# SYNTHESIS OF COUPLER-COORDINATED LINKAGES BY INVERSION 

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#### Abstract

Computer-aided graphical design procedures are introduced for the synthesis of six and eight-bar linkages with coupler coordination. The two-handled block concept is utilized, in connection with a computer graphics package, to execute inversions. Several examples are provided to illustrate the synthesis of unique six and eight-bar linkages that require coupler coordination. It is demonstrated in each case that the behavior of the resulting mechanism closely meets the design requirements. Keywords: block, computer-aided, coordination, coupler, design, graphics, handle, inversion, linkage, mechanism, synthesis.


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## INTRODUCTION

Virtually all machinery makes use of some sort of a linkage in the production, delivery or application of power, motion, or function. A given linkage may be broken down into simpler sub-linkages, or basic linkages, which, when put together, form the linkage itself. The four-bar linkage and its inversions and the slider-crank mechanism and its inversions represent the most common of the basic linkages.

One of the applications of linkages arises when the motion of an output link is to be coordinated with that of an input link. Of particular interest is the multi-step coordination of four-bar linkages, slider-crank mechanisms, and FISC (first inversion of the slider-crank) mechanisms. The method of inversion may be employed in the classical sense [1-6] to synthesize mechanisms with input-output coordination. Utilization of computer graphics, however, suggests a more pragmatic and resilient approach to design. The latter possibility was scrutinized in earlier communications [4, 7], where it was shown that reasonably accurate designs are achieved after a few trials.

The block feature that is furnished in modern design packages can be used effectively during synthesis operations. A block may be set by setting a point at a point of interest, to form one corner of the block, and a second point at the diagonal corner. One or two more points, set at strategic points on the drawing, may be selected to be used as handles. Once a block has been set in this manner, then it is possible to make copies of it, to drag the block, to rotate it, and to manipulate it, all without altering the relative dimensions of the blocked entity. The two handles are utilized to insert copies of the block at precise locations and configurations.

In what follows we extend the method introduced in references [4] and [7] to the scrutiny of those design situations where it is desired to coordinate the motion of a given coupler with the positions of a different member in the same mechanism. We utilize the two-handled blocking method of inversion to synthesize six- and eight-bar linkages with coordinated members. The computer graphics package utilized for the synthesis must necessarily support two handles on block.

The suitability of the resulting linkages is verified in each case with the aid of Al-Yaseer [8-11]. Al-Yaseer is a software package for the displacement and kinematic, as well as dynamic, analyses of plane mechanisms and machinery. It was developed at King Abdulaziz University, Jeddah, for the purpose of introducing computers into the undergraduate teaching of mechanisms and dynamics of machinery. The package is written in BASIC, and easily fits modern pocket computers. A PC version of the same is also available for the IBM PC and compatibles. The latter version also possesses animation capabilities. Such important considerations as dead points and reversals are readily addressed by Al-Yaseer.

## COORDINATION OF COUPLER AND ROCKER

Let it be required to coordinate the motions of a floating member (coupler) $A B$ and the output rocker $F_{o} F$ of a six-bar linkage. In particular let three significant relative positions each of coupler $A B$ (Figure 1a) of unit length and a rocker $F_{o} F$ of length 0.5 units be selected so as to represent the essential characteristics of the motion of each. The task of synthesis, then, is the determination of the locations of fixed pivots $A_{o}$ and $B_{o}$ as well as the ascertainment of the location of a moving pivot $E$ on coupler $A B$, and the length of member $E F$. Table 1 lists the specifications which the mechanism to be synthesized must satisfy.

The locations of the fixed pivots $A_{o}$ and $B_{o}$, with reference to Figure $1 b$, are determined promptly as centers of the circles passing through the three locations of $A$ and $B$, respectively. Figure $1 h$ depicts the ensuing un-crossed four-bar linkage $A_{o} A B B_{o}$ in its first configuration.

