DEPTH DETERMINATION USING MOVING AVERAGE RESIDUAL MAGNETIC ANOMALIES DUE TO THIN DIKES

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الخلاصـة :

نُقدِّم في هذا البحث طريقة سهلة وسريعة لتعيين العمق للتراكيب الجيولوجية المدفونة تحت سطح الأرض والتي تكون في صورة جدد رفيعة وذلك من البيانات المغناطيسية.

وتعتمد الطريقة على تطبيق نفس مرشح المتوسط المتحرك على النموذج النظري لجدة رفيعة والذي طُبق على البيانات الحقلية المغناطيسية. ونتيجة لذلك يمكن تطبيق الطريقة ليس فقط على المتبقيات الحقيقية المغناطيسية، ولكن أيضاً على البيانات الحقلية التي تتكون من شذات محلية ناتجة عن التركيب الجيولوجي وأخرى أقليمية خطية ناتجة عن التأثير المغناطيسي للصخور العميقة.

هذا وقد تم اختبار صلاحية الطريقة على بيانات حقلية من أريزونا والبرازيل وجمهورية مصر العربية ووجد أن العمق الناتج من الطريقة المستخدمة يتفق تماماً مع العمق الناتج من معلومات الحفر أو المقدَّر بطرق أخرى.

ABSTRACT

A simple and rapid method is developed to estimate the depth to a buried dike-like structure using the moving average residual magnetic anomaly. The method involves a thin dike model convolved with the same moving average filter as applied to the observed magnetic data. The validity of the method is tested in detail on three field examples from Arizona, Brazil, and Egypt.

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INTRODUCTION

Several excellent methods have been developed by various authors for interpreting magnetic anomalies (total, vertical, or horizontal) caused by a thin dike model to find the depth, index parameter (effective angle of magnetization), and the amplitude coefficient of igneous rocks in the form of dikes and veins. An excellent review is given by Nettleton [1] and Blakely [2]. The methods include, for example, matching standardized curves [3], characteristic points and distances approaches [4–6], nomograms [7], Hilbert transforms [8], Fourier transforms techniques [9], correlation factors between successive least-squares residual anomalies [10], and least-squares minimization methods [11].

In addition to the above-mentioned methods, several other methods have been reported in the geophysical literature. The most common techniques are the methods based on Werner deconvolution [12–15] and Euler deconvolution [16–19]. In these methods, the depth determination problem is transformed into the problem of finding a solution of a system of linear equations. The most important characteristic of Werner and Euler deconvolution methods is that any four to seven data points of the anomaly profile are sufficient to produce an estimate of the thin dike parameters.

In the present paper, a simpler and more rapid method than the existing ones is developed to estimate the depth to a buried dike-like structure. The method is based on using analytical expression of simple moving average residual magnetic anomalies. The method is tested on three field examples: (1) the Pima copper mine anomaly, Arizona; (2) the Parnaiba dike anomaly, Brazil; and (3) the Gabal Abu Khruq dike anomaly, Southeastern Desert, Egypt.

THEORY

The general expression, T, for the magnetic anomaly in either the total, vertical, or horizontal field at a point P along the x-axis (Figure 1) of an arbitrary magnetized thin dike (2-D) is given as [20]:

$$T(x_i, z, \theta) = zA\frac{x_i \sin\theta + z\cos\theta}{(x_i^2 + z^2)}, i = 1, 2, 3, \ldots N, \tag{1}$$

where z is the depth to the top of the body, x_i is a discrete point along x where the observed anomaly is located, A is the amplitude coefficient, and θ is the index parameter. The values of A and θ for the anomalies in the total, vertical, and horizontal fields are given in Table 1. The index parameter θ is related to the effective inclination of polarization I_o' and the angle of the dip of the dike.

In Table 1, t is the thickness of the dike, k is the magnetic susceptibility contrast, d is the dip angle of the dike, T_o' and I_o' are the values of effective total intensity and effective inclination of magnetic polarization in the vertical plane normal to the strike of the body, and α is the strike of the dike measured clockwise from magnetic north. T_o' and I_o' are related to the true total intensity T_o and true inclination I_o by

$$\tan I_o' = \frac{\tan I_o}{\sin\alpha} \quad ,$$

and

$$\frac{T_o'}{T_o} = \frac{\sin I_o}{\sin I_o'} \quad .$$

The moving average (grid) method is an important and simple technique for the separation of magnetic anomalies into residual and regional components. The basic theory of the moving average method is described by Griffin [21] and Abdelrahman and El-Araby [22]. The moving average residuals are proportional to second derivative values and hence have high resolving power in separating the residual anomaly [23,24].

