THE EFFECTS OF SELF-GENERATED MAGNETIC FIELDS ON THE CONVECTIVE RAMAN INSTABILITY

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الخلاصـة :

درسنا تأثير المجالات المغناطيسية المتولدة ذاتيا في البلازما غير المتجانسة والناشئة بسبب سقوط أشعة الليزر على هدف صلب .

لقد استخدمنا في دراستنا معادلة الزخم الخطي بالاضافة إلى معادلات ماكسويل . وكانت نتائج الدراسة تشير إلى أنّ معدل نمو تشتت (رامان) العكسي يزداد بينما يقل الحد الأدنى لشدة الموجة الساقطة التي تثير هذه الحالة من عدم الاستقرار عندما تدخل المجالات المغناطيسية المتولدة ذاتيا في حساباتنا .

ABSTRACT

Self-generated magnetic fields effects, in an inhomogeneous laser produced plasmas, are investigated by solving the slow coupled equations. It is found that the Raman backscattering growth rate is increased while its threshold is decreased.

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1. INTRODUCTION

The basic idea of the inertial confinement approach to fusion is to compress some fuel as much as possible in order to ignite it and get more energy out than was used in the compression. For this process to succeed it is vital to prevent preheating of the fuel [1–3]. In order to achieve this goal the fuel target is irradiated by an intense electromagnetic wave (laser). The outer part of the target becomes plasma almost immediately. Near the critical density ($\omega_o = \omega_p$, where ω_o is the frequency of the incident wave, and ω_p is the plasma frequency) the electromagnetic wave may decay into an electron plasma wave (plasmon) and an ion wave (phonon) leading to energy deposition into the plasma by resonance absorption. However, on its way to the critical density region, the incident electromagnetic wave passes through a region of less density known as the underdense region or corona. In this region the incident wave suffers from scattering and filamentation.

The stimulated Raman scattering (SRS) is the decay of the incident electromagnetic wave into a plasma wave and a scattered electromagnetic wave, in accordance with the frequency and wave number matching conditions:

$$\omega_o = \omega_1 + \omega_2, \vec{k}_o = \vec{k}_1 + \vec{k}_2 \ , \tag{1}$$

where ω_1 and ω_2 are the angular frequencies of the scattered electromagnetic wave and the electron plasma wave respectively; k_o, k_1, and k_2 are the propagation wave numbers of the incident electromagnetic wave, the scattered electromagnetic wave, and the electron plasma wave respectively. From the frequency matching condition it can be easily shown that this process occurs near the quarter critical density ($n_c/4$, where n_c is the critical density). The terms backward and forward refer to the direction of the scattered electromagnetic wave.

The forward scattered wave still has a chance to reach the critical surface and subsequently be absorbed there [4]. However, the plasma wave can generate hot electrons that may preheat the irradiated target and inhibit the proper isentropic compression required for the success of laser fusion [5, 6].

SRS has been observed experimentally [7] and theoretically as well as in computer simulations [8].

Since the scattering process affects the amount of energy that will be absorbed by the target, it is necessary to investigate the conditions which govern this scattering.

Forslund *et al.* [9] addressed this question and showed that for a large system, *i.e.* a long region of underdense plasma, the ratio of backscattered to incident laser energy flux can be the ratio of their frequency.

Laham *et al.* [10] investigated the effect of collision on the convective amplification of Raman backscattering in the underdense inhomogeneous plasma. They found that the collision damping reduces the growth rate and increases the threshold intensity.

The effect of bandwidth on Raman backscattering in an inhomogeneous plasma was studied [11–13] and it was shown that the amplification factor depends on the bandwidth but this dependence disappears if the collision frequency is neglected. It was found as well that the growth rate is reduced in the existence of bandwidth.

In an attempt to explain some physical phenomena in plasmas produced in laser fusion experiments, Stamper *et al.* [14] proposed the presence of self-generated magnetic fields. Since then experimental work [15] has confirmed the existence of such fields, and several theoretical reasons were given [16–18] to explain why these fields might have an important effect on the plasma dynamics. The source of these fields has been attributed [19] to the $\nabla n x \nabla T_e$ term, where T_e is the temperature of the electrons. These fields peak at typically 2–3 megagauss in the vicinity of the quarter-critical density of the plasma.

