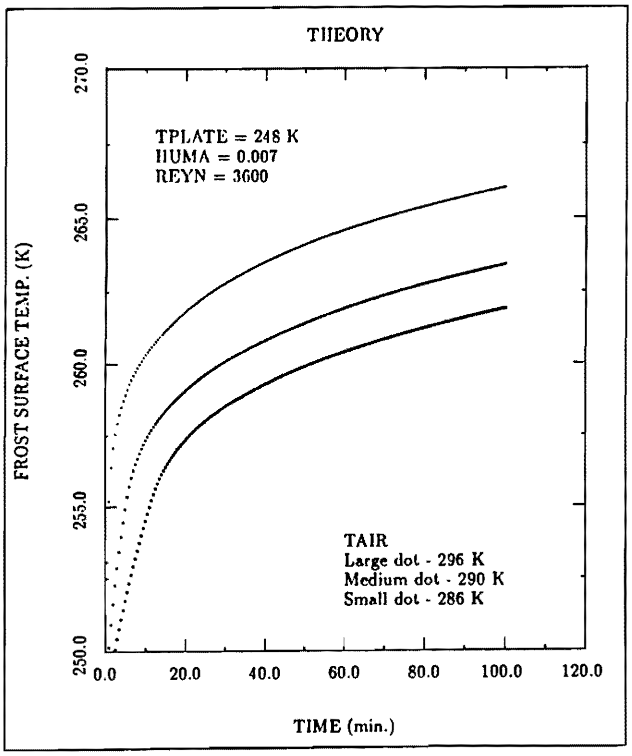

Figure 5. The Effect of Air Temperature on Frost Surface Temperature.

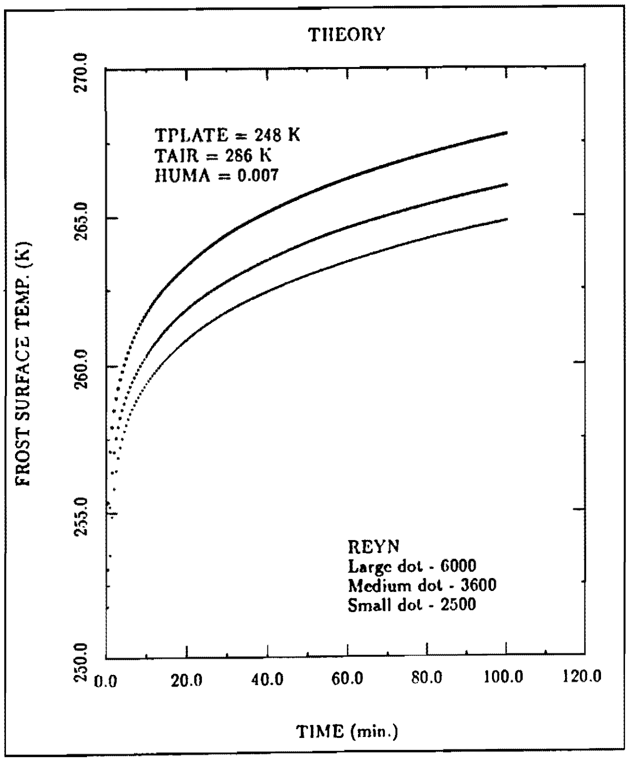

Figure 7. The Effect of Reynolds Number on Frost Surface Temperature.

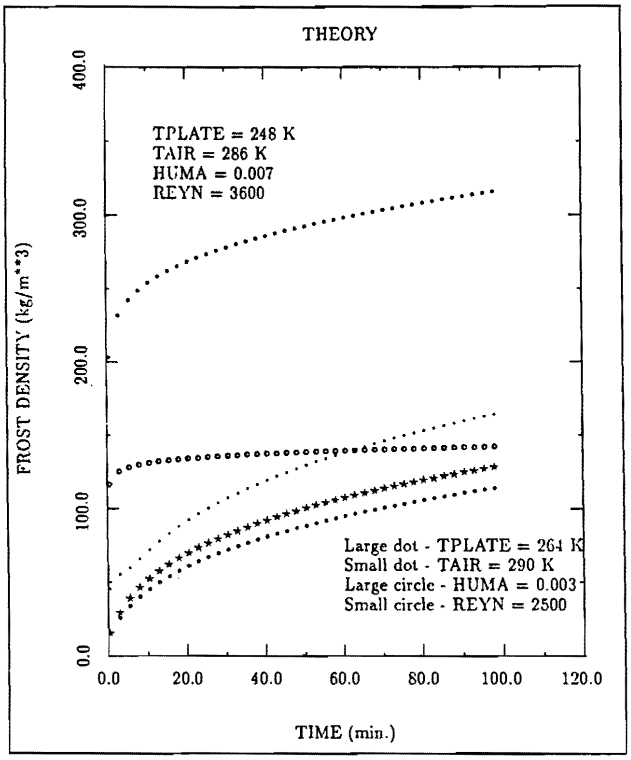

Figure 8. The Effect of Changing the Parameters on the Average Frost Density. (Stars indicate base conditions given at the top.)

