EFFECTS OF AXIAL SELF-GENERATED MAGNETIC FIELDS ON BACKWARD RAMAN SCATTERING IN A HOMOGENEOUS PLASMA

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الخلاصة :

تمّتْ دراسة تأثير وجود مجال مغناطيسي محوري على معدل النمو وعتبة الشدة اللازمة لحصول تفاعل (رامان) الخلفي في البلازما المتجانسة. فحصلنا على معادلات رياضية تَحْكُم معدل النمو وعتبة الشدة لكل من موجتيْن كهرومغناطيستيْن الأولى مستقطبة دائريا باتجاه اليد اليمني، والاخرى مستقطبة دائريا باتجاه اليد اليسرى. لقد تبيَّن إنّ وجود المجال المغناطيسي يؤثر بوضوح على النمو وكذلك على عتبة الشدة اللازمة لحصول تفاعل (رامان) الخلفي.

ABSTRACT

The effects of the presence of axial DC-magnetic fields on the growth rate and the threshold intensity of backward Raman scattering in a homogeneous plasma have been investigated. Analytical expressions for the growth rates and threshold intensities for the right-hand and left-hand circularly polarized pump waves have been obtained. It has been shown that the presence of axial DC-magnetic fields affects significantly the growth rates and threshold intensities for backward Raman scattering.

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1. INTRODUCTION

Laser-produced plasmas of scalelengths approaching those estimated for laser fusion reactor targets but obtained at a fraction of the estimated laser energy are now routinely achieved by exploding thin pellets. It is expected that reproducing suitable local conditions for such pellet experiments can be used to extrapolate to reactor-like targets.

Since energy absorption is necessary for the operation of laser pellet fusion [1–3], it is necessary to know the amount of incident laser energy scattered and the conditions governing this scattering. Forslund *et al.* [4] addressed this question and showed that for a large system, *i.e.* a long region of underdense plasma, the ratio of backscattered to incident laser energy flux can be the ratio of their frequency.

Stimulated Raman scattering (SRS) is a three-wave interaction process in which a laser photon decays into a scattered photon and a plasmon, such that they satisfy the frequency and wave-number matching conditions:

$$\omega_o = \omega_1 + \omega_2, \quad k_o = k_1 + k_2 \tag{1}$$

where $\omega_o(\omega_1)$ and $k_o(k_1)$ are the frequency and wave number of incident (scattered) photon, respectively, and $\omega_2(k_2)$ is the frequency (wave number) of the plasmon.

SRS is of considerable concern for fusion reactors [4–6] because, in addition to its radiative nature, *i.e.* it scatters energy out of the plasma, the electron plasma waves produced have the ability to accelerate electrons to high energies such that they penetrate the cold fuel, causing a considerable reduction in the fusion gain; so control and suppression of SRS is important for the success of laser-fusion.

Self-generated magnetic fields have been the motivation for many experimental and theoretical studies because of their impact on pellet design in inertial confinement fusion [7–10]. It is well established that both toroidal [11, 12] and axial [13] magnetic fields are generated in laser-produced plasmas.

Grebogi and Liu [7] studied the effects of the toroidal self-generated magnetic field on the scattering phenomena; they found that the SRS growth rates are practically unmodified as compared with the unmagnetized plasma. Barr *et al.* [14] have also investigated the effects of toroidal magnetic fields on Raman and two-plasmon decay instabilities in the region of quarter-critical density; they concluded that the Raman growth rate increases in a magnetized plasma. Sharma and Dragila [15] have considered forward Raman scattering in the presence of a background DC-magnetic field; they demonstrated that when the magnetic field is perpendicular to the direction of propagation of an elliptically polarized extraordinary wave the growth rate increases and that if the direction of propagation of the incident wave is parallel to the magnetic field the growth rate increases if the wave is right-hand circularly polarized, and decreases if the wave is left-hand circularly polarized.

Recently Laham [16] examined the effect of a toroidal magnetic field on the growth rate of backward Raman scattering (BRS) instability in an underdense homogeneous plasma, and derived analytically a general expression for the growth of this instability which showed a reduction in the growth rate due to the presence of the toroidal self-generated magnetic field.

In this paper we use the full set of Maxwell's equations in addition to the continuity and the electron momentum equations to derive coupled nonlinear equations that describe the BRS in an underdense region taking the presence of an axial self-generated magnetic field into consideration. By solving these equations, we obtain analytical expressions for the growth rates and the threshold intensities for both the right-hand circularly polarized waves and the left-hand circularly polarized waves.

In Section 2 we derive the nonlinear dispersion relations for BRS. We analyze these dispersion relations to obtain the growth rates and the threshold intensities. In Section 3 we present the results and our conclusions.

2. THEORY

We consider the propagation of an electromagnetic wave (pump) in a homogeneous underdense plasma parallel to a magnetic field $\mathcal{B}_o = (O, O, \mathcal{B}_o)$. We differentiate between two cases (1) the incident electromagnetic wave is right-hand circularly polarized; and (2) the incident electromagnetic wave is left-hand circularly polarized.

