DIGITAL MEASUREMENT OF SYMMETRICAL COMPONENTS FOR POWER SYSTEM PROTECTION

S. A. Soliman*, A. M. Mahdi, A. M. Al-Kandari, and Mahdi M. El-Arini

Power Systems Research Group
Electrical Engineering Department
Faculty of Technological Studies
P. O. Box 42325 Shuwaikh
70654 Kuwait

الخلاصـة :

تفترض نظرية المركبات المتماثلة أن النظام غير المتماثل يكون جيبي الشكل وذو تردد أساسي ولايتلوث بأي توافقيات . واليوم نتيجة انتشار استخدام أجهزة الكترونيات القوى في النقل والتحكم في نظم القوى فإن شبكات نظم القوى باتت مُعـرضة لوجود توافقيات بها وفي هذه الحالة لا تطبق نظرية المركبات المتماثلة ولابد من البحث عن طريقة دقيقة لحساب المركبات المتماثلة .

يقدم هذا البحث تطبيقات لخوارزم استنباط العناصر لقياس المركبات المتماثلة والتي تستخدم في وقاية نظم القوى . وهو خوارزم مربع أقل خطأ لاستنباط العناصر هو الخوارزم المستخدم في استنباط المركبات من العينات الرقمية للمنظومة غير المتماثلة . وهنا نفترض منظومة جهود ثلاثية الطور غير متماثلة وتحتوي على توافقيات . المركبات الثلاثة المتماثلة والمسماة المركبات موجبة التتابع ، وسالبة التتابع ، وصفرية التتابع قد استنبطت باستخدام الخوارزم المقترح . وقد تم دراسة تأثير معدل أخذ العينات ، حجم شبكات المعلومات ، وانحراف التردد على المركبات المستنبطة .

هذا وقد تم اختبار الخوارزم المقترح باستخدام بيانات مصنعة وبيانات حقيقية مسجلة . ونورد بدايـةً النتائج التي تم الحصول لتشكل الأساس لاستنتجاتنا في نهاية البحث .

* To whom correspondence should be addressed.

ABSTRACT

The theory of symmetrical component assumes that the original unsymmetrical system is sinusoidal with fundamental frequency, and is contaminated with no harmonics. Nowadays, power system networks are subjected to harmonics injection due to the widespread use of power electronic equipments in power systems transmission and control. In this case, the symmetrical components theory is not applicable, and an accurate method is needed to calculate the symmetrical components. This paper presents the applications of parameter estimation techniques for measuring the symmetrical components for power system protection. The least errors square (LES) is the parameter estimation algorithm used to estimate these components from the digitized samples of the unsymmetrical system. Here, we assume a three phase unsymmetrical system of voltages which is contaminated with harmonics. The three symmetrical components, namely the positive, negative, and zero sequence components are estimated using the proposed technique. Effects of sampling rate and data window size, as well as, the frequency drift are examined. The algorithm is tested using simulated and actual recorded data. The results obtained are reported in the text which form the basis to our conclusions.

DIGITAL MEASUREMENT OF SYMMETRICAL COMPONENTS FOR POWER SYSTEM PROTECTION

1. INTRODUCTION

The symmetrical components theory developed earlier assumed that the N unsymmetrical system of voltages and currents, having the fundamental frequency, can be resolved into N symmetrical system of voltages or currents with the same fundamental frequency. In the three phase unsymmetrical systems, these components are the positive, negative, and zero sequence components. Nowadays, power system networks are subjected to harmonics injection due to the widespread use of power electronic equipments in the transmission and control of electric energy. The symmetrical components theory has many application in the protection and analysis of power system in the abnormal operation, therefore, an accurate method is needed to measure these components, especially if the voltage and/or current waveforms are contaminated with harmonics.

References [1, 4, 6] presented algorithms for fast estimation of symmetrical components in real time. The algorithms were based on the space-phasor, which can be evaluated by analog devices.

This paper presents the application of the parameter estimation techniques for measuring the symmetrical components of an unsymmetrical three phase voltage contaminated with harmonics. The least errors square (LES) is the parameter estimation algorithm used to estimate these components from the digitized samples of the unsymmetrical system. The three symmetrical components, namely the positive, negative, and zero sequence components are estimated. Effects of the sampling rate, data window size, and the frequency drifts are examined. The proposed algorithm is tested using simulated and actual recorded data.

