ON COMPUTER-AIDED PATH SYNTHESIS OF MECHANISMS

Mehmet Akyurt

Mechanical Engineering Department
King Abdulaziz University
Jeddah 21413, Saudi Arabia

الخلاصـــة :

يبسط هذا البحث إجراءات استخدام التصميم التخطيطي بمساعدة الحاسب في تراكيب الوصلات الميكانيكية المولدة لمسار معين سواء أكانَ هذا المسار عاماً أو خاصاً مثل المسار الدائري أو مسار الخط المستقيم . وكذلك عالج البحث تراكيب الوصلات الميكانيكية ذات الحركة المتقطعة وأُعطيت أمثلة لشرح هذه الحالات .

لقد استخدمت فكرة الوحدة (ثنائية المناولة) في مصفوفة تخطيطية ذات بُعدين وآستعمل برنامج اليسير لدراسة أداء هذه التركيب واتضح في كل حالة أنَّ المسار المولَّد بهذه التراكيب يُماثل المسار الموصوف أصـــلاً .

ABSTRACT

Computer-aided graphical design procedures are introduced for the synthesis of mechanisms that generate specific paths. The generation of general trajectories is treated as well as the special cases of circular-path and straight-line path generation. The treatment is further extended to address the synthesis of dwell mechanisms. Several examples are provided to illustrate the procedure of synthesis in each case.

The two-handled block concept is employed, in connection with a two-dimensional graphical matrix. The Al-Yaseer software package is utilized to study the behavior of the resulting mechanisms. It is shown in each case that the path generated by the resulting mechanism closely matches that of the initial specifications.

ON COMPUTER-AIDED PATH SYNTHESIS OF MECHANISMS

INTRODUCTION

One of the areas of application where mechanisms have a wide variety of employment is that of packaging, where certain points on linkages must execute prescribed trajectories or paths [1–4]. Other instances of extensive use of trajectories arise in textile machinery, printing equipment, and processing machinery.

Aside from trajectories of a general nature, two specific types of paths, namely circular and straight line, deserve particular emphasis. The circular path finds applications in machinery with dwells. Mechanisms that generate straight lines arise where a point needs to follow a straight path during a certain period of the cycle.

Here we present a pragmatic and powerful technique [5] that can be utilized for the synthesis of mechanisms to generate all three types of trajectories. A computer graphics package that utilizes the *blocking* concept is employed. The software must necessarily support *two handles* on the block. A two-dimensional graphical matrix [5, 6] is made use of, with the first handle located at the center of the small circle located between the letters O and R (Figure 1), and the second handle on the circle located between E and G. The individual elements in the matrix may be color-coded for easy identification.

The suitability of the resulting linkages is verified in each case with the aid of the Al-Yaseer [7–10] software package. Al-Yaseer is suitable for the kinematic and force analyses of plane mechanisms.

SYNTHESIS OF GENERAL TRAJECTORIES

Figure 2*a* shows the part of a chain-type sludge remover that is used in animal shelters. A pusher finger (not shown) is moved intermittently along the dashed path, engaging the rollers of the chain, and hence pushing the chain and the sludge in the channel, to the left. The part of the path of the finger that is considered essential for the operation is portrayed by the four points shown in Figure 2*b*. Let the design task be to synthesize a mechanism that will guide the pusher finger such that the finger traces a path that passes through the four gauging points. It must be emphasized at this point that selection of *four* points on the path of interest is a rational choice; the method that is being presented, however, does not coerce an upper limit to the number of points.

We propose to select the four-bar linkage as the basic driving mechanism for the present task of

A4	C4	E4	G4	I4	K4	M4	O3	R3	U4	Y4
A3	C3	E3	G3	I3	K3	M3	O2	R2	U3	Y3
A2	C2	E2	G2	I2	K2	M2	O1	R1	U2	Y2
A	C	E	G	I	K	M	O	R	U	Y
a1	c1	e1	g1	i1	k1	m1	o1	r1	u1	y1
a2	c2	e2	g2	i2	k2	m2	o2	r2	u2	y2
a3	c3	e3	g3	i3	k3	m3	o3	r3	u3	y3

Figure 1. Two-Dimensional Graphical Matrix.

synthesis. The reader will further recognize that it would be necessary to locate the drive train above the sludge level. To this end lay off the four gauging points to a suitable scale (Figure 3*a*). Locate the center B_0 of a circle to the upper left side of the four points. Let the diameter of the circle be somewhat larger than the distance between point 1 and point 4. Point B_0 is one of the fixed pivots of the four-bar linkage. Select a common length l to span the distance between each gauging point and a corresponding primed (′) point on the circumference of the circle. Then draw the four lines of length l between the point sets $1-1'$, $2-2'$, $3-3'$, and $4-4'$, utilizing the *intersect* utility for accuracy.