Consider three observation points $x_i - x, x_i$, and $x_i + s$ on the anomaly profile where $s = 1, 2, 3, \ldots, M$ spacing units and is called the window length or graticule spacing. The moving average regional magnetic field $Z(x_i, z, \theta, s)$ is defined as the average of $T(x_i - s, z, \theta)$ and $T(x_i + s, z, \theta)$ which for the thin dike mentioned above can be written as

$$Z(x_i, z, \theta, s) = \frac{zA}{2}\left[\frac{(x_i - s)\sin\theta + z\cos\theta}{((x_i - s)^2 + z^2)} + \frac{(x_i + s)\sin\theta + z\cos\theta}{((x_i + s)^2 + z^2)}\right]. \tag{2}$$

The moving average residual magnetic anomaly $R(x_i, z, \theta, s)$ at the point x_i is

$$R(x_i, z, \theta, s) = \frac{zA}{2}\left[\frac{2x_i\sin\theta + 2z\cos\theta}{(x_i^2 + z^2)} - \frac{(x_i - s)\sin\theta + z\cos\theta}{((x_i - s)^2 + z^2)} - \frac{(x_i + s)\sin\theta + z\cos\theta}{((x_i + s)^2 + z^2)}\right]. \tag{3}$$

The normalized moving average residual magnetic anomalies $R(s, z)_n$ and $R(-s, z)_n$ at the points $x_i = s$ and $x_i = -s$, respectively, are obtained from Equation (3) as

$$R(s, z)_n = \frac{R(s, z)}{R(0)} = \frac{z^2 - 2s^2 + 3zs\tan\theta}{(4s^2 + z^2)}, \tag{4}$$

and

$$R(-s, z)_n = \frac{R(-s, z)}{R(0)} = \frac{z^2 - 2s^2 - 3zs\tan\theta}{(4s^2 + z^2)}, \tag{5}$$

where $R(0)$ is the anomaly value at the origin ($x_i = 0$).

Table 1. Characteristic Amplitude Coefficient A and Index Parameter Θ in Vertical (ΔT), Horizontal (ΔH), and Total (ΔT) Magnetic Anomalies due to Thin Dikes (After Parker Gay [3]).

Anomaly (T)	Amplitude coefficient (A)	Index parameter (Θ)
ΔV	$2ktT_o'/z$	$I_o' - d$
ΔH	$2ktT_o'\sin\alpha/z$	$I_o' - d - 90°$
ΔT	$\dfrac{2ktT_o'\sin I_o}{z\sin I_o'}$	$2I_o' - d - 90°$

Let the sum of $R(s,z)_n$ and $R(-s,z)_n$ be represented by F. Then from (4) and (5), we obtain

$$F = \frac{2z^2 - 4s^2}{4s^2 + z^2}, \tag{6}$$

which when solved for the depth, z, gives the following relationship

$$z = 2s\sqrt{\frac{F+1}{2-F}}. \tag{7}$$

Thus knowing F and the window length (s), the depth, z, can be determined from Equation (7).

To this stage, we have assumed knowledge of the origin of the magnetic anomaly profile. Otherwise, the origin of the profile ($x_i = 0$) can be determined using Stanley's method [5]. A straight line joining the maximum (M) to the minimum (m) of the profile will intersect the curve at the point $x = 0$ (Figure 2). Stanley's method [5] works not only for the total intensity anomaly but also for the anomalies in vertical and horizontal fields. In the present method, the base line is not needed because the moving average filter minimizes its significance. The filter also removes the regional effect.

FIELD EXAMPLES

To examine the applicability of the present method, the following three field examples are presented.

The Pima Copper Mine Anomaly

Figure 3 shows a vertical magnetic anomaly from the Pima copper mine, Arizona (Parker Gay [3], Figure 10, p.198). A magnetic profile 750 meters long was digitized at an interval of 25 meters. The magnetic data were subjected to a separation technique using the moving average method applied for three successive window sizes (Table 2, Figure 4). The method, Equation (7), was then applied to each of the three residual profiles, yielding three depth solutions (Table 3). The average depth is 62.1 meters. The depth obtained agrees very well with the depth of 64 meters obtained from drilling.

The Parnaiba Dike Anomaly

Figure 5 gives the total magnetic anomaly above a Mesozoic diabase dike intruded into Paleozoic sediments in the Parnaiba basin, Brazil (Silva [11], Figure 10, p.120). The sensor height is 1.9 meters (Figure 5). This anomaly profile of 24.64 meters length was digitized at an interval of 1.54 meters. Three successive moving average windows were applied (Table 4, Figure 6). The depth obtained from each residual anomaly profile is given in Table 5. The average depth is 3.8 meters. The depth according to Silva [11] is 3.5 meters. The depth to the top is over estimated by the present method and Silva's method. This is not unreasonable because the upper part of the dike was weathered and the magnetite present was oxidized, losing most of its magnetic property [11].

The Gabal Abu Khruq Dike Anomaly

Figure 7 shows a total magnetic anomaly over a dike-like structure, Gabal Abu Khruq district, the Southeastern Desert, Egypt (Shaaban and Sabri Ahmad [25], Figure 4, p.302). This anomaly was digitized at an interval of 0.5 km. The present method was applied to the anomaly values thus obtained and the results are shown in Tables 6 and 7 and Figure 8. The average depth is 1.233 km. The depth to the top of the dike according to Shaaban and Sabri Ahmed [25] using Powell's technique [26] is 1.2 km.