Few authors have addressed the scattering phenomena in the presence of self-generated magnetic fields. It has been shown [20] that the Raman scattering by upper hybrid wave has a substantial growth rate because of the reduction of linear damping, due to the self-generated magnetic fields. Barr *et al.* [21] investigated the Raman instability at the reflection point of the scattered extraordinary wave and they concluded that the red shift of the backscattered radiation due to plasma temperature is reduced by any magnetic field present and can be changed to a blue shift if the field is large enough.

In this paper we investigate theoretically the effects of self-generated magnetic fields on Raman backscattering instability in inhomogeneous underdense laser produced plasmas. We use the momentum equation in addition to the full set of Maxwell's equations to derive the nonlinear coupled equations. By solving these equations we obtain some expressions that govern the growth rate and the threshold of the instability in the presence of the self-generated magnetic fields.

This paper has been structured as follows: in Section 2 we derive the nonlinear coupled equations, and in Section 3 we develop a solution of these equations. In Section 4 we obtain and discuss our results. Finally in Section 5 we present our conclusions.

2. NONLINEAR COUPLED EQUATIONS

Let us consider an intense extraordinary electromagnetic pump wave

$$\vec{E} = (\hat{x}E_{ox} + \hat{y}E_{oy})\, e^{i(k_o x - \omega_o t)} + c.c. \tag{2}$$

incident on an underdense plasma, where it decays into two waves, an extraordinary electromagnetic wave whose dominant polarization will be the same as the incident pump wave

$$\vec{E}_1 = (\hat{x}E_{1x} + \hat{y}E_{1y})\, e^{-i(k_1 x + \omega_1 t)} + c.c. \tag{3}$$

and an upper hybrid electrostatic wave

$$\vec{E}_2 = \hat{x}E_{2x} e^{i(k_2 x - \omega_2 t)} + c.c. \tag{4}$$

where $\omega_o, \omega_1, \omega_2, k_o, k_1$ and k_2 are related by Equation (1).

The self-generated (dc) magnetic field direction will be perpendicular to the polarization of the incident laser radiation such that $\vec{B} = B_o\hat{z}$. In addition, we will assume that $E_{ox} \ll E_{oy}$, implying that the pump wave is almost perpendicular to the direction of propagation.

The peak (quiver) velocities of the electrons due to the above mentioned fields will be modified by the presence of the magnetic field according to the momentum equation

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}.\nabla)\vec{v} = -\frac{e}{m}\vec{E} - \frac{e}{mc}\vec{v}x\vec{B}, \tag{5}$$

where c is the speed of light.

Within the linear theory, the peak velocity of the electrons due to the pump field can be obtained as follows:

Let

$$\vec{v}_o = (v_{ox}\hat{x} + v_{oy}\hat{y})\, e^{i(k_o x - \omega_o t)}; \tag{6}$$

then from Equation (5), we get for the x-component

$$\frac{\partial v_{ox}}{\partial t} = -\frac{e}{m}E_{ox} - \frac{e}{mc}v_{oy}B_o \tag{7}$$

and for the y-component

$$\frac{\partial v_{oy}}{\partial t} = -\frac{e}{m}E_{oy} + \frac{e}{mc}v_{ox}B_o\ . \tag{8}$$

Taking the partial derivative of Equation (7) with respect to time, and using Equation (8), we obtain

$$\frac{\partial^2 v_{ox}}{\partial t^2} = -\frac{e}{m}\frac{\partial E_{ox}}{\partial t} + \Omega_e\frac{e}{m}E_{oy} - \Omega_e^2 v_{ox}\ , \tag{9}$$

where $\Omega_e = eB_o/m_oc$, is the cyclotron frequency of the electron. Using Equations (2) and (6), Equation (8) becomes

$$-\omega_o^2 v_{ox} = i\frac{\omega_o e}{m}E_{ox} + \frac{e}{m}\Omega_e E_{oy} - \Omega_e^2 v_{ox}\ ; \tag{10}$$

rearranging, we get:

$$v_{ox} = -\frac{ie}{m}\left[\frac{\omega_o E_{ox} - i\Omega_e E_{oy}}{\omega_o^2 - \Omega_e^2}\right] . \tag{11}$$