development of ice crystals and phase change of water molecules is faster. For this reason, large ice crystals do not have time to complete their development with branches and extensions. In fact, water molecules passing through the boundary layer encounter the temperature gradients and arrive at the frost surface to take their places in the ice crystal lattice where the temperature is continuously changing. This means that every molecule that arrives at the surface comes across at a different temperature causing a build up of different ice crystal structure. It is known that the crystal structure can be quite different even with only a degree or two change in temperature. For small temperature gradients in both space and time, a small crystal structure can complete its normal development, however, a large crystal structure requires more time and molecules to attain its ultimate shape, and in case of large temperature gradients a complex crystal structure is likely to develop due to the shortage of time. For example, when needles grown at temperatures between −3 and −5°C are moved up in the environment to about −2°C, plates develop on their ends.

Higher humidity ratio yields lower frost density (Figure 8). However, the densification over time is faster. This can also be explained by the similar reasonings stated above. In this case, concentration gradient over the boundary layer is greater. Therefore, the number of molecules which travel through the boundary layer is bigger. Since the temperature gradients are not in consideration anymore, more molecules build up more crystals and therefore thicker and less dense frost layers.

Frost density is higher under high Reynolds number, although the effect is small compared with the other parameters given in Figure 8. The reason for this is basically mechanical. For high stream velocities, high frost columns cannot develop. This increases the number of frost columns in the crystal growth period. Therefore, the distance between the frost columns becomes short, and the interference of each frost column occurs quickly to form a more uniform frost layer.

A comparison of the results of the preset model with the experimental data [1] is given in Figures 9 through 13.

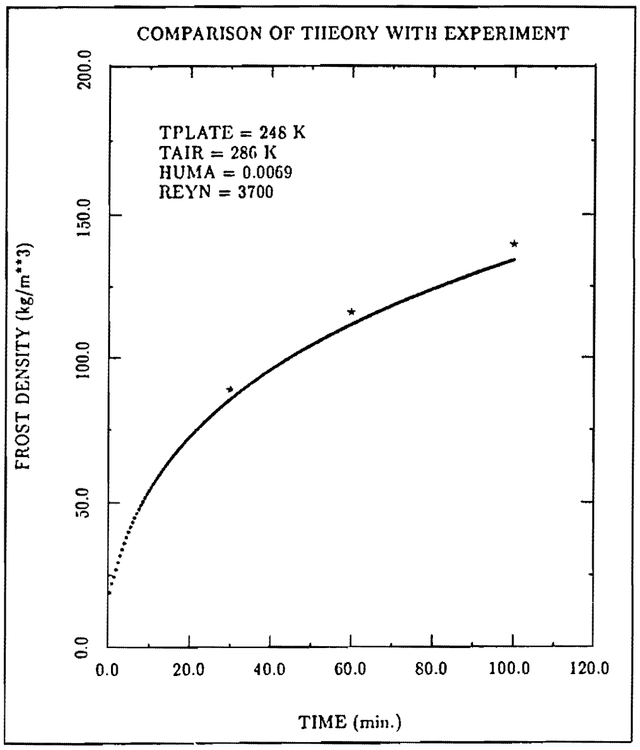

Figure 9. Frost Density Variation for Fixed Parameters: Comparison of Theory with Experiment. (Stars indicate the experimental data in [1].)

