THE DAMPING OF INTERNAL GRAVITY WAVES IN A CONTINUOUSLY STRATIFIED TURBULENT BOUSSINESQ FLUID IN NON-DARCY FLOW

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الخلاصة :

تدرس وفق نموذج (Ergum-Darcy) توجيه تقدم أمواج الجاذبية الباطنية في سائل ضعيف الترسيب متخابط وينساب في مجال أسفنجي . يرمز إلى أثر التخابط بعامل مزج دوري ونجد أن الطول المهديء وازدياد الدورة بشكل عام يعتمدان على عامل (Permeability) لـ $\left(\frac{1}{P_t}\right)$ ، ورقم براندت (P_0) ، ورقم ريليه (Rayleigh number) ، سماكة المجال الأسفنجي وأيضاً على التسريب الحراري في المزج الدوري . وبناء على اختيارنا لمقاييس هذه العوامل نتوصل إلى النتائج التالية .

أولاً : إن تأثير عامل الإسفنجية $\left(\frac{P_0}{P_t}\right)$ هو توجيه الأمواج الباطنية وتقصير الطول المهدي نتيجة لخسارة في الطاقة بسبب الاحتكاك بالقصر.

ثانياً : إن التهدئة بالضرورة سريعة عندما تكبر العوامل $\left(H, K, \frac{P_0}{P_t}\right)$.

ثالثاً : إن إعتماد الطول النسبي على عامل السماكة (H) هو عكسي لكلا الأمواج الطويلة والقصيرة .

رابعاً : إن عامل الطول المهديء النسبي وازدياد الدورة يشيران إلى وجود نهايات متناهية في الأمواج المتوسطة . وهذا التواجد يعتمد على العوامل (H, K, P_0, P_t) .

خامساً : إن تأثير الإسفنجية ونسبة قدرات الطاقة (E) و (m) مشابه إلى تأثير العامل $\left(\frac{P_0}{P_t}\right)$ في توجيه الأمواج الباطنية لإنهم جميعاً يرمزون إلى خواص المجال الإسفنجي .

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ABSTRACT

The attenuation of internal gravity waves propagating in a weakly stratified turbulent Boussinesq fluid flowing through a porous medium is studied using the Ergun–Darcy model. It is assumed that the action of turbulence can be parametrically represented by an eddy mixing coefficient. It is shown that the damping-length and period-increase are, in general, dependent on the permeability parameter P_t^{-1}, the Prandtl number P_0, the Rayleigh number R, the thickness of the porous layer H, and the eddy thermal diffusivities $K_{hr} (r=1,3)$. Subject, quantitatively, to the choice made for these parameters, the following conclusions are drawn:

(1) the effect of the porous parameter $P_0 P_t^{-1}$ is to attenuate the internal waves and to decrease the damping length as a result of energy dissipation owing to friction in the bed;
(2) the damping is more rapid when H, K_{h1}, and $P_0 P_t^{-1}$ are large;
(3) the dependence of the relative damping-length on H is opposite for long and short waves;
(4) the period-increase and relative damping-length exhibit a minimum and a maximum, respectively, at intermediate wavelengths; the position of these extrema depends on H, K_{hr}, and $P_0 P_t^{-1}$;
(5) the influence of porosity, ε, and ratio of heat capacities, M, is similar to the influence of $P_0 P_t^{-1}$ in attenuating internal waves, because all of them represent the intrinsic properties of the porous medium.

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NOMENCLATURE

a	Dimensionless wave number
C_b	Inertial parameter
g	Gravitational acceleration
H	Depth of the porous layer
k	Wave number
K^*	Effective thermal conductivity
K'	Effective thermal diffusivity
K_{h1}, K_{h2}, K_{h3}	Eddy heat diffusivities along (x, y, z) axes
K_m	Eddy viscosity
$\kappa_L, \kappa_T, \kappa$	Permeability owing to laminar flow, permeability owing to turbulent flow, and effective permeability
L	Wavelength of gravity waves
M	Ratio of heat capacities of the fluid saturated porous layer and the fluid $(=\{(1-\varepsilon)(\rho C_p)_s+\varepsilon(\rho C_p)_f\}/(\rho C_p)_f)$
n	Mode number
P	Hydrostatic pressure
P_0	Prandtl number $(=\nu/K_h)$
P_t^{-1}	Permeability parameter $(=H^2/\kappa)$
$\lvert q_i \rvert$	$=(u^2+v^2+w^2)^{1/2}$
$\mathbf{r}$	Position vector $(=(x, y, z))$
R	Rayleigh number $(=\alpha\beta g H^4/\nu K_h)$
t	Time
T	Temperature
(u, v, w)	Components of velocity along the respective coordinate axes
X_e	Damping length
(x, y, z)	Coordinate system
α	Volumetric expansion coefficient
β	Temperature gradient $(=-dT_0/dz)$
ε	Porosity of the porous layer
η_1	Dimensionless eddy heat diffusivity
η_0	$=(\pi^2 n^2+a^2)$
ν	Kinematic viscosity
ρ	Density of fluid
σ	Dimensionless growth-rate
τ	Time-constant characterizing damping-rate
ω	Growth-rate