2. MODELING OF THE VOLTAGE SIGNALS

2.1. The Zero Sequence Voltage

Assume a system of three phase voltage $v_a(t)$, $v_b(t)$, and $v_c(t)$. These voltages are unequal and contaminated with harmonics. According to the symmetrical components theory [2], these voltages can be resolved into three sets of symmetrical voltages, namely the positive, the negative, and the zero sequence voltages, *i.e.*

$$v_a(t) = v_{a1}(t) + v_{a2}(t) + v_{a0}(t) \tag{1}$$

$$v_b(t) = v_{a1}(t - 2\pi/3\omega_0) + v_{a2}(t + 2\pi/3\omega_0) + v_{a0}(t) \tag{2}$$

$$v_c(t) = v_{a1}(t + 2\pi/3\omega_0) + v_{a2}(t - 2\pi/3\omega_0) + v_{a0}(t)\ . \tag{3}$$

Adding the above three equations, we obtain:

$$[v_a(t) + v_b(t) + v_c(t)]/3 = v_{a0}(t)\ . \tag{4}$$

Define the zero sequence voltage v_{a0} (t) as:

$$v_{a0}(t) = \widehat{V}_{a0} \sin(\omega_0 t + \phi_0)\ , \tag{5}$$

where

$\widehat{V}_{a0}$ is the amplitude of the zero sequence voltage
ϕ_0 is the phase angle of this voltage,

Equation (4) becomes:

$$\{v_a(t) + v_b(t) + v_c(t)\}/3 = \widehat{V}_{a0} \sin(\omega_0 t + \phi_0) \ . \tag{6}$$

Using the trigonometric identity

$$\sin(\omega_0 t + \phi_0) = \cos\phi_0 \sin\omega_0 t + \sin\phi_0 \cos\omega_0 t$$

Equation (6) can be rewritten as:

$$\{v_a(t) + v_b(t) + v_c(t)\}/3 = \widehat{V}_{a0} \cos\phi_0 \sin\omega_0 t + \widehat{V}_{a0} \sin\phi_0 \cos\omega_0 t \tag{7}$$

Define the states x_0, y_0 as

$$x_0 = \widehat{V}_{a0} \cos\phi_0 \tag{8a}$$

$$y_0 = \widehat{V}_{a0} \sin\phi_0 \tag{8b}$$

and the coefficients

$$h_{11}(t) = 3 \sin\omega_0 t \tag{9a}$$

$$h_{12}(t) = 3 \cos\omega_0 t \ . \tag{9b}$$

Equation (7) can be rewritten as:

$$[v_a(t) + v_b(t) + v_c(t)] = x_0 h_{11}(t) + y_0 h_{12}(t) \ . \tag{10}$$

If the three voltages are sampled at a preselected rate, say Δt, m samples would be obtained at $t_1,\ t_1 + \Delta t,\ t_1 + 2\Delta t, \ldots t_1 + (m-1)\,\Delta t$, where t_1 is the initial sampling time. Equation (10) can be written as

$$\begin{vmatrix} v_a(t_1) + v_b(t_1) + v_c(t_1) \\ v_a(t_2) + v_b(t_2) + v_c(t_2) \\ \vdots \\ v_a(t_m) + v_b(t_m) + v_c(t_m) \end{vmatrix} = \begin{vmatrix} h_{11}(t_1) & h_{12}(t_1) \\ h_{11}(t_2) & h_{12}(t_2) \\ \vdots & \vdots \\ h_{11}(t_m) & h_{12}(t_m) \end{vmatrix} \begin{vmatrix} x_0 \\ y_0 \end{vmatrix} \tag{11}$$

The above equation can be written as

$$z(t) = H(t)\Theta + \omega(t) \tag{12}$$

where

$z(t)$	is a m by 1 measurements vector of the digitized voltage
$H(t)$	is a m by 2 coefficients matrix of measurements
Θ	is a 2 by 1 parameters vector to be estimated
$\omega(t)$	is a m by 2 errors vector associated with the measurements to be minimized.

