# A SEMI-INFINITE ELASTIC STRIP CONTAINING A TRANSVERSE CRACK

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الخلاصـــة :

يتناول هذا البحث دراسة مشكلة المستويات المرنة الساكنة لشريحة مرنة شبه نهائية بها شرخ عرضي وسطي . وقد تركت هذه الشريحة دون تأثير خارجي ، وجعل طرفها القريب ثابتاً بينما وقع طرفها البعيد تحت تأثير قوة شدّ منتظمة .

ولحلِّ هذه المشكلة تُـؤخذ شريحة نهائية بها شرخان عرضيان متناظران حول محتوى عرضي صلب في الوسط ، ثم إنه عند مايصل المحتوى الصلب إلى جانبي الشريحة النهائية فإن نصفه يصبح مساوياً للشريحة شبه النهائية .

وبذلك فقد اختصرت المسألة الى ثلاث معادلات تكاملية مفردة يتم حلها حسابيا . أما نتائج الاجهادات وعوامل شدتها وزاوية انشقاق الشرخ وسرعة اطلاق طاقة الجهد فقد تم عرضها على شكل منحنيات بيانية .

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**ABSTRACT**

This paper considers the elastostatic plane problem of a semi-infinite strip which contains a transverse central crack. Short end of the strip is fixed while the sides are free and the far end is subjected to uniform tension. Solution for the cracked semi-infinite strip is obtained by considering an infinite strip containing a transverse rigid inclusion at the middle and two symmetrically located transverse cracks. In the limiting case when the rigid inclusion approaches the sides of the infinite strip, one-half of it becomes equivalent to the semi-infinite strip. Formulation is reduced to a system of three singular integral equations. Numerical results for stresses, stress intensity factors, probable cleavage angle and strain energy release rate are given in graphical form.

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## 1. INTRODUCTION

The semi-infinite strip problem has attracted considerable attention due to its possible applications to many structural problems. Horvay used first Goodier's minimum strain energy methods [1] (reference [2]) and then biharmonic Papkovich–Fadle eigenfunctions [3, 4], which are trigonometric sidewise and exponential lengthwise, for the semi-infinite strip with prescribed end tractions [5]. Gaydon and Shepherd [6] and Johnson and Little [7] also used the Papkovich–Fadle eigenfunctions. Theocaris [8] used conformal mapping in conjunction with the minimum strain energy theorem for the problem of compression of a semi-infinite strip by a concentrated end load. Benthem [9] used the Laplace transform in the longitudinal direction and the Fourier series for the stress function at the end. He reduced the problem to an infinite system of algebraic equations for the prescribed end tractions. He treated also the fixed end problem but his procedure could not reveal the stress singularities. Appropriate singular terms had to be derived separately and added. Vorovich and Kopasenko [10] considered the semi-infinite strip problem in which the end is fixed and the strip is compressed laterally. They reduced the problem to a singular integral equation but they had to determine the singularity separately and introduce it into their equation.

The problem of the tensile semi-finite strip with fixed end was considered also by Gupta [11]. Gupta's solution seems to be the first elegant treatment of the problem. He reduced the problem to a singular integral equation and could extract the stress singularity directly. Bogy used the same method in [12] for a general treatment of the problem of a semi-infinite strip with traction and displacement end conditions subjected to tension and/or bending at infinity. The authors used Gupta's method in [13] for the problem of a finite strip with free sides and one end fixed.

The present paper is concerned with the problem of a semi-infinite strip with free sides. The short end of the strip is perfectly bonded to a rigid support while the far end is subjected to uniform tension. There is a transverse central crack in the strip. The solution of the problem is obtained from the solution for an infinite strip which contains a rigid inclusion at the middle and two symmetrical transverse cracks. When the rigid inclusion approaches the sides of the strip, one-half of the infinite strip becomes equivalent to the cracked semi-infinite strip. The infinite strip solution is obtained by the superposition of several infinite strip and infinite plane solutions.

## 2. FORMULATION OF THE PROBLEM

Consider the isotropic, linearly elastic, semi-infinite strip of width $2h$, shown in Figure 1. The