2.1. The Incident Electromagnetic Wave is Right-Hand Circularly Polarized

The electric field of the pump wave can be written as follows:

$$\mathbf{E}_{oR} = E_{oR}\,(\hat{x} + i\hat{y})\,e^{i(k_o z - \omega_o t)} + c.c. \tag{2}$$

Let us consider the parametric decay of this wave into a scattered electromagnetic wave whose dominant polarization will be the same as the incident pump wave:

$$\mathbf{E}_{1R} = E_{1R}\,(\hat{x} + i\hat{y})\,e^{-i(k_1 z + \omega_1 t)} + c.c. \tag{3}$$

and an electron plasma wave

$$\mathbf{E}_2 = \hat{z}E_2 e^{i(k_2 z - \omega_2 t)} + c.c. \tag{4}$$

The total electron density N consists of the background density N_o, and perturbations due to the pump wave n_o, the scattered electromagnetic wave n_1, and the plasmon wave n_2. The perturbation n_2 can be expressed as:

$$n_2 = n_2 e^{i(k_2 z - \omega_2 t)} + c.c. \tag{5}$$

The peak velocities of the electrons, due to the above-mentioned fields, will be modified by the presence of the magnetic field according to the momentum equation:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}.\nabla)\,\mathbf{v} = -\frac{e}{m}\mathbf{E} - \frac{e}{mc}\,\mathbf{v}\times\mathbf{B} \tag{6}$$

where c is the speed of light.

Within the limits of the linear theory, the peak velocity of the electrons due to the pump field can be obtained as follows, let

$$\mathbf{v}_o = (v_{ox}\hat{x} + v_{oy}\hat{y})\,e^{i(k_o z - \omega_o t)} + c.c. \tag{7}$$

noting that $\nabla = \hat{z}\frac{\partial}{\partial z}$, then the x-component of Equation (6) gives:

$$\frac{\partial v_{ox}}{\partial t} = -\frac{e}{m}E_{ox} - \frac{e}{m}\, v_{oy}\mathcal{B}_o, \tag{8}$$

and the y-component gives

$$\frac{\partial v_{oy}}{\partial t} = -\frac{e}{m}E_{oy} + \frac{e}{mc}\, v_{ox}\mathcal{B}_o. \tag{9}$$

Solving Equations (8) and (9) for v_{ox}, v_{oy} and noting that for the right circular polarization we have $E_{oy} = iE_{ox} = iE_{oR}$, we obtain:

$$v_{ox} = -\frac{ieE_{oR}}{m\,(\omega_o - \omega_c)} \tag{10}$$

$$v_{oy} = \frac{eE_{oR}}{m\,(\omega_o - \omega_c)}. \tag{11}$$

Similarly for the scattered electromagnetic wave, we have:

$$v_{1x} = -\frac{ieE_{1R}}{m\,(\omega_1 - \omega_c)} \tag{12}$$

$$v_{1y} = \frac{eE_{1R}}{m\,(\omega_1 - \omega_c)}. \tag{13}$$

From Equation (10) and (11), we have

$$\mathbf{v}_{oR} = -\frac{ie\mathbf{E}_{oR}}{m\,(\omega_o - \omega_c)} \tag{14}$$

and from Equations (12) and (13), we have

$$\mathbf{v}_{1R} = -\frac{ie\mathbf{E}_{1R}}{m\,(\omega_1 - \omega_c)}. \tag{15}$$

To derive the general wave equation we use Faraday's law of induction:

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{16}$$

and the generalized Ampere's law

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}. \tag{17}$$

Taking the curl of Equation (16), we obtain

$$\nabla^2 \mathbf{E} - \nabla(\nabla . E) = \frac{4\pi}{c^2}\frac{\partial \mathbf{J}}{\partial t} + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} \tag{18}$$

then, for the scattered electromagnetic wave, using the phase-matching condition, we have

$$\left(\omega_1^2 - c^2 k_{1R}^2\right)\mathbf{E}_{1R} = 4\pi i \omega_1 e\left[N_o \mathbf{v}_{1R} + n_2^* \mathbf{v}_{oR}\right]. \tag{19}$$

Substituting for $\mathbf{v}_{oR}$ and $\mathbf{v}_{1R}$ from Equations (14) and (15) into Equation (19), we obtain:

$$\left[\omega_1^2 - c^2 k_{1R}^2 - \frac{\omega_1 \omega_p^2}{\omega_1 - \omega_c}\right]\mathbf{E}_{1R} = \frac{\omega_1 \omega_p^2}{\omega_o - \omega_c}\frac{n_2^*}{N_o}\mathbf{E}_{oR}. \tag{20}$$

Using the continuity equation and the phase matching condition, we have:

$$\frac{\partial n_2}{\partial t} + \nabla.\left(N_o \mathbf{v}_2 + n_2 \mathbf{v}_{1R} + N_1^* \mathbf{v}_{oR}\right) = 0 \tag{21}$$

but since $\nabla = \hat{z}\frac{\partial}{\partial z}$, this equation reduces to

$$\frac{\partial n_2}{\partial t} + N_o \frac{\partial v_2}{\partial z} = 0. \tag{22}$$

