APPLICATION OF THE SEMI-CLASSICAL FERMION μ-SPACE DENSITY TO NUCLEI USING WOODS–SAXON POTENTIAL

O. B. Dabbousi*

Department of Physics, University of Petroleum and Minerals, Dhahran, Saudi Arabia

الـخلاصــة :

لقد أستعمل نموذج بالاش وزيفل لكثافة الفراغ الفرميوني النصف تقليدى لحساب توابع الكثافة الموضعية لمختلف مضاريب العزم p^n . فوجدنا أن توابع التوزع لـ p^n تتناسب مع تحويل وايل في المرتبة $(n+3)/2$ لتابع أيرى . أى

$$D_n(r) = \frac{4\pi}{(2\pi\hbar)^3} \frac{\Gamma^{\{(n+5)/2\}}}{n+1} \left(\frac{2}{\sigma_{E_F}(r)}\right)^{(n+1)/2} \mathscr{W}_{(n+3)/2}\{\mathrm{Ai}(r), B(E_F, r)\}$$

حيث $\mathscr{W}_n$ تمثل تحويل وايل و

$$B(E_F, r) = \sigma_{E_F}(r) \cdot (V(r) - E_F)$$

وتمثل E_F طاقة فرمى و $\sigma_{E_F}(r)$ عامل للتناسب محسوبا عند الطاقة E_F وتمثل $V(r)$ تابع الكمون . ولذافان تابع توزيع الكثافة الرقمية هو $D_2(r)$ وتابع توزيع كثافة الطاقة الحركية هو $\frac{1}{2m} D_4(r)$

لقد حسبت القيم العددية للتكامل في تحويل وايل من أجل $n = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots, 8$ ولـ $B \in [-7.9, 2]$ بدقه خمسة مراتب وجدولت في أخر هذا الموضوع .

لقد طبقنا هذه العملية على حالة جسيم وحيد عديم الالتفاف في كمون وود — ساكسون . وقد قورنت نتائج استخدام عوامل تناسب ثابته مع نتائج استخدام نموذج توماس — فرمى التقليدى .

* Work performed at: *Material and Molecular Research Division, Lawrence Berkeley Laboratory and Department of Physics, University of California, Berkeley, California 94720, U.S.A.*

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ABSTRACT

The Balazs and Zipfel model for the semi-classical fermion μ-space density is used to calculate spatial density functions for different powers of the momentum. We find that the distribution functions for p^n, are to a multiplicative factor, the Weyl transform of order $(n+3)/2$ of the Airy function. Or,

$$D_n(r)=\frac{4\pi}{(2\pi h)^3}\,\frac{\Gamma[(n+5)/2]}{n+1}\left(\frac{2}{\sigma_{E_F}(r)}\right)^{(n+1)/2}\quad {}_{(n+3)/2}\{\mathrm{Ai}(r),\,B(E_F,\,r)\}$$

where $\mathscr{W}_u$ is the Weyl transform, and

$$B(E_F,r)=\sigma_{E_F}(r)\cdot(V(r)-E_F);$$

E_F is the Fermi energy, $\sigma_{E_F}(r)$ is a scalling factor evaluated at E_F and $V(r)$ is the potential. Thus, the number density dsitribution function is $D_2(r)$ and the kinetic energy density function is $\frac{1}{2}m\ D_4(r)$.

Tables of numerical values of the integral in the Weyl Transform for $n=0$, 1/2, 0, 3/2, ... 8 and $B\in[-7.9,2]$, accurate to five significant figures are presented.

We apply this procedure for the case of a spin-independent single-particle Woods–Saxon potential. Results for constant scaling factors are presented and compared to those of the classical Thomas–Fermi model.

APPLICATION OF THE SEMI-CLASSICAL FERMION μ-SPACE DENSITY TO NUCLEI WOODS–SAXON POTENTIAL

1. INTRODUCTION

The Wigner transform of the singlet-density matrix [1–3] has had a wide range of applications in nuclear physics in recent years. For example, the Wigner density function, as it is usually referred to, has been used in a model for the relativistic transport in multiparticle production [4] to obtain a generalized master equation for relative and internal motions in deep-inelastic heavy-ion collisions [5] and to obtain semi-classical μ-space for a dense Fermi gas [6]. The Wigner distribution function associated with the singlet-density matrix, $\langle x_1|\rho|x_2\rangle$ is given by [7]

$$f(\mathbf{r},\mathbf{p})=\frac{1}{(2\pi)^3}\int d^3x\langle \mathbf{r}-\frac{\mathbf{x}}{2}|\rho|\mathbf{r}+\frac{\mathbf{x}}{2}\rangle \exp(-i\mathbf{p}\cdot\mathbf{x}),\ (m=1,\ h=1) \qquad (1).$$

The Wigner transform, $f(\mathbf{r},\mathbf{p})$, corresponds to the semi-classical μ-space, and can be regarded as a probability distribution for the coordinate $\mathbf{r}$ and the momentum $\mathbf{p}$. Balazs and Zipfel [6] have obtained an approximate form for this function for a three-dimensional Fermi gas in a spherically symmetric potential well. Their result for an Airy-type approximation of the wave function is

$$f(r,p)=\frac{1}{(2\pi)^3}\int_{(H-E_F)\sigma}^{\infty} dt\ \mathrm{Ai}(t) \qquad (2)$$

where $H(r,p)=\frac{1}{2}p^2+V(r)$ and σ is a scaling factor.

In Section 2 we use the form of the Wigner function given in Equation (2) to calculate the distribution functions for the different powers of the momentum p. In Section 3 we use a single-particle Woods–Saxon potential in the Hamiltonian to get the density functions for p^n using constant scaling factor σ. In Section 4 we compare the results for the number and the kinetic energy densities with those of the classical Thomas–Fermi theory.

2. SEMI-CLASSICAL DENSITY DISTRIBUTION

The Wigner transform, $f(r,p)$, when multiplied by $d^3\mathbf{r}d^3\mathbf{p}$, represents the probability distrituion for the coordinate and momenta. Hence, we may regard the integral

$$\int_0^{\infty} d^3p\ p^n f(r,p) \qquad (3)$$

as a semi-classical distribution function for the nth power of the momentum. Using $f(r,p)$ in Equation (2), and $d^3p=4\pi p^2dp$, the energy distribution functions for the $(n-2)$th power of the momentum will be given by:

$$D_n(r)=\frac{4\pi}{(2\pi)^3}\int_0^{\infty} dp\ p^n\int_{\sigma(1/2p^2+V(r)-E_F)}^{\infty} dt\ \mathrm{Ai}(t) \qquad (4)$$

where E_F is the Fermi energy. Interchanging the order of integration in Equation (4), we get

$$D_n(r)=\frac{4\pi}{(2\pi)^3}\int_{B(E_F,r)}^{\infty} dt\ \mathrm{Ai}(t)\int_0^{\sqrt{\{2/\sigma(t-B)\}}} dp\ p^n \qquad (5)$$

where $B(E_F,r)=\sigma_{E_F}(r)(V(r)-E_F)$. Or,

$$D_n(r)=\frac{4\pi}{(2\pi)^3}\frac{1}{n+1}\left(\frac{2}{\sigma_E(r)}\right)^{(n+1)/2}\int_B^{\infty} dt\ \mathrm{Ai}(t)(t-B)^{(n+1)/2} \qquad (6)$$

The Weyl (fractional) transform of order μ, of a function $f(x)$, is defined as [8]

$$\mathscr{W}_\mu\{f(x);y\}=h(y;v)=\frac{1}{\Gamma(v)}\int_y^{\infty} f(x)\ (x-y)^{v-1}\ dx \qquad (7)$$

The integral in Equation (6) has the same form as the Weyl transform of order $\frac{1}{2}(n+3)$ of the Airy function. Thus, to a multiplicative factor the number density is proportional to the Weyl transform of order 5/2 of the Airy function and the kinetic energy density is proportional to the Weyl transform of order 7/2. Or,

$$D_n(r)=\frac{4\pi}{(2\pi h)^3}\frac{\Gamma[(n+5)/2]}{n+1}\left(\frac{2}{\sigma_{E_F}(r)}\right)^{(n+1)/2}{}_{(n+3)/2}\{\mathrm{Ai}(t);\ B(E_F,r)\} \qquad (8)$$

The Weyl transforms are connected with differentiation [8]:

$$-\frac{d}{dy}h(y;\mu)=h(y;\mu-1) \qquad (9)$$

Thus, as in the classical Thomas–Fermi theory, the differentiation of the kinetic energy density with respect to the energy, $\sigma(E_F - V(r))$, would result in a term proportional to the number density [9]. This result is consistent with the fact that should we replace the Airy function in the Wigner transform with a Heaviside step function, we would have the results of the classical Thomas–Fermi theory [6]. Other properties of the Wyle transform are [8]:

$$h(y;1)=\int_y^{\infty} f(x)\,\mathrm{d}x \tag{10}$$

and

$$\mathscr{W}_{\mu}\mathscr{W}_{\nu}=\mathscr{W}_{\mu+\nu} \tag{11}$$

or

$$\frac{1}{\Gamma(\mu+\nu)}\int_y^{\infty} f(x)\,(x-y)^{\mu+\nu-1}\,\mathrm{d}x$$

$$=\frac{1}{\Gamma(\mu)}\frac{1}{\Gamma(\nu)}\int_y^{\infty}\left[\int_x^{\infty} f(t)(t-x)^{\nu-1}\,\mathrm{d}t\right](x-y)^{\mu-1}\,\mathrm{d}x$$

to relate the energy density to the number density we set $\nu=5/2$ and $\mu=1$. We get

$$D_4(r)=\frac{21}{5}\frac{1}{\sigma_{E_F}(r)}\int_{B(E_F,r)}^{\infty} D_2(t)\,\mathrm{d}t \tag{12}$$

as the relation between the energy density and the number density.