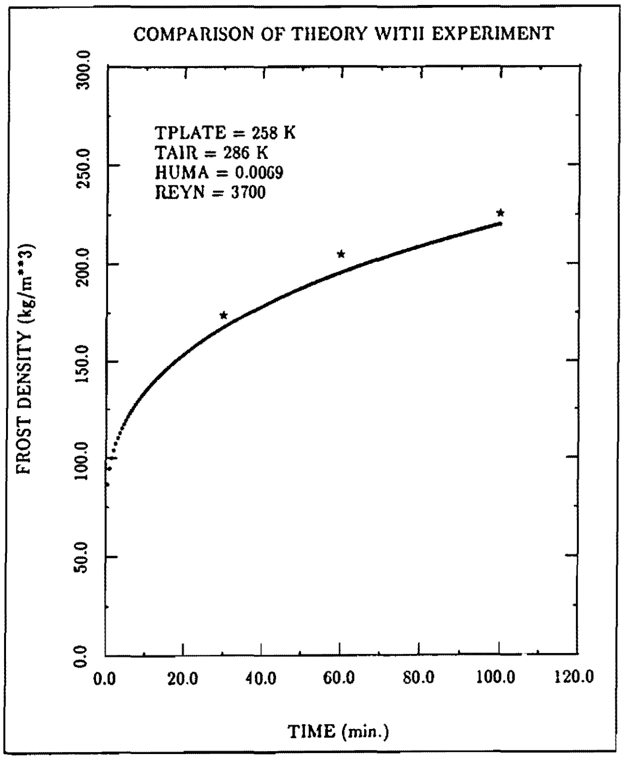

Figure 10. Frost Density Variation for Higher Plate Temperature: Comparison of Theory with Experiment. (Stars indicate the experimental data in [1].)

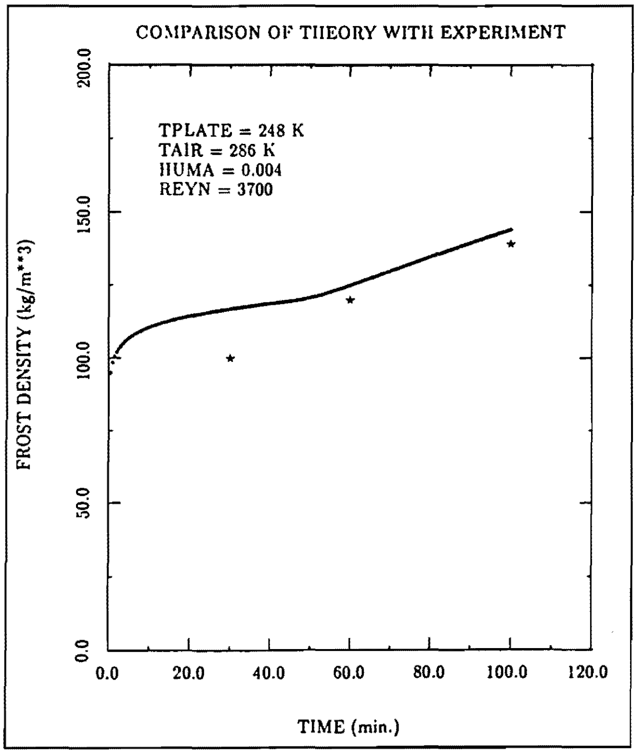

Figure 12. Frost Density Variation for Lower Air Humidity Ratio: Comparison of Theory with Experiment. (Stars indicate the experimental data in [1].)

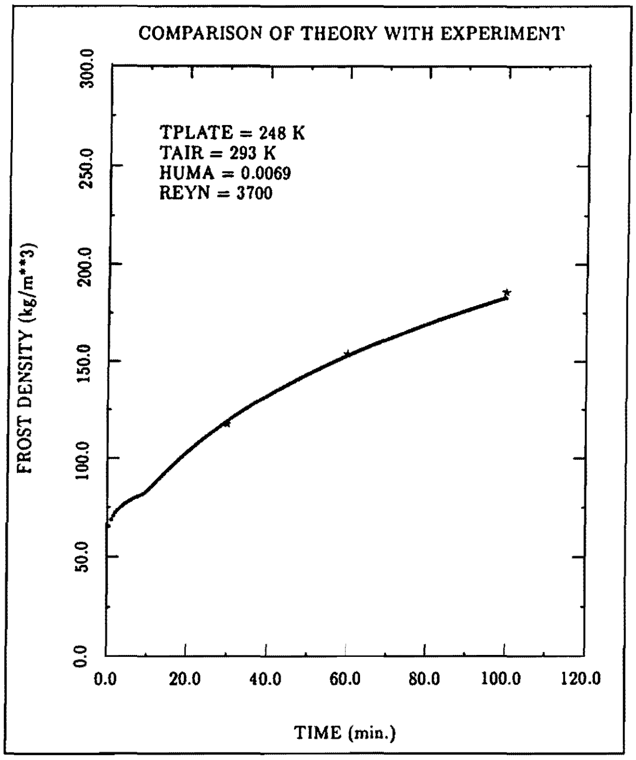

Figure 11. Frost Density Variation for Higher Air Temperature: Comparison of Theory with Experiment. (Stars indicate the experimental data in [1].)