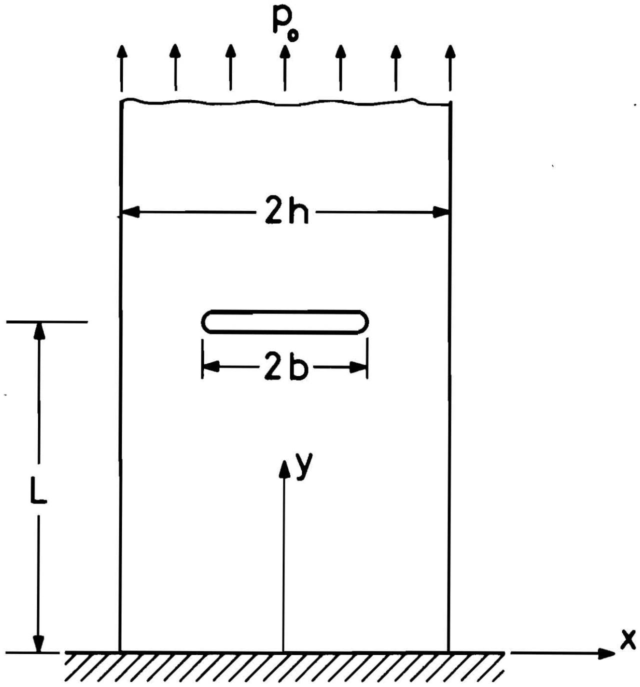


*Figure 1. Cracked Semi-Infinite Strip.*

short end of the strip is fixed along a rigid support while the far end is under the action of a uniformly distributed tensile load of intensity $p_o$ and the sides are free of traction. The strip contains a transverse crack of length $2b$ with stress-free surfaces at a distance of $L$ from the fixed end. Therefore, the field equations of plane elasticity theory must be solved under the following boundary conditions:

$$\sigma_x(\pm h, y) = 0\ , \quad \tau_{xy}(\pm h, y) = 0\ , \quad (0<y<\infty)\ , \qquad (1a-d)$$

$$u(x,0) = 0\ , \quad v(x,0) = 0\ , \quad (|x|<h)\ , \qquad (2a,\ b)$$

$$\sigma_y(x, L) = 0\ , \quad \tau_{xy}(x, L) = 0\ , \quad (|x|<b)\ , \qquad (3a,\ b)$$

$$\tau_{xy} \to 0\ , \quad (y \to \infty)\ , \qquad (4)$$

$$\sigma_y \to p_o\ , \quad (y \to \infty)\ , \qquad (5)$$

where $u$ and $v$ are the $x$- and $y$-components of the displacement vector.

A solution for the semi-infinite strip problem may be obtained conveniently by considering a symmetric infinite strip containing a rigid inclusion of length $2a$ at $y=0$ and two cracks of length $2b$ with stress-free surfaces at $y=\pm L$. In the limiting case when the rigid inclusion approaches the sides of the strip (*i.e.* when $a \to h$), one-half of the infinite strip turns out to be the semi-infinite strip shown in Figure 1. Boundary conditions for the infinite strip can be written as:

$$\sigma_x(\pm h, y)=0 \,, \quad \tau_{xy}(\pm h, y)=0 \,, \quad (|y|<\infty) \,, \qquad (6a\text{–}d)$$

$$u(x,0)=0 \,, \quad v(x,0)=0 \,, \quad (|x|<a) \,, \qquad (7a,\ b)$$

$$\sigma_y(x, \pm L)=0 \,, \quad \tau_{xy}(x, \pm L)=0 \,, \quad (|x|<b) \,, \qquad (8a\text{–}d)$$

$$\sigma_y \to p_o \,, \quad \tau_{xy} \to 0 \,, \quad (|y| \to \infty) \,. \qquad (9a,\ b)$$

Due to symmetry about both $x=0$ and $y=0$ planes, it is sufficient to consider one-quarter of the problem $(0 \leqslant x \leqslant h,\ 0 \leqslant y < \infty)$ only.

A general solution for the tensile infinite strip problem defined by the conditions (6–9) may be obtained by the superposition of solutions for the following four subproblems: (*i*) an infinite strip subjected to arbitrary symmetric loads, (*ii*) another infinite strip subjected to uniform tensile loads at infinity, (*iii*) an infinite plane containing two parallel cracks, and (*iv*) another infinite plane containing a central rigid inclusion. When the auxiliary solutions for these subproblems, which already satisfy the conditions (7*b*) and (9), are added and substituted in (6), the necessary stress and displacement derivative expressions can be written in the form (see [13]):

$$\frac{\partial u}{\partial x} = \sum_{i=1}^{3} \int_{-a_i}^{a_i} M_{1i}(x,y,t) G_i(t)\,dt - \frac{3-\kappa}{8\mu} p_o \,, \qquad (10a)$$

$$\tau_{xy} = \sum_{i=1}^{3} \int_{-a_i}^{a_i} M_{2i}(x,y,t) G_i(t)\,dt \,, \qquad (10b)$$

$$\sigma_y = \sum_{i=1}^{3} \int_{-a_i}^{a_i} M_{3i}(x,y,t) G_i(t)\,dt + p_o \,, \qquad (10c)$$

where $\mu$ is the shear modulus, $\kappa = 3-4\nu$ for plane strain, $\kappa = (3-\nu)/(1+\nu)$ for plane stress, $\nu$ being the Poisson's ratio, $G_1$ is the odd shear stress on the rigid inclusion,

$$G_1(x) = \tau_{xy}(x, 0^+) \,, \quad (|x|<a) \,, \qquad (11)$$

$G_2$ and $G_3$ are, respectively, the even and odd dislocation densities on the cracks,

$$G_2(x) = \frac{\partial}{\partial x} u(x, L^+) - \frac{\partial}{\partial x} u(x, L^-), \quad (|x|<b), \qquad (12)$$

$$G_3(x) = \frac{\partial}{\partial x} v(x, L^+) - \frac{\partial}{\partial x} v(x, L^-), \quad (|x|<b), \qquad (13)$$