Similarly, we can get

$$v_{1x} = -\frac{ie}{m}\left[\frac{\omega_1 E_{1x} - i\Omega_e E_{1y}}{\omega_1^2 - \Omega_e^2}\right] \tag{12}$$

$$v_{oy} = -\frac{ie}{m}\left[\frac{\omega_o E_{oy} + i\Omega E_{ox}}{\omega_o^2 - \Omega_e^2}\right] \tag{13}$$

$$v_{1y} = -\frac{ie}{m}\left[\frac{\omega_1 E_{1y} + i\Omega_e E_{1x}}{\omega_1^2 - \Omega_e^2}\right] \tag{14}$$

and

$$v_{2x} = -\frac{ie}{m}\frac{\omega_2 E_{2x}}{\omega_2^2 - \Omega_e^2} . \tag{15}$$

To get the slow coupling equations for the interacting waves, we start by the generalized wave equation of an inhomogeneous plasma slab with linearly increasing density such that $n_e(x) = n_i(x) = n_o(1 + x/L)$, where $n_e(x)$ is the electron density, $n_i(x)$ is the ion density, L is the inhomogeneity scale length of the plasma and x is the distance along the density gradient, namely [10]

$$\nabla^2\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} - \nabla\left(\nabla \bullet \vec{E}\right) - \frac{4\pi e^2 n_e(x)}{mc^2}\vec{E} - \frac{4\pi e n_e(x)}{c^2}\nu\vec{v} + \frac{3T}{mc^2}\nabla\left(\nabla \bullet \vec{E}\right)$$
$$= \frac{1}{c^2}\frac{\partial}{\partial t}\left[\vec{v}\left(\nabla \bullet \vec{E}\right)\right] + \frac{4\pi e n_e(x)}{c^2}\left[\frac{1}{2}\nabla\left(\vec{v} \bullet \vec{v}\right)\right] - \frac{4\pi e n_e(x)}{c^2}\left[\vec{v}x\left(\nabla x\vec{v} - \frac{e\vec{B}}{mc}\right)\right] , \tag{16}$$

where T is the electron thermal energy and ν is the electron–ion collision frequency.

The magnetic field in Equation (16) is composed of two parts, the oscillating magnetic field (B_{osc}) and the static field (B_o) generated by the plasma itself.

In the limit when $\omega \gg \Omega_e$, it can be easily shown that

$$\nabla x\vec{v} = \frac{e\vec{B}_{osc}}{mc} \tag{17}$$

and the generalized wave equation becomes:

$$\nabla^2\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} - \nabla\left(\nabla\bullet\vec{E}\right) - \frac{4\pi e^2 n_e(x)}{m\ c^2}\vec{E} - \frac{4\pi e n^e(x)}{c^2}\nu\vec{v} + \frac{3\ T}{m\ c^2}\nabla\left(\nabla\bullet\vec{E}\right)$$

$$= \frac{1}{c^2}\frac{\partial}{\partial t}\left[\vec{v}\left(\nabla\bullet\vec{E}\right)\right] + \frac{4\pi e n_e(x)}{c^2}\left[\frac{1}{2}\nabla\left(\vec{v}\bullet\vec{v}\right)\right] - \frac{4\pi e n_e(x)}{c^2}\vec{v}x\Omega_e\,. \tag{18}$$

Separating this equation into its components, we get for the x-component

$$-\frac{1}{c^2}\frac{\partial^2 E_x}{\partial t^2} - \frac{4\pi e^2 n_e(x)}{mc^2}E_x - \frac{4\pi e n_e(x)}{c^2}\nu v_x + \frac{3T}{mc^2}\frac{\partial^2 E}{\partial x^2} + \frac{4\pi e n_e(x)}{c^2}\Omega_e v_y$$

$$= \frac{1}{c^2}\frac{\partial}{\partial t}\left[v_x\frac{\partial E_x}{\partial_x}\right] + \frac{2\pi e n_e(x)}{c^2}\frac{\partial}{\partial_x}\left(v_x^2 + v_y^2\right)\ ; \tag{19}$$

and for the y-component we get

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} - \frac{4\pi e^2 n_e(x)}{mc^2}E_y - \frac{4\pi e n_e(x)}{c}\nu v_y$$

$$-\frac{4\pi e n_e(x)}{c^2}\Omega_e v_x = \frac{1}{c^2}\frac{\partial}{\partial t}\left[v_y\frac{\partial E_x}{\partial x}\right]\ , \tag{20}$$

where the linear terms appear on the left hand sides of Equation (19) and (20), whereas the nonlinear terms appear on the right hand sides of these equations.