Block the graphical matrix of Figure 1, and set the first handle at the dot located between O and R, and the second handle on the dot located between E and G. Next insert this matrix into Figure 3*a*, with the first handle at point 1, and the second handle at point 1′. Repeat the same for the point sets $2-2'$, $3-3'$, and $4-4'$. Figure 3*b* illustrates a part of the resulting pattern, where each character in the matrix of Figure 1 is to be sighted at four different locations.

Note in Figure 3*b* that it is possible to spot a number of matrix elements the successive locations of which lie in patterns that resemble circular arcs. Zoom to investigate closely. Pass a circular arc through any three locations of a chosen character, and see if the fourth location of the character lies close to the circumference of the circle. Thus the fourth position (and possibly others) of a given character serves as a check as to the goodness of fit to a circular arc. Any of the four handles of a given character may be utilized during the construction of the three-point arc for increased accuracy. Repeat this search several times until an adequately-fitting circle is found. Figure 3*b* displays several (dashed) circles obtained in this manner. Several circular arcs, including one that is constructed by utilizing three positions of the lower-left handle of character $A2$ are depicted in Figure 3*c*.

The center A_0 of the circular arc due to character $A2$ in Figure 3*c* becomes the second fixed pivot of the four-bar linkage in case this circular arc is selected. Joining points B_0 to point 4 (Figure 3*a*), and A_0 (Figure 3*c*) to the fourth (extreme left) location of $A2$ determines members B_0B and A_0A, respectively, at the position of the mechanism that corresponds to point 4. Lines drawn between A and B, and between A and point 4 resolve the remaining dimensions of the mechanism. The resulting linkage is depicted in Figure 4*a* and 4*b* at its point 4 configuration and a general configuration, respectively. The dimensions of the uncrossed crank-lever mechanism are $B_0A_0 = 9.039$ units, $\measuredangle xB_0A_0 = 116.81°$, $A_0A = 3.902$, $AB = 7.301$, $B_0B = 5.805$, $AC = 21.110$ units, and $\measuredangle BAC = 10.64°$. Also, Figure 4*a* depicts the four gauging points (solid circles) as well as the actual path generated by the mechanism. It is clear from the figure that the mechanism shown in Figure 4 fully satisfies the design objective.

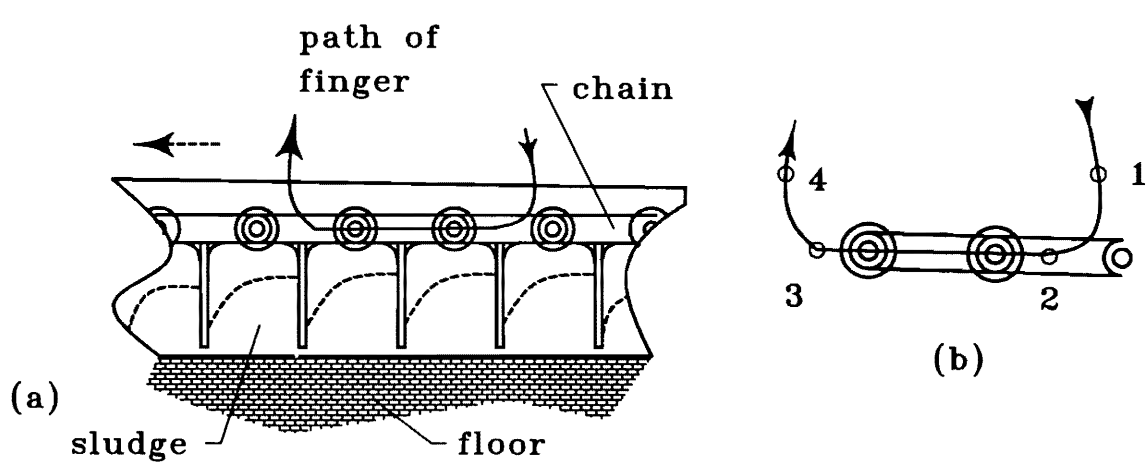

Figure 2. Path for a Sludge-Pushing Device.