$a_1 = a$, $a_2 = a_3 = b$, $M_{ij}(i,j = 1-3)$ are given in [13] and for brevity will not be repeated here.

In order to obtain the expressions corresponding to the semi-infinite strip problem, one must consider the limiting case when $a \to h$. Hence the expressions in (10) can be used for the semi-infinite strip if $a_1 = a$ is replaced by $h$.

### 3. THE INTEGRAL EQUATIONS

The three new unknown functions $G_1 - G_3$ can be determined by using the remaining boundary conditions (7*a*) and (8). Note that (7*a*) is a displacement type condition whereas (8) are stress type conditions. In order to have the same type of conditions (*e.g.* stress type), (7*a*) may be replaced by

$$\frac{\partial}{\partial x} u(x,0) = 0 \,, \quad (|x|<h) \,. \qquad (14)$$

By this replacement we disregard some divergent integrals and obtain integral equations with Cauchy-type singularity [14]. Now if (10) are substituted in (8) and (14), the following singular integral equations are obtained

$$\frac{1}{\pi\mu} \int_{-h}^{h} \left[ \frac{\kappa}{t-x} + N_{11}(x,t) \right] G_1(t)\,dt + \frac{1}{\pi} \sum_{i=2}^{3} \int_{-b}^{b} N_{1i}(x,t) G_i(t)\,dt = \frac{(\kappa-3)(\kappa+1)}{8\mu} p_o \,, \quad (|x|<h) \qquad (15a)$$

$$\frac{1}{\pi\mu} \int_{-h}^{h} N_{n1}(x,t) G_1(t)\,dt + \frac{1}{\pi} \sum_{i=2}^{3} \int_{-b}^{b} \left[ \frac{\delta_{ni}}{t-x} + N_{ni}(x,t) \right] G_i(t)\,dt = -\frac{1+\kappa}{2\mu} p_o\, \delta_{n3} \,, \quad (n=2,3),\ (|x|<b), \qquad (15b)$$

where $\delta_{ni}$ is the Kronecker delta and the kernels $N_{ij}(i,j=1-3)$ are given in [13]. It can be shown that, in addition to the simple Cauchy kernel $(t-x)^{-1}$, $N_{11}$ has singular terms when $t=h$ and $x=\pm h$, due to the behavior of the integrand of the integral giving $N_{11}$ as

$s \to \infty$*. These singular terms can be separated as

$$N_{11s}(x,t) = \left[\frac{3-\kappa^2}{2} - 6(h-x)\frac{d}{dx} + 2(h-x)^2\frac{d^2}{dx}\right]\left[\frac{1}{t-(2h-x)}\right] + \left[\frac{3-\kappa^2}{2} + 6(h+x)\frac{d}{dx} + 2(h+x)^2\frac{d^2}{dx}\right]\left[\frac{1}{t-(2h+x)}\right], \quad (16)$$

and after somewhat routine manipulations (see, for example [11–13, 17]) it can be shown that the unknown functions $G_1 - G_3$ may be written in the form

$$\left.\begin{aligned} G_1(x) &= \phi_1(x)(h^2-x^2)^{-\alpha}, \quad (|x|<h), \\ G_i(x) &= \phi_i(x)(b^2-x^2)^{-1/2}, \\ &\qquad (i=2,3),\ (|x|<b), \end{aligned}\right\} \quad (17a-c)$$

where $\phi_1 - \phi_3$ are Hölder-continuous functions in the respective intervals and $\alpha$ satisfies the characteristics equation

$$2\kappa\cos\pi\alpha + 4(\alpha-1)^2 - \kappa^2 - 1 = 0, \quad Re(\alpha) < 1. \quad (18)$$

This equation is in perfect agreement with previously reported results for the power of stress singularity at the 90° corner of a fixed-free wedge (*e.g.* [11–13, 18–20]). According to (17) and (18) $G_1 - G_3$ have integrable singularities at the end points. Therefore, the index of the integral equations (15) is +1 [14]. Consequently, their solution will contain three arbitrary constants which can be determined from the conditions

$$\int_{-h}^{h} G_1(x)dx = 0, \qquad \int_{-b}^{b} G_i(x)dx = 0, \qquad (i=2,3), \quad (19a-c)$$

which are required for zero resultant shear force at the rigid support and continuous displacements outside the crack.