As for the determination of the location of $E$ and of the length of member $E F$, use is made of inversion. Of the three given configurations or positions of the mechanism, we decide to invert to the first position (Table 1 and Figure $1 a$ ). To this end the mechanism is laid out in its three positions (Figure $1 c, 1 d$, and $1 e$ ). The configuration of Figure $1 c$ is blocked, with handles at $A$ and $B$, and then inserted on the configuration of Figure $1 e$, using the same
handles. The process is then repeated for the configuration of Figure $1 d$ such that it is also inserted on the configuration of Figure $1 e$. Figure $1 f$ represents the resulting pattern.

The center $E$ of the circle that passes through the numbered end of member $F_{o} F$ in its three configurations (Figure $1 g$ ) resolves the location of the moving pivot $E$. The synthesis of the mechanism is concluded by drawing rays from $B$ and $F$ (position 1 in Figure $1 h$ ) to $E$. Figure $1 i$ displays the completed mechanism. It is to be noticed that the moving pivot $E$ is on a rigid extension of the coupler $A B$. The resulting dimensions of the mechanism of Figure $1 i$ are $B_{o} A_{o}=0.664$ units, $\angle x B_{o} A_{o}=157.15^{\circ}, A_{o} A=0.324, A B=0.999, B_{o} B=0.623, A E=2.095$ units, $\angle B A E=97.74^{\circ}, F_{o} A_{o}=2.089, \angle x F_{o} A_{o}=172.54^{\circ}, E F=3.012$, and $F_{o} F=0.5$ units.

The behavior of the mechanism of Figure $1 i$ can be investigated conveniently by the use of the software package Al-Yaseer [8-11]. Table 2 is a typical listing of a program for this purpose to demonstrate the briefness of the programs involved. The results are summarized in Table 1 and in Figure 2. It is found that the motion of input

(a)




(f)

(c)

(d)


(h)

(i)

Table 1. Summary of Specifications and Attainment for Coupler-Rocker Coordination.

| Position of | Inclination of coupler $A B^{\circ}$ |  |  | Inclination of rocker $F_{0} F^{\circ}$ |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| mechanism | Required | 24.41 | Generated |  |  |
| First | 2.97 | 24.49 |  | 60.00 | Generated |
| Second | 341.45 | 2.36 |  | 110.00 | 64.67 |
| Third |  | 339.75 |  | 130.00 | 111.80 |



Figure 2. Successive Displacements of the Coupler and the Rocker.
crank $A_{o} A$ is confined to a rocking motion of about $180^{\circ}$, as indicated in Figure 2, while it swings the output member $F_{o} F$ by about $60^{\circ}$. The maximum error is observed from Table 1 to occur at the first position of rocker $F_{o} F$, and corresponds to about $8 \%$ of the range.

It is also possible to utilize the above approach for the synthesis of six-link dwell mechanisms [4, 12]. For this purpose, the above design task may be recast as follows. Let three significant relative positions each of coupler $A B$ (Figure $3 a$ ) of unit length and rocker $F_{0} F$ of length 0.5 units be selected so as to represent the essential parts of the motion of each. It is noticed in Figure $3 a$ that the second and third positions of the coupler correspond to the same position of the rocker. This indicates that the rocker is to dwell while the coupler is displaced from its second position to its third position. The task of synthesis is, as before, the determination of the locations of fixed pivots $A_{o}$ and $B_{o}$ as well as the location of a moving pivot $E$ on coupler $A B$, and the length of member $E F$. Table 3 is a list of specifications which the mechanism to be synthesized must satisfy.

Following the same procedure as before, it may be verified that the mechanism of Figure $3 b$ results, where $A E=$ $7.786, E F=9.297$ units and $\angle B A E=136.95^{\circ}$. Figure $3 c$ and 4 summarize the behavior of the coupler $A B$ and output rocker $F_{o} F$ within the range of operation of this dwell mechanism. A distinct dwell is visible in Figure 4 . It may be concluded from Table 3, which lists the generated data alongside the required data at the three positions, that the performance of the mechanism of Figure $3 b$ is quite acceptable.

As an application that utilizes the coordination of a coupler and a rocker, we may cite a device for trimming paper. The paper cutter would be attached to the coupler of a six-bar linkage, and the rocker would press on the paper during the cutting process. Figure 5 [5] illustrates a machine as a second example, where the motion of coupler $B C$ needs to be coordinated with rocker $E_{o} E$ to which the needle is attached.