Taking the z-component of the momentum equation:

$$\frac{\partial \mathbf{v}_2}{\partial t} + (\mathbf{v}.\nabla)\,\mathbf{v} = -\frac{eE_2}{m} - \frac{3v_{th}^2}{2N_o}\nabla n_2 - \frac{e}{mc}\mathbf{v} \times \mathbf{B} \tag{23}$$

we get

$$\frac{\partial v_2}{\partial t} = -\frac{eE_2}{m} - \frac{3v_{th}^2}{2N_o}\frac{\partial n_2}{\partial z} - \frac{e}{mc}\left[\mathbf{v}_1^* \times \mathbf{B}_o + \mathbf{v}_o \times \mathbf{B}_1^*\right]_z. \tag{24}$$

Separating v_2 into two parts linear part $\left(v_2^{Ln}\right)$ and nonlinear part $\left(v_2^{nL}\right)$, we obtain

$$\frac{\partial v_2^{Ln}}{\partial t} = -\frac{eE_2}{m} - \frac{3v_{th}^2}{2N_o}\frac{\partial n_2}{\partial z} \tag{25}$$

$$\frac{\partial v_2^{nL}}{\partial t} = -\frac{e}{mc}\left[v_{1x}B_{oy} - v_{1y}^* B_{ox} + v_{ox}B_{1y}^* - v_{oy}B_{1x}^*\right]. \tag{26}$$

Substituting for $v_2 = v_2^{Ln} + v_2^{nL}$ into Equation (22) and differentiating with respect to time, we have:

$$\frac{\partial^2 n_2}{\partial t^2} + N_o \frac{\partial}{\partial z}\frac{\partial v_2^{Ln}}{\partial t} + N_o \frac{\partial}{\partial z}\frac{\partial v_2^{Ln}}{\partial t} = 0. \tag{27}$$

Substituting Equations (25) and (26) into Equation (27), we obtain

$$\frac{\partial^2 n_2}{\partial t^2} - \frac{eN_o}{m}\frac{\partial E_2}{\partial z} - \frac{3}{2}v_{th}^2 \frac{\partial^2 n_2}{\partial z^2} = \frac{N_o e}{mc}\frac{\partial}{\partial z}\left[v_{1x}^* B_{oy} - v_{1y}^* B_{ox} + v_{ox}B_{1y}^* - v_{oy}B_{1x}^*\right] \tag{28}$$

but from Poisson's equation we have

$$\frac{\partial E_2}{\partial z} = -4\pi e n_2. \tag{29}$$

Substituting Equation (29) into Equation (28), we obtain

$$\left[\frac{\partial^2}{\partial t^2} + \omega_p^2 - \frac{3}{2}v_{th}^2 \frac{\partial^2}{\partial z^2}\right] n_2 = \frac{iN_o e k_2}{mc}\left[v_{1x}^* B_{oy} - v_{1y}B_{ox} + v_{ox}B_{1y}^* - v_{oy}B_{1x}^*\right]. \tag{30}$$

For the right-circular polarization, we have

$$B_{oy} = iB_{ox} = iB_{oR}$$

$$B_{1y} = iB_{1x} = iB_{1R}. \tag{31}$$

Using Equation (31) and substituting for v_{ox}, v_{1y} into Equation (30), we obtain

$$\left(\omega_2^2 - \omega_p^2 - \frac{3}{2}v_{th}^2 k_2^2\right)\frac{n_2}{N_o} = \frac{2ie^2 k_2}{m^2 c}\left[\frac{B_{oR}E_{1R}^*}{\omega_1 - \omega_c} + \frac{B_{1R}^* E_{oR}}{\omega_o - \omega_c}\right]. \tag{32}$$

Applying Faraday's law of induction, Equation (32) transforms to:

$$\left(\omega_2^2 - \omega_p^2 - \frac{3}{2}k_2^2 v_{th}^2\right)\frac{n_2}{N_o} = \frac{2e^2 k_2 E_{oR}}{m^2}\left[\frac{k_{1R}}{\omega_1(\omega_o - \omega_c)} + \frac{k_{oR}}{\omega_o(\omega_1 - \omega_c)}\right] E_{1R}^*. \tag{33}$$

Taking the complex conjugates of Equations (20) and (33), we obtain the nonlinear coupled equations:

$$\left[\omega_1^2 - c^2 k_{1R}^2 - \frac{\omega_1 \omega_p^2}{\omega_1 - \omega_c}\right] E_{1R}^* = \frac{\omega_1 \omega_p^2}{\omega_1 - \omega_c}\frac{n_2}{N_o} E_{oR}^* \tag{34}$$

$$\left[\omega_2^2 - \omega_{ek}^2\right] \frac{n_2}{N_o} = \frac{2e^2 k_2 E_{oR}}{m^2} \left[\frac{k_{1R}}{\omega_1 (\omega_o - \omega_c)} + \frac{k_{oR}}{\omega_o (\omega_1 - \omega_c)}\right] E_{1R}^* \tag{35}$$

where

$\omega_{ek}^2 = \omega_p^2 + \frac{3}{2} v_{th}^2 k_2^2$ is the Bohm-Gross frequency. These are two simultaneous equations for E_{1R}^* and n_2; they have a nontrivial solution if the determinant of the coefficients vanishes, hence we obtain:

$$\left[\omega_2^2 - \omega_{ek}^2\right] \left[\omega_1^2 - c^2 k_{1R}^2 - \frac{\omega_1 \omega_p^2}{\omega_1 - \omega_c}\right]$$

$$= \frac{2e^2 k_2 |E_{oR}|^2 \omega_p^2 \omega_1 k_2}{m^2 (\omega_o - \omega_c)} \left[\frac{k_{oR}}{\omega_o (\omega_1 - \omega_c)} + \frac{k_{1R}}{\omega_o (\omega_{o1} - \omega_c)}\right]. \tag{36}$$