* Note that as $s \to 0$, $N$ behaves as $s^{-1}$ where $s$ is the integration variable in the improper integral giving $N_{11}$. In this case, one must examine the integral $\int_{-h}^{h} N_{11}(x,t)\ G_1(t)dt$ with the technique employed in [15, 16]. If the integral giving $N_{11}$ is separated into two parts so that in the first part the asymptotic expansion of the integrand around $s=0$ can be used, it can be shown that because of the condition $\int_{-h}^{h} G_1(t)dt = 0$ ($G_1$ is an odd function), $N_{11}$ is regular around $s=0$ and the singular terms are due to the behavior of the integrand as $s \to \infty$ only.

Now using appropriate Gauss–Jacobi integration formulas [21], the integral Equations (15) and (19) can be replaced by a system of linear algebraic equations. From the viewpoint of fracture, particularly important are the stress intensity factors at the tips of the crack and the corners of the strip on the rigid support. The normal and the shear components of the stress intensity factors, $k_1$ and $k_2$, may be defined as

$$\left.\begin{aligned} k_{1b} &= \lim_{x\to b}\,[2(x-b)]^{1/2}\,\sigma_y(x,L), \\ k_{2b} &= \lim_{x\to b}\,[2(x-b)]^{1/2}\,\tau_{xy}(x,L), \\ k_{1h} &= \lim_{x\to h}\,2^{1/2}(h-x)^{\alpha}\,\sigma_y(x,0^+), \\ k_{2h} &= \lim_{x\to h}\,2^{1/2}(h-x)^{\alpha}\,\tau_{xy}(x,0^+), \end{aligned}\right\} \quad (20)$$

and obtained as

$$\left.\begin{aligned} k_{1b} &= -\frac{2\mu}{\kappa+1}\frac{\phi_3(b)}{\sqrt{b}}, \\ k_{2b} &= -\frac{2\mu}{\kappa+1}\frac{\phi_2(b)}{\sqrt{b}}, \\ k_{1h} &= \frac{2^{1/2-\alpha}\,\phi_1(h)}{h^{\alpha}(\kappa-1)\sin\pi\alpha} \\ &\qquad [(1-\kappa)\cos\pi\alpha - 3\kappa - 5 + (\kappa+3)\alpha - 4\alpha^2], \\ k_{2h} &= 2^{1/2-\alpha}\,h^{-\alpha}\,\phi_1(h). \end{aligned}\right\} \quad (21)$$

## 4. RESULTS

The cracked semi-infinite strip problem is completely defined by the dimensionless parameters $\kappa$, $p_o/\mu$, $L/h$, and $b/h$. Numerical results presented are obtained with a value of unity for the load factor $p_o/\mu$. However, since the stresses and the stress intensity factors are normalized with $p_o$, the results are valid for all values of $p_o/\mu$.

Some of the calculated results are shown in Figures 2–11. Figures 2–4 show the normalized stress intensity factors at the tips of the crack defined by

$$\bar{k}_{ib} = k_{ib}/p_o\sqrt{b}, \qquad (i=1,2). \quad (22)$$

From these figures one may conclude that, for example, both $\bar{k}_{1b}$ and $\bar{k}_{2b}$ increase with increasing $b/h$ ratio, $\bar{k}_{2b}$ decreases with increasing $\kappa$. With increasing $L/h$ ratio, $\bar{k}_{2b}$ decreases monotonically, whereas $\bar{k}_{1b}$ increases for relatively small $L/h$ and/or $b/h$ ratios and decreases for relatively large values of these ratios. As $L/h \to \infty$, the problem for the crack becomes that of an infinite strip with a central crack.

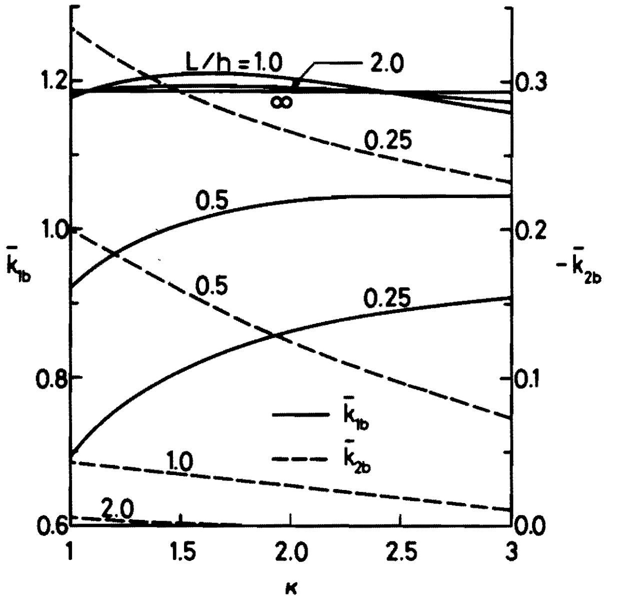


*Figure 2. Normalized Stress Intensity Factors at the Crack Tips When* b = *0.5* h.