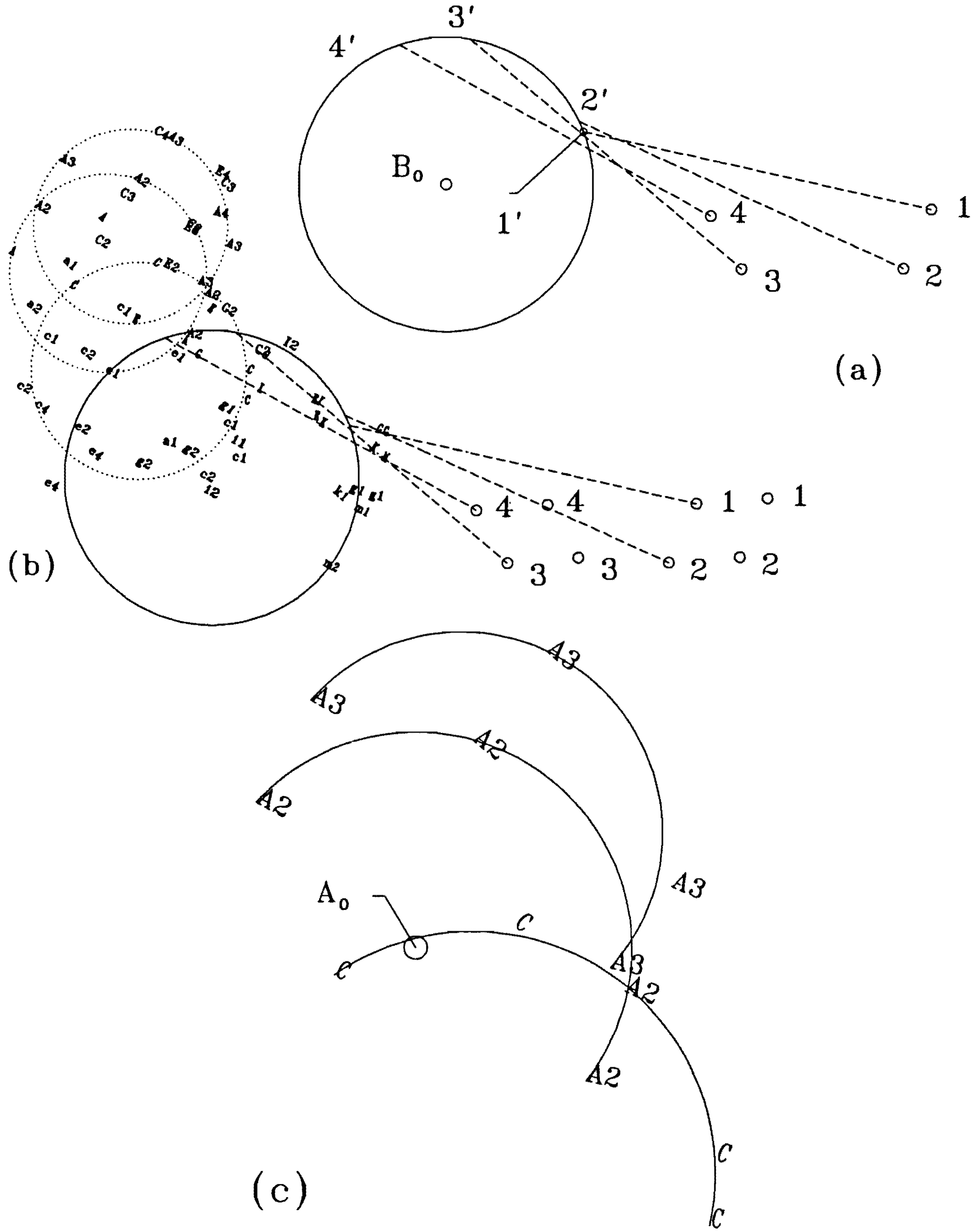

Figure 3. Synthesis of a Mechanism for Pushing Sludge.

STRAIGHT-LINE MECHANISMS

Let there be four significant points (Figure 5*a*) on a straight-line that needs to be generated by a four-bar linkage. The location of a fixed pivot A_0 (Figure 5*b*) is also specified. Start the process of synthesis by letting the diameter of a circle with center at A_0 be somewhat larger than the distance between points 1 and 4. Select a distance L that can span the distance from the given points to the circumference of the circle, and lay off the corresponding rays.

Use the two handles of the two-dimensional graphical matrix to insert the matrix at the four positions of the ray of length L (Figure 5*c*). Observe that there are a number of matrix elements, including $C2$, $C3$, $C4$, and $E2$, that lie on circular arcs. Figure 5*d* shows the mechanism that results when the circle that passes through $a1$ is picked, thus determining the location of fixed pivot B_0 and length AB. The pertinent dimensions of this un-crossed double-lever mechanism are $A_0B_0 = 5.356$, $B_0B = 6.312$, $AB = 6.195$, $A_0A = 10.043$, $AC = 17.711$ units, and $\measuredangle xA_0B_0 = 176.81°$ and $\measuredangle ABC = 11.85°$. Figure 5*d* exhibits the actual path generated by point C of the mechanism along with the specified points of significance. It is observed that the mechanism satisfies the design requirement.