INTRODUCTION

In recent years considerable interest has been evinced in studying internal gravity waves in a porous medium (see Rudraiah [1]) because of its application to many geophysical problems, particularly in the case of energy and momentum transfer from the deep interior of the Earth to shallower geothermal regions (see Rudraiah and Srimani [2]) and in the case of landslides during earthquakes. Earthquakes occur in the interior of the Earth; landslides occur at the surface. The physical connection is that earthquakes may generate internal gravity waves in a stratified porous reservoir that are capable of transferring energy and momentum from the interior of the Earth to the surface, possibly causing landslides. The study of internal gravity waves in a porous medium is also of interest in understanding mixing processes in many industrial problems, particularly in fixed-bed chemical reactors and in filters.

Recently, Venkatachalappa and Sekar [3] have studied these internal waves in a porous medium using Darcy's law under the assumption of laminar flow. In a densely packed porous medium, the resistance to fluid motion is so large that one may neglect inertial effects and hence a linear relationship exists between the pressure gradient and the velocity, which is Darcy's law:

$$\nabla P = -\frac{\mu}{\kappa_L}\mathbf{q} + \rho \mathbf{g}. \tag{1}$$

This is true only for very small values of the Reynolds number so that laminar flow, i.e. creeping flow is valid.

Many practical problems connected with those cited above, however, involve a special type of porous medium of porosity close to unity (see Rudraiah [4]) in which the flow is curvilinear and the curvature of paths of fluid elements gives rise to inertial effects that cannot be neglected (because these are responsible for generating turbulent motion at high flow velocities, i.e. at high Reynolds number). Available experimental data on flow through this special type of porous medium reveal the existence of turbulent flow at high Reynolds number (see, for example, Stark and Volker [5]).

Therefore, recently Rudraiah [1] has studied internal gravity waves in turbulent flow through a porous medium using a modified Darcy equation of motion in the form (see Rudraiah [4]):

$$\rho\left[\frac{1}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t} + \frac{1}{\varepsilon^2}(\mathbf{q}\cdot\nabla)\mathbf{q}\right] = -\nabla P - \frac{\mu}{\kappa_L}\mathbf{q} + \rho\mathbf{g}. \tag{2}$$

This equation is not general in the sense that it fails to give any inertial drag effect for the case of steady unidirectional flow, since the left-hand side of Equation (2) is then identically zero (see Joseph and others [6]). An essential feature of flow in a porous medium is that the term that is quadratic in the velocity is associated with a pressure drag of the type that always exists in flow around blunt bodies, and hence acts in the direction $-\mathbf{q}$. Therefore, in the general form of the momentum equation for flow through a porous medium, the nonlinear inertial effects appear as a drag proportional to the square of the velocity. This drag arises from the inertial term $(\mathbf{q}\cdot\nabla)\mathbf{q}$ in the Navier–Stokes equation for flow of a viscous fluid with velocity $\mathbf{q}$ as an average taken over many pores of the medium. Therefore, to study the propagation of internal gravity waves in turbulent flow through a porous medium we use the modified Darcy law, where the inertial effects are significant and are expressed as quadratic in the velocity. These inertial effects cannot be neglected because they are responsible for generating turbulent motion at high flow velocities (see Stark and Volker [5]). This suggests that, in a stably stratified fluid flowing through such a porous medium, the respective strengths of inertial acceleration and buoyancy forces are important in characterizing the dynamical behavior of turbulence.

MATHEMATICAL FORMULATION

Consider a Boussinesq fluid flowing through a porous layer of thickness H and of infinite lateral extent. Let (x, y, z) be a right-handed coordinate system with z-axis vertically upwards such that $z=0$ is at the lower surface of the layer and gravity acts vertically downwards. The corresponding mean filter velocity components are denoted by $\mathbf{q}=(u, v, w)$. The fluid in the porous medium is, on time average, at rest and is endowed with a weak density stratification, $\rho_0(z)$, maintained by an unspecified but adequate source of heat. This zero-order state is described by