Noting that:

$$\begin{aligned} E_x &= E_{ox}e^{i\theta_o} + E_{1x}e^{i\theta_1} + E_{2x}e^{i\theta_2} + \text{c.c.} \\ E_y &= E_{oy}e^{i\theta_o} + E_{1y}e^{i\theta_1} + \text{c.c.} \\ V_x &= v_{ox}e^{i\theta_o} + v_{1x}e^{i\theta_1} + v_{2x}e^{i\theta_2} + \text{c.c.} \\ v_y &= v_{oy}e^{i\theta_o} + v_{1y}e^{i\theta_1} + \text{c.c.}\ , \end{aligned} \tag{21}$$

where

$$\begin{aligned} \theta_o &= k_{ox} - \omega_o t \\ \theta_1 &= -\left(k_{1x} + \omega_1 t\right) \\ \theta_2 &= k_2 x - \omega_2 t\ , \end{aligned} \tag{22}$$

and using the slow coupling limit:

$$\frac{\partial^2}{\partial x^2}(E_{ox},\ E_{1x},\ E_{2x}) = 0$$

$$\frac{\partial^2}{\partial t^2}(E_{ox},\ E_{1x},\ E_{2x}) = 0\ , \tag{23}$$

we obtain:

$$\frac{\partial E_{2x}}{\partial t} + \frac{1}{2i\omega_2}\left[\omega_2^2 - \omega_p^2(x) - 3v_{th}^2 k_2^2\right] E_{2x} + \frac{3v_{th}^2 k_2}{\omega_2}\frac{\partial E_{2x}}{\partial x} + i\frac{2\pi e n_e(x)}{\omega_2}\nu v_{2x}$$

$$= -\frac{i}{2}\left(k_1 v_{ox} E_{1x}^* + i k_o E_{ox} v_{1x}^*\right) + \frac{2\pi e n_e(x)}{\omega_2} k_2\left(v_{ox} v_{1x}^* + v_{oy} v_{1y}^*\right)\ . \tag{24}$$

Using Equations (14) and (15) and the fact that $E_{oy} \gg E_{ox}$ and $E_{1y} \gg E_{1x}$, the product terms such as $v_{ox}E_{1x}^*, E_{ox}v_{1x}^*$ and $v_{ox}v_{1x}^*$ can be ignored and Equation (24) takes the following form:

$$\frac{\partial E_{2x}}{\partial t} + \frac{1}{2i\omega_2}\left[\omega_2^2 - \omega_p^2(x) - 3v_{th}^2 k_2^2\right] E_{2x} + \frac{3v_{th}^2}{\omega_2}\frac{\partial E_{2x}}{\partial x} + \frac{2\pi e^2 n_e(x) v}{m\left(\omega_2^2 - \Omega^2\right)} E_{2x} = i\frac{\omega_p^2 \omega_1 k_2 v_{oy}}{2\omega_2\left(\omega_1^2 - \Omega^2\right)} E_{1y}^*\ . \tag{25}$$

For E_{1y} we have:

$$\frac{\partial E_{1y}}{\partial t} + \frac{1}{2i\omega_1}\left[\omega_1^2 - \omega_p^2(x) - k_1^2 c^2\right] E_{1y} - \frac{k_1 c^2}{\omega_1}\frac{\partial E_{1y}}{\partial x} + i\frac{2\pi e n_e(x)\nu v_{1y}}{\omega_1} + i\frac{2\pi e n_e(x)\Omega}{\omega_1} v_{1x} = \frac{ik_2}{2} v_{oy} E_{2x}^*\ . \tag{26}$$

Using Equations (12) and (14), Equation (26) takes the form:

$$\frac{\partial E_{1y}}{\partial t} - \frac{k_1 c^2}{\omega_1}\frac{\partial E_{1y}}{\partial x} + \frac{1}{2i\omega_1}\left[\omega_1^2 - \omega_p^2(x) - k_1^2 c^2\right] E_{1y} + \frac{2\pi e^2 n_e(x)\nu}{m\left(\omega_1^2 - \Omega^2\right)} E_{1y} - i\frac{2\pi e^2 n_e(x)}{m\omega_1\left(\omega_1^2 - \Omega^2\right)} E_{1y} = \frac{ik_2}{2} v_{oy} E_{2x}^*\ . \tag{27}$$