At least two measurements are required to identify the parameters Θ in Equation (12), we assume m, $m > 2$, measurements are available. The solution for the overdetermined set of equations given in Equation (12), based on the least errors square algorithm, is given by [3]:

$$\Theta = [H^T(t)H(t)]^{-1}H^T(t)\,z(t)\,. \tag{13}$$

Having obtained the parameter Θ, $(x_0,\ y_0)$, then the magnitude and phase angle of the zero sequence voltage can be obtained as

$$\widehat{V}_{a0}^2 = (x_0^2 + y_0^2) \tag{14}$$

$$\phi_0 = \tan^{-1}\frac{x_0}{y_0}\,. \tag{15}$$

2.2. The Positive And Negative Sequence Components

Having obtained the zero sequence component of the voltage, Equations (1) to (3) can be rewritten as

$$v_a(t) - v_{a0}(t) = v_{a1}(t) + v_{a2}(t) \tag{16}$$

$$v_b(t) - v_{a0}(t) = v_{a1}(t - 2\pi/3\omega_0) + v_{a2}(t + 2\pi/3\omega_0) \tag{17}$$

$$v_c(t) - v_{a0}(t) = v_{a1}(t + 2\pi/3\omega_0) + v_{a2}(t - 2\pi/3\omega_0)\,. \tag{18}$$

Define the positive sequence voltage $v_{a1}(t)$ and the negative sequence voltage $v_{a2}(t)$ as

$$v_{a1}(t) = \widehat{V}_{a1}\sin(\omega_0 t + \phi_1) \tag{19}$$

and

$$v_{a2}(t) = \widehat{V}_{a2}\sin(\omega_0 t + \phi_2) \tag{20}$$

where

$\widehat{V}_{a1}, \widehat{V}_{a2}$	are the magnitudes of the positive and negative components respectively
ϕ_1, ϕ_2	are the phase angles of the positive and negative components respectively.

Then, Equations (16) to (18) become

$$v_a(t) - v_{a0}(t) = \widehat{V}_{a1}\sin(\omega_0 t + \phi_1) + \widehat{V}_{a2}\sin(\omega_0 t + \phi_2) \tag{21}$$

$$v_b(t) - v_{a0}(t) = \widehat{V}_{a1}\sin(\omega_0 t + \phi_1 - 2\pi/3) + \widehat{V}_{a2}\sin(\omega_0 t + \phi_2 + 2\pi/3) \tag{22}$$

$$v_c(t) - v_{a0}(t) = \widehat{V}_{a1}\sin(\omega_0 t + \phi_1 + 2\pi/3) + \widehat{V}_{a2}\sin(\omega_0 t + \phi_2 - 2\pi/3)\ . \tag{23}$$

The above three equations can be written as:

$$\begin{vmatrix} v_a(t) - v_{a0}(t) \\ v_b(t) - v_{b0}(t) \\ v_c(t) - v_{c0}(t) \end{vmatrix} = \begin{vmatrix} \sin\omega_0 t & 0 & \cos\omega_0 t & 0 \\ \sin\omega_0 t\cos 2\pi/3 & -\cos\omega_0 t\sin 2\pi/3 & \cos\omega_0 t\cos 2\pi/3 & \sin\omega_0 t\sin 2\pi/3 \\ \sin\omega_0 t\cos 2\pi/3 & \cos\omega_0 t\sin 2\pi/3 & \cos\omega_0 t\cos 2\pi/3 & -\sin\omega_0 t\sin 2\pi/3 \end{vmatrix} \begin{vmatrix} x_1 + x_2 \\ x_1 - x_2 \\ y_1 + y_2 \\ y_1 - y_2 \end{vmatrix} \tag{24}$$

where, we define

$$x_1 = \widehat{V}_{a1}\cos\phi_1, \quad y_1 = \widehat{V}_{a1}\sin\phi_1 \tag{25}$$

$$x_2 = \widehat{V}_{a2}\cos\phi_2, \quad y_2 = \widehat{V}_{a2}\sin\phi_2\ . \tag{26}$$

If the three voltages are sampled at a preselected rate, say Δt, m samples would be obtained and Equation (24) can be written as:

$$z(t) = H(t)\Theta + \omega(t) \tag{27}$$

where

$z(t)$ is a $3m$ by 1 vector of measurements
$H(t)$ is a $3m$ by 4 matrix of measurement
Θ is a 4 by 1 parameter vector to be estimated
$\omega(t)$ is a $3m$ by 1 errors vector to be minimized.