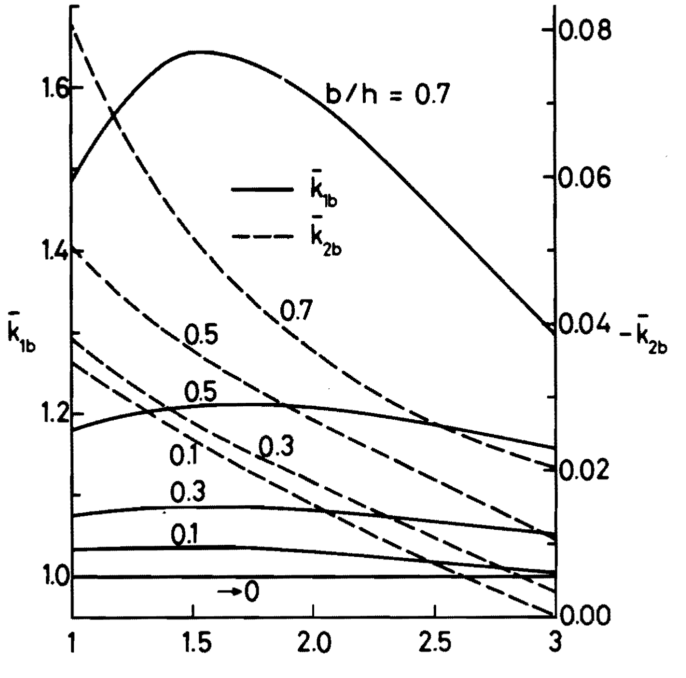


*Figure 3. Normalized Stress Intensity Factors at the Crack Tips When* L = h.

Results for this special case seem to be in very good agreement with those of references [22–24]. Note that the stress intensity factor $\bar{k}_{1b}$ is independent of κ and $\bar{k}_{2b}$ vanishes in this case. One may notice also that ν decreases as κ increases. When κ = 3 (*i.e.*, when ν = 0), there is no Poisson's effect and the constraint due to the rigid support simply disappears.

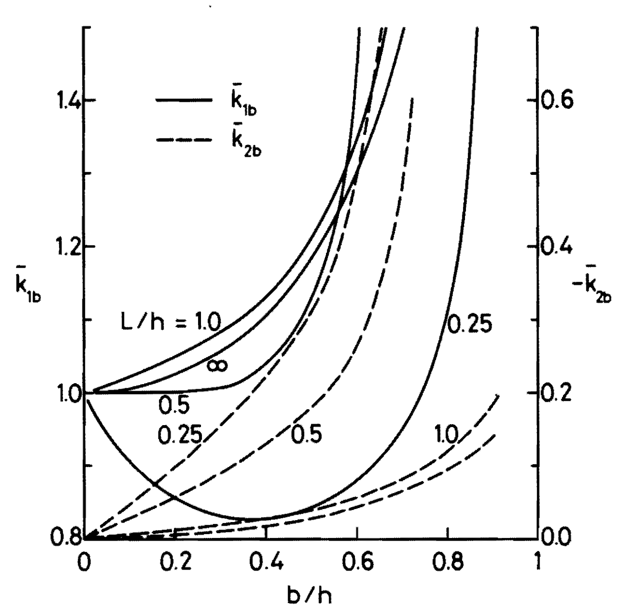


*Figure 4. Normalized Stress Intensity Factors at the Crack Tips When* κ = *1.6.*

Then the problem becomes that of an infinite strip with two parallel transverse central cracks which has been treated by Civelek and Erdogan [25]. Results of the present analysis seem to be indistinguishable from those given in reference [25].

If an energy balance type criterion is used for estimation of the crack propagation load, one has to calculate the strain energy release rate given by [26, 27]:

$$\frac{\partial U}{\partial b} = \frac{\pi(\kappa+1)}{4\mu}\,(k_{1b}^2 + k_{2b}^2)\;. \tag{23}$$

Figure 5 shows the dimensionless strain energy release rate defined in the form

$$\overline{w} = \frac{4\mu}{\pi(\kappa+1)\,b\,p_0^2}\,\frac{\partial U}{\partial b}\;. \tag{24}$$

The rate $\overline{w}$ increases with increasing $b/h$ except for relatively small values of $L/h$. When $L/h$ is small, the crack is close to the rigid support near which the normal stress $\sigma_y$ is very small around the central part. Therefore, when both $L/h$ and $b/h$ are small, the crack lies in a low-stress region so that the stress intensity factors and consequently the strain energy release rate are relatively small.