The densities in Equation (8) have a functional dependence on the potential which appears both in the limits of the integration $B(E_F,r)=\sigma_{E_F}(r)(V(r)-E_F)$ and in the sealing factor σ as will be discussed in Section 3. The dependence of the number density on the potential warrant further consideration. For inasmuch as the potential is the result of the fermions and their distribution, self-consistency may be achieved by repeated application of the Weyl transform to the number density [10].

3. RESULTS FOR WOODS–SAXON POTENTIAL

The scaling factor in Equation (4) is given by [6]

$$\sigma(E,r)=\frac{2^{5/3}}{k_{\parallel}^2}\left(\frac{3S_{\parallel}}{2}\right)^2 \tag{13}$$

where

$$S_{\parallel}(r)=\int_r^{r_t}\mathrm{d}\xi\;k_{\parallel}(\xi),$$

and

$$k_{\parallel}(\xi)^2=2\left(E_{nl}-V(\xi)-\frac{(l+\frac{1}{2})^2}{2\xi^2}\right) \tag{14}$$

r_t is the turning point and E_{nl} is the energy specified by the Bohr–Sommerfeld quantization rules.

Using a Woods–Saxon potential, $V(r)=\dfrac{V_0}{1+\exp[(r-a)/R]}$, in Equation (14), we find that,

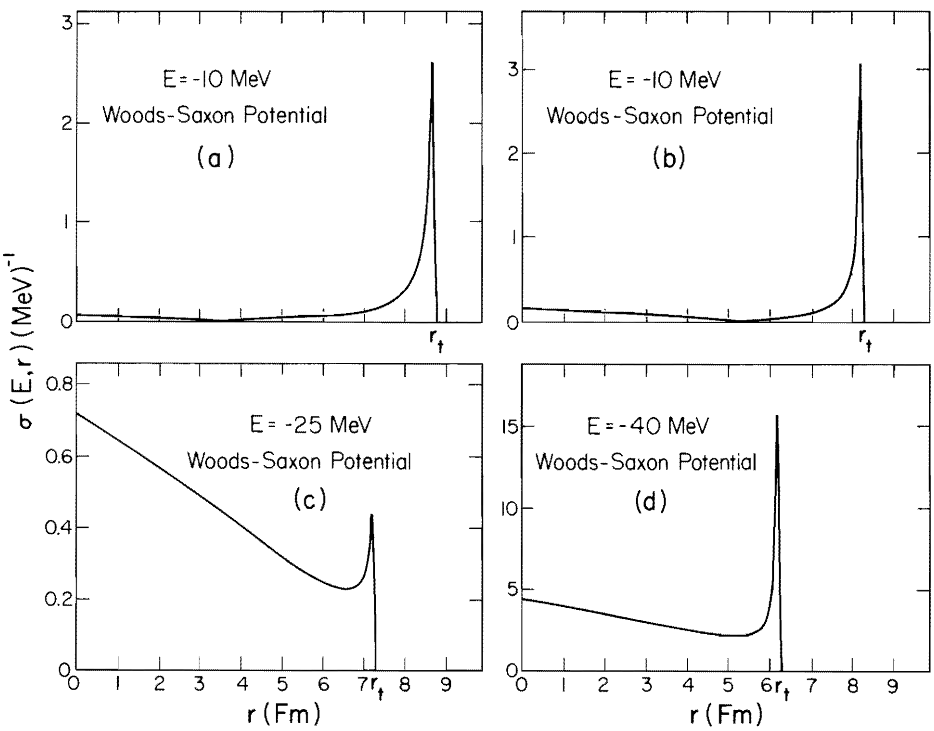

Figure 1. Scaling Factors of Equation (15) *for a Woods–Saxon potential*

should we ignore the $(l+\frac{1}{2})^2/r^2$ term, the integral reduces to:

$$\sigma(E,r)=2^{5/3}\left\{\frac{3a}{(-2E)y^3}\left(\tan^{-1} y+\frac{1}{2y_0}\ln\frac{y_0-y}{y_0+y}\right)\right\}^{2/3} \quad (15)$$

where

$$y(E,r)=\sqrt{\left(\frac{V(r)}{E}-1\right)} \quad \text{and} \quad y_0=\sqrt{\left(\frac{V_0}{E}-1\right)}$$

In Figure 1 we present plots of the scaling factor σ, Equation (15), as a function of the coordinates. In these plots we used $V_0=-47.1$ MeV, $R=7.36$ Fm, $a=0.66$ Fm in the Woods–Saxon potential. These numbers are consistent with neutrons in ^{208}Pb [11]. Since the scaling factors presented in Figure 1 vanish at some coordinate position for the higher energies, we used a constant scaling factor $\sigma_F=2/(V_F')^{2/3}$, where V_F' is the derivative of the potential

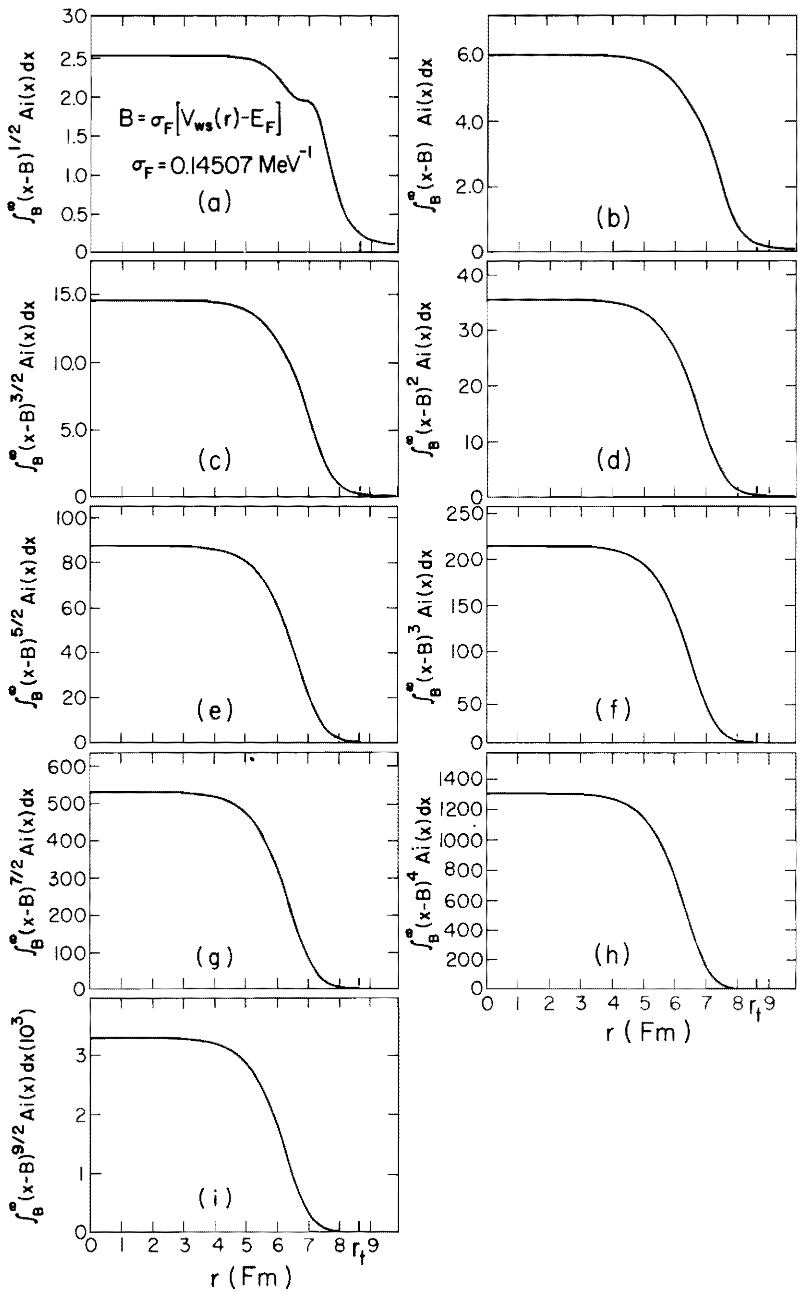

Figure 2. The Integral $I_v(r)=\int_B^\infty (x-B)^v$ Ai(x)dx, $B=\sigma_F[V_{WS}(r)-E_F]$ with $\sigma_F=0.14506$ MeV^{-1} for $v=1/2, 1, \ldots, 9/2$

