$c\alpha$-CONTINUITY IN BITOPOLOGICAL SPACES

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الخلاصـــة :

قدمنا في هذا البحث تجمعا جديدا من الرواسم في الفراغات ثنائية التوبولوجي يسمى بالرواسم المتصلة من النوع $c\alpha$ ، كما درسنا بعض خواص هذه الرواسم . كذلك تم استخدام هذا النوع من الرواسم لإعطاء بعض التحليلات للاتصال في الفراغات ثنائية التوبولوجي .

ABSTRACT

In this paper we introduce a new class of mappings in bitopological spaces, called pairwise $c\alpha$-continuous mappings, and investigate some of its properties.

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1. INTRODUCTION

The study of bitopological spaces was initiated with the paper of Kelly [1] in 1963 and thereafter a large number of papers which has been generalize topological concepts to the bitopological setting. The present paper is devoted to introducing and investigating the concepts of $c\beta$-continuity, c-precontinuity, c-semicontinuity, and $c\alpha$-continuity in bitopological spaces and making use of them to give some decompositions of continuity in such spaces.

Throughout the paper, by a space (X, τ_1, τ_2) or simply by X we shall mean a bitopological space. For a subset A of X, i-Int(A) and i-Cl(A) will denote the interior and the closure of A with respect to τ_i, respectively. The complement of A in X will be denoted by A^c. Also, $i, j = 1, 2$ and $i \neq j$.

A subset A of a space (X, τ_1, τ_2) is called τ_i-semiopen with respect to τ_j (we denote that by ij-semiopen) [2] if there exists a τ_i-open set U such that $U \subset A \subset j$-Cl(U). A mapping f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called pairwise continuous (resp. pairwise open) [3] if the induced mappings f: $(X, \tau_i) \to (Y, \sigma_i)$, $i = 1, 2$ are continuous (resp. open). f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called ij-semicontinuous [2] if the inverse image of each i-open set in Y is ij-semiopen in X, and is called ij-weakly continuous [4] if for each $x \in X$ and each i-open set V, in Y, containing $f(x)$, there exists an i-open set U, in X, containing x such that $f(U) \subset j$-Cl(V). A space X is called pairwise connected [3] iff X cannot expressed as the union of two nonempty disjoint sets A and B such that $(A \cap i\text{-Cl}(B)) \cup (j\text{-Cl}(A) \cap B) = \varnothing$ iff X cannot expressed as the union of two disjoint nonempty sets one of which is i-open and the other is j-open. X is called pairwise T_2 [1] (resp. pairwise T_2' [5]) iff for each pair of distinct points x and y of X, there are an i-open set A and a j-open set B containing x and y, respectively, such that $A \cap B = \varnothing$ (resp. $j\text{-Cl}(A) \cap i\text{-Cl}(B) = \varnothing$).

2. $c\alpha$-OPEN SETS IN BITOPOLOGICAL SPACES

Definition 2.1. Let X be a space and $A \subset X$. A is said to be ij-$c\beta$-open (resp. ij-c-preopen, ij-c-semiopen, ij-$c\alpha$-open) if $A \subset [j\text{-Cl}(i\text{-Int}(j\text{-Cl}(A)))]^c \cup [i\text{-Int}(A)]$ (resp. $A \subset [i\text{-Int}(j\text{-Cl}(A))]^c \cup [i\text{-Int}(A)]$, $A \subset [j\text{-Cl}(i\text{-Int}(A))]^c \cup [i\text{-Int}(A)]$, $A \subset [i\text{-Int}(j\text{-Cl}(i\text{-Int}(A)))]^c \cup [i\text{-Int}(A)]$). The complement of an ij-$c\beta$-open (resp. ij-c-preopen, ij-c-semiopen, ij-$c\alpha$-open) set is called ij-$c\beta$-closed (resp. ij-c-preclosed, ij-c-semiclosed, ij-$c\alpha$-closed). The family of all ij-$c\beta$-open (resp. ij-c-preopen, ij-c-semiopen, ij-$c\alpha$-open) sets of X will be denoted by ij-$C\beta O(X)$ (resp. ij-$CPO(X)$, ij-$CSO(X)$, ij-$C\alpha O(X)$). A subset A of a space X is called pairwise Q iff it is ij-Q and ji-Q, where $Q = c\beta$-open, c-preopen, c-semiopen, $c\alpha$-open.