Using the dispersion relation for the electron plasma wave:

$$\omega_2^2 - \omega_p^2(0)\left(1 + \frac{x}{L}\right) - 3v_{th}^2 k_2^2 = \Omega^2 - \omega_p^2(0)\frac{x}{L}\ ,$$

and the relations:

$$V_1 = \frac{k_1 c^2}{\omega_1} and\ V_2 = \frac{3v_{th}^2 k_2}{\omega_2}\ ,$$

Equations (25) and (27) become:

$$\frac{\partial E_{2x}}{\partial t} - \frac{i}{2\omega_2}\left(\Omega^2 - \omega_p^2(0)\frac{x}{L}\right) E_{2x} + V_2\frac{\partial E_{2x}}{\partial X} + \omega_p^2(x)\frac{\nu}{2}\frac{1}{\omega_2^2 - \Omega^2} E_{2x} = ia\ E_{1y}^*\ , \tag{28}$$

and

$$\frac{\partial E_{1y}}{\partial t} - V_1\frac{\partial E_{1y}}{\partial x} - \frac{i}{2\omega_1}\left[\omega_1^2 - \omega_p^2(x) - k_1^2c^2 - \frac{\omega_p^2(x)\Omega^2}{\omega_1^2-\Omega^2}\right] E_{1y} + \frac{\nu}{2}\frac{\omega_p^2(x)}{\omega_1^2-\Omega^2}E_{1y} = ib\ E_{2x}^* \ , \tag{29}$$

where:

$$a = \frac{\omega_p^2\omega_1 k_2 v_{oy}}{2\omega_2\left(\omega_1^2-\Omega^2\right)} \ \ and \ \ b = \frac{k_2 v_{oy}}{2} \ .$$

3. SOLUTION OF THE COUPLED EQUATIONS

We now proceed to find a simultaneous solution for the system of the nonlinear coupled Equations (28) and (29).

Since $V_1 \gg V_2$, Equation (28) reduces to the form

$$\frac{\partial E_{2x}}{\partial t} - \frac{i}{2\omega_2}\left(\Omega^2 - \omega_p(0)\frac{x}{L}\right) E_{2x} + \frac{\omega_p^2(x)\nu}{2\left(\omega_2^2-\Omega^2\right)}E_{2x} = ia\ E_{1y}^* \ . \tag{30}$$

The solution of this equation is

$$E_{2x}(x,t) = ia\int_{-oo}^{t} dt' E_1^*(x,t') exp - \left[\frac{\nu\omega_p^2(x)}{2\left(\omega_2^2-\Omega^2\right)} - \frac{i}{2\omega_2}\left(\Omega^2 - \omega_p^2(0)\frac{x}{L}\right)\right](t-t'). \tag{31}$$

Substituting from this equation into Equation (29), we get

$$\frac{\partial E_{1y}}{\partial t} - V_1\frac{\partial E_{1y}}{\partial x} - \frac{i}{2\omega_1}\left[\omega_1^2 - \omega_p^2(x) - k_1^2c^2 - \omega_p^2(x)\frac{\Omega^2}{\omega_1^2-\Omega^2}\right] E_{1y} + \frac{\nu\omega_p^2(x)}{2\left(\omega_1^2-\Omega^2\right)}E_{1y}$$

$$= ab\int_{-\infty}^{t} dt' E_{1y}^*(x,t') \exp - \left[\frac{\nu\omega_p^2}{2\left(\omega_2^2-\Omega^2\right)} + \frac{i}{2\omega_2}\left(\Omega^2 - \omega_p^2(0)\frac{x}{L}\right)\right](t-t'). \tag{32}$$

Taking the complex conjugate of this equation we obtain:

$$\frac{\partial E_{1y}^*}{\partial t} - V_1\frac{\partial E_{1y}^*}{\partial x} + \frac{i}{2\omega_1}\left[\omega_1^2 - \omega_p^2(x) - k_1c^2 - \omega_p^2(x)\frac{\Omega^2}{\omega_1^2-\Omega^2}\right] E_{1y}^* + \frac{\nu}{2}\frac{\omega_p^2(x)}{\omega_1^2-\Omega^2}E_{1y}^*$$

$$= +\frac{\nu}{2}\frac{\omega_p^2(x)}{\omega_1^2-\Omega^2}E_{1y}^* = ab\int_{-\infty}^{t} dt' E_{1y}^*(x,t') \exp - \left[\frac{\nu\omega_p^2}{2\left(\omega_2^2-\Omega^2\right)} - \frac{i}{2\omega_2}\left(\Omega^2 - \omega_p^2(0)\frac{x}{L}\right)\right](t-t'). \tag{33}$$

