A LEAST-SQUARES APPROACH FOR INTERPRETATION OF SELF-POTENTIAL ANOMALY OVER A TWO-DIMENSIONAL INCLINED SHEET

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الخلاصـة :

نعرض في هذا البحث طريقة أقلِ المربعات لتعيين عنصرٍ مركبٍ يتعلق بالعمق والميل لصفيحةٍ صخريةٍ مائلةٍ وذلك باستخدام النقط والمسافات المميزة لنمط بيانات الجهد الذاتي؛ وذلك بحلٍّ معادلة غير خطية كما أُعطيت طرق أخرى لتعيين بقية عناصر النمذجة.

تم تطبيق الطريقة على نماذج نظرية بأخطاء نسبيه قدرها ٥٪، ووُجد أن الخطأ في جميع الحالات المدروسة لايزيد عن ١٢,٧٪ كما تـمَّ تطبيق الطريقة بنجاح على مثال حقلي من ألمانيا لتعيين عمقٍ لخام جرافيت.

ABSTRACT

A least squares approach has been applied to the self-potential profile of an inclined sheet to determine a composite parameter related to its depth and dip. By defining a few characteristic distances and points on the anomaly profile, the composite parameter determination is transformed into a method of finding a solution of a nonlinear equation. Knowing the numerical value of the composite parameter, procedures are then formulated to estimate the model parameters of the sheet.

Error response of the parameters determined due to errors in the anomaly profile was studied through imposing 5% errors in the characteristic distances and points in four synthetic profiles. A maximum error of 12.7% is displayed by the electric dipole constant.

Finally the method is applied to interpret self-potential measurements which were carried out above a graphite deposit in the southern Bavarian woods, Germany.

Key Words: SP Interpretation, Inclined Sheet, Iterative Methods.

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INTRODUCTION

Estimating the depth to the upper and lower edges, as well as the dip, of a 2-D inclined sheet from self-potential anomalies measured perpendicular to the strike is a tedious task confronting the mineral industry searching for both sulfides and graphites. Several methods have been proposed and discussed in the literature for interpreting the self-potential anomalies of a two-dimensional inclined sheet, including, for example, logarithmic curve matching [1, 2], the characteristic distances, points, and curves [3–5], analysis in the frequency domain using Fourier transform [6], and finally graphical methods and nomograms [7, 8].

In the present paper, a numerical approach to determine the parameters of the inclined sheet from the characteristic distances and points is presented. The method involves iterative calculation of a composite parameter related to the depths to the upper and lower edges of the sheet and to its dip. The method is applied to theoretical and field data to illustrate its efficiency.

FORMULATION OF THE PROBLEM

Following Roy and Chowdhury [9], the general expression for the SP anomaly at point p along the x-axis (Figure 1) of an arbitrary polarized inclined sheet (2-D) is given by:

$$V(x) = K \ln \frac{x^2 + h^2}{(x-b)^2 + H^2}, \tag{1}$$

in which

$$b = \frac{H - h}{\tan\theta}, \tag{2}$$

where b is a composite parameter related to h and H, the depths to the upper and lower edges, respectively, θ is the angle of inclination, K is the electric dipole constant, and x is the position coordinate.

The following equations were obtained by Roy and Chowdhury [9] and Atchuta Rao *et al.* [6] using the characteristic properties of Equation (1) (see Appendix 1):

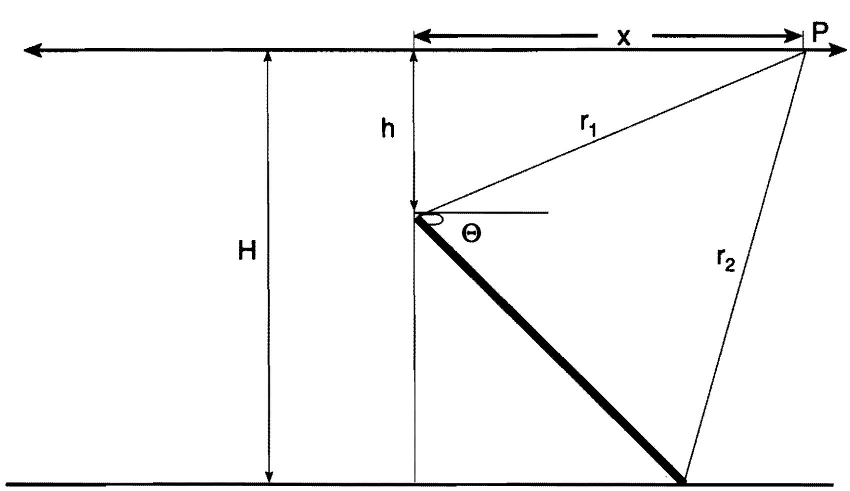

Figure 1. Cross section of an inclined sheet of finite depth extent.