Table 2. Input and Output Data of the Pima Copper Mine Field Example.

x_i	Input (nT)	Output (nT)		
	$T(x_i)$	$R(x_i,z,\theta,1)$ ($s = 1$ unit)	$R(x_i,z,\theta,2)$ ($s = 2$ units)	$R(x_i,z,\theta,3)$ ($s = 3$ units)
−15	82.41			
−14	92.37	−0.23		
−13	102.79	−1.04	−3.01	
−12	115.29	−0.69	−3.82	−9.26
−11	129.18	−1.39	−5.56	−11.45
−10	145.85	−2.08	−6.25	−15.51
−9	166.68	−0.69	−7.87	−19.45
−8	188.90	−4.40	−12.50	−25.00
−7	219.93	−3.01	−12.73	−25.47
−6	256.97	−2.32	−9.95	−27.78
−5	298.64	−2.32	−12.73	−27.32
−4	344.94	−5.79	−15.05	−1.16
−3	402.81	−1.16	17.36	$R(-3,z)$ 94.91
−2	463.00	25.47	$R(-2,z)$ 111.12	226.87
−1	472.26	$R(-1,z)$ 61.35	184.04	299.79
0	358.83	$R(0)$ 35.88	$R(0)$ 127.33	$R(0)$ 177.42
1	173.63	$R(1,z)$ −5.79	−42.51	−25.93
2	0.00	−66.81	$R(2,z)$ −147.47	−196.31
3	−40.00	−8.05	−86.99	$R(3,z)$ −174.27
4	−63.89	−4.08	−18.75	−103.25
5	−79.64	−2.55	−12.18	−31.48
6	−90.29	−3.01	−10.19	−23.99
7	−94.92	−1.62	−8.80	−20.14
8	−96.30	−2.55	−8.33	−16.21
9	−92.60	−1.62	−4.86	−11.58
10	−85.66	0.93	−1.62	−6.02
11	−80.56	−1.85	−2.08	−3.94
12	−71.77	0.69	−0.46	−1.16
13	−64.36	0.00	0.23	
14	−56.95	−0.46		
15	−48.62			

In the three field examples, individual depth solutions vary much from the actual depth because the data contain measurement errors whereas the average depth is in good agreement with the actual one. Averaging the results tends to cancel the effect of noise in the data. Generally, the error of depth estimation is independent of the window length (Tables 3 & 7).

Table 3. Interpreted Depth from Moving Average Residuals of Pima Copper Mine Magnetic Anomaly Using Present Method.

Window length (s) (spacing units)	$R\ (0)$ (nT)	$R\ (-s,z)$ (nT)	$R(s,z)$ (nT)	$R\ (-s,z)_n$ (nT)	$R(s,z)_n$ (nT)	F	Computed depth (meters)
1	35.88	61.35	-5.79	1.71	-0.16	1.55	59.5
2	127.33	111.12	-147.47	0.87	-1.16	-0.29	55.7
3	177.42	94.91	-174.27	0.53	-0.98	-0.45	71.1
Average values							62.1

Table 4. Input and Output Data of the Parnaiba Field Example.

x_i	Input (nT)	Output (nT)		
	$T(x_i)$	$R(x_i,z,\theta,1)$ ($s=1$ unit)	$R(x_i,z,\theta,2)$ ($s=2$ units)	$R(x_i,z,\theta,3)$ ($s=3$ units)
-7	14.5			
-6	16.5	2.00		
-5	15.0	2.00	6.00	
-4	9.5	0.00	1.25	4.50
-3	4.0	-0.75	-1.50	$R(-3,z)$ $\underline{15.00}$
-2	0.0	0.00	$R(-2,z)$ $\underline{14.50}$	17.00
-1	-4.0	$R(-1,z)$ $\underline{15.25}$	18.50	12.00
0	-38.5	$R(0)$ $\underline{-12.00}$	$R(0)$ $\underline{-17.75}$	$R(0)$ $\underline{-23.00}$
1	-49.00	$R(1,z)$ $\underline{-9.00}$	-30.25	-40.00
2	-41.5	-0.25	$R(2,z)$ $\underline{-13.25}$	-35.00
3	-33.5	-3.75	-4.50	$R(3,z)$ $\underline{-12.25}$
4	-18.0	3.25	4.75	8.00
5	-9.0	2.00	9.25	12.50
6	-4.0	2.00	5.75	12.25
7	-3.0	-0.25	1.00	
8	-1.5	-0.50		
9	1.0			

Table 5. Interpreted Depth from Moving Average Residuals of Parnaiba Magnetic Anomaly Using Present Method.

Window length (s) (spacing units)	R (0) (nT)	R ($-s,z$) (nT)	$R(s,z)$ (nT)	R $(-s,z)_n$ (nT)	$R(s,z)_n$ (nT)	F	Computed depth (meters)
1	−12.00	15.25	−9.00	−1.27	0.75	−0.52	1.34
2	−17.75	14.50	−13.25	−0.82	0.75	−0.07	4.13
3	−23.00	15.00	−12.25	−0.65	0.53	−0.12	5.95
Average values							3.81

Table 6. Input and Output Data of the Gabal Abu Khruq Field Example.