Multiplying Equation (32) by $E_{1y}^{*}\ (x,t)$ and Equation (33) by $E_{1y}\ (x,t)$ and adding, we get:

$$\frac{\partial |E_{1y}|^2}{\partial t} - V_1 \frac{\partial |E_{1y}|^2}{\partial x} + \frac{\nu\omega_p^2(x)}{\omega_1^2 - \Omega^2}|E_{1y}|^2$$

$$= \ ab \int_{-\infty}^{t} dt' E_{1y}(x,t) E_{1y}^{*}(x,t) \exp - \left[\frac{\nu\omega_p^2(x)}{2\left(\omega_2^2 - \Omega^2\right)} + \frac{i}{2\omega_2}\left(\Omega^2 - \omega_p^2(0)\frac{x}{L}\right)\right](t-t')$$

$$+ ab \int_{-\infty}^{t} dt' E_{1y}^{*}(x,t') E_{1y}(x,t) \exp - \left[\frac{\nu\omega_p^2(x)}{2\left(\omega_2^2 - \Omega^2\right)} - \frac{i}{2\omega_2}\left(\Omega^2 - \omega_p^2(0)\frac{x}{L}\right)\right](t-t'). \tag{34}$$

Assuming that the phases on the right hand side of this equation change with respect to time much faster than the change of amplitudes, then we get:

$$\frac{\partial |E_{1y}|^2}{\partial t} - \frac{V_1}{L}\frac{\partial |E_{1y}|^2}{\partial x} + \nu\frac{\omega_p^2(0)}{\omega_1^2 - \Omega^2}|E_{1y}|^2$$

$$= - \frac{a\ b\nu\omega_p^2(x)/\left(\omega_2^2 - \Omega^2\right)}{\left[\frac{\nu}{2}\frac{\omega_p^2(x)}{\omega_2^2 - \Omega^2}\right]^2 + \left[\frac{1}{2\omega_2}\left(\Omega^2 - \omega_p^2(0)\frac{x}{L}\right)\right]^2} \tag{35}$$

changing to the variable $u = 1 + x/L$, this equation becomes:

$$\frac{\partial |E_{1y}|^2}{\partial t} - \frac{V_1}{L}\frac{\partial |E_{1y}|^2}{\partial u} + \nu\frac{\omega_p^2(0)}{\omega_1^2 - \Omega^2}|E_{1y}|^2$$

$$= - \frac{a\ b\nu\omega_p^2(0)\ u/\left(\omega_2^2 - \Omega^2\right)}{\left[\frac{\nu\omega_p^2(0)}{2\left(\omega_2^2 - \Omega^2\right)}u\right]^2 + \left[\frac{1}{2\omega_2}\left[\Omega^2 - \omega_p^2(0)\ (u-1)\right]\right]^2}. \tag{36}$$

Now, let

$$\alpha_1 = \frac{a\ b\nu\omega_p^2(0)}{\omega_2^2 - \Omega^2}$$

$$\alpha_2 = \frac{\nu\omega_p^2(0)}{2\left(\omega_2^2 - \Omega^2\right)}$$

$$\alpha_3 = \frac{\Omega^2}{2\omega_2} \quad \textit{and} \quad \alpha_4 = \frac{\omega_p^2(0)}{2\omega_2}$$

and, substituting into Equation (36), we get:

$$\frac{\partial |E_{1y}|^2}{\partial t} - \frac{V_1}{L} + \frac{\partial |E_{1y}|^2}{\partial u} + \frac{\nu \omega_p^2(0) u}{\omega_1^2 - \Omega^2} |E_{1y}|^2$$

$$= \frac{\alpha_1 \; u}{\alpha_2^2 \; u^2 + [\alpha_3 - \alpha_4(u-1)]^2} \tag{37}$$