$$K = \frac{V(0)}{\ln \frac{h^2}{(b^2 + H^2)}}, \tag{3}$$

$$h = \sqrt{x_M^2 - 2x_0 x_M}, \tag{4}$$

$$K = \frac{V_{Mm}}{\ln \frac{h^2}{H^2}}, \tag{5}$$

and

$$H = \sqrt{h^2 - b^2 + 2bx_0}, \tag{6}$$

where $V(0)$ is the anomaly value at the origin ($x = 0$), V_{Mm} is the sum of the maximum and minimum anomaly values, occur at x_M and x_m, respectively, and x_0 is the zero-anomaly distances (Figure 2).

From Equation (3–6), it can be shown that the composite parameter (b) can take the following form:

$$b = \frac{\left(2bx_0 - b^2 + x_M^2 - 2x_0 x_M\right)^s - \left(x_M^2 - 2x_0 x_M\right)^s}{2x_0\left(x_M^2 - 2x_0 x_M\right)^{s-1}}. \tag{7}$$

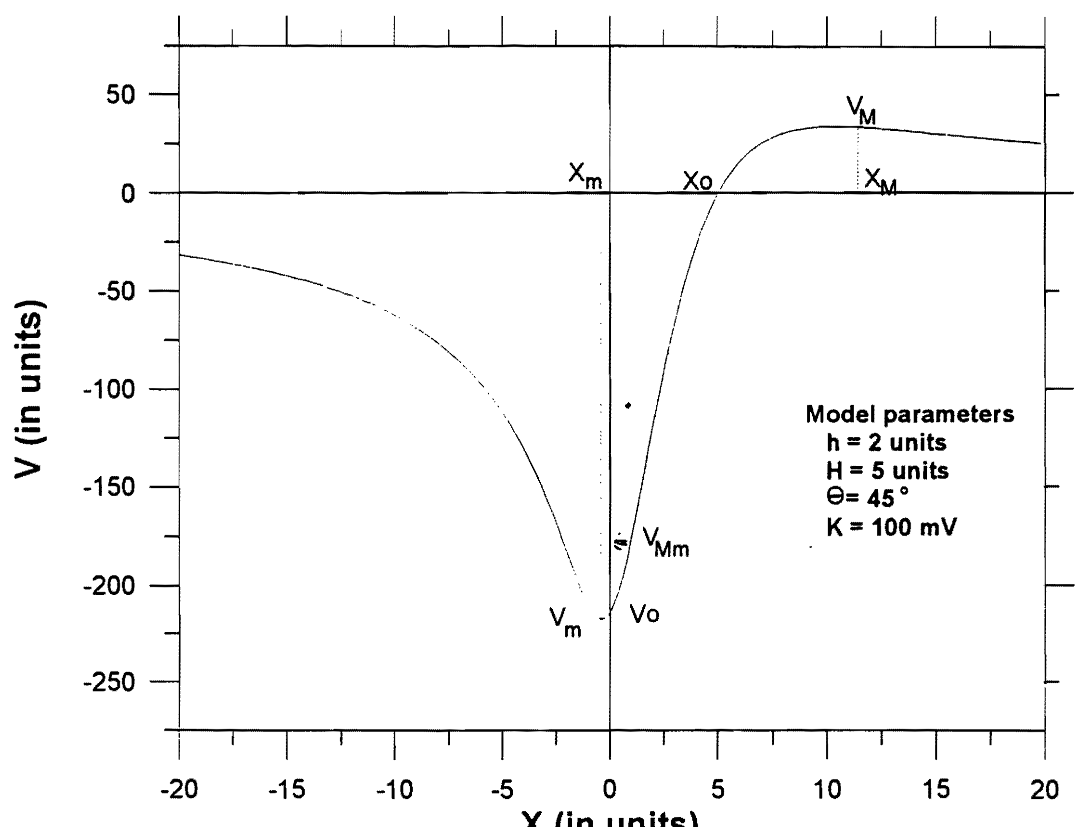

Figure 2. A typical self- potential anomaly profile over an inclined sheet. The characteristic distances (x_0, x_M, x_m) and the zero-anomaly value $V(0)$ are illustrated.