Assume $m \geq 4$; then the solution to Equation (27) based on the least errors square is

$$\Theta = [H^T(t)H(t)]^{-1}H(t)^T z(t) \tag{28}$$

having identified the parameter Θ, we may define

$$\begin{aligned} \Theta_{sx} &= x_1 + x_2 \\ \Theta_{dx} &= x_1 - x_2 \\ \Theta_{sy} &= y_1 + y_2 \\ \Theta_{dy} &= y_1 - y_2 \end{aligned} \tag{29}$$

where

$$\Theta = \text{col.}\ (\Theta_{sx}, \Theta_{dx}, \Theta_{sy}, \Theta_{dy})\ .$$

Then, the values of x_1, x_2, y_1, and y_2 can be determined as

$$x_1 = 0.5(\Theta_{sx} + \Theta_{dx}) \tag{30}$$

$$x_2 = 0.5(\Theta_{sx} - \Theta_{dx}) \tag{31}$$

$$y_1 = 0.5(\Theta_{sy} + \Theta_{dy}) \tag{32}$$

$$y_2 = 0.5(\Theta_{sy} - \Theta_{dy})\ . \tag{33}$$

Having obtained the states x_1, x_2, y_1, and y_2, we can calculate the positive sequence phasor as well as the negative sequence phasor using Equations (25) and (26) as

$$\widehat{V}_{a1}^2 = (x_1^2 + y_1^2) \tag{34}$$

$$\phi_1 = \tan^{-1}(y_1/x_1) \tag{35}$$

and

$$\widehat{V}_{a2}^2 = (x_2^2 + y_2^2) \tag{36}$$

$$\phi_2 = \tan^{-1}(y_2/x_2) \tag{37}$$

3. TESTING THE ALGORITHM

3.1. Simulated Data

The proposed algorithm is tested using simulated data, where we assume a three phase system of unbalanced voltages given in the time domain by:

$$\begin{aligned} v_a(t) &= 212.132 \sin(\omega_0 t + 45^\circ) \\ v_b(t) &= 353.55 \sin(\omega_0 t + 150^\circ) \\ v_c(t) &= 141.42 \sin(\omega_0 t + 300^\circ)\ . \end{aligned} \tag{38}$$

The above three voltages are sampled using a sample frequency of 500 Hz, a number of samples equal to 12 and data window size = 24 ms. Table 1 gives the results obtained, where we compare the proposed algorithm and the symmetrical component theory. It can be noticed, from this table, that the proposed algorithm estimates exactly the positive, negative, and zero sequence components of the three phase unsymmetrical voltages.

The algorithm was also tested, when these voltages are contaminated with an unsymmetrical third harmonic having a magnitude equal to half the fundamental magnitude. The three voltage signals, in this case, become

$$\begin{aligned} v_a(t) &= 212.132\sin(\omega_0 t + 45°) + 106.067\sin(3\omega_0 t + 90°) \\ v_b(t) &= 353.55\sin(\omega_0 t + 150°) + 176.78\sin(3\omega_0 t - 60°) \\ v_c(t) &= 141.42\sin(\omega_0 t - 60°) + 70.71\sin(3\omega_0 t + 120°)\ . \end{aligned}$$

It has been shown, through extensive runs, that third harmonics contaminating the wave signal have no effect on the estimation of the symmetrical components of the fundamental unsymmetrical voltages. However, if the matrix $H(t)$ is modified to take into account this harmonics contamination, (this can easily be done by replacing ω_0 by $3\omega_0$),we can obtain the symmetrical components of the unsymmetrical third harmonic voltages. We found also, through extensive runs, that the proposed algorithm behaves as a digital filter.

3.1.1. Frequency Drift

Effects of the frequency drift on the estimated components of the fundamental voltage have been tested, where the frequency drift changes from a small value, $\Delta f = -0.1$ Hz to a large value $\Delta f = -1.0$ Hz. Table 2 gives the results obtained for these two values of the frequency drift. Examining this table reveals the following:

The small frequency drift, $\Delta f = -0.1$ Hz, has no effect on the estimated parameters either on the magnitudes or on the phase angles.