It must be pointed out that each of the other candidate circles of Figure 5*c* will yield other mechanisms

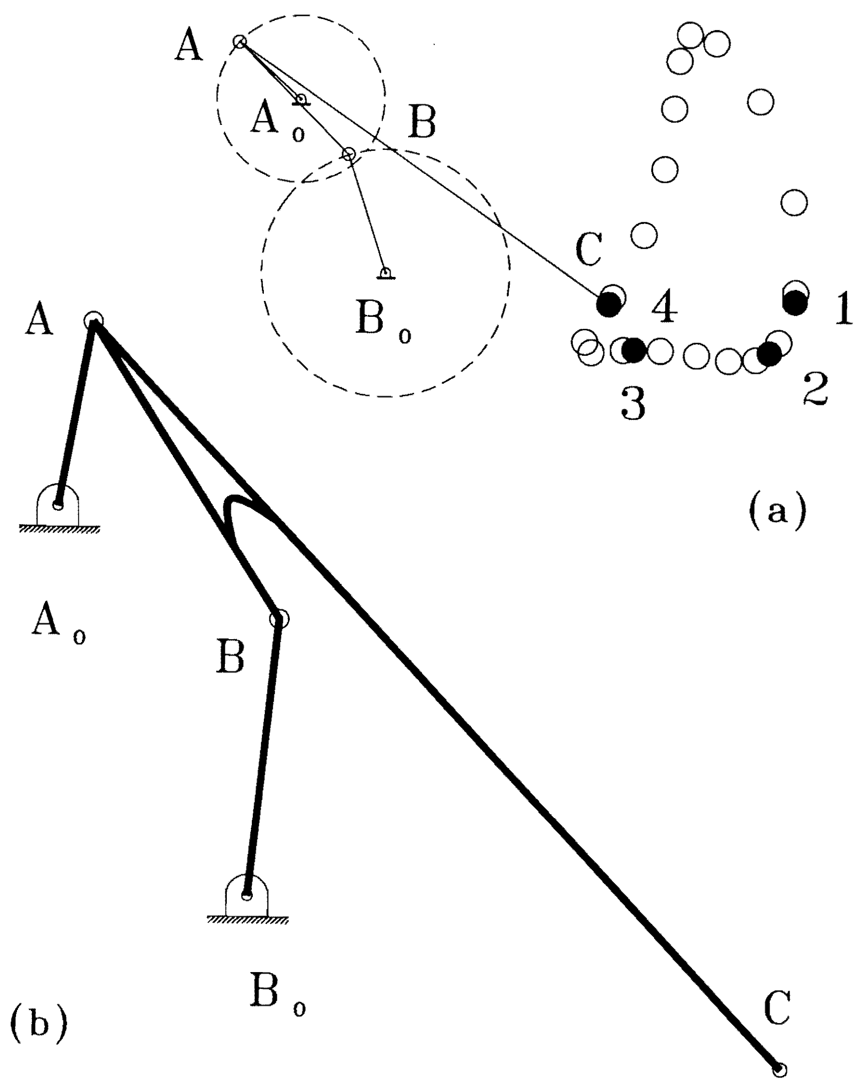

Figure 4. The Sludge Pusher.

with characteristics of their own. The designer has the option to go for an alternate design or accept the present one before making further analyses.

GENERATION OF CIRCULAR PATHS

Four-bar linkages may be utilized also to generate paths that feature circular arcs. To illustrate this, consider Figure 6*a*, where four significant points lying on a circular arc are shown. It is desired to synthesize a four-bar linkage to generate this circular path. One of the fixed pivots of the mechanism is constrained to be located at A_o.

Proceeding in the usual manner, adopt a suitable radius for a circle at A_o, and a length L (Figure 6*b*) to span the distance between each of the given points and the circumference of the circle. Insert the graphical matrix (Figure 6*c*), and inspect the resulting motif for elements that lie on a circular arc. Elements *i*4, *O*1, *O*2, and *U*2 are found to exhibit good fits. Figure 6*d* shows the un-crossed double-lever