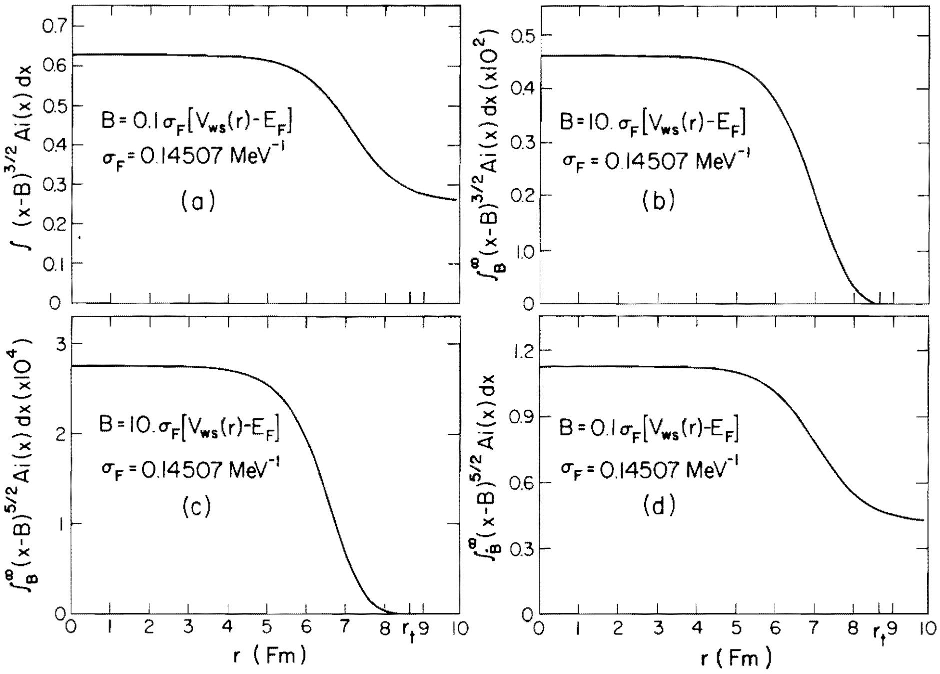

Figure 3. The integral $I_{3/2}$ for the Number Density, and the integral $I_{5/2}$ for the Kinetic Energy Density, Using $\sigma = 0.1\ \sigma_F$ and $\sigma = 10.0\ \sigma_F$, and using a Woods–Saxon Potential

evaluated at the Fermi energy, $E_F = -6$ MeV [11]. The value of σ_F used was 0.14507 MeV^{-1}.

In Figure 2 we present plots of the integrals

$$I_v(r) = \int_B^{\infty} (x - B)^v\ \mathrm{Ai}(x)\ dx \qquad B = \sigma_F[V_{WS}(r) - E_F] \tag{16}$$

for $v = 1/2, 1, 3/2, \ldots, 9/2$, for Woods–Saxon potential and constant $\sigma = \sigma_F$. In Figure 3 we used $\sigma_1 = 0.1\ \sigma_F$ and $\sigma_2 = 10.0\ \sigma_F$ to calculate the integrals I_v in the number and energy densities. Other density distributions show similar trends as those in Figure 3.

Equations (9) and (12) represent the relation between integrals I_v with powers differing by one. In Figure 4 we plot the ratio of the integrals I_v/I_{v-1} for $v = 1, 3/2$ using the same Woods–Saxon potential.

4. DISCUSSION

In the Thomas–Fermi (TF) theory, the Wigner function is a step function which has a constant value, $1/(2\pi\hbar)^3$, in the classically allowed region and vanishes for higher energies. In Figure 5 we present the TF number and energy distributions for a Woods–Saxon potential. The Balazs and Zipfel (BZ) model replaces the step function with the integral of the Airy function. The integral of the Airy function, Figure 6, has a lower

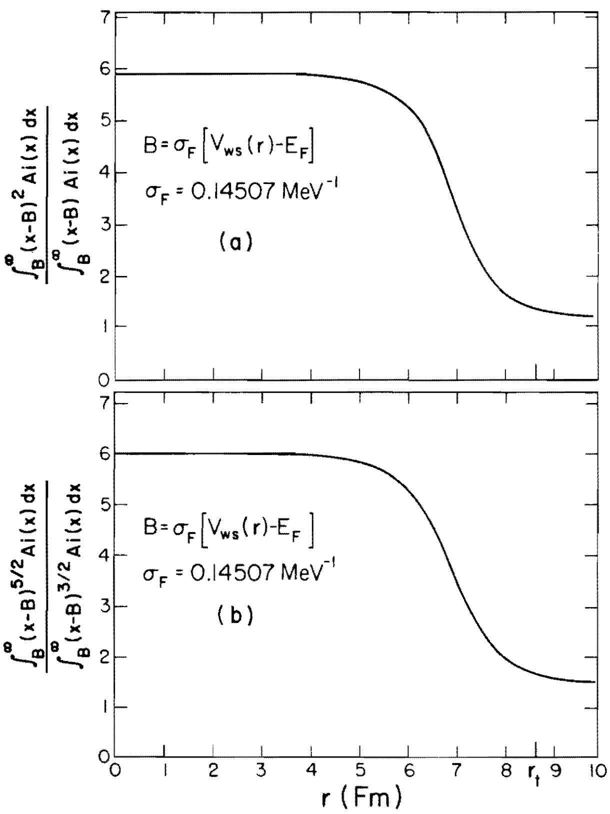

Figure 4. Ratios of the Integrals $I_v^{(r)}/I_{v-1}^{(r)}$ for $v = 1, 3/2$ Using a Woods–Saxon potential

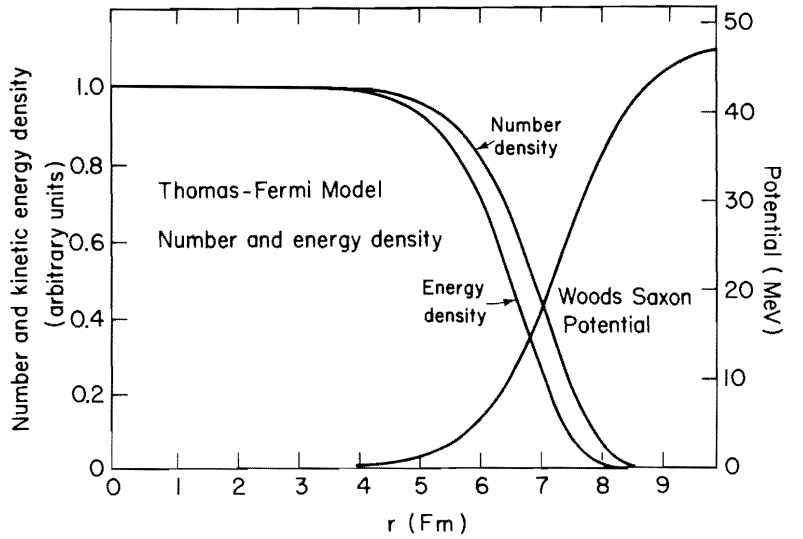

Figure 5. The Thomas–Fermi Number and Kinetic Energy distributions for a Woods–Saxon Potential, r_t = Turning Point

value, 1/3, at the boundary of the classically forbidden region and oscillates about one in the interior region. In Table 1 we compare $[E_F - V_{WS}(r)]^{3/2}$ and $I_{3/2}(r)$ using $B = \sigma_F[V_{WS}(r) - E_F]$, which are involved in the number density in the TF and BZ cases, respectively. In Table 2 we compare $[E_F - V_{WS}(r)]^{5/2}$ and the $I_{5/2}$ integral. The term $[E_F - V_{WS}(r)]^{5/2}$ represents the uncorrected kinetic energy density in the Thomas–Fermi theory [12] and $I_{5/2}$ is proportional to the kinetic energy density in the Balazs and Zipfel model.

In the above we have used a single-particle Woods–Saxon potential to obtain the distribution functions for p^n. The addition of velocity-dependent terms to the potential could be handled easily provided an inverse to the limit of the integration in Equation (4) could be found. It is interesting to note that should the dependence on the velocity be proportional to p^2, its effect will be the same as a change in the scaling factor σ.

Balazs and Pauli [13], have recently extended the model used above to include spin-dependent forces. Thus, for the case of spin one-half particles with spin-orbit interaction, the Wigner function we used would be replaced by a matrix in the spin indices. This matrix, the μ-matrix, is a function of the semiclassical coordinate and momentum variables.

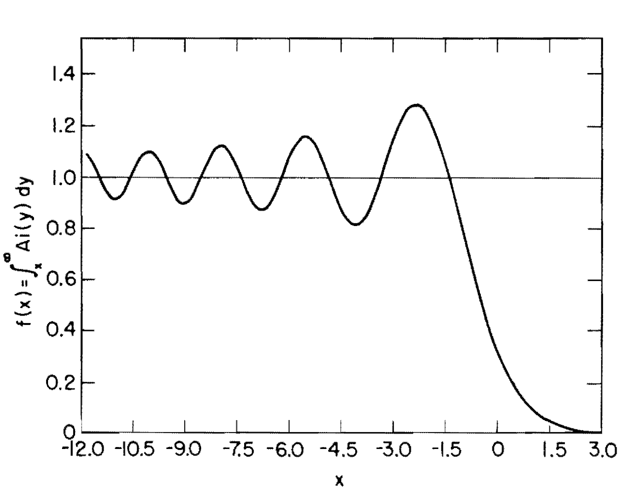

Figure 6. The Integral of the Airy Function

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APPENDIX

In Table 3, we present values of the integral

$$I_\nu(x) = \int_x^\infty (y-x)^\nu \, \mathrm{Ai}(y)\mathrm{d}y$$

for $\nu = 1/2, 1, 3/2, \ldots, 8$ and $x \in [-7.9, 2]$.

The Airy function was generated from an expansion with an accuracy of 1×10^{-10}, and we used the Romberg integration method with a relative error of 1×10^{-5}. Plots of the integrals are presented in Figure 7.