Remark 2.1. One may deduce the following diagram:

	$\nearrow$	ij-$c\beta$-open	$\Longrightarrow$	ij-c-semiopen
i-open		$\Downarrow$		$\Downarrow$
	$\searrow$	ij-c-preopen	$\Longrightarrow$	ij-$c\alpha$-open

The converse of these implications need not be true as is shown by the following counterexample.

Counterexample 2.1.
Let $X = \{a, b, c, d\}$, $\tau_1 = \{\varnothing, X, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $\tau_2 = \{\varnothing, X, \{b\}, \{d\}, \{b, d\}\}$. Then, the subset $\{a, d\}$ is 12-$c\beta$-open but not 1-open. The subset $\{c, d\}$ is 12-c-preopen but not 12-$c\beta$-open. The subset $\{b, d\}$ is 12-c-semiopen but not 12-$c\beta$-open. The subset $\{b, c, d\}$ is 12-$c\alpha$-open but not 12-c-preopen. The subset $\{a, c, d\}$ is 12-$c\alpha$-open but not 12-c-semiopen.

Theorem 2.1. Let X be a space and $A \subset X$, the following statements are equivalent.

(1) A is ij-$c\beta$-open (resp. ij-c-preopen, ij-c-semiopen, ij-$c\alpha$-open).

(2) $j\text{-Cl}(i\text{-Int}(j\text{-Cl}(A)))$ (resp. $i\text{-Int}(j\text{-Cl}(A))$, $j\text{-Cl}(i\text{-Int}(A))$, $i\text{-Int}(j\text{-Cl}(i\text{-Int}(A)))$) $\subset i\text{-Int}(A) \cup A^c$.

(3) $j\text{-Cl}(i\text{-Int}(j\text{-Cl}(A))) \cap A$ (resp. $i\text{-Int}(j\text{-Cl}(A)) \cap A$, $j\text{-Cl}(i\text{-Int}(A)) \cap A$, $i\text{-Int}(j\text{-Cl}(i\text{-Int}(A))) \cap A$) $= i\text{-Int}(A)$.

Proof. We shall prove this theorem for ij-$c\beta$-open sets only, since the other cases are similar.

(1) $\Longrightarrow$ (2): Let A be ij-$c\beta$-open and $x \in j\text{-Cl}(i\text{-Int}(j\text{-Cl}(A)))$. Suppose that $x \notin i\text{-Int}(A) \cup A^c$, then $x \notin i\text{-Int}(A)$ and $x \in A$. Since $A \subset i\text{-Int}(A) \cup (j\text{-Cl}(i\text{-Int}(j\text{-Cl}(A))))^c$ and $x \notin i\text{-Int}(A)$, then $x \notin A$, a contradiction. Then $x \in i\text{-Int}(A) \cup A^c$ and so $j\text{-Cl}(i\text{-Int}(j\text{-Cl}(A))) \subset i\text{-Int}(A) \cup A^c$.

(2) $\Longrightarrow$ (3): Let $x \in A \cap j\text{-Cl}(i\text{-Int}(j\text{-Cl}(A)))$, since $j\text{-Cl}(i\text{-Int}(j\text{-Cl}(A))) \subset i\text{-Int}(A) \cup A^c$, then $x \in i$-

Int(A). Thus $A \cap j\text{-Cl}(i\text{-Int}(j\text{-Cl}(A))) \subset i\text{-Int}(A)$ which implies that $A \cap j\text{-Cl}(i\text{-Int}(j\text{-Cl}(A))) = i\text{-Int}(A)$.

(3) $\Longrightarrow$ (1): If $x \notin i\text{-Int}(A) = A \cap j\text{-Cl}(i\text{-Int}(j\text{-Cl}(A)))$ and $x \in A$, then $x \notin j\text{-Cl}(i\text{-Int}(j\text{-Cl}(A)))$ which implies that $x \in [j\text{-Cl}(i\text{-Int}(j\text{-Cl}(A)))]^c$ and so $x \in i\text{-Int}(A) \cup [j\text{-Cl}(i\text{-Int}(j\text{-Cl}(A)))]^c$. Hence $A \subset i\text{-Int}(A) \cup [j\text{-Cl}(i\text{-Int}(j\text{-Cl}(A)))]^c$ and the result follows.