The instability growth rate can be obtained from Equation (36) by taking $\omega_2 = \omega_{ek} + i\gamma_R$ where $\omega_{ek} \gg \gamma_R$ and making use of the frequency matching condition $\omega_1 = \omega_o - \omega_2$, then Equation (36) transforms to:

$$2i\gamma_R \omega_{ek} \left[-2i\gamma_R (\omega_o - \omega_{ek}) + (\omega_o - \omega_{ek})^2 - c^2 k_{1R}^2 - \frac{\omega_p^2 \omega_1}{\omega_1 - \omega_c}\right]$$

$$= \frac{2e^2 k_2 E_{oR}^2 \omega_p^2 \omega_1 k_2}{m^2 (\omega_o - \omega_c)} \left[\frac{k_{oR}}{\omega_o (\omega_1 - \omega_c)} + \frac{k_{1R}}{\omega_1 (\omega_o - \omega_c)}\right] \tag{37}$$

hence

$$\gamma_R^2 = \gamma_R^2(0) \frac{\omega_o}{\omega_o - \omega_c} \frac{1}{\alpha_R} \tag{38}$$

where $\gamma_R(0)$ is the growth rate of BRS in a homogeneous unmagnetized plasma and α_R is given by the relation:

$$\alpha_R = \frac{\dfrac{k_{oR}}{\omega_o \omega_1} + \dfrac{k_{1R}}{\omega_o \omega_1}}{\dfrac{k_{oR}}{\omega_o(\omega_1 - \omega_c)} + \dfrac{k_{1R}}{\omega_1(\omega_o - \omega_c)}} < 1. \tag{39}$$

It is obvious from Equations (38) and (39) that the growth rate of BRS increases in the presence of an axial magnetic field parallel to the propagation of a right-hand circularly polarized pump wave.

To calculate the intensity threshold of the instability we must introduce the damping term into the momentum equation of the scattered electromagnetic wave, hence

$$\frac{\partial \mathbf{v}_1}{\partial t} = -\frac{e\mathbf{E}_1}{m} - \frac{e}{mc} v_1 \times \mathcal{B}_o - \nu \mathbf{v}_1 \tag{40}$$

where ν is the damping rate for the scattered wave.

Taking the x-component of Equation (40), we get

$$\frac{\partial v_{1x}}{\partial t} = -\frac{eE_{1x}}{m} - \frac{e\mathcal{B}_o}{mc}v_{1y} - \nu v_{1x} \tag{41}$$

and from the y-component, we have:

$$\frac{\partial v_{1y}}{\partial t} = -\frac{eE_{1y}}{m} + \frac{e\mathcal{B}_o}{mc}v_{1x} - \nu v_{1y}. \tag{42}$$

Solving Equations (41) and (42) for v_{1y} and v_{1x}, we obtain:

$$v_{1x} = -\frac{ieE_{1R}}{m\left[(\omega_1 + i\nu) - \omega_c\right]} \tag{43}$$

$$v_{1y} = \frac{eE_{1R}}{m\left[(\omega_1 + i\nu) - \omega_c\right]} \tag{44}$$

hence

$$\mathbf{v}_{1R} = -\frac{ie}{m\left[(\omega_1 - \omega_c + i\nu)\right]}\mathbf{E}_{1R}. \tag{45}$$

This equation can be simplified if we consider the case $\omega_1 \gg \omega_c$ and $\omega_1 \gg \nu$, then we obtain

$$\mathbf{v}_{1R} \approx -\frac{ie}{m\left[(\omega_1 - \omega_c)\right]}\left[1 + \frac{i\nu}{\omega_1}\right]\mathbf{E}_{1R}. \tag{46}$$

Substituting for $\mathbf{v}_{1R}$ in Equation (20), we get:

$$\left[\omega_1^2 - c^2k_{1R}^2 - \frac{\omega_1\omega_p^2}{\omega_1 - \omega_c} + \frac{i\nu\omega_p^2}{\omega_1 - \omega_c}\right]E_{1R} = \frac{\omega_1\omega_p^2}{(\omega_o + \omega_c)}\frac{n_2^*}{N_o}\mathbf{E}_{oR}. \tag{47}$$

Taking the complex conjugate of this equation, we get:

$$\left[\omega_1^2 - c^2k_{1R}^2 - \frac{\omega_1\omega_p^2}{\omega_1 - \omega_c} + \frac{i\nu\omega_p^2}{\omega_1 - \omega_c}\right]E_{1R}^* = \frac{\omega_1\omega_p^2}{\omega_o - \omega_c}\frac{n_2}{N_o}E_{oR}^*. \tag{48}$$