x_i	Input (nT)	Output (nT)		
	$T(x_i)$	$R(x_i,z,\theta,1)$ ($s = 1$ unit)	$R(x_i,z,\theta,2)$ ($s = 2$ units)	$R(x_i,z,\theta,3)$ ($s = 3$ units)
−6	6926			
−5	6963	37.5		
−4	6926	74.5	260.0	
−3	6740	74.0	223.0	$R(-3,z)$ <u>705.5</u>
−2	6406	0.5	$R(-2,z)$ <u>371.5</u>	576.0
−1	6071	$R(-1,z)$ <u>296.5</u>	352.5	129.5
0	5143	$R(0)$ <u>−241.0</u>	$R(0)$ <u>−538.5</u>	$R(0)$ <u>−1167.0</u>
1	4697	$R(1,z)$ <u>−353.0</u>	−1278.5	−1523.0
2	4957	−331.5	$R(2,z)$ <u>−631.5</u>	−1151.5
3	5880	384.5	458.5	$R(3,z)$ <u>180.0</u>
4	6034	12.0	427	
5	6146	0.5		
6	6257			

Table 7. Interpreted Depth from Moving Average Residuals of the Gabal Abu Khruq Magnetic Anomaly Using Present Method.

Window length (s) (spacing units)	R (0) (nT)	R ($-s,z$) (nT)	$R(s,z)$ (nT)	R $(-s,z)_n$ (nT)	$R(s,z)_n$ (nT)	F	Computed depth (meters)
1	−241.0	296.5	−353.0	−1.23	1.47	0.24	0.84
2	−538.5	371.5	−631.5	−0.69	1.17	0.48	1.97
3	−1167.0	705.5	180.0	−0.61	−0.15	−0.76	0.88
Average values							1.23

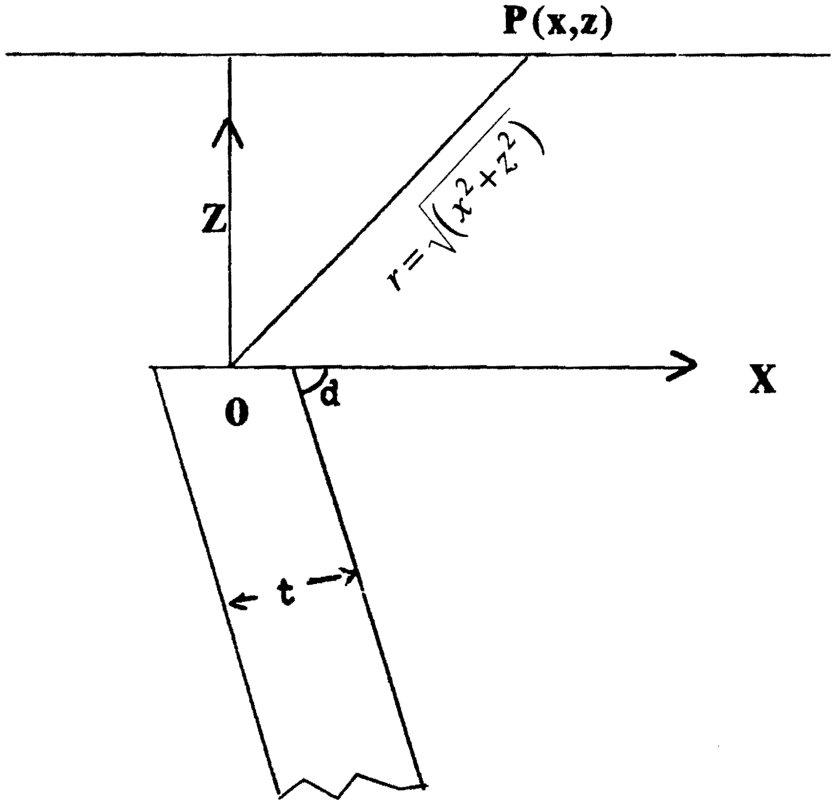

Figure 1. The Thin Dike Model (2-D).

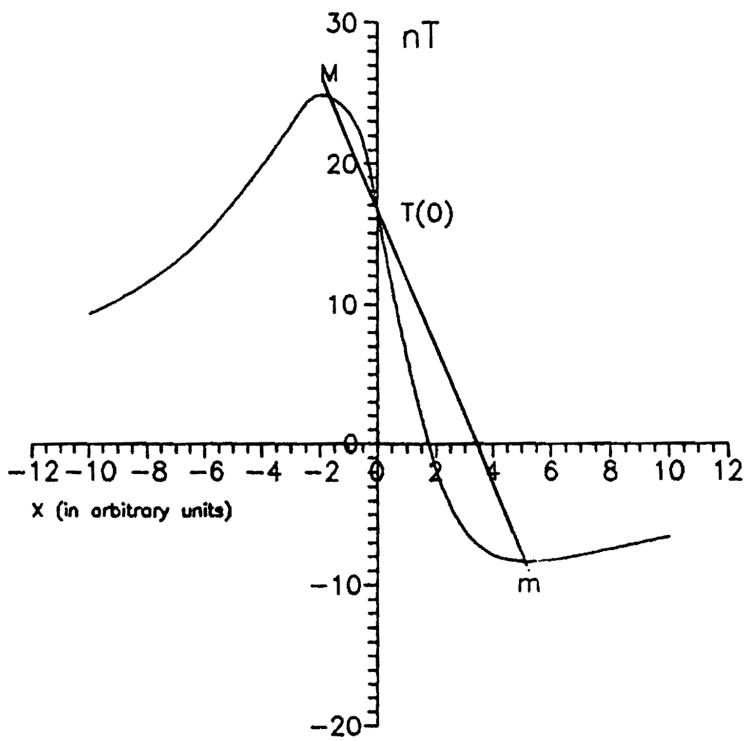

Figure 2. A Typical Total Magnetic Anomaly over a Thin Dike.
The positions of maximum value (M) and minimum value (m) are illustrated.