In the close vicinity of the crack tips, the cleavage stress can be expressed in terms of the stress intensity factors as [26]:

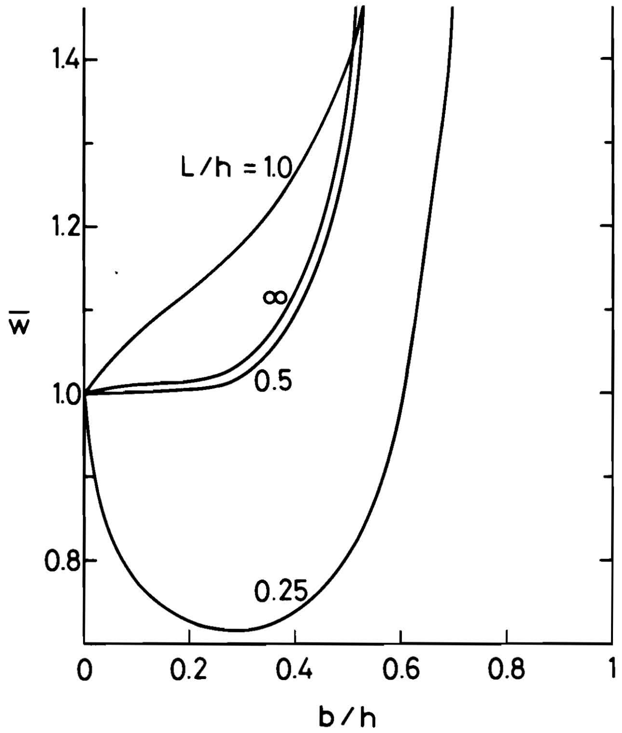


*Figure 5. Normalized Strain Energy Release Rate $\bar{w}$ When $\kappa = 1.6$.*

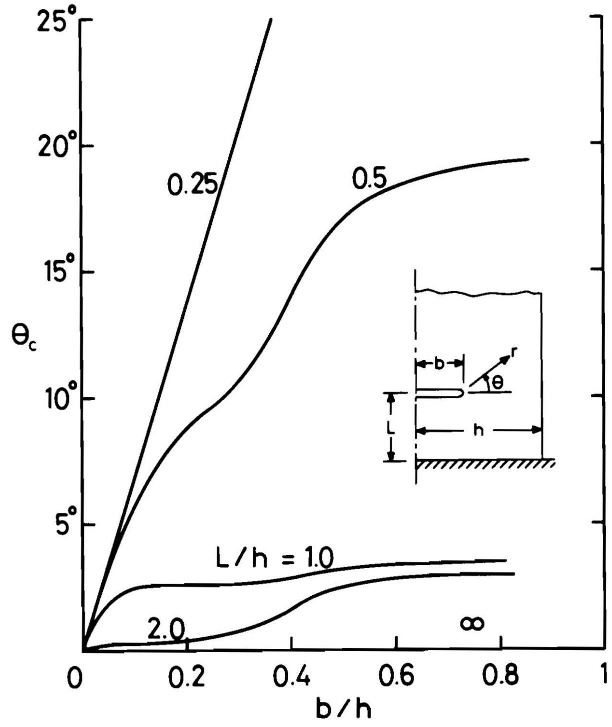


*Figure 6. Probable Crack Propagation Angle $\theta_c$ When $\kappa = 1.6$.*

$$\sigma_\theta = \frac{\cos(\theta/2)}{\sqrt{(2r)}}$$

$$\times [k_{1b}\cos^2(\theta/2) - (3/2)k_{2b}\sin\theta] + 0(\sqrt{r}) \;, \qquad (25)$$

where $(r, \theta)$ are the polar coordinates at the crack tips. For brittle solids the probable crack propagation angle, $\theta_c$, may be proposed to be the angle of the radial plane corresponding to the maximum cleavage stress and may be determined from

$$k_{2b}(1 - 3\cos\theta_c) - k_{1b}\sin\theta_c = 0 \;,$$

$$3k_{2b}\sin\theta_c - k_{1b}\cos\theta_c < 0 \;. \qquad (26)$$

Figure 6 shows the probable crack propagation angle $\theta_c$. It seems that the crack has a tendency to escape from the rigid support which is more pronounced for larger cracks and/or cracks closer to the support. Similar behaviors had been reported by Erdogan and Gupta [28] and Geçit [29]. When $L/h \to \infty$, the crack propagates in its own plane.