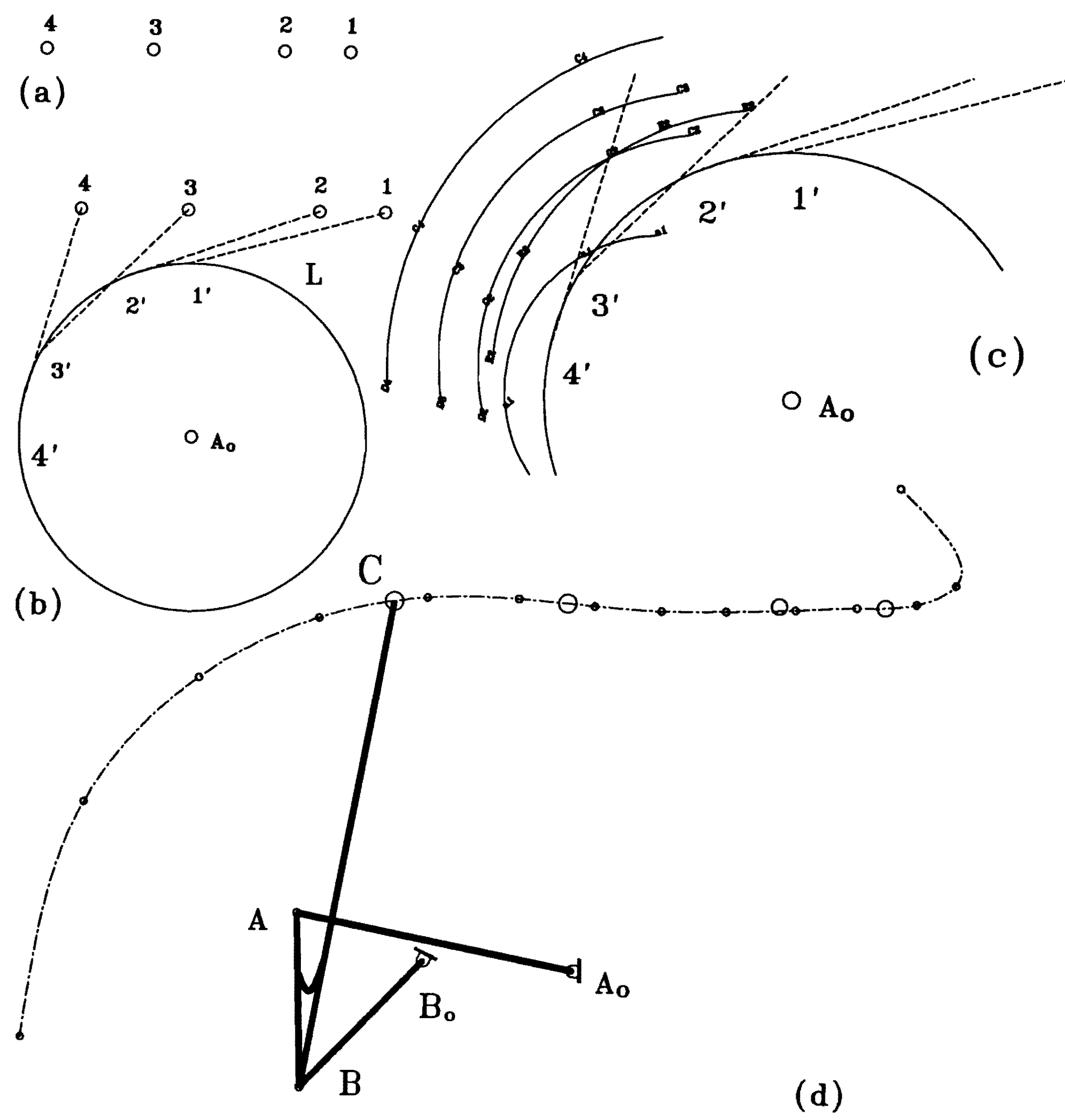

Figure 5. Synthesis of a Straight-Line Mechanism.

mechanism that results when the circle passing through $U2$ is utilized. The associated dimensions are read from the screen to be $B_0A_0 = 13.208$, $A_0A = 10.375$, $AB = 4.171$, $B_0B = 26.242$, $AC = 10$ units, and $\measuredangle xB_0A_0 = 121.94°$ and $\measuredangle BAC = 134.96°$. A number of points lying on the path generated by this mechanism are plotted in Figure 6*d* along with the specified significant points (solid circles). It is acknowledged that the path generated by the mechanism just synthesized is quite flawless.

PATH GENERATION BY THE USE OF SLIDER-CRANK MECHANISMS

The two-dimensional graphical matrix and the associated procedure for synthesis may be utilized effectively in connection with slider-crank mechanisms also for the generation of all three kinds of trajectories. We illustrate below the synthesis of a dwell mechanism that is based on a slider-crank mechanism.

Let it be required for a rocker DD_0 of a dwell mechanism to hesitate while crank A_0A of the driving offset slider-crank mechanism rotates by 160°, and that the total swing of D_0D will be restricted to 25°. Length of crank A_0A, the amount of offset, as well as the locations of four significant points on the crank circle are also specified (Figure 7*a*). Although the driving mechanism is to be a slider-crank arrangement, it is clear that the slider can be eliminated [6, 11] if service conditions call for it.