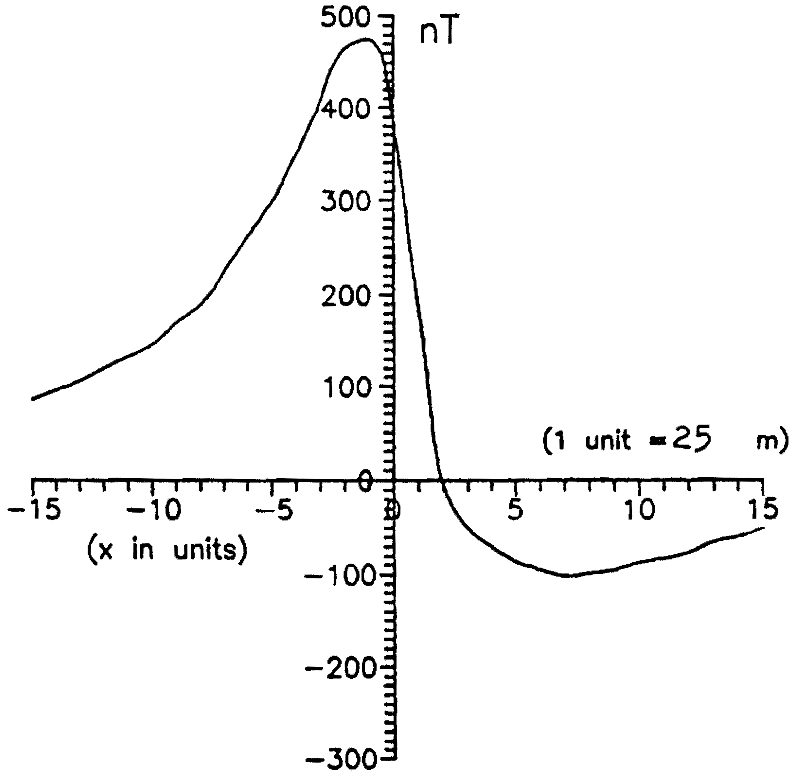

Figure 3. Vertical Magnetic Anomaly over Pima Copper Mine in Arizona. The origin is the same as that given by Parker Gay [3].

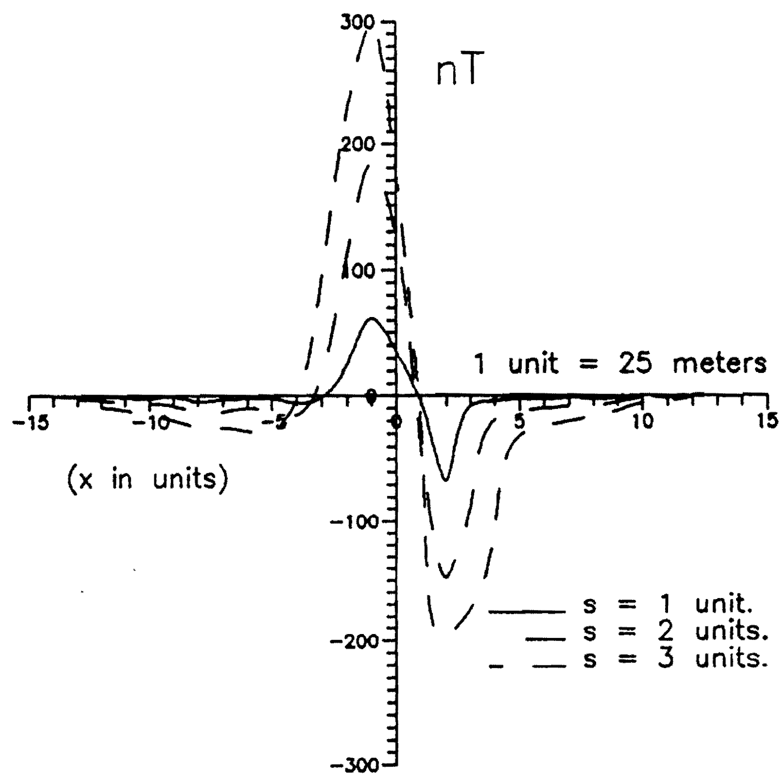

Figure 4. Moving Average Residual Magnetic Anomalies over Pima Copper Mine in Arizona. The moving average filter has high resolving power in separating residual anomalies when the graticule spacing (window length) is large.