let

$$f_1 = \frac{\alpha_4^2 + \alpha_3 \alpha_4}{\alpha_2^2 + \alpha_4^2} \quad \text{and} \quad f_2^2 = \frac{\alpha_3 + \alpha_4 + 2\alpha_3\alpha_4}{\alpha_2^2 + \alpha_4^2} - f_1^2 \, .$$

then Equation (37) can be written as:

$$\frac{\partial |E_{1y}|^2}{\partial t} - \frac{V_1}{L}\frac{\partial |E_{1y}|^2}{\partial u} + \frac{\nu \omega_p^2(0) \; u}{\omega_1^2 - \Omega^2} |E_{1y}|^2$$

$$= - \frac{\alpha_1}{\alpha_2^2 + \alpha_4^2} \frac{u}{(u - f_1)^2 + f_2^2} |E_{1y}|^2 \, . \tag{38}$$

Since we are interested in the convective instability *i.e.* the spatial variation ($\partial/\partial t = 0$) then Equation (38) reduces to:

$$\frac{V_1}{L}\frac{d|E_{1y}|^2}{d\,u} - \frac{\nu \omega_p^2(0)}{\omega_1^2 - \Omega^2} E_{1y} = \frac{\alpha_1}{\alpha_2^2 + \alpha_4^2} \frac{u}{(u - f_1)^2 + f_2^2} |E_{1y}|^2 \, . \tag{39}$$

Assuming that the interaction region is symmetric and extends from $-u_I$ to $+u_I$, then integrating Equation (39), we obtain:

$$1n \left[\frac{|E_{1y}(v_+)|^2}{|E_{1y}(v_-)|^2} \right] = \frac{\alpha_1 L}{V_1 \, (\alpha_2^2 + \alpha_4^2)} \frac{f_1}{f_2} \left[\tan^{-1} \frac{v_+}{f_2} - \tan^{-1} \frac{v_-}{f_2} \right] , \tag{40}$$

where $v_\pm = \pm(u_I - f_1)$.

Extending the interaction region from $-\infty \; to + \infty$, we can write

$$1n \left[\frac{|E_{1y}(\infty)|^2}{|E_{1y}(-\infty)|^2} \right] = \frac{\alpha_1 L}{V_1 \, (\alpha_2^2 + \alpha_4^2)} \frac{f_1}{f_2} \pi \, . \tag{41}$$

Writing this equation in a compact form, we get:

$$I = I_0 e^{2\pi A} , \tag{42}$$

where A is the amplification factor, given by:

$$A = \frac{\alpha_1 L}{2\, V_1 \left(\alpha_2^2 + \alpha_4^2\right)} \frac{f_1}{f_2} . \tag{43}$$

Substituting for $\alpha_1, \alpha_2, \alpha_4, f_1$, and f_2 in this equation, we get:

$$A = \frac{2|\gamma_0|^2 \omega_2 \omega_1^2 L}{V_1 \omega_p^2(0) \left(\omega_1^2 - \Omega^2\right) \left[1 + \frac{\nu^2 \omega_2^2}{\left(\omega_2^2 - \Omega^2\right)^2}\right]} , \tag{44}$$

where $|\gamma_0| = \sqrt{\frac{k_2^2 |v_{oy}|^2 \omega_p^2(0)}{4\omega_1 \omega_2}}$ is the Raman homogeneous growth rate for the unmagnetized plasma.

4. RESULTS AND DISCUSSION

From Equation (44), we can see that there is an increase in scattering as the density scale length increases, and collision suppression of the instability; a result that agrees with recent experiments [7, 22] and with our previous work [10], which shows that collision provides another mechanism of energy absorption. However, in collisionless plasmas ($\nu = 0$), Equation (44) reduces to:

$$A = \frac{2|\gamma_0|^2 \omega_2 \omega_1^2 L}{V_1 \omega_p^2 \left(\omega_1^2 - \Omega^2\right)} , \tag{45}$$

which shows that there is an increase in scattering with the increase of the self-generated magnetic fields. This result can be explained on the basis that self-generated magnetic fields may rotate the heated electrons velocities into the reverse direction where they cause less damping.