Figures 7 and 8 show the normalized axial and shear stresses along the rigid support. These stresses tend to $+\infty$ and $-\infty$, respectively, towards the corners if $\kappa < 3$. Stress distributions become smoother as $\kappa$ increases. When $\kappa = 3$ (*i.e.*, when $\nu = 0$), the

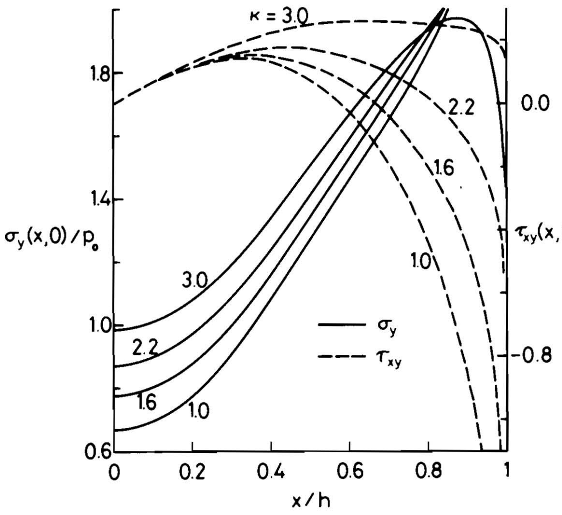


*Figure 7. Axial and Shear Stresses Along the Rigid Support When $b = 0.5\,h$, $L = h$.*

disturbance due to the rigid support disappears and the stresses become finite at the corners. When the crack is close to the rigid support, the stress distributions are very complicated and the axial stress $\sigma_y$ assumes very small values around the center. As $L/h$ increases, stress distributions become smoother. When $L/h \to \infty$ the effect of the crack disappears and the results for an uncracked semi-infinite strip [11] are recovered.

Figures 9–11 show the normalized stress intensity factors at the corners of the strip on the rigid support defined by

$$\bar{k}_{ih} = k_{ih}/p_0\, h^{\alpha}\,, \qquad (i = 1,2)\,. \tag{27}$$

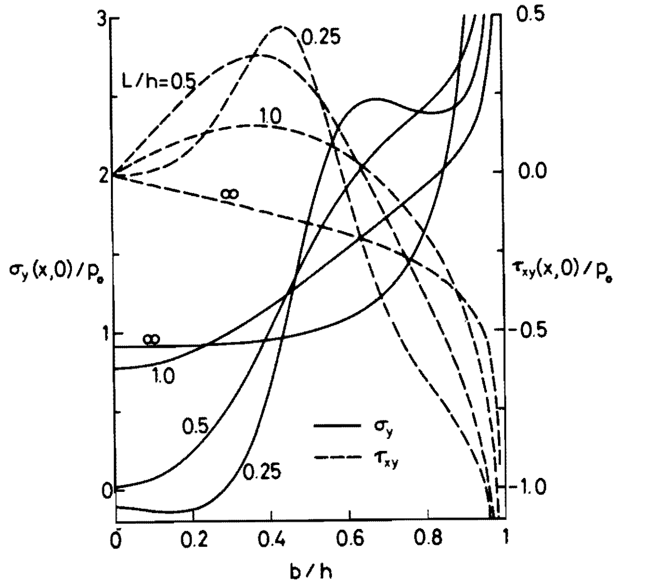


*Figure 8. Axial and Shear Stresses Along the Rigid Support When $\kappa = 1.6$.*

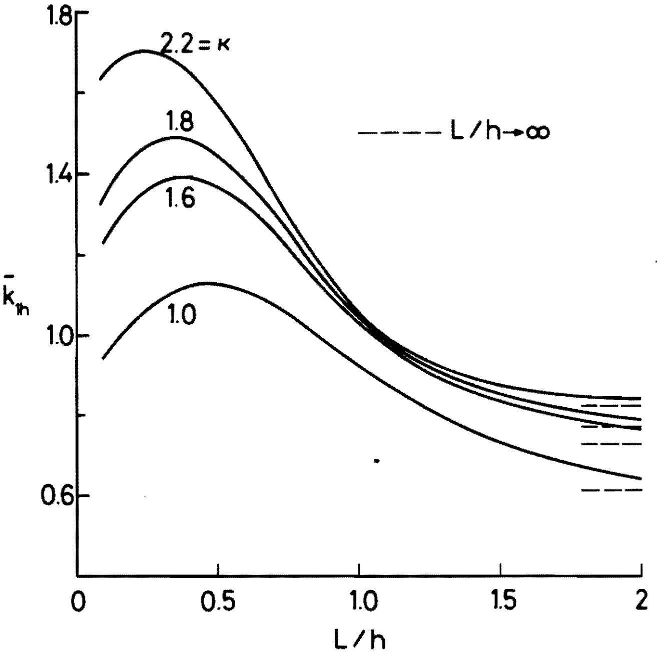


*Figure 9. Normalized Stress Intensity Factor $\bar{k}_{1h}$ at the Corners of the Strip When b = 0.5 h.*