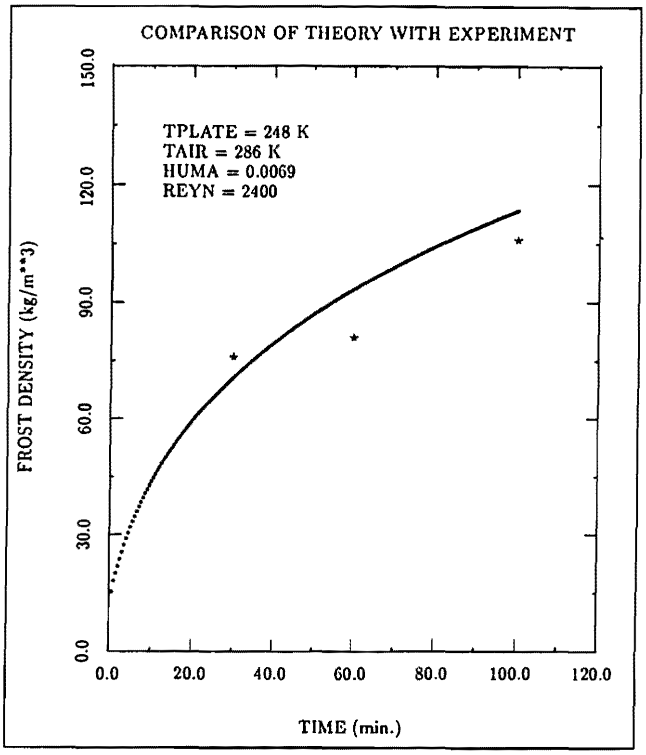

Figure 13. Frost Density Variation for Lower Reynolds Number: Comparison of Theory with Experiment. (Stars indicate the experimental data in [1].)

5. CONCLUSIONS

One of the most critical inputs to the frost formation model is known to be the frost density. Instead of using the experimental and empirical correlations of frost density found in the literature on frost formation, ice crystal density results under different ambient conditions measured by cloud physicists [5, 6] are found to be more suitable to be used in the theory of frost nucleation and growth.

Frost density primarily depends on frost surface temperature, besides other external parameters. Higher plate and air temperatures produce denser frost layers. Higher humidity ratio yields lower frost density and frost density is higher under high Reynolds number.

Frost column density, on the other hand, is found to closely follow the sublimation density of ice crystals at the time of formation under the crystal temperature, that is the frost surface temperature. However, the number of the ice crystal columns of frost increases with increasing Reynolds number, decreasing air humidity or increasing air temperature which are all external parameters to the frost layer.

These external parameters determine the timewise variation of the frost surface temperature, the key parameter in frost formation, therefore affect the overall density of frost through frost column density and the density of ice crystals in the frost columns during the crystal growth period of frost formation. The results and agreement with experiment are found to be satisfactory.

NOMENCLATURE

D Diffusivity of water vapor in air (m^2/s)
h_c Heat transfer coefficient (W/m^2K)
h_{ig} Latent heat of sublimation (J/kg)
k_a Thermal conductivity of moist air (W/mK)
k_f Frost layer thermal conductivity (W/mK)
k_{fc} Frost column thermal conductivity (W/mK)
l_f Frost layer thickness (m)
$\dot{m}$ Total mass flux (kg/s)
$\dot{m}_d$ Mass diffusion rate inside the frost layer (kg/m^2s)
n Number of frost columns per unit area
P_g Partial pressure of water vapor (N/m^2)
P_o Partial pressure of water vapor at $T_o = 0°C$ (N/m^2)
$\dot{q}_s$ Heat flux through frost layer surface per unit of time (W/m^2s)
r Radius of cylindrical frost columns (m)
R Gas constant
T Temperature (K)
T_a Air temperature (K)
T_c Crystal temperature (K)
T_o 273.15 K
T_p Plate temperature (K)
T_s Frost surface temperature (K)
y Distance from the plate surface (m)

β Volumetric ratio of frost columns
β_o Initial volumetric ratio of frost columns
ρ_a Density of air (kg/m^3)
ρ_c Sublimation density of ice crystals (kg/m^3)
ρ_f Frost density (kg/m^3)
ρ_{fc} Density of frost column (kg/m^3)
ω_a Humidity ratio of air stream
ω_s Humidity ratio of air at the frost surface

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