When we take the damping into consideration, Equation (25) transforms to:

$$\frac{\partial v_2^{Ln}}{\partial t} = \frac{-eE_2}{m} - \frac{3}{2}v_{th}^2\frac{\partial n_2}{\partial z} - \nu v_2; \tag{49}$$

differentiating this equation with respect to z we get:

$$\frac{\partial}{\partial z}\frac{\partial v_2^{Ln}}{\partial t} = -\frac{e}{m}\frac{\partial E_2}{\partial z} - \frac{3v_{th}^2}{2N_o}\frac{\partial^2 n_2}{\partial z^2} - \nu\frac{\partial v_2}{\partial z}. \tag{50}$$

Substituting for $\frac{\partial v_2}{\partial z}$ from Equation (22), we obtain

$$\frac{\partial}{\partial z}\frac{\partial v_2^{Ln}}{\partial t} = -\frac{e}{m}\frac{\partial E_2}{\partial z} - \frac{3v_{th}^2}{2N_o}\frac{\partial^2 n_2}{\partial z^2}\nu + \frac{\nu}{N_o}\frac{\partial n_2}{\partial t}. \tag{51}$$

Now, substituting Equations (26) and (51) into Equation (27) and repeating the same procedure we followed earlier to arrive at Equation (33), we find that:

$$\left[\omega_2^2 - \omega_p^2 - \frac{3}{2}v_{th}^2 + i\nu\omega_2\right]\frac{n_2}{N_o} = \frac{2e^2k_2E_{oR}}{m^2}\left[\frac{k_{1R}}{\omega_1(\omega_o-\omega_c)} + \frac{k_{oR}}{\omega_o(\omega_1-\omega_c)}\right]E_{1R}^*. \tag{52}$$

Substituting Equation (48) into Equation (52) yields:

$$\left[\omega_2^2 - \omega_{ck}^2 - i\nu\omega_2\right]\left[\omega_1^2 - c^2k_{iR}^2 - \frac{\omega_1\omega_p^2}{\omega_1-\omega_c} - \frac{i\nu\omega_p^2}{\omega_1-\omega_c}\right]$$

$$= \frac{2e^2|E_{oR}|^2\omega_p^2\omega_1k_2}{m^2(\omega_o-\omega_c)}\left[\frac{k_{oR}}{\omega_o(\omega_1-\omega_c)} + \frac{k_{1R}}{\omega_1(\omega_o-\omega_c)}\right]. \tag{53}$$

Since the lowest threshold occurs at resonance condition, *i.e.* $\omega_2^2 = \omega_{ek}^2$ and $\omega_1^2 = c^2k_{1R}^2 + \frac{\omega_1\omega_p^2}{\omega_1-\omega_c}$, then

$$|E_{oR}|_{Th}^2 = \frac{\nu^2m^2(\omega_o-\omega_c)\omega_2}{2e^2\omega_1k_2(\omega_1-\omega_c)}\frac{1}{\dfrac{k_{oR}}{\omega_o(\omega_1-\omega_c)} + \dfrac{k_{1R}}{\omega_1(\omega_o-\omega_c)}}; \tag{54}$$

this equation can be written as:

$$|E_{oR}|_{Th}^2 = |E_{oR}|_{Th}^2(0)\frac{\omega_o-\omega_c}{\omega_o}\,\frac{\omega_1}{\omega_1-\omega_c}\,\alpha_R, \tag{55}$$

where α_R is given by Equation (39) and $|E_{oR}|_{Th}^2$ is the threshold intensity in a homogeneous unmagnetized plasma.

2.2. The Incident Electromagnetic Wave is Left-Hand Circularly Polarized

In this case we have $E_{oy} = -iE_{ox} = --iE_{oL}$, so using Equation (14), we get

$$\mathbf{v}_{oL} = -\frac{ie}{m(\omega_o+\omega_c)}\,\mathbf{E}_{oL} \tag{56}$$

and from Equation (15), we get

$$\mathbf{v}_{1L} = -\frac{ie}{m(\omega_1 + \omega_c)} \mathbf{E}_{1L} \tag{57}$$

and Equations (34) and (35) transform to:

$$\left[\omega_1^2 - c^2 k_{1L}^2 - \frac{\omega_1 \omega_p^2}{\omega_1 + \omega_c}\right] \mathbf{E}_{1L}^* = \frac{\omega_1 \omega_p^2}{\omega_o + \omega_c} \frac{n_2}{N_o} \mathbf{E}_{oL}^* \tag{58}$$

$$\left[\omega_2^2 - \omega_p^2 - \frac{3}{2} v_{th}^2 k_2^2\right] \frac{n_2}{N_o} = \frac{2e^2 k_2 E_{oL}}{m^2} \left[\frac{k_{1L}}{\omega_1(\omega_o + \omega_c)} + \frac{k_{oL}}{\omega_o(\omega_1 + \omega_c)}\right] E_{1L}^* . \tag{59}$$