## COORDINATION OF COUPLER AND SLIDER

The method of inversion can be readily extended to cases of coordination that involve six-bar linkages with a slider. Thus let three positions of coupler $A B$ be specified, along with the three required displacements of a slider (Table 4 and Figure $6 a$ ). To determine the dimensions of the mechanism that will achieve the constraints of
coordination, the configurations of the mechanism at the third and second positions (Figure 6a) are blocked, with handles at $A$ and $B$, and both blocks are inserted on the first position (Figure $6 b$ ). The center $E$ of the circle that passes through the three slider positions in Figure $6 b$ corresponds to the location of the moving pivot on the coupler. Figure $6 c$ displays the completed mechanism where $B_{o} A_{o}=0.872$ units, $\angle x B_{o} A_{o}=159.35^{\circ}$, $\angle B A E=15.92^{\circ}, A_{o} A=0.419, A B=1, B_{o} B=0.729, A E=2.658, E F=0.568$, and eccentricity of the slider crank is 0.037 units.


Figure 3. Synthesis of a Six-Bar Dwell Mechanism.

Table 2. AI-Yaseer Program for the Mechanism of Figure $1 i$.
10 FOR QQ = 0.1 TO 361 STEP 20: $T(12)=$ QQ: GOSUB 20: GOSUB 30: NEXT QQ: END
$20 \mathrm{~L}(10)=0: \mathrm{L}(11)=0.664: \mathrm{T}(11)=157.15: \mathrm{L}(12)=0.34: \mathrm{L}(13)=0.999: \mathrm{L}(14)=0.623: \mathrm{R}(11)=2.095: \mathrm{H}(11)=-97.72$ :
R $(9)=L(12)$ : GOSUB \#2: PRINT QQ; X(3); Y(3); T(13); : RETURN
$\left.30 \mathrm{~L}(10)=0: \mathrm{L}(11)=2.089: \mathrm{T}(11)=172.54: \mathrm{L}(12)=\operatorname{SQR}\left(\mathrm{X}(4)^{\wedge} 2\right)+\mathrm{Y}(4)^{\wedge} 2\right): \mathrm{T}(12)=\operatorname{ATN}(\mathrm{Y}(4) / \mathrm{X}(4)): \operatorname{IF~X}(4)<0$ THEN T $(12)=\mathrm{T}(12)+180$
$32 \mathrm{~L}(13)=3.012: \mathrm{L}(14)=0.5$ : GOSUB \#2: PRINT T(14): RETURN

Table 3. Summary of Specifications and Attainment for the Dwell Mechanism.

| Position of <br> mechanism | Inclination of coupler $A B-^{\circ}$ |  |  | Inclination of rocker $F_{0} F-^{\circ}$ |  |
| :--- | :---: | :---: | :---: | :---: | ---: |
|  | Required | Generated |  | Required | Generated |
| First | 24.41 | 24.49 |  | 60.00 | 65.64 |
| Second | 2.97 | 2.36 |  | 110.00 | 111.65 |
| Third | 341.45 | 339.75 |  | 110.00 | 110.08 |

Table 4. Coordination Data for Coupler and Slider.

| Position of <br> mechanism | Inclination of coupler $A B-^{\circ}$ |  |  | Location of slider-units |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | Required | Generated |  | Required | Generated |
| First | 23.58 | 23.57 |  | 2.86 | 2.68 |
| Second | 6.85 | 6.80 |  | 3.07 | 2.88 |
| Third | -1.25 | -1.25 |  | 3.19 | 3.00 |

Table 5. Coordination Data for Couplers AB and FH.

| Position of <br> mechanism | Inclination of coupler $A B-^{\circ}$ |  |  | Inclination of coupler $F H-^{\circ}$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | Required | 23.58 | Generated |  | Required |

Table 4 lists the data that is generated by the mechanism of Figure $6 c$ along with the initial requirements. It may be observed that the deviations from the requirements are generally less than $7 \%$. Figure 7 displays the variation of the inclination of coupler $A B$ and displacement of slider with crank displacement.