Table 1. Number Densities

r(Fm)	Thomas = Fermi $[E_F - V_{WS}(r)]^{3/2}$	Balazs = Zipfel $\int_B^{\infty} (x-B)^{3/2}\ \mathrm{Ai}(x)\mathrm{d}x;$ $B = \sigma_F[V_{WS}(r) - E_F]$
0	2.6348×10^{2}	1.4554×10^{1}
1.0	2.6160×10^{2}	1.4553×10^{1}
2.0	2.6336×10^{2}	1.4547×10^{1}
3.0	2.6288×10^{2}	1.4520×10^{1}
4.0	2.6072×10^{2}	1.4396×10^{1}
5.0	2.5125×10^{2}	1.3855×10^{1}
6.0	2.1401×10^{2}	1.1755×10^{1}
7.0	1.1624×10^{2}	6.4754×10^{0}
7.2	9.2076×10^{1}	5.042×10^{0}
7.4	6.9085×10^{1}	3.6511×10^{0}
7.6	4.8585×10^{1}	2.4693×10^{0}
7.8	3.1520×10^{1}	1.5921×10^{0}
8.0	1.8321×10^{1}	1.0087×10^{0}
8.2	8.9324×10^{0}	6.4759×10^{-1}
8.4	2.9843×10^{0}	4.3161×10^{-1}
8.6	1.1910×10^{-1}	3.0294×10^{-1}
8.8	0	2.2508×10^{-1}
9.0	0	1.7674×10^{-1}
9.2	0	1.4584×10^{-1}
9.4	0	1.2553×10^{-1}
9.6	0	1.1184×10^{-1}
9.8	0	1.0243×10^{-1}

$r_t = 8.63$ Fm

Table 2. Kinetic Energy Density

r(Fm)	TF $[E_F - V_{WS}(r)]^{3/2}$	BZ $\int_B (x-B)^{5/2}\ \mathrm{Ai}(z)\mathrm{d}z$; $B=\sigma_F[V_{WS}(r)-E_F]$
0	1.0829×10^4	8.7003×10^1
1.0	1.0827×10^4	8.6991×10^1
2.0	1.0820×10^4	8.6933×10^1
3.0	1.0788×10^4	8.6672×10^1
4.0	1.0641×10^4	8.5495×10^1
5.0	1.0004×10^4	8.0400×10^1
6.0	7.6569×10^3	6.1667×10^1
7.0	2.7686×10^3	2.2421×10^1
7.2	1.8775×10^3	1.5267×10^1
7.4	1.1631×10^3	9.6809×10^0
7.6	6.4689×10^2	5.7978×10^0
7.8	3.1452×10^2	3.3658×10^0
8.0	1.2733×10^2	1.9553×10^0
8.2	3.8454×10^1	1.1711×10^0
8.4	6.1858×10^0	7.3911×10^{-1}
8.6	2.8832×10^{-2}	4.9740×10^{-1}
8.8	0	3.5789×10^{-1}
9.0	0	2.7427×10^{-1}
9.2	0	2.2222×10^{-1}
9.4	0	1.8868×10^{-1}
9.6	0	1.6640×10^{-1}
9.8	0	1.5124×10^{-1}

$r_t = 8.63$ Fm

Table 3a. The Integral $I_\nu(x)=\int_x^\infty(z-x)^\nu \mathrm{Ai}\,(z)dz$ ($\nu=1/2, 1, 3/2, 2$)

x	$I_{1/2}(x)$	$I_1(x)$	$I_{3/2}(x)$	$I_2(x)$
-7.9	2.84706(0)	7.89113(0)	2.21704(+1)	6.23816(+1)
-7.8	2.81371(0)	7.77970(0)	2.17458(+1)	6.08145(+1)
-7.7	2.77835(0)	7.66960(0)	2.13263(+1)	5.92696(+1)
-7.6	2.74233(0)	7.56162(0)	2.09123(+1)	5.77466(+1)
-7.5	2.70700(0)	7.45641(0)	2.05036(+1)	5.62448(+1)
-7.4	2.67367(0)	7.35439(0)	2.01001(+1)	5.47638(+1)
-7.3	2.64347(0)	7.25576(0)	1.97013(+1)	5.33028(+1)
-7.2	2.61728(0)	7.16047(0)	1.93069(+1)	5.18613(+1)
-7.1	2.59568(0)	7.06823(0)	1.89159(+1)	5.04384(+1)
-7.0	2.57893(0)	6.97850(0)	1.85279(+1)	4.90338(+1)
-6.9	2.56690(0)	6.89061(0)	1.81420(+1)	4.76469(+1)
-6.8	2.55914(0)	6.80373(0)	1.77576(+1)	4.62775(+1)
-6.7	2.55494(0)	6.71697(0)	1.73741(+1)	4.49254(+1)
-6.6	2.55326(0)	6.62944(0)	1.69910(+1)	4.35908(+1)
-6.5	2.55298(0)	6.54027(0)	1.66080(+1)	4.22737(+1)
-6.4	2.55282(0)	6.44874(0)	1.62251(+1)	4.09748(+1)
-6.3	2.55155(0)	6.35425(0)	1.58422(+1)	3.96944(+1)
-6.2	2.54796(0)	6.25641(0)	1.64597(+1)	3.84333(+1)
-6.1	2.54101(0)	6.15502(0)	1.50780(+1)	3.71921(+1)
-6.0	2.52987(0)	6.05012(0)	1.46977(+1)	3.59716(+1)
-5.9	2.51394(0)	5.94194(0)	1.43193(+1)	3.47723(+1)
-5.8	2.49290(0)	5.83092(0)	1.39437(+1)	3.35950(+1)
-5.7	2.46678(0)	5.71767(0)	1.35717(+1)	3.24401(+1)
-5.6	2.43570(0)	5.60292(0)	1.32040(+1)	3.13080(+1)
-5.5	2.40030(0)	5.48749(0)	1.28412(+1)	3.01990(+1)
-5.4	2.36134(0)	5.37223(0)	1.24841(+1)	2.91130(+1)
-5.3	2.31972(0)	5.25801(0)	1.21329(+1)	2.80500(+1)
-5.2	2.27649(0)	5.14559(0)	1.17882(+1)	2.70097(+1)
-5.1	2.23276(0)	5.03570(0)	1.14500(+1)	2.59916(+1)
-5.0	2.18966(0)	4.92888(0)	1.11184(+1)	2.49952(+1)
-4.9	2.14824(0)	4.82556(0)	1.07930(+1)	2.40198(+1)
-4.8	2.10948(0)	4.72597(0)	1.04738(+1)	2.30647(+1)
-4.7	2.07419(0)	4.63017(0)	1.01600(+1)	2.21292(+1)
-4.6	2.04299(0)	4.53802(0)	9.85129(0)	2.12124(+1)
-4.5	2.01631(0)	4.44924(0)	9.54690(0)	2.03137(+1)
-4.4	1.99434(0)	4.36336(0)	9.24616(0)	1.94325(+1)
-4.3	1.97707(0)	4.27982(0)	8.94836(0)	1.85682(+1)
-4.2	1.96424(0)	4.19792(0)	8.65281(0)	1.77205(+1)
-4.1	1.95544(0)	4.11691(0)	8.35887(0)	1.68890(+1)
-4.0	1.95005(0)	4.03599(0)	8.06600(0)	1.60737(+1)
-3.9	1.94730(0)	3.95438(0)	7.77372(0)	1.52746(+1)
-3.8	1.94633(0)	3.87129(0)	7.48171(0)	1.44920(+1)
-3.7	1.94622(0)	3.78602(0)	7.18974(0)	1.37263(+1)
-3.6	1.94599(0)	3.69795(0)	6.89785(0)	1.29778(+1)
-3.5	1.94470(0)	3.60653(0)	6.60603(0)	1.22473(+1)
-3.4	1.94140(0)	3.51137(0)	6.31454(0)	1.15355(+1)
-3.3	1.93525(0)	3.41219(0)	6.02375(0)	1.08430(+1)
-3.2	1.92548(0)	3.30885(0)	5.73416(0)	1.01709(+1)
-3.1	1.91146(0)	3.20134(0)	5.44633(0)	9.51978(0)
-3.0	1.89265(0)	3.08980(0)	5.16096(0)	8.89060(0)

Table 3a (continued)