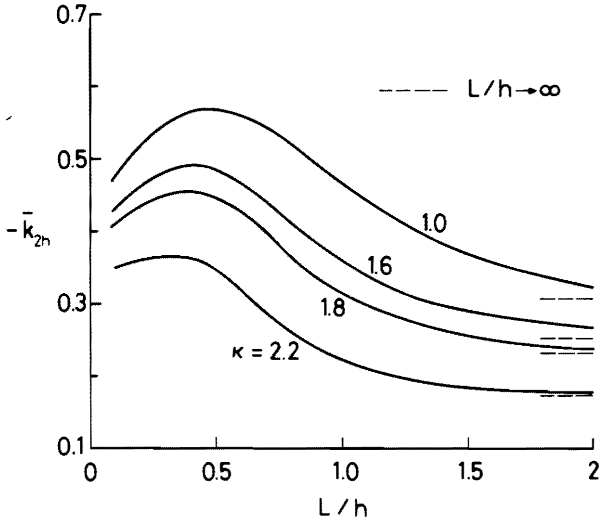


*Figure 10. Normalized Stress Intensity Factor $\bar{k}_{2h}$ at the Corners of the Strip When b = 0.5 h.*

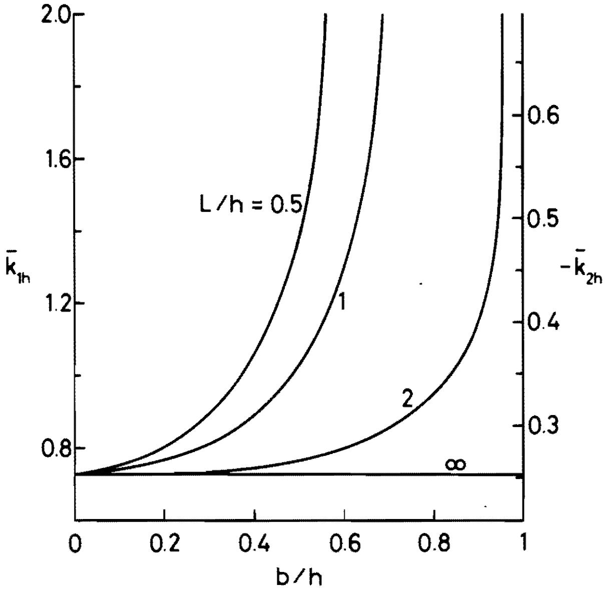


*Figure 11. Normalized Stress Intensity Factors $\bar{k}_{1h}$ and $\bar{k}_{2h}$ at the Corners of the Strip When $\kappa = 1.6$.*

As can be realized from these figures, both $\bar{k}_{1h}$ and $\bar{k}_{2h}$ decrease with increasing $L/h$ except for relatively small values of $L/h$, $\bar{k}_{1h}$ increases whereas $\bar{k}_{2h}$ decreases with increasing $\kappa$, both $\bar{k}_{1h}$ and $\bar{k}_{2h}$ increase with increasing $b/h$. Note that in Figure 11, $\bar{k}_{1h}$ and $\bar{k}_{2h}$ are given by the same curve (scales are different) since $\kappa = 1.6$ is fixed. From (20) and (27) it can be noticed that the ratio

$$\bar{k}_{1h}/\bar{k}_{2h} = \frac{1}{(\kappa+1)\sin\pi\alpha} \times[(1-\kappa)\cos\pi\alpha-3\kappa-5+(\kappa+3)\alpha-4\alpha^2] \tag{28}$$

has a constant value for a fixed $\kappa$. Typical values for the power of stress singularity $\alpha$ and the stress intensity factor ratio are given in Table 1.

**Table 1. Power of Stress Singularity $\alpha$ and Stress Intensity Factor Ratio at the Corners of the Strip.**

| $\kappa$ | $\nu$ Plane Strain | $\nu$ Plane Stress | $\alpha$ | $\bar{k}_{1h}/\bar{k}_{2h}$ |
|---|---|---|---|---|
| 1.0 | 0.50 | – | 0.4053883559 | −1.983315612 |
| 1.6 | 0.35 | – | 0.3203048030 | −2.869620924 |
| 1.67 | 0.33 | 0.5 | 0.3100164640 | −3.003946518 |
| 1.8 | 0.30 | 0.43 | 0.2888270670 | −3.308237825 |
| 2.0 | 0.25 | 0.33 | 0.2552503014 | −3.887930020 |
| 2.2 | 0.20 | 0.25 | 0.2189265635 | −4.709176079 |
| 3.0 | 0 | 0 | 0 | – |

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**Paper Received 25 March 1987; Revised 29 September 1987.**