where

$$s = \frac{V(0)}{V_{Mm}}.$$

Solution of this nonlinear equation can be done by using standard methods for solving nonlinear equations. Here, it is solved by the simple iterative method [10]. The iterative form of Equation (7) is given as:-

$$b_j = f(b_i) \tag{8}$$

where b_i is the initial value and b_j is the revised value. Thereafter b_j is used as b_i for the next iteration. The iteration stops when $|b_i - b_j| \le \varepsilon$, where ε is a small predetermined real number close to zero. However, the present method uses only a few data points of the measured anomaly profile in determining the composite parameter (b) of the sheet.

SOLUTION USING THE LEAST-SQUARES METHOD

Using Equations (3), (4), and (6), Equation (1) can be written as:

$$V(x_i, b) = V(0)\, S(b)\, W(x_i, b) \tag{9}$$

where

$$W(x_i, b) = \ln\left[\frac{x_i^2 + (x_M^2 - 2x_0 x_M)}{x_i^2 + 2b(x_0 - x_i) + (x_M^2 - 2x_0 x_M)}\right],$$

and

$$S(b) = \frac{1}{\ln\left[\dfrac{(x_M^2 - 2x_0 x_M)}{x_M^2 + 2bx_0 - 2x_0 x_M}\right]}.$$

Equation (9) is a nonlinear equation in one unknown (b). The unknown composite parameter (b) can be obtained by minimizing:

$$\varphi(b) = \sum_{i=1}^{N} \left[L(x_i) - V(0) S(b)\, W(x_i, b)\right]^2 \tag{10}$$

where $L(x_i)$ denotes the observed SP anomaly at x_i.

Setting the derivative with respect to b equal to zero leads to

$$f(b) = \sum_{i=1}^{N} \left[L(x_i) - V(0) S(b) W(x_i, b)\right] . \left[S^*(b) W(x_i, b) + W^*(x_i, b) S(b)\right] = 0, \tag{11}$$

where

$$S^*(b) = \mathrm{d}S(b)/\mathrm{d}b$$

and

$$W^{*}(x_i, b) = \mathrm{d}W(x_i, b)/\mathrm{d}b$$

Equation (11) can be solved for (*b*) using standard methods for solving nonlinear equations. Here, it is also solved by the simple iterative method [10].

The composite parameter (*b*) is determined by solving one nonlinear equation in (*b*). Any initial guess for (*b*) works well because there is only one global minimum. The experience with the minimization technique for two or more unknowns is that it always produce good results from synthetic data with or without random noise. In the case of field data, good results may only be obtained when using very good initial guesses on the model parameters. The optimization problem for depth parameters and dip angle is highly nonlinear; increasing the number of parameters to be solved simultaneously also increases the dimensionally of the energy surface, thereby increasing the probability of the optimization stalling in a local minimum on that surface. Thus, common sense dictates that the nonlinear optimization should be restricted to as few parameters as are consistent with obtaining useful results. This is why we propose a solution for only one unknown (*b*).

Because x_M and x_0 are known, the depth to the upper edge of the sheet (*h*) can be determined from Equation (4). Knowing (*h*), (*b*), and (x_0), the depth to the lower edge of the sheet (*H*) can be determined from Equation (6). Knowing *h*, *H*, and *b*, the dielectric constant (*K*) can be determined from Equation (3), and the dip angle (θ) can be estimated from Equation (2).

To this stage, we have assumed knowledge of the axes of the SP profile so that *V*(0), x_0 and x_M can be determined. Otherwise, *V*(0), x_0, and x_M can be determined using the method described by Meiser [1] for base line determination and the method given by Roy and Chowdhury [9] for the origin determination.

SYNTHETIC EXAMPLES

The procedure of interpretation using the present least-squares method starts with the removal of the regional trend and the effect of topography from the observed data and fixing the axes of the profile .The position of x_0, and x_M as well as the values of *V*(0), take errors from the determined origin to produce erroneous results of the lateral position and other parameters.