However, the large frequency drift, $\Delta f = -1.0$ Hz, has no appreciable effect on the estimated magnitudes, but it has appreciable effect on the phase angles estimate.

In order to avoid the errors due to the frequency drift, the proposed algorithm needs, in advance, a frequency measurement technique and this frequency is used in the matrix $H(t)$.

Table 1. Results of the Simulated Example.

	Positive Sequence		Negative Sequence		Zero Sequence	
	Magnitude	Phase	Magnitude	Phase	Magnitude	Phase
Symmetrical Components	48.0	-87.7°	163.2	40.5°	52.2	112.7°
Proposed Components	48.01	-87.6°	163.21	40.45°	52.189	112.7°

Table 2. Effects of the Frequency Drift.

	Positive Sequence		Negative Sequence		Zero Sequence	
Frequency drift	Magnitude	Phase	Magnitude	Phase	Magnitude	Phase
$\Delta f = -0.1$	48.05	-87.975°	163.22	40.535°	52.14	112.4°
$\Delta f = -1.0$	48.36	-91.393°	163.22	36.5°	51.51	105.0°

3.2. Actual Recorded Data

The proposed algorithm has also been tested using actual recorded data of a practical system in operation. The EMTP is used to compute the three phase voltages at a certain bus on a system during the switching-on of 500 kV transmission system. Figure 1 gives the waveform of the three voltages v_a, v_b, and v_c.

It can be seen from this figure that, during the first two cycles, the three voltages are unbalanced. The output data from the EMTP, the voltages magnitudes and the time, are in the form of samples. The sampling frequency used in this test was 10 000 Hz (the sampling time $\Delta t = 0.1$ ms). A computer program is written to change this sampling frequency, as well as, the number of samples.

These samples are used to calculate digitally the positive, negative, and zero sequence components of these three unbalanced voltages, using the proposed algorithm. Table 3 gives the results obtained, when the chosen number of samples equals 100 samples, with a sampling frequency equals 10 000 Hz.