For Equations (58) and (59) to have a nontrivial solution, we must have:

$$\left[\omega_2^2 - \omega_{ek}^2\right] \left[\omega_1^2 - c^2 k_{1L}^2 - \frac{\omega_1 \omega_p^2}{\omega_1 + \omega_c}\right] = \frac{2e^2 |E_{oL}|^2 \omega_p^2 \omega_1 k_2}{m^2(\omega_o + \omega_c)} \left[\frac{k_{oL}}{\omega_o(\omega_1 + \omega_c)} + \frac{k_{1L}}{\omega_1(\omega_o + \omega_c)}\right] \tag{60}$$

from this equation we find that the growth rate γ_L is given by

$$\gamma_L^2 = \frac{e^2 |E_{oL}|^2 \omega_o k_2^2}{2m^2 \omega_1 \omega_o^2} \frac{\omega_o}{\omega_o + \omega_c} \frac{1}{\alpha_L} \tag{61}$$

where

$$\alpha_L = \frac{\dfrac{k_{oL}}{\omega_1 \omega_o} + \dfrac{k_{1L}}{\omega_1 \omega_o}}{\dfrac{k_{oL}}{\omega_o(\omega_1 + \omega_c)} + \dfrac{k_{1L}}{\omega_1(\omega_o + \omega_c)}} > 1. \tag{62}$$

We can write Equation (61) in the form

$$\gamma_L^2 = \gamma_L^2(0) \frac{\omega_o}{\omega_o + \omega_c} \frac{1}{\alpha_L}, \tag{63}$$

where $\gamma_L(0)$ is the instability growth rate for BRS in a homogeneous unmagnetized plasma.

It is obvious from Equation (63) that the growth rate decreases in the presence of an axial magnetic field when the pump wave is left-hand circularly polarized.

To calculate the threshold intensity we use Equation (48) which in the case of left-circular polarization takes the form

$$\left[\omega_1^2 - c^2 k_{1L}^2 - \frac{\omega_1 \omega_p^2}{(\omega_1 + \omega_c)} - \frac{i\nu \omega_p^2}{(\omega_1 + \omega_c)}\right] = \frac{\omega_1 \omega_p^2}{(\omega_o + \omega_c)} \frac{n_2}{N_o} E_{oL}^* \tag{64}$$

and Equation (53), which takes the form:

$$\left[\omega_2^2 - \omega_{ek}^2 + i\nu\omega_2\right] \frac{n_2}{N_o} = \frac{2e^2 k_2 E_{oL}}{m^2} \left[\frac{k_{1L}}{\omega_1(\omega_o + \omega_c)} + \frac{k_{oL}}{\omega_o(\omega_1 + \omega_c)}\right] E_{1L}^{*}. \tag{65}$$

Substituting for E_{1L}^{*} from Equation (64) into Equation (65), we obtain

$$\left[\omega_2^2 - \omega_{ek}^2 + i\nu\omega_2\right] \left[\omega_1^2 - c^2 k_{1L}^2 - \frac{\omega_1 \omega_p^2}{\omega_1 + \omega_c} - \frac{i\nu\omega_p^2}{\omega_1 + \omega_c}\right]$$

$$= \frac{2e^2 |E_{oL}|^2 \omega_p^2 \omega_1 k_2}{m^2(\omega_o + \omega_c)} \left[\frac{k_{oL}}{\omega_o(\omega_1 + \omega_c)} + \frac{k_1 L}{\omega_1(\omega_o + \omega_c)}\right]. \tag{66}$$

The lowest threshold intensity occurs at resonance, then Equation (66) transforms to:

$$\frac{\omega_p^2 \omega_2 \nu^2}{(\omega_1 + \omega_c)} = \frac{2e^2 |E_{oL}|^2 \omega_p^2 k_2}{m^2(\omega_o + \omega_c)} \left[\frac{k_{oL}}{\omega_o(\omega_1 + \omega_c)} + \frac{k_{1L}}{\omega_1(\omega_o + \omega_c)}\right]. \tag{67}$$

From this equation, we have

$$|E_{oL}|^2 = |E_{oL}|^2(0) \frac{\omega_o + \omega_c}{\omega_o} \frac{\omega_1}{\omega_1 + \omega_c} \alpha_L, \tag{68}$$

where α_L is given by Equation (62), and $|E_{oL}|^2(0)$ is the threshold intensity of BRS instability of a left-hand circularly polarized wave in a homogeneous unmagnetized plasma.

3. RESULTS AND CONCLUSION

To study the effects of an axial DC-magnetic field on the BRS instability, we differentiated between two cases according to the nature of the incident pump wave; for case (1) where the pump wave is right-hand circularly polarized we have plotted Figure 1 which represents the ratio of the growth rate in the presence of the magnetic field to the value of the growth rate in the absence of the magnetic field *versus* the strength of the magnetic field for different plasma densities. It is obvious from this figure that the growth rate increases in the presence of the magnetic field and that it increases as the density of the plasma increases. For case (2) the incident pump wave is left-hand circularly polarized (Figure 2); we see that the growth rate decreases in the presence of the magnetic field and decreases more rapidly as the density of the plasma increases.

In Figure 3 we have plotted the normalized homogeneous threshold (normalized to the value of the threshold in the absence of the magnetic field) *versus* the strength of the magnetic field. For case (1), we see from this figure that the threshold decreases linearly with the magnetic field, while for case (2) we find that the threshold increases linearly with the magnetic field. Thus, we have shown analytically that the presence of a DC-magnetic field affects significantly the growth rates and the threshold intensities of the BRS instability.