Thus, in studying the error response of the least-squares method, four synthetic examples of an inclined sheet ($h = 1$ unit, $H = 6$ units, $K = 100$ mV and $\theta = 30°, 45°, 60°$, and $75°$) are considered. In each case, errors of 5 percent are imposed in *V*(0), x_0, and x_M. Following the interpretation method, values of the four parameters (*h*, *H*, *K*, and θ) of the inclined sheet were computed and the percentage of error in each parameter is tabulated (Table 1). In all cases the error in (*h*) is constant and is equal to the imposed error (5%) whereas the error in (*H*) ranges between 7.8 and -6.8% and the error in the dip angle (θ) ranges between 7.4 and 2%. Finally the maximum error obtained in determining *K* is 12.7%.

FIELD EXAMPLE

Figure 3 shows the self-potential anomaly over a graphite deposit in the southern Bavarian woods, Germany. The SP measurements were performed and described by Meiser [1]. They represent the anomaly as a result of a polarized sheet. This anomaly profile of 520.5 m length (Figure 4) was digitized at an interval of 10.41 meters. The values of *V*(0), x_0, and x_M used in the least-squares method are –500 mV, 72.87 m, and 154.07 m., respectively. The sheet parameters obtained by the least-squares method are $b = 26.03$ meters, $h = 35.8$ meters, $H = 66.3$ meters, $\theta = 49.50°$, and $K= 363.6$ mV. The depth to the center of the sheet obtained by the present method is 51.05 meters agrees with the depth obtained by Meiser [1] using a double logarithmic net method (53 meters). Also, a very good fit between the measured and calculated self-potential anomalies is observed (Figure 4).

Table 1. Numerical Results for the Synthetic Data.

Dip angle (degree)	*h* (m)	% of error in *h*	*H* (m)	% of error in *H*	θ (degree)	% of error in θ	*K* (mV)	% of error in *K*
30	1.05	5	6.47	7.80	32.21	7.40	106.2	6.2
45	1.05	5	6.15	2.50	48.12	6.90	108.5	8.5
60	1.05	5	5.79	-3.50	62.83	4.70	111.3	11.3
75	1.05	5	5.60	-6.88	76.53	2.00	112.7	12.7

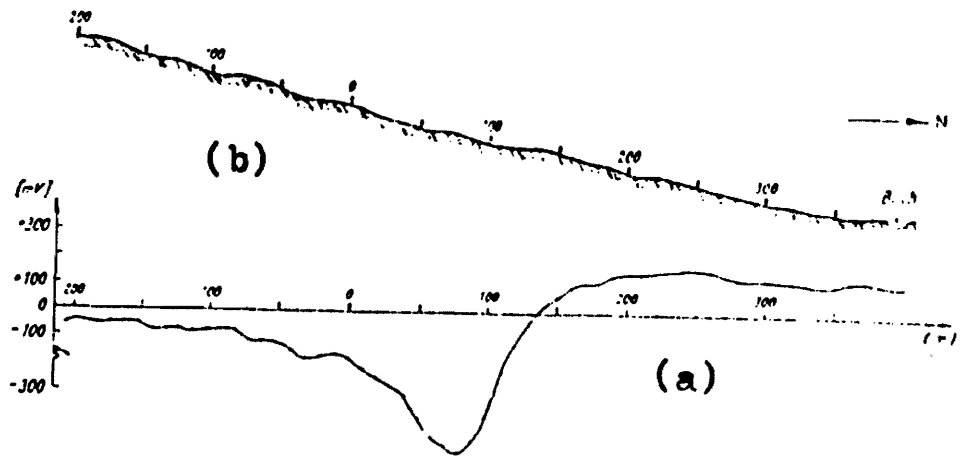

Figure 3. (a) Measured self-potential anomaly over a graphite ore body, southern Bavarian woods, Germany; (b) surface topography [1].