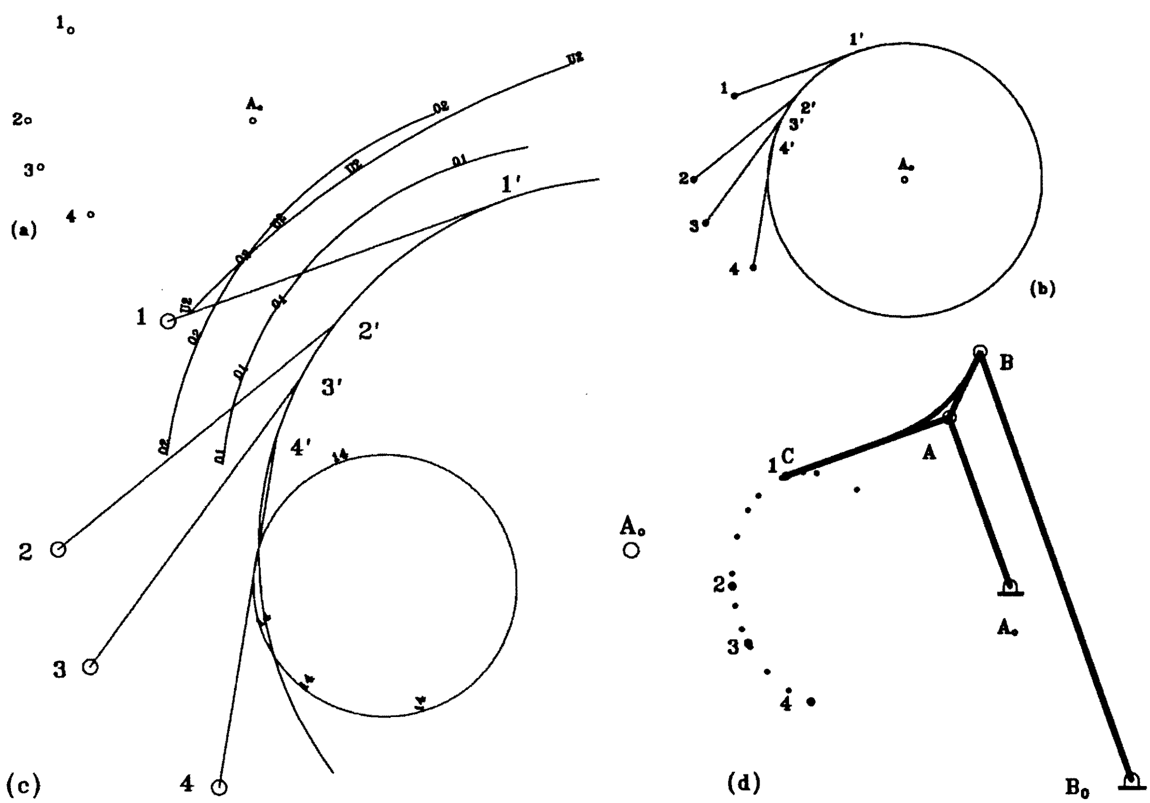

Figure 6. Synthesis of a Circular-Path Generator.

Start the synthesis by assuming a suitable length for the connecting rod AB. Draw the positions of the connecting rod (Figure 7*b*) that correspond to the given significant points. Insert the matrix on the four positions of AB (Figure 7*c*). Study the resulting motif for circular patterns. A number of such patterns are readily recognizable at the top of Figure 7*c*. Selection of element *I*3 results in the mechanism of Figure 7*d*, where the path described by point *C* is also plotted. It is recognized, however, that this