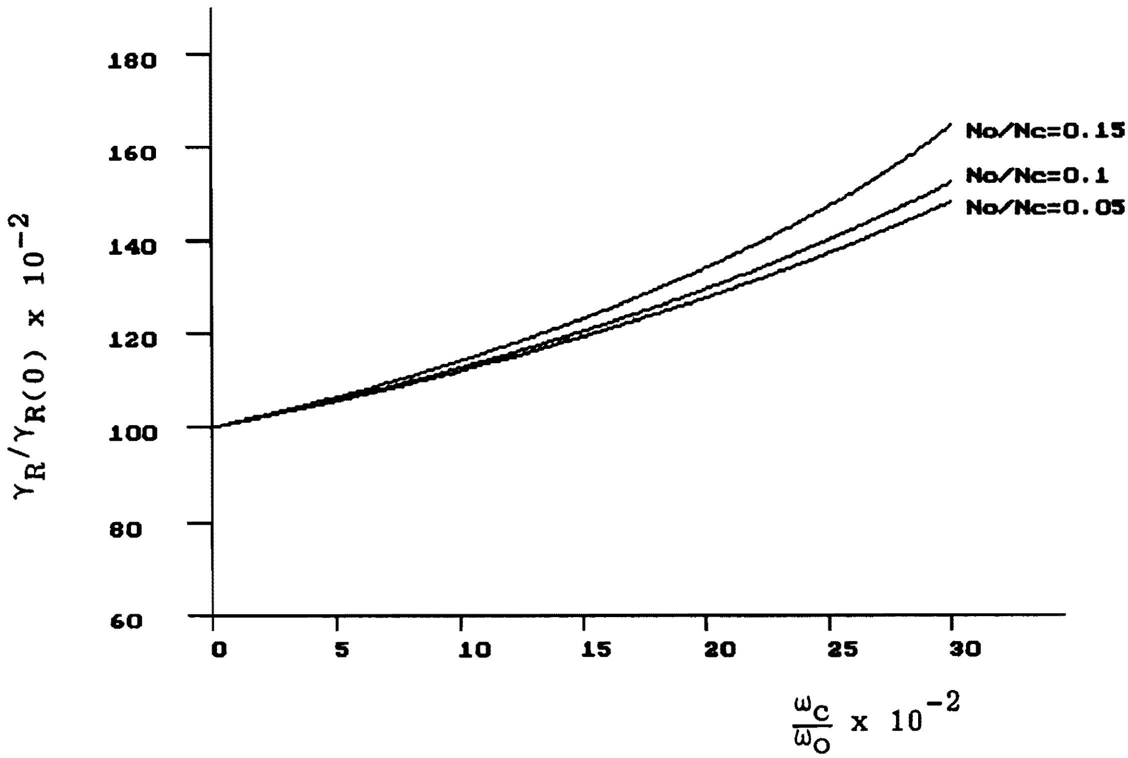

Figure 1. Relative Right-Hand Growth Rate versus *Magnetic Field Intensity.*

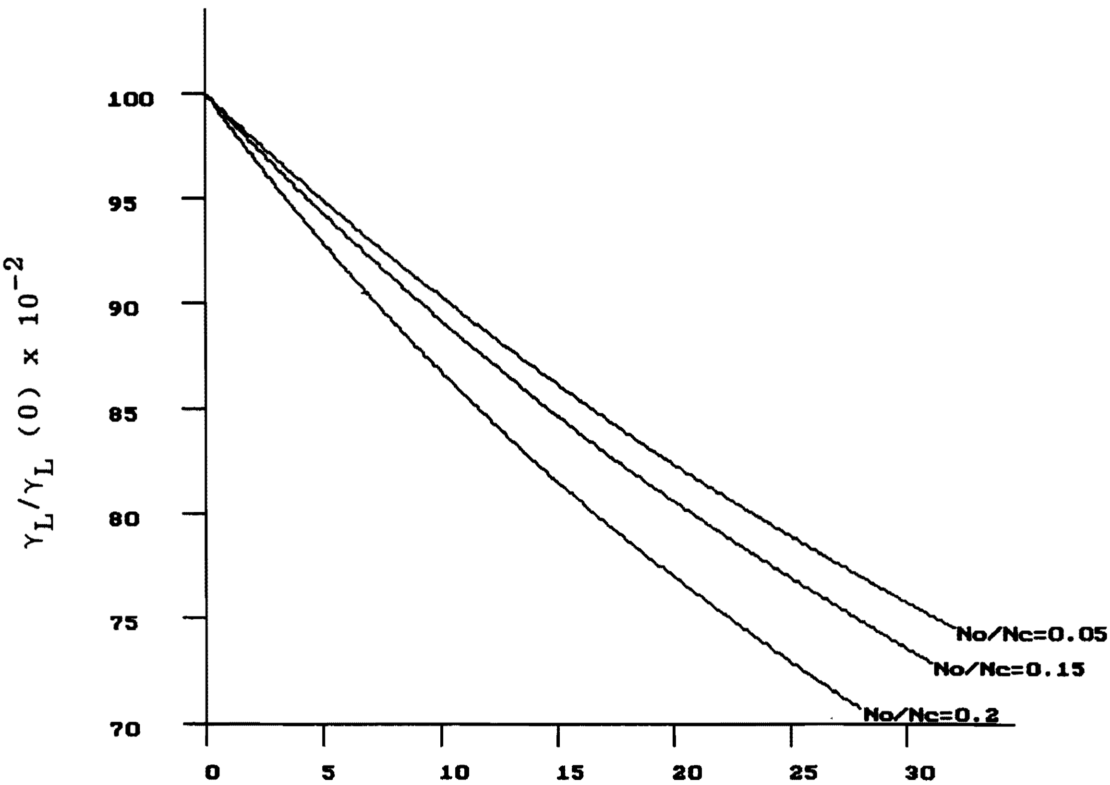

Figure 2. Relative Left-Hand Growth Rate versus *Magnetic Field Intensity.*

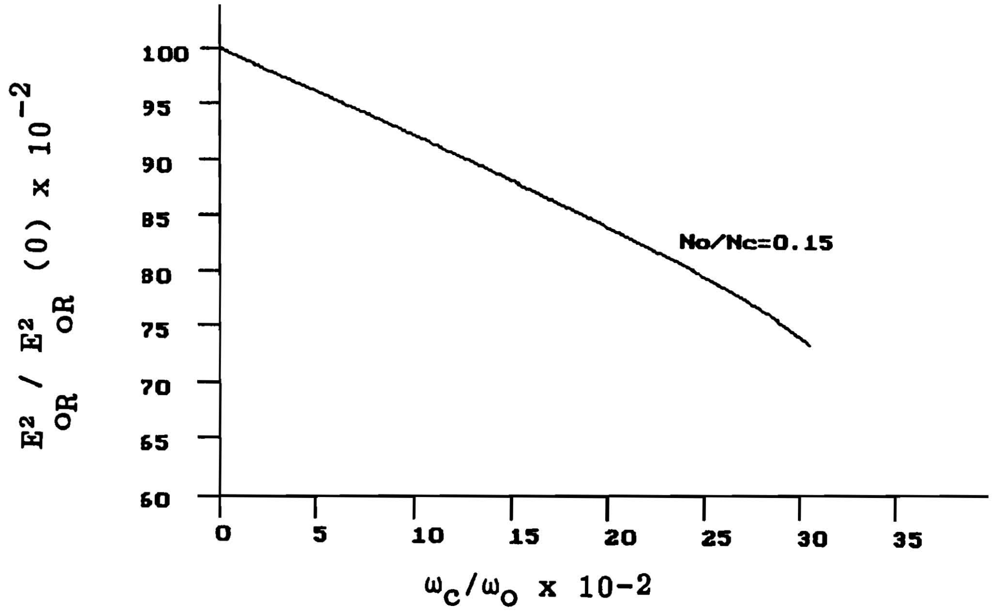