Theorem 2.2. Let X be a space and $F \subset X$, the following statements are equivalent:

(1) F is ij-$c\beta$-closed (resp. ij-c-preclosed, ij-c-semiclosed, ij-$c\alpha$-closed).

(2) $i\text{-Cl}(F) - j\text{-Int}(i\text{-Cl}(j\text{-Int}(F)))$ (resp. $i\text{-Cl}(F) - i\text{-Cl}(j\text{-Int}(F))$, $i\text{-Cl}(F) - j\text{-Int}(i\text{-Cl}(F))$, $i\text{-Cl}(F) - i\text{-Cl}(j\text{-Int}(i\text{-Cl}(F)))$) $\subset F$.

(3) $F \cup j\text{-Int}(i\text{-Cl}(j\text{-Int}(F)))$ (resp. $F \cup i\text{-Cl}(j\text{-Int}(F))$, $F \cup j\text{-Int}(i\text{-Cl}(F))$, $F \cup i\text{-Cl}(j\text{-Int}(i\text{-Cl}(F)))) = F$.

Theorem 2.3. In any space X every singleton is ij-c-semiopen.

Proof. Let $x \in X$, if $\{x\}$ is i-open, then it is ij-c-semiopen. If $\{x\}$ is not i-open, then $i\text{-Int}(\{x\}) = \emptyset$ and so $i\text{-Int}(\{x\}) \cup (j\text{-Cl}(i\text{-Int}(\{x\})))^c = X \supset \{x\}$. Hence $\{x\}$ is ij-c-semiopen.

Corollary 2.1. In any space X every singleton is ij-$c\alpha$-open.

Theorem 2.4. The finite intersection of ij-$c\beta$-open (resp. ij-c-preopen, ij-c-semiopen, ij-$c\alpha$-open) sets is ij-$c\beta$-open (resp. ij-c-preopen, ij-c-semiopen, ij-$c\alpha$-open).

Proof. Since $\bigcap_{k \in I} i\text{-Int}(A_k) = i\text{-Int}(\bigcap_{k \in I} A_k)$ and $j\text{-Cl}(\bigcap_{k \in I} A_k) \subset \bigcap_{k \in I} (j\text{-Cl}(A_k))$, where $\{A_k: k \in I \text{ (finite)}\} \subset 2^X$, thus if $\{U_k: k \in I \text{ (finite)}\} \subset ij\text{-}C\beta O(X)$, then $\bigcap_{k \in I} U_k \subset \bigcap_{k \in I} (i\text{-Int}(U_k) \cup (j\text{-Cl}(i\text{-Int}(j\text{-Cl}(U_k))))^c)$ $\subset \bigcap_{k \in I} i\text{-Int}(U_k) \cup (\bigcap_{k \in I} j\text{-Cl}(i\text{-Int}(j\text{-Cl}(U_k))))^c \subset$ $i\text{-Int}(\bigcap_{k \in I} U_k) \cup (j\text{-Cl}(i\text{-Int}(j\text{-Cl}(\bigcap_{k \in I} U_k))))^c$. Hence, $\bigcap_{k \in I} U_k$ is ij-$c\beta$-open.

Other cases can be proved similarly.

Corollary 2.2.

(1) $ij\text{-}C\beta O(X)$ (resp. $ij\text{-}CPO(X)$, $ij\text{-}CSO(X)$, $ij\text{-}C\alpha O(X)$) is a base for a topology on X denoted by $\tau_{ic\beta}$ (resp. τ_{icp}, τ_{ics}, $\tau_{ic\alpha}$).

(2) $\tau_{ic\alpha} = \tau_{ics} = D$, where D is the discrete topology on X.

(3) $\tau_i \subset \tau_{ic\beta} \subset \tau_{icp}$.

Remark 2.2. The union of two ij-$c\beta$-open (resp. ij-c-preopen, ij-c-semiopen, ij-$c\alpha$-open) sets need not be ij-$c\alpha$-open as illustrated by the following counterexample.

Counterexample 2.2. Let X, τ_1, and τ_2 as in Counterexample 2.1. The subsets $\{b, c\}$ and $\{d\}$ are 12-$c\beta$-open but $\{b, c\} \cup \{d\} = \{b, c, d\}$ is not 12-$c\beta$-open. The subsets $\{a, d\}$ and $\{c\}$ are 12-c-semiopen but $\{a, d\} \cup \{c\} = \{a, c, d\}$ is not 12-c-semiopen.