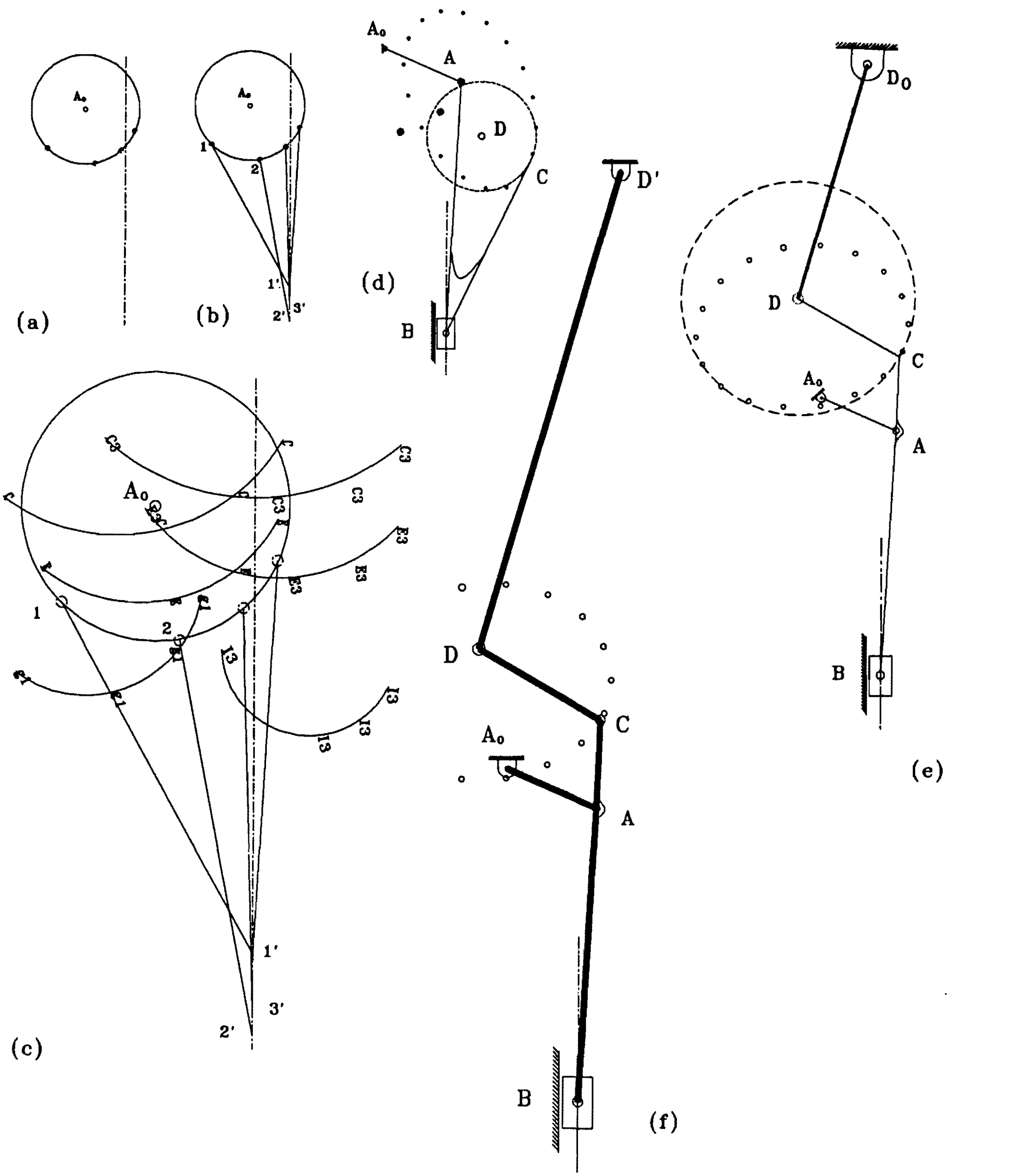

Figure 7. Synthesis of a Dwell Mechanism.

particular mechanism is not readily exploitable as a dwell mechanism since the path of C transgresses the circular path assigned for it.

A more suitable mechanism (Figure 7*e*) is obtained from the path of element C of the matrix (Figure 7*c*). Note here that point D is the center of the circular part of the path described by point C on the rigid extension AC of the connecting rod. It follows that point D will remain stationary as long as point C is on a circular path. A member D_oD would be expected to dwell during this part of the motion, while member CD would be in pure rotation. The associated dimensions for the mechanism of Figure 7*e* are eccentricity = 0.748, $A_oA = 1$, $AB = 3.006$, $BC = 3.913$, $CD = 1.432$, $D_oD = 3$, $D_oA_o = 4.132$ units, and $\measuredangle ABC = 0.22°$ and $\measuredangle xD_oA_o = 262.42°$.

Figure 8 displays the variation of the inclination of rocker D_oD with crank angle, where a dwell occurs for crank angles from 280° to 360°, as required. The swing of the rocker, however, is observed to be about 43°, and not the 25° specified for it. The amount of swing can be readily adjusted by utilizing the relationship

new rocker length $l = (3 * \sin(43°/2))/(\sin(25°/2))$

whence $l = 5.08$ units. Figure 7*f* depicts the corresponding modified mechanism, where $D'A_o = 6.197$ units and $\measuredangle xD'A_o = 259.53°$. Figure 8 illustrates the corrected dwell behavior.

CONCLUSIONS

Utilization of computer graphics permits the pragmatic synthesis of linkages for path generation. Reasonably accurate designs may be achieved in a short time. The two-handled blocking method introduced here possesses the potential to become a powerful and useful technique for the computer-aided graphical design of such mechanisms. Such powerful tools of modern design packages as zoom and intersect are effectively utilized to achieve accurate designs. One outstanding feature of the approach is that the designer would be able to synthesize the required mechanism without losing sight of the physical situation.