x	$I_{1/2}(x)$	$I_1(x)$	$I_{3/2}(x)$	$I_2(x)$
-2.9	1.86868(0)	2.97449(0)	4.87880(0)	8.28411(0)
-2.8	1.83932(0)	2.85576(0)	4.60062(0)	7.70103(0)
-2.7	1.80449(0)	2.73409(0)	4.32727(0)	7.14200(0)
-2.6	1.76421(0)	2.61002(0)	4.05956(0)	6.60756(0)
-2.5	1.71866(0)	2.48418(0)	3.79828(0)	6.09811(0)
-2.4	1.66814(0)	2.35720(0)	3.54421(0)	5.61396(0)
-2.3	1.61303(0)	2.22981(0)	3.29807(0)	5.15526(0)
-2.2	1.55381(0)	2.10267(0)	3.06051(0)	4.72202(0)
-2.1	1.49102(0)	1.97650(0)	2.83211(0)	4.31413(0)
-2.0	1.42524(0)	1.85195(0)	2.61335(0)	3.93131(0)
-1.9	1.35709(0)	1.72968(0)	2.40465(0)	3.57319(0)
-1.8	1.28720(0)	1.61027(0)	2.20631(0)	3.23925(0)
-1.7	1.21621(0)	1.49427(0)	2.01854(0)	2.92886(0)
-1.6	1.14472(0)	1.38214(0)	1.84147(0)	2.64129(0)
-1.5	1.07332(0)	1.27431(0)	1.67512(0)	2.37572(0)
-1.4	1.00258(0)	1.17111(0)	1.51944(0)	2.13125(0)
-1.3	9.32989(-1)	1.07282(0)	1.37429(0)	1.90694(0)
-1.2	8.65007(-1)	9.79657(-1)	1.23946(0)	1.70178(0)
-1.1	7.99034(-1)	8.91746(-1)	1.11469(0)	1.51473(0)
-1.0	7.35414(-1)	8.09168(-1)	9.99634(-1)	1.34473(0)
-0.9	6.74431(-1)	7.31941(-1)	8.94215(-1)	1.19071(0)
-0.8	6.16314(-1)	6.60030(-1)	7.97160(-1)	1.05160(0)
-0.7	5.61232(-1)	5.99351(-1)	7.08882(-1)	9.26346(-1)
-0.6	5.09307(-1)	5.31779(-1)	6.28631(-1)	8.13917(-1)
-0.5	4.60609(-1)	4.75153(-1)	5.55928(-1)	7.13305(-1)
-0.4	4.15164(-1)	4.23283(-1)	4.90286(-1)	6.23539(-1)
-0.3	3.72958(-1)	3.75953(-1)	4.31216(-1)	5.43689(-1)
-0.2	3.33943(-1)	3.32931(-1)	3.78238(-1)	4.72870(-1)
-0.1	2.98040(-1)	2.93971(-1)	3.30878(-1)	4.10246(-1)
0.0	2.65146(-1)	2.58819(-1)	2.88675(-1)	3.55028(-1)
0.1	2.35138(-1)	2.27218(-1)	2.51189(-1)	3.06481(-1)
0.2	2.07875(-1)	1.98909(-1)	2.17996(-1)	2.63921(-1)
0.3	1.83208(-1)	1.73637(-1)	1.88696(-1)	2.26715(-1)
0.4	1.60977(-1)	1.51155(-1)	1.62912(-1)	1.94280(-1)
0.5	1.41019(-1)	1.31221(-1)	1.40289(-1)	1.66083(-1)
0.6	1.23169(-1)	1.13604(-1)	1.20500(-1)	1.41638(-1)
0.7	1.07263(-1)	9.80865(-2)	1.03241(-1)	1.20502(-1)
0.8	9.31412(-2)	8.44618(-2)	8.82321(-2)	1.02277(-1)
0.9	8.06472(-2)	7.25366(-2)	7.52173(-2)	8.66039(-2)
1.0	6.96317(-2)	6.21315(-2)	6.39639(-2)	7.31610(-2)
1.1	5.99528(-2)	5.30804(-2)	6.42609(-2)	6.16610(-2)
1.2	5.14768(-2)	4.52308(-2)	4.59178(-2)	5.18487(-2)
1.3	4.40784(-2)	3.84437(-2)	3.87639(-2)	4.34979(-2)
1.4	3.76414(-2)	3.25922(-2)	3.26462(-2)	3.64089(-2)
1.5	3.20587(-2)	2.74521(-2)	2.74287(-2)	3.04063(-2)
1.6	2.72318(-2)	2.32604(-2)	2.29908(-2)	2.53362(-2)
1.7	2.30712(-2)	1.95649(-2)	1.92258(-2)	2.10644(-2)
1.8	1.94957(-2)	1.64235(-2)	1.60401(-2)	1.74740(-2)
1.9	1.64322(-2)	1.37531(-2)	1.33515(-2)	1.44636(-2)
2.0	1.38151(-2)	1.14892(-2)	1.10881(-2)	1.19457(-2)

Table 3b. Values of the Integral $I_\nu(x)=\int_x^\infty(z-x)^\nu \mathrm{Ai}(z)\mathrm{d}z$ ($\nu=5/2, 3, 7/2, 4$)

x	$I_{5/2}(x)$	$I_3(x)$	$I_{7/2}(x)$	$I_4(x)$
-7.9	1.75625(+2)	4.95051(+2)	1.39811(+3)	3.95825(+3)
-7.8	1.70135(+2)	4.76572(+2)	1.33761(+3)	3.76394(+3)
-7.7	1.64752(+2)	4.58560(+2)	1.27901(+3)	3.57693(+3)
-7.6	1.59472(+2)	4.41008(+2)	1.22227(+3)	3.39703(+3)
-7.5	1.54295(+2)	4.23909(+2)	1.16736(+3)	3.22406(+3)
-7.4	1.49220(+2)	4.07259(+2)	1.11425(+3)	3.05784(+3)
-7.3	1.44245(+2)	3.91049(+2)	1.06290(+3)	2.89819(+3)
-7.2	1.39369(+2)	3.75275(+2)	1.01327(+3)	2.74494(+3)
-7.1	1.34591(+2)	3.59931(+2)	9.65329(+2)	2.59792(+3)
-7.0	1.29910(+2)	3.45010(+2)	9.19044(+2)	2.45694(+3)
-6.9	1.25327(+2)	3.30509(+2)	8.74380(+2)	2.32185(+3)
-6.8	1.20839(+2)	3.16420(+2)	8.31304(+2)	2.19248(+3)
-6.7	1.16448(+2)	3.02740(+2)	7.89782(+2)	2.06866(+3)
-6.6	1.12152(+2)	2.89463(+2)	7.49779(+2)	1.95023(+3)
-6.5	1.07952(+2)	2.76584(+2)	7.11264(+2)	1.83704(+3)
-6.4	1.03848(+2)	2.64097(+2)	6.74202(+2)	1.72891(+3)
-6.3	9.98398(+1)	2.51997(+2)	6.38559(+2)	1.62571(+3)
-6.2	9.59270(+1)	2.40279(+2)	6.04303(+2)	1.52727(+3)
-6.1	9.21098(+1)	2.28935(+2)	5.71399(+2)	1.43344(+3)
-6.0	8.83879(+1)	2.17961(+2)	5.39814(+2)	1.34407(+3)
-5.9	8.47608(+1)	2.07350(+2)	5.09516(+2)	1.25902(+3)
-5.8	8.12280(+1)	1.97096(+2)	4.80471(+2)	1.17814(+3)
-5.7	7.77887(+1)	1.87191(+2)	4.52646(+2)	1.10130(+3)
-5.6	7.44418(+1)	1.77629(+2)	4.26008(+2)	1.02834(+3)
-5.5	7.11863(+1)	1.68404(+2)	4.00526(+2)	9.59147(+2)
-5.4	6.80208(+1)	1.59508(+2)	3.76167(+2)	8.93575(+2)
-5.3	6.49438(+1)	1.50934(+2)	3.52901(+2)	8.31498(+2)
-5.2	6.19537(+1)	1.42675(+2)	3.30696(+2)	7.72786(+2)
-5.1	5.90491(+1)	1.34726(+2)	3.09523(+2)	7.17316(+2)
-5.0	5.62282(+1)	1.27078(+2)	2.89352(+2)	6.64965(+2)
-4.9	5.34894(+1)	1.19727(+2)	2.70154(+2)	6.15614(+2)
-4.8	5.08312(+1)	1.12664(+2)	2.51900(+2)	5.69145(+2)
-4.7	4.82521(+1)	1.05886(+2)	2.34563(+2)	5.25444(+2)
-4.6	4.57508(+1)	9.93851(+1)	2.18115(+2)	4.84399(+2)
-4.5	4.33261(+1)	9.31566(+1)	2.02528(+2)	4.45900(+2)
-4.4	4.09770(+1)	8.71951(+1)	1.87778(+2)	4.09839(+2)
-4.3	3.87027(+1)	8.14954(+1)	1.73836(+2)	3.76109(+2)
-4.2	3.65026(+1)	7.60525(+1)	1.60677(+2)	3.44608(+2)
-4.1	3.43762(+1)	7.08615(+1)	1.48275(+2)	3.15234(+2)
-4.0	3.23231(+1)	6.59175(+1)	1.36605(+2)	2.87886(+2)
-3.9	3.03432(+1)	6.12156(+1)	1.25641(+2)	2.62467(+2)
-3.8	2.84362(+1)	5.67511(+1)	1.15356(+2)	2.38882(+2)
-3.7	2.66023(+1)	5.25187(+1)	1.05727(+2)	2.17035(+2)
-3.6	2.48413(+1)	4.85136(+1)	9.67263(+1)	1.96836(+2)
-3.5	2.31534(+1)	4.47302(+1)	8.83294(+1)	1.78195(+2)
-3.4	2.15383(+1)	4.11633(+1)	8.05104(+1)	1.61023(+2)
-3.3	1.99960(+1)	3.78070(+1)	7.32441(+1)	1.45236(+2)
-3.2	1.85263(+1)	3.46555(+1)	6.65048(+1)	1.30751(+2)
-3.1	1.71288(+1)	3.17024(+1)	6.02672(+1)	1.17485(+2)
-3.0	1.58029(+1)	2.89414(+1)	5.45062(+1)	1.05363(+2)
-2.9	1.45480(+1)	2.63658(+1)	4.91969(+1)	9.43076(+1)