Lemma 2.1. If $f: X \to Y$ is a pairwise continuous pairwise open mapping, then for each $B \subset Y$, $i\text{-Cl}(f^{-1}(B)) \subset f^{-1}(i\text{-Cl}(B))$ and $i\text{-Int}(f^{-1}(B)) = f^{-1}(i\text{-Int}(B))$.

Theorem 2.5. If $f: X \to Y$ is a pairwise continuous pairwise open mapping, then the inverse image of each ij-$c\beta$-open (resp. ij-c-preopen, ij-c-semiopen, ij-$c\alpha$-open) set in Y is ij-$c\beta$-open (resp. ij-c-preopen, ij-c-semiopen, ij-$c\alpha$-open) in X.

Proof. Follows immediately from Lemma 2.1.

Theorem 2.6. Let X be a space and $B \subset Y \subset X$ such that $Y \in \tau_1 \cap \tau_2$. If B is an ij-$c\beta$-open (resp. ij-c-preopen, ij-c-semiopen, ij-$c\alpha$-open) set in X, then B is ij-$c\beta$-open (resp. ij-c-preopen, ij-c-semiopen, ij-$c\alpha$-open) in the subspace Y.

Proof. Since $Y \in \tau_1 \cap \tau_2$ then $i\text{-Int}(B) = (i\text{-Int}(B))_Y$ and $j\text{-Cl}(B) \supset (j\text{-Cl}(B))_Y$ and then the proof is straightforward.

Theorem 2.7. Let $X \times Y$ be the product space of two spaces X and Y. Let A and B be ij-$c\beta$-open (resp. ij-c-preopen, ij-c-semiopen, ij-$c\alpha$-open) sets in X and Y, respectively. Then $A \times B$ is ij-$c\beta$-open (resp. ij-c-preopen, ij-c-semiopen, ij-$c\alpha$-open) in $X \times Y$.

Proof. Since $i\text{-Int}(A \times B) = i\text{-Int}(A) \times i\text{-Int}(B)$ and $j\text{-Cl}(A \times B) = j\text{-Cl}(A) \times j\text{-Cl}(B)$, then the proof follows immediately.

Definition 2.2. A subset A of a space X is called ij-β-open (resp. ij-preopen, ij-α-open) iff $A \subset j\text{-Cl}(i\text{-Int}(j\text{-Cl}(A)))$ (resp. $A \subset i\text{-Int}(j\text{-Cl}(A))$, $A \subset i\text{-Int}(j\text{-Cl}(i\text{-Int}(A)))$).

Remark 2.3. One can verify the following diagram:

$$\begin{array}{ccccc} & \nearrow & ij\text{-}\alpha\text{-open} & \Rightarrow & ij\text{-semiopen} \\ i\text{-open} & & \Downarrow & & \Downarrow \\ & \searrow & ij\text{-preopen} & \Rightarrow & ij\text{-}\beta\text{-open.} \end{array}$$

Counterexamples can be given to show that the converses of these implications need not be true.

Theorem 2.8. If A is a subset of a space X, then A is i-open iff it is both ij-β-open and ij-$c\beta$-open (resp. ij-preopen and ij-c-preopen, ij-semiopen and ij-c-semiopen, ij-α-open and ij-$c\alpha$-open).

Proof. If A is i-open, then it is both ij-$c\beta$-open and ij-β-open.

Conversely, let A be ij-$c\beta$-open and ij-β-open. Then, $A \subset (j\text{-Cl}(i\text{-Int}(j\text{-Cl}(A))))^c \cup i\text{-Int}(A)$ and $A \subset j\text{-Cl}(i\text{-Int}(j\text{-Cl}(A)))$. Thus $A \subset i\text{-Int}(A)$ and hence A is i-open.

The proof of the other cases is similar.

Corollary 2.3.

(1) A is i-open iff it is both ij-α-open and ij-c-semiopen (resp. ij-c-preopen, ij-$c\beta$-open).

(2) A is i-open iff it is both ij-semiopen (resp. ij-preopen) and ij-$c\beta$-open.