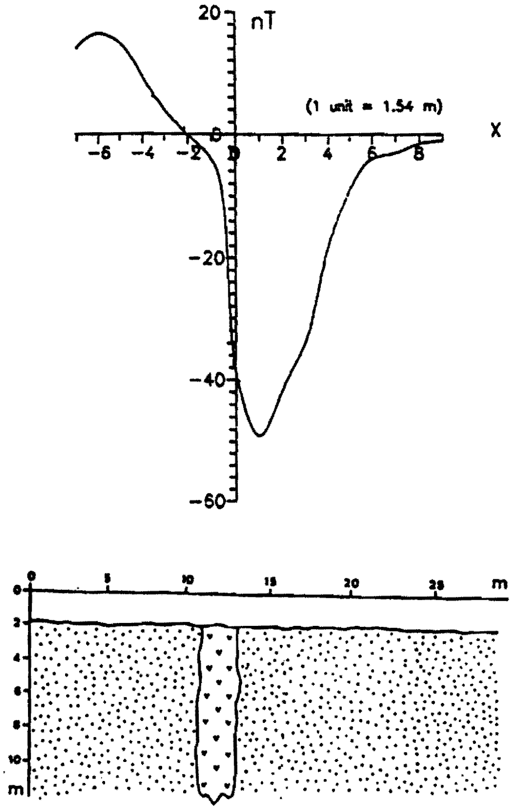

Figure 5. Total Magnetic Anomaly (top) over an Outcropping Dike (bottom) in the Parnaiba Basin, Brazil [11].

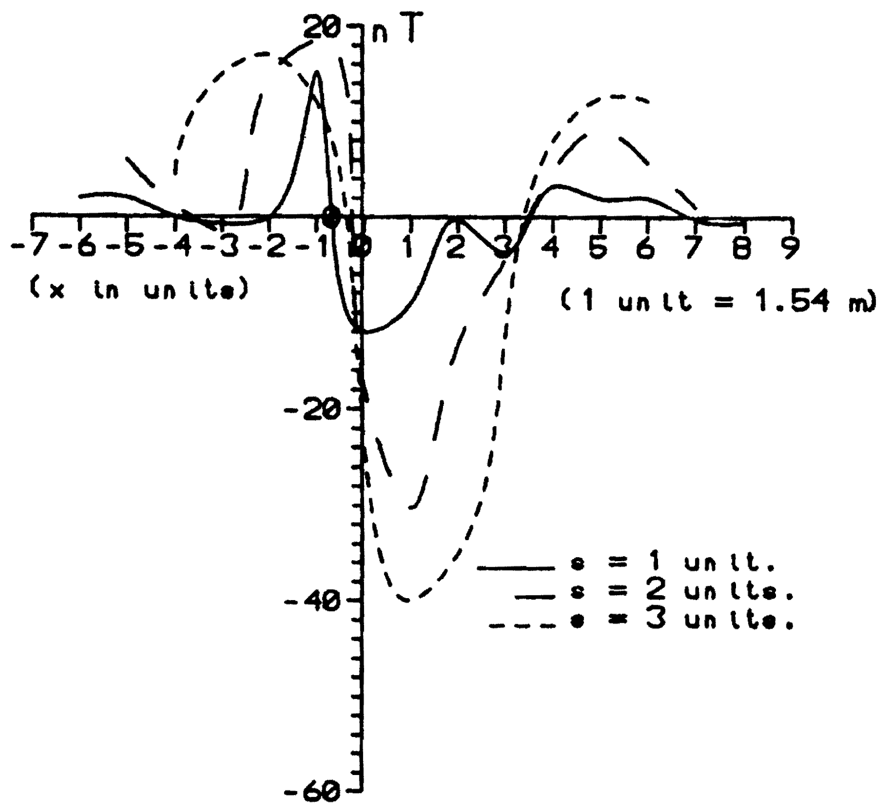

Figure 6. Moving Average Residual Magnetic Anomalies over an Outcropping Dike in the Parnaiba Basin, Brazil.

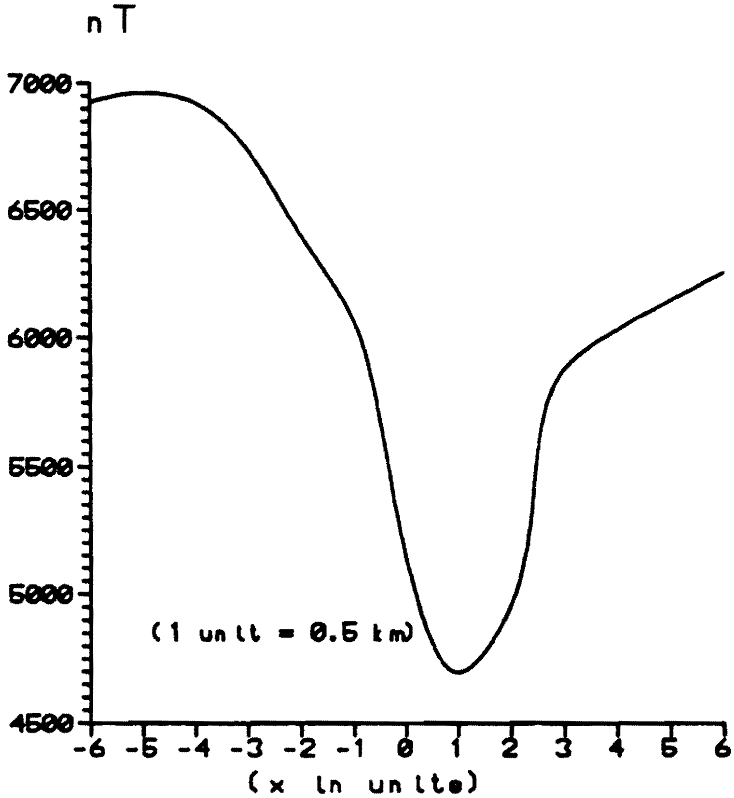

Figure 7. Total Magnetic Anomaly over a Dike, Gabal Abu Khruq District, Southeastern Desert of Egypt [25].