Figure 3. Relative Threshold Intensity of the Right-Hand Polarized Wave versus *Magnetic Field Intensity.*

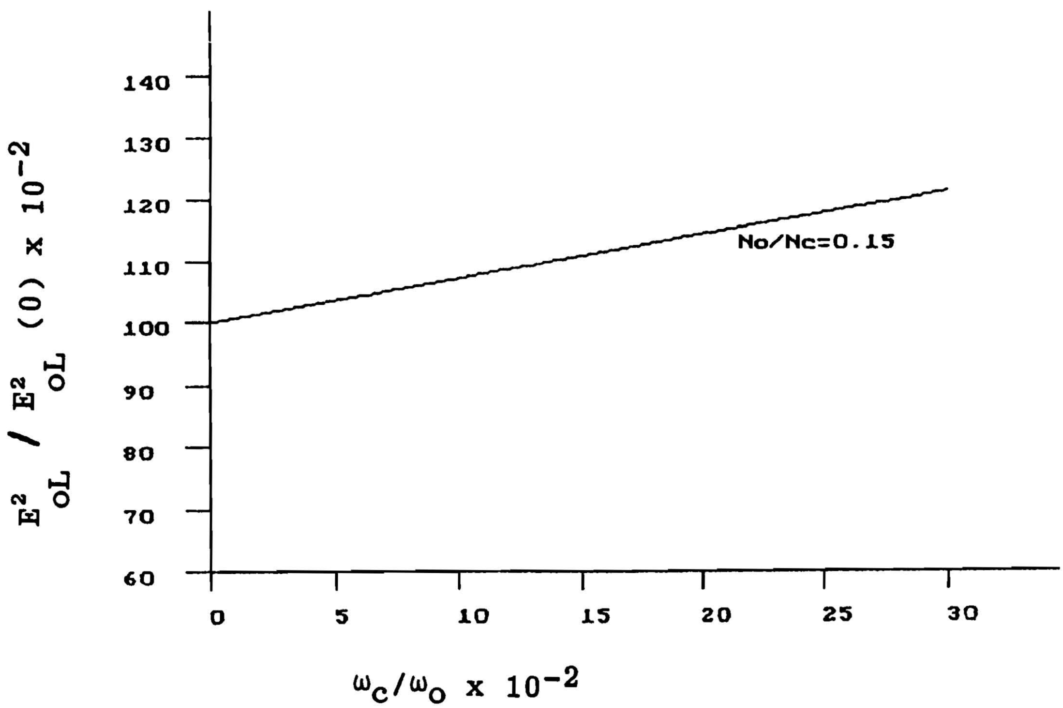

Figure 4. Relative Threshold Intensity of the Left-Hand Polarized Wave versus *Magnetic Field Intensity.*

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Paper Received 7 July 1996; Accepted 31 December 1996.