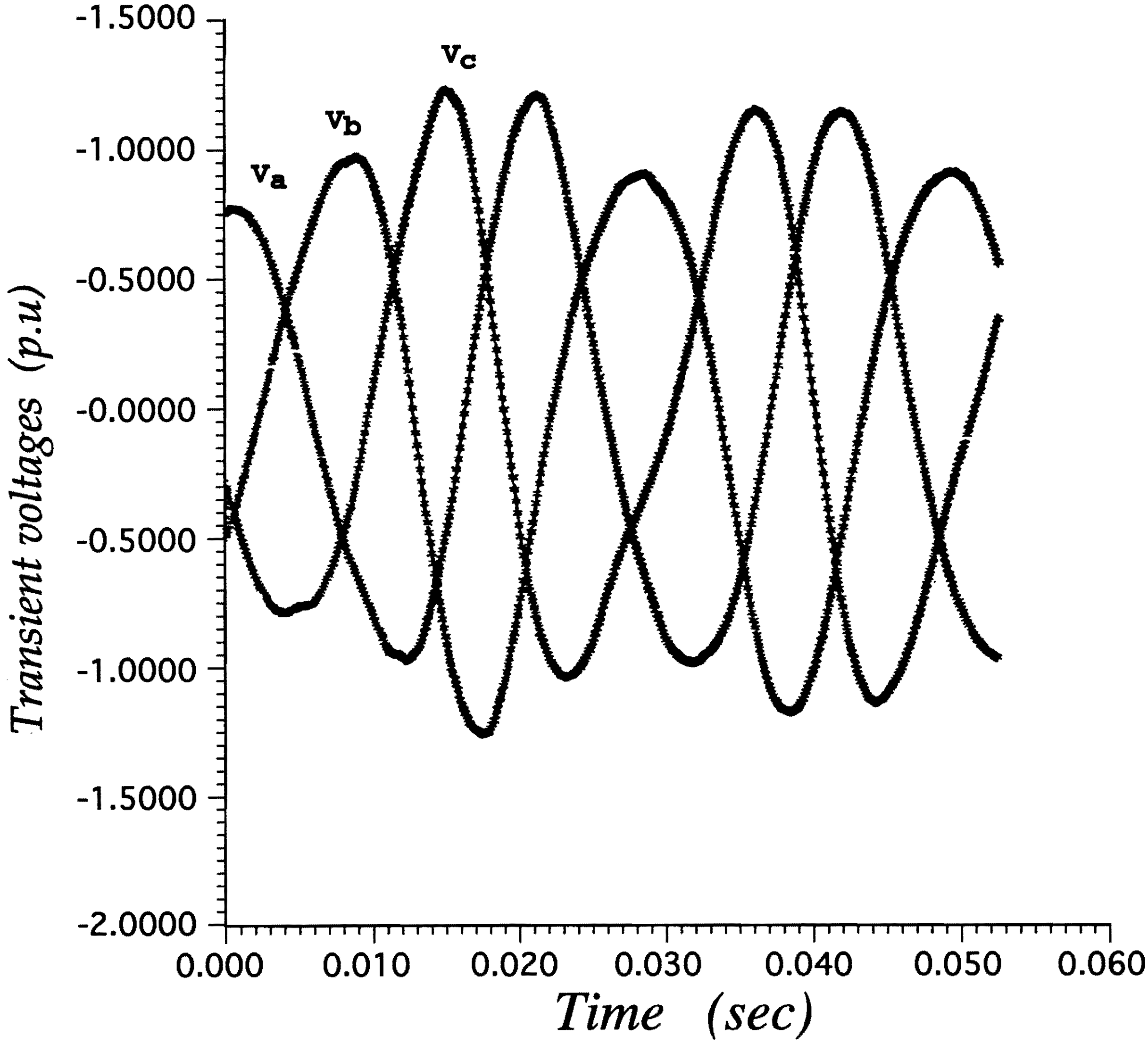

Figure 1. The Three-Phase Transient Voltage.

3.2.1. Effects of Number of Samples

The effects of the number of samples on the estimated parameters have been studied. Table 4 gives the result obtained, when the number of samples changes from $m = 25$ samples to $m = 100$ samples with a sampling frequency of 10 000 Hz. Examining this table carefully reveals the following:

variation in the number of samples at a constant sampling frequency (variable data window size) has a great effect on the estimated parameters, especially the estimated phase angles;

the positive sequence component, both magnitude and phase angle, is less affected by the variation in data window size;

the estimated parameters are not constant, since their magnitudes change as the number of samples changes, which simply means that they are time varying parameters. Indeed, this is true, since the three phase voltages are contaminated with harmonics, especially in the first few cycles, which is the transient period;

although the voltages are harmonics contaminated, the most effective component is the positive sequence component.

3.2.2. Effects of Sampling Frequency

The effects of sampling frequency on the behavior of the proposed algorithm has also been studied. Table 5 gives the results obtained when the sampling frequency varies from 2 000 Hz to 10 000 Hz, when the number of samples equals 100. (Data window size in this case varies from 2.5 cycles to 0.5 cycles). Examining this table reveals the following:

Table 3. Estimated Components for $m = 100$, $f_s = 10000$ Hz.

	Positive Sequence	Negative Sequence	Zero Sequence
Magnitude (p.u.)	0.84735	0.074915	0.017944
Phase Angle (degree)	78.938	−153.0914	−79.88

The base voltage = 500 kV

Table 4. Effects of Number of Samples, Sampling Frequency = 10 kHz.

Number of Samples m	Data Window Size	Positive Sequence		Negative Sequence		Zero Sequence	
		Magnitude (p.u.)	Phase (Degree)	Magnitude (p.u.)	Phase (Degree)	Magnitude (p.u.)	Phase (Degree)
25	1/8 Cyc.	0.774	82.20	0.1108E-3	170.02	1.950E-6	12.954
50	1/4 Cyc.	0.778	82.11	4.0200E-3	−123.0	4.225E-3	−26.460
75	3/8 Cyc.	0.816	80.32	4.9060E-2	−133.44	4.563E-3	134.29
100	1/2 Cyc.	0.848	78.94	7.4915E-2	−153.09	1.794E-2	−79.880

the estimated parameters change as the sampling frequency changes;

as the data window size increases, the negative and zero sequence magnitudes decrease. Indeed, that is the case, because as the data window size increases, *e.g.* to 2.5 cycles, the three phase voltages become almost sinusoidal, and harmonics contamination is reduced greatly;

the component parameters are nonstationary (time-varying parameters).