Table 3b (continued)

x	$I_{5/2}(x)$	$I_3(x)$	$I_{7/2}(x)$	$I_4(x)$
-2.8	1.33632(+1)	2.39686(+1)	4.43144(+1)	8.42466(+1)
-2.7	1.22473(+1)	2.17427(+1)	3.98346(+1)	7.51099(+1)
-2.6	1.11991(+1)	1.96809(+1)	3.57334(+1)	6.68305(+1)
-2.5	1.02170(+1)	1.77757(+1)	3.19875(+1)	5.93443(+1)
-2.4	9.29936(0)	1.60195(+1)	2.85740(+1)	5.25901(+1)
-2.3	8.44424(0)	1.44048(+1)	2.54706(+1)	4.65098(+1)
-2.2	7.64961(0)	1.29238(+1)	2.26560(+1)	4.10484(+1)
-2.1	6.91323(0)	1.15690(+1)	2.01091(+1)	3.61539(+1)
-2.0	6.23275(0)	1.03328(+1)	1.78102(+1)	3.17774(+1)
-1.9	5.60571(0)	9.20778(0)	1.57400(+1)	2.78729(+1)
-1.8	5.02956(0)	8.18651(0)	1.38802(+1)	2.43973(+1)
-1.7	4.50168(0)	7.26187(0)	1.22136(+1)	2.13108(+1)
-1.6	4.01940(0)	6.42691(0)	1.07237(+1)	1.85759(+1)
-1.5	3.58005(0)	5.67490(0)	9.39505(0)	1.61582(+1)
-1.4	3.18095(0)	4.99937(0)	8.21301(0)	1.40258(+1)
-1.3	2.81945(0)	4.39413(0)	7.16400(0)	1.21493(+1)
-1.2	2.49294(0)	3.85329(0)	6.23531(0)	1.05019(+1)
-1.1	2.19888(0)	3.37125(0)	5.41516(0)	9.05885(0)
-1.0	1.93479(0)	2.94274(0)	4.69260(0)	7.79775(0)
-0.9	1.69829(0)	2.56281(0)	4.05758(0)	6.69818(0)
-0.8	1.48708(0)	2.22683(0)	3.50085(0)	5.74164(0)
-0.7	1.29899(0)	1.93047(0)	3.01393(0)	4.91143(0)
-0.6	1.13197(0)	1.66974(0)	2.58909(0)	4.19252(0)
-0.5	9.84055(-1)	1.44094(0)	2.21932(0)	3.57139(0)
-0.4	8.53420(-1)	1.24067(0)	2.89824(0)	3.03596(0)
-0.3	7.38364(-1)	1.06582(0)	1.62011(0)	2.57546(0)
-0.2	6.37305(-1)	9.13555(-1)	1.37975(0)	2.18029(0)
-0.1	5.48777(-1)	7.81282(-1)	1.17253(0)	1.84195(0)
0.0	4.71436(-1)	6.66667(-1)	9.94302(-1)	1.55292(0)
0.1	4.04047(-1)	5.67598(-1)	8.41365(-1)	1.30655(0)
0.2	3.45484(-1)	4.82179(-1)	7.10439(-1)	1.09702(0)
0.3	2.94725(-1)	4.08710(-1)	5.98616(-1)	9.19212(-1)
0.4	2.50843(-1)	3.45673(-1)	5.03330(-1)	7.68660(-1)
0.5	2.13005(-1)	2.91718(-1)	4.22321(-1)	6.41464(-1)
0.6	1.80462(-1)	2.45648(-1)	3.53609(-1)	5.34235(-1)
0.7	1.52544(-1)	2.06405(-1)	2.95458(-1)	4.44035(-1)
0.8	1.28654(-1)	1.73056(-1)	2.46358(-1)	3.68325(-1)
0.9	1.08262(-1)	1.44784(-1)	2.04992(-1)	3.04914(-1)
1.0	9.08991(-2)	1.20871(-1)	1.70221(-1)	2.51917(-1)
1.1	7.61513(-2)	1.00693(-1)	1.41058(-1)	2.07720(-1)
1.2	6.36554(-2)	8.37057(-2)	1.16652(-1)	1.70938(-1)
1.3	5.30933(-2)	5.94376(-2)	9.62735(-2)	1.40393(-1)
1.4	4.41872(-2)	5.74808(-2)	7.92940(-2)	1.15080(-1)
1.5	3.66953(-2)	4.74837(-2)	6.51776(-2)	9.41470(-2)
1.6	3.04079(-2)	3.91439(-2)	5.34669(-2)	7.58721(-2)
1.7	2.51439(-2)	3.22022(-2)	4.37727(-2)	6.26457(-2)
1.8	2.07468(-2)	2.64372(-2)	3.57651(-2)	5.09576(-2)
1.9	1.70824(-2)	2.16599(-2)	2.91646(-2)	4.13644(-2)
2.0	1.40356(-2)	1.77098(-2)	2.37354(-2)	3.35156(-2)

Table 3c. The Integral $I_\nu(x)=\int_x^\infty (z-x)^\nu \mathrm{Ai}(z)dz$ ($\nu=9/2, 5, 11/2, 6$)

x	$I_{9/2}(x)$	$I_5(x)$	$I_{11/2}(x)$	$I_6(x)$
-7.9	1.12391(+4)	3.20187(+4)	9.15545(+4)	2.62849(+5)
-7.8	1.06236(+4)	3.00885(+4)	8.55439(+4)	2.44222(+5)
-7.7	1.00350(+4)	2.82536(+4)	7.98640(+4)	2.26724(+5)
-7.6	9.47223(+3)	2.65104(+4)	7.45007(+4)	2.10299(+5)
-7.5	8.93463(+3)	2.48554(+4)	6.94399(+4)	1.94894(+5)
-7.4	8.42134(+3)	2.32852(+4)	6.46681(+4)	1.80456(+5)
-7.3	7.93154(+3)	2.17964(+4)	6.01721(+4)	1.66935(+5)
-7.2	7.46447(+3)	2.03869(+4)	5.59392(+4)	1.54284(+5)
-7.1	7.01935(+3)	1.90505(+4)	5.19572(+4)	1.42457(+5)
-7.0	5.59543(+3)	1.77870(+4)	4.82141(+4)	1.31409(+5)
-6.9	6.19198(+3)	1.65925(+4)	4.46985(+4)	1.21099(+5)
-6.8	5.80825(+3)	1.54642(+4)	4.13993(+4)	1.11485(+5)
-6.7	5.44356(+3)	1.43991(+4)	3.83059(+4)	1.02529(+5)
-6.6	5.09721(+3)	1.33946(+4)	3.54080(+4)	9.41939(+4)
-6.5	4.76854(+3)	1.24480(+4)	3.26957(+4)	8.64439(+4)
-6.4	4.45686(+3)	1.15568(+4)	3.01595(+4)	7.92452(+4)
-6.3	4.16154(+3)	1.07183(+4)	2.77902(+4)	7.25652(+4)
-6.2	3.88196(+3)	9.93025(+3)	2.55789(+4)	6.63731(+4)
-6.1	3.61747(+3)	9.19026(+3)	2.35173(+4)	6.06393(+4)
-6.0	3.36749(+3)	8.49607(+3)	2.15971(+4)	5.53356(+4)
-5.9	3.13144(+3)	7.84548(+3)	1.98105(+4)	5.04353(+4)
-5.8	2.90874(+3)	7.23636(+3)	1.81500(+4)	4.59128(+4)
-5.7	2.69883(+3)	6.66666(+3)	1.66085(+4)	4.17438(+4)
-5.6	2.50118(+3)	6.13441(+3)	1.51791(+4)	3.79053(+4)
-5.5	2.31525(+3)	5.63769(+3)	1.38551(+4)	3.43754(+4)
-5.4	2.14054(+3)	5.17466(+3)	1.26302(+4)	3.11333(+4)
-5.3	1.97654(+3)	4.74354(+3)	1.14985(+4)	2.81594(+4)
-5.2	1.82277(+3)	4.34260(+3)	1.04542(+4)	2.54350(+4)
-5.1	1.67876(+3)	3.97021(+3)	9.49168(+3)	2.29426(+4)
-5.0	1.54405(+3)	3.62477(+3)	8.60583(+3)	2.06654(+4)
-4.9	1.41819(+3)	3.30475(+3)	7.79161(+3)	1.85878(+4)
-4.8	1.30077(+3)	3.00867(+3)	7.04427(+3)	1.66949(+4)
-4.7	1.19135(+3)	2.73514(+3)	6.35930(+3)	1.49729(+4)
-4.6	1.08953(+3)	2.48279(+3)	5.73240(+3)	1.34085(+4)
-4.5	9.94913(+2)	2.25032(+3)	5.15950(+3)	1.19896(+4)
-4.4	9.07125(+2)	2.03648(+3)	4.63674(+3)	1.07044(+4)
-4.3	8.25792(+2)	1.84009(+3)	4.16047(+3)	9.54229(+3)
-4.2	7.50556(+2)	1. 66000(+3)	3.72725(+3)	8.49305(+3)
-4.1	6.81069(+2)	1.49513(+3)	3.33381(+3)	7.54724(+3)
-4.0	6.16998(+2)	1.34443(+3)	2.97708(+3)	6.69606(+3)
-3.9	5.58019(+2)	1.20692(+3)	2.65418(+3)	5.93129(+3)
-3.8	5.03819(+2)	1.08166(+3)	2.36238(+3)	5.24531(+3)
-3.7	4.54100(+2)	9.67746(+2)	2.09916(+3)	4.63104(+3)
-3.6	4.08571(+2)	8.64345(+2)	1.86211(+3)	4.08191(+3)
-3.5	3.66956(+2)	7.70650(+2)	1.64901(+3)	3.59188(+3)
-3.4	3.28988(+2)	6.85905(+2)	1.45779(+3)	3.15534(+3)
-3.3	2.94413(+2)	6.09396(+2)	1.28650(+3)	2.76715(+3)
-3.2	2.62989(+2)	5.40452(+2)	1.13335(+3)	2.42256(+3)
-3.1	2.34484(+2)	4.78442(+2)	9.96678(+2)	2.11722(+3)
-3.0	2.08677(+2)	4.22776(+2)	8.74928(+2)	1.84716(+3)
-2.9	1.85360(+2)	3.72901(+2)	7.66677(+2)	1.60873(+3)