The Dobbie-McGinnis engine indicator [6] of Figure 8 may be cited as an application where coordination is required between the coupler and a slider. Here coordination is required between coupler $A B C$, which carries the indicator pen, and piston $G$, when $A_{\theta} A$ is considered to be the crank.

## COORDINATION OF TWO COUPLERS

Let there be two couplers, $A B$ and $F H$, the displacements of which need to be coordinated. Let three significant locations of coordination between the two couplers be given as listed in Table 5 and exhibited in Figure $9 a$. The process of synthesis is commenced by determining the coordinates of the fixed pivots $A_{o}, B_{o}, F_{o}$, and $H_{o}$ of the couplers (Figure $9 b$ ). Blocking the second and third positions of the couplers, with handles at $A$ and $B$, and inserting on the first position yields Figure $9 c$. The center of the circle that passes through the three $F$ positions (Figure $9 c$ ) corresponds to the location of the moving pivot $E$ on coupler $A B$. The resulting crank-rocker type eight-
bar linkage is shown in its first-position configuration in Figure $9 d$. The associated dimensions are $B_{o} A_{o}=0.872$, $F_{o} A_{o}=1.604, H_{o} F_{o}=0.986, A E=2.208$ units, $\angle x B_{o} A_{o}=159.35^{\circ}, \angle x F_{o} A_{o}=182.51^{\circ}, \angle x H_{o} F_{o}=182.36^{\circ}$ and $\angle B A E=8.95^{\circ}$. Also, $A_{o} A=0.419, A B=1, B_{o} B=0.729, E F=0.775, F_{o} F=0.554, F H=1.2$, and $H_{o} H=0.666$ units.

It may be pointed out that the above procedure applies equally well to cases when coupler $F H$ needs to be connected to pivot $E v i a$ a point $G$ on the coupler that does not coincide with pivot $F$ (See Figure 10). The latter possibility allows considerable freedom in design.


Figure 4. Variations of the Inclinations of the Coupler and the Rocker with Crank Displacements of the Dwell Mechanism.


Figure 5. A Six-Bar Linkage Used in a Shoe-Stitching Machine [5].

The coupler inclinations generated by the mechanism of Figure $9 d$ are listed in Table 5 along with the original requirements of the same. It is observed that the generation errors are small, generally being $1 \%$ of the range in each case. Variations of the inclinations of the couplers with crank angle are plotted in Figure 11 for a complete cycle.

Figure 12 [5] shows a mechanism where coordination of couplers is required for the automatic assembly of parts. Member $A_{o} A$ is the input crank. The translatory motion of manipulator $G$ is modulated by the parallellinkage mechanisms featuring the superimposed couplers. Notice that Figure 12 is a special case of Figure 9 where member $E F$ is missing.

## CONCLUSIONS

Utilization of computer graphics permits the pragmatic synthesis of six- and eight-bar mechanisms where the motions of two members need to be coordinated. Reasonably accurate designs are achieved in a short time. The two-handled blocking method introduced here possesses the potential to become a powerful technique for the computer-aided graphical design of such mechanisms.

The method allows the rapid synthesis of mechanisms where the displacements of two selected links are to be coordinated at three significant locations. The resulting linkage is unique in each case. Such powerful tools of modern design packages as zoom and intersect are effectively utilized to achieve accurate designs. One outstanding feature of the approach is that the designer is able to synthesize the mechanism without losing sight of the physical situation.



Figure 6. Synthesis for the Coordination of a Coupler and a Slider.


Figure 7. Variation of Coupler Inclination and Slider Displacement with Crank Displacement .


Figure 8. Dobbie-McGinnis Engine Indicator [6].

(b)

(d)

Figure 9. Synthesis for the Coordination of Two Couplers.


Figure 10. Eight-Bar Linkage for the Coordination of Couplers.


Figure 11. Variations of Coupler Indications with Crank Angle in the Eight-Bar Mechanism.


Figure 12. Mechanism for the Automatic Assembly of Parts [5].

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