3. cα-CONTINUITY IN BITOPOLOGICAL SPACES

Definition 3.1. A mapping $f: X \to Y$ is said to be ij-$c\beta$-continuous (resp. ij-c-precontinuous, ij-c-semicontinuous, ij-$c\alpha$-continuous) if the inverse image of each i-open set in Y is ij-$c\beta$-open (resp. ij-c-preopen, ij-c-semiopen, ij-$c\alpha$-open) in X. f is called pairwise Q iff it is ij-Q and ji-Q, where $Q = c\beta$-continuous, c-precontinuous, c-semicontinuous, or $c\alpha$-continuous.

Remark 3.1. One can deduce the following diagram:

$$\begin{array}{ccccc} & \nearrow & ij\text{-}c\beta\text{-continuity} & \Rightarrow & ij\text{-}c\text{-semi continuity} \\ i\text{-continuity} & & \Downarrow & & \Downarrow \\ & \searrow & ij\text{-}c\text{-pre continuity} & \Rightarrow & ij\text{-}c\alpha\text{-continuity.} \end{array}$$

Each implication of the above diagram may not be reversible as shown by the following counterexample:

Counterexample 3.1.

Let $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\varnothing, X, \{a\}, \{b, c\}, \{a, b, c\}\}$, $\tau_2 = \{\varnothing, X, \{b\}, \{d\}, \{b, d\}\}$, $\sigma_1 = \{\varnothing, Y, \{a\}, \{c\}, \{a, c\}\}$, and $\sigma_2 = \{\varnothing, Y, \{a, b\}, \{c, d\}\}$. Let $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be defined as in the following cases:

(a) f be the identity mapping, then f is 12-c-precontinuous but not 12-$c\beta$-continuous and 12-$c\alpha$-continuous but not 12-c-semicontinuous.

(b) $f(a) = a$, $f(b) = d$, $f(c) = b$, $f(d) = c$, then f is 12-$c\beta$-continuous but not 1-continuous.

(c) $f(a) = b$, $f(b) = d$, $f(c) = d$, $f(d) = c$, then f is 12-c-semicontinuous but not 12-$c\beta$-continuous and 12-c-precontinuous but not 1-continuous.

(d) $f(a) = c$, $f(b) = a$, $f(c) = d$, $f(d) = b$, then f is 12-$c\alpha$-continuous but not 12-c-precontinuous.

Definition 3.2. A mapping $f: X \to Y$ is called ij-β-continuous (resp. ij-precontinuous, ij-α-continuous) iff the inverse image of each i-open set in Y is ij-β-open (resp. ij-preopen, ij-α-open) in X. f is called pairwise Q iff it is ij-Q and ji-Q, where $Q = \beta$-continuous, precontinuous, or α-continuous.

Remark 3.2. For these types of continuity, a diagram similar to that given in Remark 2.3 can be deduced.

The proofs of the following four theorems are straightforward and thus omitted.

Theorem 3.1. A mapping $f: X \to Y$ is i-continuous iff it is both ij-β-continuous and ij-$c\beta$-continuous (resp. ij-precontinuous and ij-c-precontinuous, ij-semicontinuous and ij-c-semicontinuous, ij-α-continuous and ij-$c\alpha$-continuous).

Theorem 3.2. For a mapping $f: X \to Y$, the following statements are equivalent:

(1) f is ij-$c\beta$-continuous (resp. ij-c-precontinuous, ij-c-semicontinuous, ij-$c\alpha$-continuous).

(2) The inverse image of each i-closed set in Y is ij-$c\beta$-closed (resp. ij-c-preclosed, ij-c-semiclosed, ij-$c\alpha$-closed) in X.

(3) $i\text{-Cl}(f^{-1}(i\text{-Cl}(B))) - j\text{-Int}(i\text{-Cl}(j\text{-Int}(f^{-1}(i\text{-Cl}(B)))))$ (resp. $i\text{-Cl}(f^{-1}(i\text{-Cl}(B))) - j\text{-Int}(f^{-1}(i\text{-Cl}(B)))$, $i\text{-Cl}(f^{-1}(i\text{-Cl}(B))) - j\text{-Int}(i\text{-Cl}(f^{-1}(i\text{-Cl}(B))))$, $i\text{-Cl}(f^{-1}(i\text{-Cl}(B))) - i\text{-Cl}(j\text{-Int}(i\text{-Cl}(f^{-1}(i\text{-Cl}(B)))))) \subset f^{-1}(i\text{-Cl}(B))$.