Table 3c (continued)

x	$I_{9/2}(x)$	$I_5(x)$	$I_{11/2}(x)$	$I_6(x)$
-2.8	1.64336(+2)	3.28303(+2)	6.70611(+2)	1.39862(+3)
-2.7	1.45417(+2)	2.88501(+2)	5.85521(+2)	1.21381(+3)
-2.6	1.28428(+2)	2.53050(+2)	5.10299(+2)	1.05155(+3)
-2.5	1.13204(+2)	2.21538(+2)	4.43927(+2)	9.09359(+2)
-2.4	9.95894(+1)	1.93584(+2)	3.85479(+2)	7.84991(+2)
-2.3	8.74406(+1)	1.68836(+2)	3.34110(+2)	6.76418(+2)
-2.2	7.66225(+1)	1.46971(+2)	2.89051(+2)	5.81812(+2)
-2.1	6.70101(+1)	1.27693(+2)	2.49604(+2)	4.99535(+2)
-2.0	5.84872(+1)	1.10731(+2)	2.15140(+2)	4.28118(+2)
-1.9	5.09466(+1)	9.58367(+1)	1.85088(+2)	3.66245(+2)
-1.8	4.42896(+1)	8.27862(+1)	1.58937(+2)	3.12745(+2)
-1.7	3.84254(+1)	7.13746(+1)	1.36225(+2)	2.66574(+2)
-1.6	3.32708(+1)	6.14168(+1)	1.16539(+2)	2.26805(+2)
-1.5	2.87499(+1)	5.27459(+1)	9.95106(+1)	1.92617(+2)
-1.4	2.47933(+1)	4.52111(+1)	8.48106(+1)	1.63283(+2)
-1.3	2.13382(+1)	3.86774(+1)	7.21460(+1)	1.38163(+2)
-1.2	1.83277(+1)	3.30237(+1)	6.12571(+1)	1.16694(+2)
-1.1	1.57102(+1)	2.81415(+1)	5.19136(+1)	9.83806(+1)
-1.0	1.34394(+1)	2.39345(+1)	4.39123(+1)	8.27893(+1)
-0.9	1.14737(+1)	2.03168(+1)	3.70743(+1)	6.95414(+1)
-0.8	9.77582(0)	1.72125(+1)	3.12422(+1)	5.83065(+1)
-0.7	8.31247(0)	1.45542(+1)	2.62780(+1)	4.87972(+1)
-0.6	7.05398(0)	1.22825(+1)	2.20609(+1)	4.07642(+1)
-0.5	5.97403(0)	1.03453(+1)	1.84859(+1)	3.39914(+1)
-0.4	5.04929(0)	8.69684(0)	1.54611(+1)	2.82921(+1)
-0.3	4.25917(0)	7.29690(0)	1.29070(+1)	2.35055(+1)
-0.2	3.58553(0)	6.11050(0)	1.07546(+1)	1.94931(+1)
-0.1	3.01243(0)	5.10714(0)	8.94449(0)	1.61363(+1)
0.0	2.52591(0)	4.26033(0)	7.42511(0)	1.33333(+1)
0.1	2.11377(0)	3.54712(0)	6.15235(0)	1.09972(+1)
0.2	1.76538(0)	2.94765(0)	5.08829(0)	9.05402(0)
0.3	1.47151(0)	2.44482(0)	4.20045(0)	7.44072(0)
0.4	1.22414(0)	2.02390(0)	3.46112(0)	6.10387(0)
0.5	1.01637(0)	1.67227(0)	2.84664(0)	4.99820(0)
0.6	8.42212(-1)	1.37911(0)	2.33695(0)	4.08547(0)
0.7	6.96538(-1)	1.13519(0)	1.91499(0)	3.33343(0)
0.8	5.74943(-1)	9.32660(-1)	1.56634(0)	2.71497(0)
0.9	4.73657(-1)	7.64821(-1)	1.27883(0)	2.20731(0)
1.0	3.89462(-1)	6.26012(-1)	1.04219(0)	1.79138(0)
1.1	3.19618(-1)	5.11438(-1)	8.47795(-1)	1.45125(0)
1.2	2.61797(-1)	4.17057(-1)	6.88409(-1)	1.17362(0)
1.3	2.14027(-1)	3.39462(-1)	5.57978(-1)	9.47431(-1)
1.4	1.74642(-1)	2.75793(-1)	4.51444(-1)	7.63487(-1)
1.5	1.42234(-1)	2.23653(-1)	3.64594(-1)	6.14176(-1)
1.6	1.15621(-1)	1.81037(-1)	2.93925(-1)	4.93201(-1)
1.7	9.38114(-2)	1.46273(-1)	2.36531(-1)	3.95364(-1)
1.8	7.59731(-2)	1.17969(-1)	1.90005(-1)	3.16383(-1)
1.9	6.14120(-2)	9.49694(-2)	1.52360(-1)	2.52741(-1)
2.0	4.95494(-2)	7.63152(-2)	1.21958(-1)	2.01552(-1)

Table 3d. The Integral $I_\nu(x)=\int_x^\infty (z-x)^\nu \, Ai(z)dz$ ($\nu=13/2, 7, 15/2, 8$)

x	$I_{13/2}(x)$	$I_7(x)$	$I_{15/2}(x)$	$I_8(x)$
-7.9	7.57887(+5)	2.19525(+6)	6.38910(+6)	1.86873(+7)
-7.8	7.00348(+5)	2.01785(+6)	5.84251(+6)	1.70029(+7)
-7.7	6.46608(+5)	1.85308(+6)	5.33763(+6)	1.54554(+7)
-7.6	5.96456(+5)	1.70018(+6)	4.87170(+6)	1.40348(+7)
-7.5	5.49692(+5)	1.55842(+6)	4.44210(+6)	1.27321(+7)
-7.4	5.06122(+5)	1.42711(+6)	4.04636(+6)	1.15386(+7)
-7.3	4.65563(+5)	1.30557(+6)	3.68216(+6)	1.04461(+7)
-7.2	4.27841(+5)	1.19319(+6)	3.34731(+6)	9.44720(+6)
-7.1	3.92788(+5)	1.08938(+6)	3.03974(+6)	8.53473(+6)
-7.0	3.60245(+5)	9.93573(+5)	2.75750(+6)	7.70206(+6)
-6.9	3.30060(+5)	9.05237(+5)	2.49878(+6)	6.94302(+6)
-6.8	3.02090(+5)	8.23872(+5)	2.26186(+6)	6.25183(+6)
-6.7	2.76197(+5)	7.49004(+5)	2.04512(+6)	5.62309(+6)
-6.6	2.52250(+5)	6.80187(+5)	1.84707(+6)	5.05181(+6)
-6.5	2.30126(+5)	6.16996(+5)	1.66629(+6)	4.53329(+6)
-6.4	2.09707(+5)	5.59036(+5)	1.50146(+6)	4.06322(+6)
-6.3	1.90882(+5)	5.05932(+5)	1.35133(+6)	3.63754(+6)
-6.2	1.73546(+5)	4.57331(+5)	1.21473(+6)	3.25252(+6)
-6.1	1.57597(+5)	4.12903(+5)	1.09067(+6)	2.90470(+6)
-6.0	1.42943(+5)	3.72336(+5)	9.78044(+5)	2.59085(+6)
-5.9	1.29492(+5)	3.35339(+5)	8.75954(+5)	2.30801(+6)
-5.8	1.17162(+5)	3.01638(+5)	7.83526(+5)	2.05343(+6)
-5.7	1.05872(+5)	2.70978(+5)	6.99951(+5)	1.82458(+6)
-5.6	9.55465(+4)	2.43120(+5)	6.24477(+5)	1.61912(+6)
-5.5	8.61159(+4)	2.17839(+5)	5.56408(+5)	1.43490(+6)
-5.4	7.75134(+4)	1.94927(+5)	4.95096(+5)	1.26994(+6)
-5.3	6.96764(+4)	1.74190(+5)	4.39946(+5)	1.12243(+6)
-5.2	6.25464(+4)	1.55446(+5)	3.90405(+5)	9.90708(+5)
-5.1	5.60683(+4)	1.38527(+5)	3.45964(+5)	8.73235(+5)
-5.0	5.01906(+4)	1.23276(+5)	3.06153(+5)	7.68620(+5)
-4.9	4.48652(+4)	1.09549(+5)	2.70540(+5)	6.75587(+5)
-4.8	4.00470(+4)	9.72100(+4)	2.38728(+5)	5.92972(+5)
-4.7	3.56941(+4)	8.61358(+4)	2.10353(+5)	5.19714(+5)
-4.6	3.17674(+4)	7.62112(+4)	1.85080(+5)	4.54848(+5)
-4.5	2.82303(+4)	6.73300(+4)	1.62605(+5)	3.97498(+5)
-4.4	2.50491(+4)	5.93946(+4)	1.42646(+5)	3.46868(+5)
-4.3	2.21925(+4)	5.23151(+4)	1.24950(+5)	3.02239(+5)
-4.2	1.96312(+4)	4.60090(+4)	1.09283(+5)	2.62959(+5)
-4.1	1.73384(+4)	4.04007(+4)	9.54358(+4)	2.28438(+5)
-4.0	1.52893(+4)	3.54208(+4)	8.32149(+4)	1.98149(+5)
-3.9	1.34609(+4)	3.10061(+4)	7.24467(+4)	1.71614(+5)
-3.8	1.18321(+4)	2.70986(+4)	6.29736(+4)	1.48404(+5)
-3.7	1.03836(+4)	2.36459(+4)	5.46534(+4)	1.28135(+5)
-3.6	9.09756(+3)	2.06000(+4)	4.73576(+4)	1.10462(+5)
-3.5	7.95769(+3)	1.79174(+4)	4.09706(+4)	9.50783(+4)
-3.4	6.94911(+3)	1.55589(+4)	3.53883(+4)	8.17081(+4)
-3.3	6.05824(+3)	1.34887(+4)	3.05175(+4)	7.01072(+4)
-3.2	5.27273(+3)	1.16747(+4)	2.62746(+4)	6.00580(+4)
-3.1	4.58132(+3)	1.00879(+4)	2.25849(+4)	5.13672(+4)
-3.0	3.97381(+3)	8.70236(+3)	1.93816(+4)	4.38636(+4)
-2.9	3.44099(+3)	7.49454(+3)	1.66055(+4)	3.73960(+4)