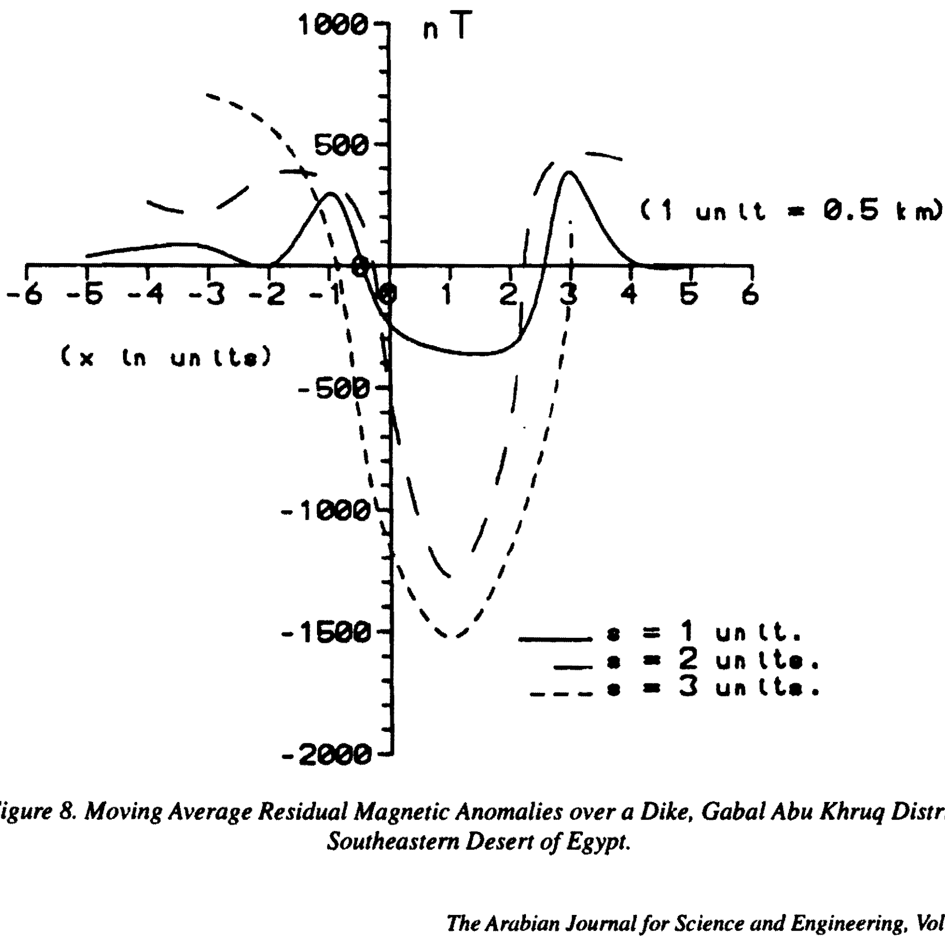

Figure 8. Moving Average Residual Magnetic Anomalies over a Dike, Gabal Abu Khruq District, Southeastern Desert of Egypt.

DISCUSSION AND CONCLUSION

The depth determination problem using moving average residual magnetic anomalies over thin dikes has been transformed into finding the sum of the two anomaly values located at a distance equal to the graticule spacing of the moving average filter from the origin of the profile. The method uses a simple dike model convolved with the same moving filter applied to the observed data. As a result, our method can be applied not only to "true residuals" but also to a real-world composite magnetic field consisting of a residual component due to a thin dike-like structure and a quasi-linear regional component due to deep seated structures.

The advantages of the present algorithm over previous graphical techniques, which use characteristic points and distances, standardized curves, and nomograms, are (1) the method is automatic, (2) the problem of the effects of sampling interval and length of the data (profile) can be solved, and (3) the method is not subject to human errors in computing the depth. The advantages of the present algorithm over the Werner and Euler deconvolution methods and least-squares methods as well as the other numerical techniques are (1) the method is simpler and easier, (2) measured or calculated horizontal and vertical gradients are not required, and (3) reduction of data to the pole is not required. Finally, our method does not need the value of magnetic field inclination in estimating the depth whereas most depth estimation techniques do. The technique can be extended to gravity and self-potential anomalies due to spheres and long horizontal cylinders.