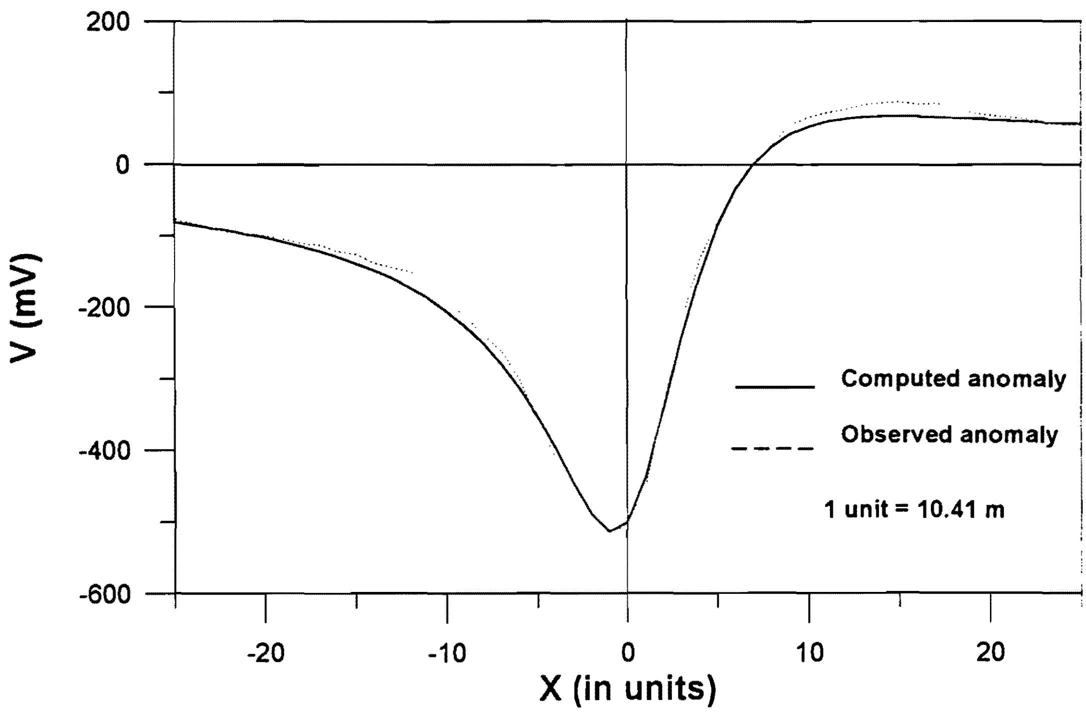

Figure 4. Measured and calculated self-potential anomalies over a graphite ore body, southern Bavarian woods, Germany.

CONCLUSIONS

The least squares method has been used to determine the sheet parameters from self potential data. The method is easy to apply and does not require any other graphical aids to interpret the self potential anomaly profile.

APPENDIX 1

Derivation of Equation (7)

At the origin ($x = 0$), Equation (1) gives the following relationship:

$$K = \frac{V(0)}{\ln \dfrac{h^2}{\left(b^2 + H^2\right)}} \tag{3a}$$

The turning points of Equation (1) may be found by equating the horizontal derivative with zero. The maximum (V_M) and minimum (V_m) anomaly values are found to occur at x_M and x_m, respectively, where

$$x_M = x_0 + \sqrt{x_0^2 + h^2}, \tag{4a}$$

and

$$x_m = x_0 - \sqrt{x_0^2 + h^2}. \tag{4b}$$

However, because of the fact that the origin is very close to x_m with a slight displacement towards x_M, Equation (4*a*) only will be used in this work, from which we obtain:

$$h = \sqrt{x_M^2 - 2x_0 x_M} \tag{4c}$$

Substituting Equation (4*a*) and (4*b*) in Equation (1), we obtain the sum of the maximum and the minimum anomaly values as:

$$V_M + V_m = V_{Mm} = K \ln \frac{h^2}{H^2} \tag{5b}$$

from which,

$$K = \frac{V_{Mm}}{\ln \dfrac{h^2}{H^2}}. \tag{5b}$$

Setting Equation (1) to be equal to zero, we obtain

$$H = \sqrt{h^2 - b^2 + 2bx_0}, \tag{6a}$$

From Equations (3*a*), (4*c*), (5*b*), and (6*a*), the composite parameter (*b*) is obtained after simplification as:

$$b = \frac{\left(2bx_0 - b^2 + x_M^2 - 2x_0 x_M\right)^s - \left(x_M^2 - 2x_0 x_M\right)^s}{2x_0\left(x_M^2 - 2x_0 x_M\right)^{s-1}}, \tag{7}$$

where

$$s = \frac{V(0)}{V_{Mm}}.$$

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