In collisionless inhomogeneous plasmas with self-generated magnetic fields, the threshold condition that we obtain from Equation (45) is

$$|\gamma_0|^2 > \frac{v_1 \omega_p^2 \left(\omega_1^2 - \Omega^2\right)}{2\omega_2 \omega_1^2 L} , \tag{46}$$

which shows that the Raman backscattering instability appears at lower intensities if the existence of self-generated magnetic fields were taken into consideration. This result might help in bringing theory and experiment into unison, since it has been commonly observed that thresholds are typically an order of magnitude lower than those predicted by a long-accepted theory [23] which does not take these fields into account.

5. CONCLUSIONS

The effects of self-generated magnetic fields on the convective Raman instability have been investigated in the quarter critical density region where the condition for the occurrence of this instability is satisfied. We have used the momentum equation in addition to the full set of Maxwell's equations. We found that the Raman backscattering growth rate is increased, while the instability threshold is decreased when the self-generated magnetic fields were taken into consideration.

REFERENCES

[1] R. A. Cairns (ed.), *Proceedings of the Scottish University Summer School on Physics No. 24*, 1983.

[2] H. Hora and G. H. Miley (eds.), *Laser Interact. Relat. Plasma*, **6** (1984), p.204.

[3] T. W. Johnston, *Can. J. Phys.*, **64** (1986), p.1006.

[4] K. Estabrook and W. L. Kruer, *Phys. Fluids*, **26** (1983), p.1892.

[5] B. B. Afeyan and E. A. Williams, *Phys. Fluids*, **28** (1985), p.3397.

[6] K. A. Brueckner and S. Jorna, *Rev. of Modern Phys.* **46** (1974), p.325.

[7] H. Figueroa, C. Joshi, H. Azechi, N. A. Ebrahim, and K. G. Esabrook, *Phys. Fluids*, **27** (1984), p.1887 and references therein.

[8] K. Estabrook and W. L. Kruer, *Phys. Rev. Lett.*, **53** (1984), p.465 and references therein.

[9] D. W. Forslund, J. M. Kindel, and E. Lindman, *Phys. Fluids*, **18** (1975), p.1002.

[10] N. M. Laham, A. M. Khateeb, and N. Y. Ayoub, *AJSE*, **19:2A** (1994), p.175.

[11] N. M. Laham, A. M. Khateeb, N. Y. Ayoub, A. K. Abdallah, I. M. Odeh, and M. S. Dababneh, *Jpn. J. Appl. Phys.*, **34** (1995), to be published.

[12] P. N. Guzdar, C. S. Liu, and R. H. Lehmberg, *Phys. Fluids*, **B3(10)** (1991), p.2882.

[13] J. J. Thomson and J. I. Karush, *Phys. Fluids*, **17** (1974), p.1608; J. J. Thomson, *Nuclear Fusion*, **15** (1975), p.237.

[14] J. A. Stamper, K. Papadopoulos, R. N. Sudan, S. O. Dean, E. A. Mclean, and J. M. Dawson, *Phys. Rev. Lett.*, **26** (1971), p.1012.

[15] A. Raven, O. Willi, and P. T. Rumsby, *Phys. Lett.*, **71A** (1979), p.435 and references therein.

[16] D. A. Tidman, *Phys. Rev. Lett.*, **35** (1975), p.1228.

[17] D. A. Tidman and R. A. Shanny, *Phys. Fluids*, **17** (1974), p.1207.

[18] R. S. Craxton and M. G. Haines, *Phys. Rev. Lett.*, **35** (1975), p.1336.

[19] T. J. M. Boyd, D. Cooke, and G. J. Humphreys-Jones, *Phys. Lett.*, **A88** (1982), p.140; D. G. Colombant and N. K. Winsor, *Phys. Rev. Lett.*, **28** (1977), p.697.

[20] C. Grebogi and C. S. Liu, *Phys. Fluids*, **23** (1980), p.1330.

[21] H. C. Barr, T. J. M. Boyd, L. R. T. Gardner, and R. Rankin, *Phys. Fluids*, **27** (1984), p.2730.

[22] C. L. Shephard, J. A. Tarven, R. L. Berger, Gar. E. Busch, R. R. Johnson, and R. J. Schroeder, *KMSFU*, (1985), p.1562; C. J. Walsh, D. M. Villeuve, and H. A. Baldis, *Phys. Rev. Lett.*, **53** (1984), p.1445, and references therein.

[23] N. M. Laham, H. C. Barr, T. J. M. Boyd, and A. P. Mackwood, *Dirasat*, **16** (1989), p.27.

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