ACKNOWLEDGMENTS

I would like to thank the Editor and the two capable reviewers for their keen interest, valuable comments on the manuscript, improvements to this work, and excellent suggestions. Thanks are also due to Dr. S.M. Sharafeldin, Geophysics Department, Faculty of Science, for his assistance.

REFERENCES

[1] L. L. Nettleton, *Gravity and Magnetics in Oil Prospecting.* New York: McGraw-Hill, 1976.

[2] R. S. Blakely, *Potential Theory in Gravity and Magnetic Applications.* London: Cambridge University Press, 1994.

[3] S. Parker Gay, "Standard Curves for Interpretation of Magnetic Anomalies Over Long Tabular Bodies", *Geophysics*, **28** (1963), p.161.

[4] R. S. Grant and G. F. West, *Interpretation Theory in Applied Geophysics.* New York: McGraw-Hill, 1965.

[5] J. M. Stanley, "Simplified Magnetic Interpretation of the Geologic Contact and Thin Dike", *Geophysics*, **42** (1977), p.1236.

[6] E. M. Abdelrahman, "A Rapid Approach to Depth Determination from Magnetic Anomalies due to Simple Geometrical Bodies", *J. Univ. of Kuwait (Science)*, **21** (1994), p.109.

[7] T. K. S. Prakasa Rao, M. Subrahmantam, and A. Srikrishna Murthy, "Nomograms for Direct Interpretation of Magnetic Anomalies due to Long Horizontal Cylinders", *Geophysics*, **51** (1986), p.1236.

[8] N. L. Mohan, N. Sundararajan, and S. V. Seshagiri Rao, "Interpretation of Some Two-Dimensional Magnetic Bodies Using Hilbert Transforms", *Geophysics*, **47** (1982), p.376.

[9] B. K. Bhattacharyya, "Two-Dimensional Harmonic Analysis as a Tool for Magnetic Interpretation", *Geophysics*, **30** (1965), p.829.

[10] E. M. Abdelrahman, "Magnetic Interpretation of Long Horizontal Cylinders Using Correlation Factors Between Successive Least-Squares Residual Anomaly Profiles", *PAGEOPH*, **132(3)** (1990), p.521.

[11] J. B. C. Silva, "Transformation of Nonlinear Problems Into Linear Ones Applied to the Magnetic Field of a Two-Dimensional Prism", *Geophysics*, **54** (1989), p.114.

[12] S. Werner, "Interpretation of Magnetic Anomalies of Sheet-Like Bodies", *Sveriges Geologiska Undersok.; Series C*, **43(6)** (1953), p.508.

[13] R. R. Hartman, D. J. Tesky, and J. L. Friedberg, "A System for Rapid Aeromagnetic Interpretation", *Geophysics*, **36** (1971), p.891.

[14] S. Jain, "An Automatic Method of Direct Interpretation of Magnetic Profiles", *Geophysics*, **41** (1976), p.531.

[15] R. O. Hansen and M. Simmonds, "Multiple-Source Werner Deconvolution", *Geophysics*, **58** (1993), p.1792.

[16] P. Hood, "Gradient Measurements in Aeromagnetic Surveying", *Geophysics*, **30** (1965), p.891.

[17] H. A. Lynch, V. M. Lynch, and L. Langan, "The Geomagnetic Gradiometer'", *Geophysics*, **55** (1989), p.877.

[18] D. T. Thompson, "EULDPH A New Technique for Making Computer-Assisted Depth Estimates from Magnetic Data", *Geophysics*, **47** (1982), p.31.

[19] A. B. Reid, J. M. Allsop, H. Gramser, A. J. Millett, and I. W. Somerton, "Magnetic Interpretation in Three Dimensions Using Euler Deconvolution", *Geophysics*, **55** (1990), p.80.

[20] D. Atchuta Rao, H. V. Ram Babu, and P. V. Sankar Narayan, "Relationship of Magnetic Anomalies Due to Subsurface Features and Interpretation of Solving Contacts", *Geophysics*, **45** (1980), p.32.

[21] W. R. Griffin, "Residual Gravity in Theory and Practice", *Geophysics*, **14** (1949), p.39.

[22] E. M. Abdelrahman and T. M. El-Araby, "Gravity Interpretation Using Correlation Factors Between Moving Average Residual Anomaly Profiles", *Arabian Journal for Science and Engineering*, **19(4A)** (1994), p.629.

[23] R. S. B. Rao and Radhakrishnamurthy, "Some Remarks Concerning Residuals and Derivatives", *PAGEOPH*, **61** (1965), p.5.

[24] E. M. Abdelrahman and T. M. El-Araby, "A Least-Squares Minimization Approach to Depth Determination from Moving Average Residual Gravity Anomalies", *Geophysics*, **59** (1993), p.1779.

[25] M. A. Shaaban and A. M. Sabri Ahmed, "Interpretation of the Total Intensity Map of the Gabel Abu Khrug Area in the Southeastern Desert of Egypt", *Arabian Journal for Science and Engineering*, **9** (1984), p.297.

[26] D. W. Powell, "Fitting Observed Profiles to Magnetized Dyke or Fault Step Model", *Geophysical Prospecting*, **XV** (1967), p.208.

Paper Received 1 April 1995; Revised 24 June 1995; Accepted 6 November 1995.