4. DISCUSSION

The proposed algorithm has been tested on simulated and actual data. It is clear, from the results obtained, that the three voltages have time-varying magnitudes, we therefore recommend that a dynamic state estimation algorithm, such as Kalman Filtering (KF) algorithm or dynamic least absolute value (DLAV) algorithm developed by the first author, be used to estimate these parameters. This is the current research interest of the authors.

The proposed algorithm can be implemented as a harmonics identification and measurement algorithm. We tested, through extensive runs, this implementation for the actual recorded data. It has been found that the voltage signal waveforms are contaminated with the 2nd, 3rd, 4th, and 5th harmonics.

It has been shown that the three phase harmonic voltages are unsymmetrical, and hence we can obtain, for each harmonics component, the corresponding symmetrical components.

The proposed algorithm can be used as a digital filter algorithm. This can easily be done by changing the frequency ω_0 in the matrix $H(t)$ to the required harmonic frequency to be filtered.

5. CONCLUSIONS

In this paper, we have discussed the implementation of the least errors square parameter estimation algorithm for measuring the symmetrical components for power system protection. The proposed algorithm has been tested on simulated and actual recorded data. Effects of sampling frequency, number of samples as well as the frequency drift are discussed in the body of the text. It has been found that the magnitude as well as the phase angle of these components are non-stationary (time-varying), and a dynamic state estimation algorithm is needed, which is the authors current area of research.

Table 5. Effects of the Sampling Frequency, Number of Samples = 100.

Sampling frequency Hz	Window size Cycle	Positive Sequence		Negative Sequence		Zero Sequence	
		Magnitude (p.u.)	Phase (Degree)	Magnitude (p.u.)	Phase (Degree)	Magnitude (p.u.)	Phase (Degree)
2000	2.5	1.01380	87.975	1.115E-2	−147.78	5.708E-3	20.312
2500	2.0	1.0144	86.415	2.352E-2	−144.34	1.076E-2	24.965
3300	1.5	0.9854	86.787	2.023E-2	−151.239	1.704E-2	02.134
500	1.0	0.9934	83.374	4.020E-2	−143.580	2.549E-2	−41.00
1000	0.5	0.8474	78.938	7.492E-2	−153.091	1.794E-2	−79.88

REFERENCES

[1] T. Lobos, "Fast Estimation of Symmetrical Components In Real Time", *IEE Proced.-C*, **139(1)** (1992), pp. 27–30.

[2] M. E. El-Hawary, *Electrical Power Systems, Design and Analysis.* Reston, Virginia, U.S.A.: Reston Publishing Company, 1983.

[3] S. A. Soliman, G. S. Christensen, D. H. Kelly, and K. M. El-Naggar, "A State Estimation Algorithm For power System Harmonics Identification and Measurements", *Electrical Power Systems Research J.*, **19** (1990), pp. 195–206.

[4] T. Lobos, "Nonrecursive Methods for On-line Estimation of Voltages or Currents and Symmetrical Components Using Kalman Filter Theory", *Elect. Power Syst. Res.*, **9** (1985), pp. 243–252.

[5] D. L. Waikar, S. Elangovan, and A. C. Liew, "Symmetrical Component Based Improved Fault Impedance Estimation Methods for Digital Distance Protection: Part I and Part II", *Elect. Power Syst., Res.*, **26** (1993), pp. 143–154.

[6] T. Lobos, "Nonrecursive Methods for Real-Time Determination of Basic Waveforms of Voltages and Currents", *IEE Proc. Part c*, **136** (1989), pp. 347–351.

[7] A. G. Phadke and J.S.Thorp, *Computer Relaying For Power Systems.* New York, N.Y.: John Wiley, 1988.

Paper Received 14 March 1993; Revised 9 November 1993; Accepted 10 January 1994.