Table 3d (continued)

x	$I_{13/2}(x)$	$I_7(x)$	$I_{15/2}(x)$	$I_8(x)$
-2.8	2.97449(+3)	6.44353(+3)	1.42036(+4)	3.18306(+4)
-2.7	2.56681(+3)	5.53057(+3)	1.21290(+4)	2.70496(+4)
-2.6	2.21118(+3)	4.73894(+3)	1.03404(+4)	2.29493(+4)
-2.5	1.90151(+3)	4.05373(+3)	8.80080(+3)	1.94389(+4)
-2.4	1.63236(+3)	3.46168(+3)	7.47797(+3)	1.64385(+4)
-2.3	1.39885(+3)	2.95154(+3)	6.34335(+3)	1.38785(+4)
-2.2	1.19665(+3)	2.51139(+3)	5.37187(+3)	1.16979(+4)
-2.1	1.02187(+3)	2.13342(+3)	4.54153(+3)	9.84373(+3)
-2.0	8.71082(+2)	1.80956(+3)	3.83307(+3)	8.26978(+3)
-1.9	7.41232(+2)	1.53205(+3)	3.22968(+3)	6.93603(+3)
-1.8	6.29622(+2)	1.29486(+3)	2.71667(+3)	5.80776(+3)
-1.7	5.33869(+2)	1.09250(+3)	2.28128(+3)	4.85497(+3)
-1.6	4.51876(+2)	9.20164(+2)	1.91243(+3)	4.05176(+3)
-1.5	3.81793(+2)	7.73670(+2)	1.60049(+3)	3.37582(+3)
-1.4	3.22007(+2)	6.49368(+2)	1.33717(+3)	2.80797(+3)
-1.3	2.71099(+2)	5.44091(+2)	1.11527(+3)	2.33176(+3)
-1.2	2.27832(+2)	4.55089(+2)	9.28610(+2)	1.93309(+3)
-1.1	1.91130(+2)	3.79983(+2)	7.71879(+2)	1.59992(+3)
-1.0	1.60054(+2)	3.16721(+2)	6.40510(+2)	1.32196(+3)
-0.9	1.33792(+2)	2.63532(+2)	5.30595(+2)	1.09049(+3)
-0.8	1.11640(+2)	2.18894(+2)	4.38795(+2)	8.98032(+2)
-0.7	9.29891(+1)	1.81500(+2)	3.62261(+2)	7.38318(+2)
-0.6	7.73164(+1)	1.50233(+2)	2.98568(+2)	6.05999(+2)
-0.5	6.41709(+1)	1.24137(+2)	2.45655(+2)	4.96567(+2)
-0.4	5.31657(+1)	1.02395(+2)	2.01777(+2)	4.06220(+2)
-0.3	4.39696(+1)	8.43150(+1)	1.65455(+2)	3.31759(+2)
-0.2	3.62996(+1)	6.93070(+1)	1.35441(+2)	2.70497(+2)
-0.1	2.99145(+1)	5.68718(+1)	1.10684(+2)	2.20182(+2)
0.0	2.46088(+1)	4.65871(+1)	9.02998(+1)	1.78930(+2)
0.1	2.02084(+1)	3.80963(+1)	7.35450(+1)	1.45165(+2)
0.2	1.65656(+1)	3.10994(+1)	5.97980(+1)	1.17578(+2)
0.3	1.35555(+1)	2.53438(+1)	4.85386(+1)	9.50756(+1)
0.4	1.10729(+1)	2.06179(+1)	3.93330(+1)	7.67533(+1)
0.5	9.02903(0)	1.67445(+1)	3.18197(+1)	6.18599(+1)
0.6	7.34955(0)	1.35755(+1)	2.56985(+1)	4.97744(+1)
0.7	5.97201(0)	1.09874(+1)	2.07200(+1)	3.99844(+1)
0.8	4.84419(0)	8.87753(0)	1.66781(+1)	3.20673(+1)
0.9	3.92253(0)	7.16060(0)	1.34022(+1)	2.56757(+1)
1.0	3.17070(0)	5.76592(0)	1.07519(+1)	2.05245(+1)
1.1	2.55853(0)	4.63500(0)	8.61132(0)	1.63800(+1)
1.2	2.06099(0)	3.71959(0)	6.88547(0)	1.30511(+1)
1.3	1.65734(0)	2.97994(0)	5.49639(0)	1.03818(+1)
1.4	1.33045(0)	2.38334(0)	4.38029(0)	8.24509(0)
1.5	1.06620(0)	1.90298(0)	3.48508(0)	6.53752(0)
1.6	8.52973(-1)	1.51689(0)	2.76826(0)	5.17521(0)
1.7	6.81225(-1)	1.20711(0)	2.19526(0)	4.09017(0)
1.8	5.43131(-1)	9.58990(-1)	1.73802(0)	3.22742(0)
1.9	4.32296(-1)	7.60600(-1)	1.37376(0)	2.54255(0)
2.0	3.43496(-1)	6.02251(-1)	1.08407(0)	1.99980(0)

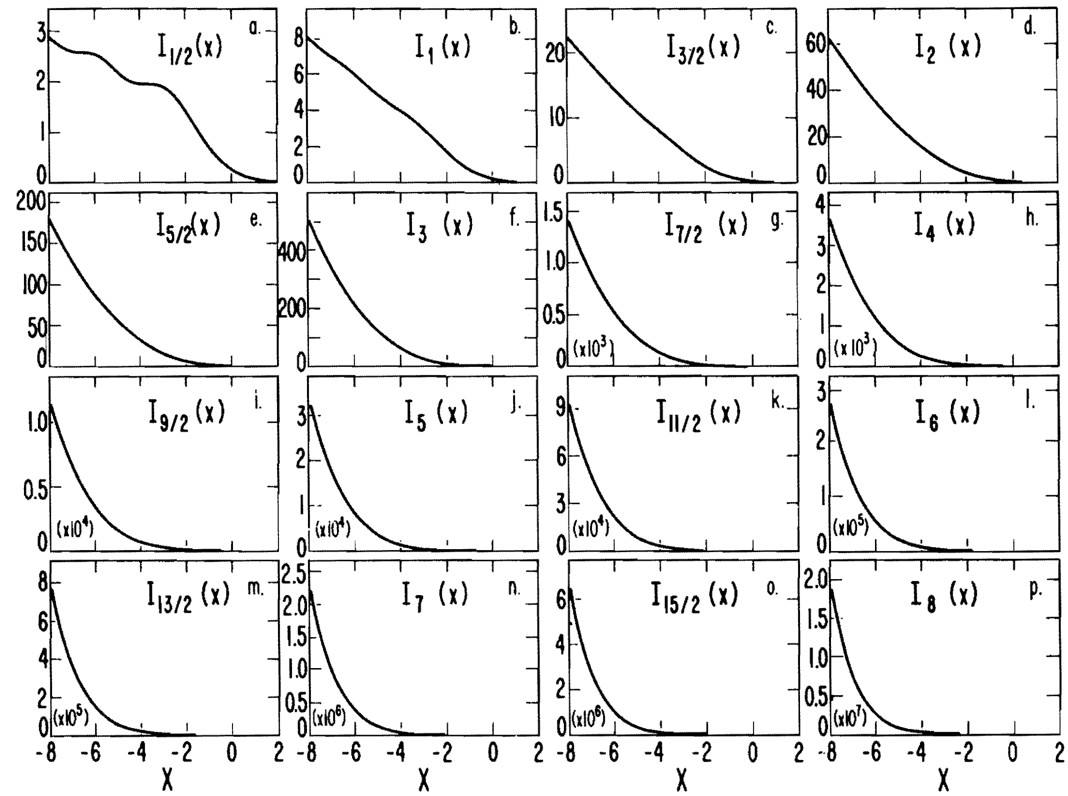

Figure 7. The Integrals $I_\nu(x)=\int_x^\infty (y-x)^\nu\,\mathrm{Ai}(y)\mathrm{d}y$ *for* $x\in[-7.9,2]$ *and* $\nu=1/2, 1, 3/2, \ldots, 8$

A similar table has been published recently [14] using an interpolative technique for generating the Airy function. The numbers presented in Table 3 were generated independently of those in Reference [14] and are included here for completeness.

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