Theorem 3.3. Let $f: X \to Y$ be an ij-$c\beta$-continuous (resp. ij-c-precontinuous, ij-c-semicontinuous, ij-$c\alpha$-

continuous) mapping. For any point $x \in X$ and any i-open set V in Y containing $f(x)$, there exists an ij-$c\beta$-open (resp. ij-c-preopen, ij-c-semiopen, ij-$c\alpha$-open) set U in X such that $x \in U$ and $f(U) \subset V$.

Theorem 3.4. If f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is an ij-$c\beta$-continuous (resp. ij-c-precontinuous, ij-c-semicontinuous, ij-$c\alpha$-continuous) mapping, then f: $(X, \tau_{ic\beta})$ (resp. (X, τ_{icp}), (X, τ_{ics}), $(X, \tau_{ic\alpha})) \to (Y, \sigma_i)$ is continuous.

Theorem 3.5. Let f: $X \to Y$ be an ij-$c\beta$-continuous (resp. ij-c-precontinous, ij-c-semicontinuous, ij-$c\alpha$-continuous) mapping. Then, $f \mid U$ is ij-$c\beta$-continuous (resp. ij-c-precontinuous, ij-c-semicontinuous, ij-$c\alpha$-continuous), where U is an ij-$c\beta$-open (resp. ij-c-preopen, ij-c-semiopen, ij-$c\alpha$-open) set in X.

Proof. Follows immediately from Theorem 2.4.

Theorem 3.6. Let f: $X \to Y$ be an ij-$c\beta$-continuous (resp. ij-c-semicontinuous) surjective mapping such that $(j\text{-Cl}(i\text{-Int}(j\text{-Cl}(f^{-1}(V)))))^c$ (resp. $(j\text{-Cl}(i\text{-Int}(f^{-1}(V))))^c) \subset f^{-1}(j\text{-Cl}(V))$, for each i-open set V in Y. If X is pairwise connected, then Y is pairwise connected.

Proof. Suppose that Y is not pairwise connected, then there exist an i-open set V_1 and a j-open set V_2 such that $V_1 \cup V_2 = Y$ and $V_1 \cap V_2 = \varnothing$. Since f is ij-$c\beta$-continuous then $f^{-1}(V_1) \subset i\text{-Int}(f^{-1}(V_1)) \cup (j\text{-Cl}(i\text{-Int}(j\text{-Cl}(f^{-1}(V_1)))))^c$. Since $(j\text{-Cl}(i\text{-Int}(j\text{-Cl}(f^{-1}(V_1)))))^c \subset f^{-1}(j\text{-Cl}(V_1)) = f^{-1}(V_1)$, then $f^{-1}(V_1) \subset i\text{-Int}(f^{-1}(V_1))$ and so $f^{-1}(V_1)$ is i-open in X. Similarly $f^{-1}(V_2)$ is j-open into X. Also, $f^{-1}(V_1) \cap f^{-1}(V_2) = f^{-1}(V_1 \cap V_2) = \varnothing$ and $f^{-1}(V_1) \cup f^{-1}(V_2) = f^{-1}(V_1 \cup V_2) = f^{-1}(Y) = X$. Therefore, X is not pairwise connected, a contradiction. Then, Y is pairwise connected.

The rest of the proof is similar.

Theorem 3.7. Let f: $X \to Y$ be an ij-$c\beta$-continuous (resp. ij-c-semicontinuous) injective mapping and $(j\text{-Cl}(i\text{-Int}(j\text{-Cl}(f^{-1}(V)))))^c$ (resp. $(j\text{-Cl}(i\text{-Int}(f^{-1}(V))))^c) \subset f^{-1}(j\text{-Cl}(V))$ for each i-open set V of Y. If Y is pairwise T_2', then X is T_2.

Theorem 3.8. Let f: $X \to Y$ be an ij-$c\beta$-continuous (resp. ij-c-semicontinuous) mapping such that $(j\text{-Cl}(i\text{-Int}(j\text{-Cl}(f^{-1}(V)))))^c$ (resp. $(j\text{-Cl}(i\text{-Int}(f^{-1}(V))))^c) \subset f^{-1}(j\text{-Cl}(V))$ for each i-open set V of Y. Then f is ij-weakly continuous.

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Paper Received 16 June 1990; Revised 3 June 1991.

ANNOUNCEMENTS