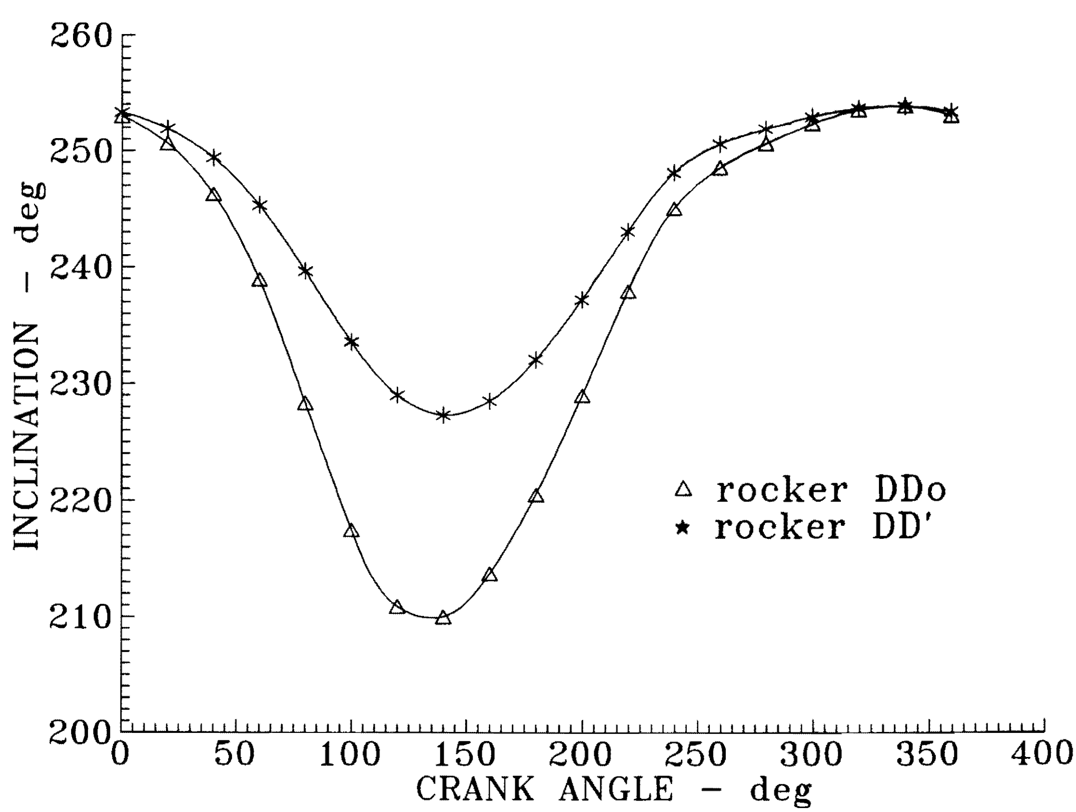

Figure 8. Variation of the Inclination of the Dwelling Link with Crank Angle.

REFERENCES

[1] R. Beyer, *Kinematische Getriebelehre.* Berlin: Springer–Verlag, 1953, p. 217.
[2] W. Lichtenheldt, *Konstrucktionslehre der Getriebe.* Berlin: Akademie Verlag, 1970, p. 254.
[3] S. Molian, *Mechanism Design.* Cambridge: Cambridge University Press, 1982, p. 120.
[4] J. S. Rao and R. V. Dukkipaji, *Mechanism and Machine Theory.* New Delhi: Wiley, 1988.
[5] M. Akyurt, *Computer-Aided Design of Mechanisms.* King Abdulaziz University, Jeddah: Mechanical Engineering Department, 1991, p. 203.
[6] M. Akyurt, "Replacement of Sliders Using Computer Graphics", *Computers in Industry,* **21** (1993), p. 87.
[7] A. R. Mannaa, M. A. Mansour, and M. Akyurt, "Mechanism Analysis by the Aid of Al-Yaseer", *Proc. 10th National Computer Conf., Feb. 28–March 3.* Jeddah: Scientific Publication Center, KAU, 1988, vol. 2, p. 857.
[8] A. R. Mannaa, F. Dehlawi, and M. Akyurt, "Kinematic Analysis *via* Virtual Links", *Computers in Industry,* **10** (1988), p. 261.
[9] M. Akyurt, *Analysis of Mechanisms and Machinery.* Jeddah: Scientific Publication Center, KAU, 1992, p. 584.
[10] M. Akyurt, "On the Use of Computers During the Undergraduate Teaching of Mechanisms and Machine Dynamics", *J. Mech. Design Prod. (METU-Ankara),* **2** (1991), p. 92.
[11] M. Akyurt, "Transportation of Linkage Pivots by Computer Graphics", *Trans. Mechanical Eng., I.E. Australia,* **ME18** (1993), accepted for publication.

Paper Received 12 April 1992; Revised 14 October 1992.