$$\mathbf{q}_0=0; \qquad P_b=-\rho_0 gz+\text{constant}. \tag{3}$$

Superimposed on this is a horizontally homogeneous and nearly stationary field of 'external' turbulence characterized by velocity and density fluctuations $\mathbf{q}_t(\tau, t)$, $\rho_t(\tau, t)$, $\mathbf{r}(x, y, z)$, with $\rho_t \ll \rho_0$. Because of homogeneity, time averages are independent of horizontal coordinates; and because of the nearly stationary behavior, all non-zero space averages are only slowly-varying functions of time. Since we deal with small Prandtl numbers, the turbulent flow is governed by Ergun–Darcy equations wherein the effective-resistance causes the turbulent field to decay (no sources need be postulated to maintain it). Here the increased resistance is due to a pressure drag and is essentially quadratic in the velocity. It arises from the inertial term $(\mathbf{q}\cdot\nabla)\mathbf{q}$ in the Navier–Stokes equation for flow of a viscous fluid with velocity $\mathbf{q}$ as an average taken over many pores in the medium. It is assumed that the turbulence decay time is much longer than the period of the waves. Under these assumptions (which means we deal with small values of the coefficient of volume expansion; for example, for gases and liquids it is usually in the range 10^{-3} to 10^{-4}) the governing equations for a Boussinesq fluid are

$$\frac{1}{\varepsilon}\frac{\partial q_i}{\partial t}+C_b|q_i|q_j=\frac{-1}{\rho_0}\frac{\partial P}{\partial x_i}-\frac{\rho}{\rho_0}g\delta_{i3}-\frac{\nu}{\kappa_L}q_i, \tag{4}$$

$$(\rho C_p)^*\frac{\partial T}{\partial t}+(\rho C_p)_f q_j\frac{\partial T}{\partial x_j}=\frac{\partial}{\partial x_j}\left(K^*\frac{\partial T}{\partial x_j}\right), \tag{5}$$

$$\frac{\partial q_i}{\partial x_i}=0, \tag{6}$$

$$\rho=\rho_0(0)-\alpha(T-T_0), \tag{7}$$

where q_i are the velocity components, P the pressure, ρ the density, T the temperature, ρ_0 the density at $T=T_0$, g the gravity, ε the porosity, κ_L the permeability owing to laminar flow, ν the kinematic viscosity, α the volumetric coefficient of expansion, $(\rho C_p)^*=(1-\varepsilon)(\rho C_p)_s+\varepsilon(\rho C_p)_f$ is the relative heat capacity, suffixes f and s denote 'fluid' and 'solid' respectively, and K^* is the effective thermal conductivity of the fluid in the presence of a solid matrix (K^* is the sum of the stagnation thermal conductivity owing to molecular diffusivity and the thermal dispersion coefficient owing to mechanical dispersion). Equation (4) is the Ergun–Darcy equation. The experimental work of Ward [7] and Beavers and Sparrow [8] supports this equation. This law depicts the effect of inertial drag even in steady unidirectional flow.

We assume that flat-crested internal waves propagate in turbulent flow through a porous medium in the positive x-direction with the angular frequency ω and wave number k. The amplitude of these waves is assumed to be small enough (because of small internal Reynolds number in the presence of a stratified fluid) that we may neglect squares of velocities as well as density fluctuations owing to the waves themselves when compared with the mean density field. The turbulent field will be separated from the waves by averaging in the y-direction. Since the turbulent variables are random in phase and homogeneous in space, the y-average will leave only the wave field. In other words, the fluid variables q_i, ρ, and P are split into mean and fluctuating parts:

$$\begin{aligned} q_i &= \bar{q}_i + q_1' \\ \rho &= \rho_0 + \bar{\rho} + \rho' \\ P &= P_b + \bar{P} + P' \end{aligned} \tag{8}$$

so that, by definition, $\bar{q_i'} = \bar{\rho'} = \bar{p'} = 0$.

The equations for the mean flow field are readily obtained from Equations (4) and (5) using Equation (8) and taking y-averages in the following form:

$$\frac{1}{\varepsilon}\frac{\partial \bar{q}_i}{\partial t} + \frac{\bar{\rho}}{\rho_0} g\,\delta_{i3} + \frac{1}{\rho_0}\frac{\partial \bar{P}}{\partial x_i} = \frac{-\nu}{\kappa_L}\bar{q}_i - C_b \overline{|q_i'|q_j'}, \tag{9}$$

$$M\frac{\partial \bar{T}}{\partial t} + \bar{W}\frac{dT_0}{dz} = \frac{\partial}{\partial x_j}\left(K'\frac{\partial \bar{T}}{\partial x_j} - \overline{q_j' T'}\right), \tag{10}$$

where

$K' = \dfrac{K^*}{(\rho C_p)_f}$ is the effective thermal diffusivity,

and

$$M = \frac{(\rho C_p)^*}{(\rho C_p)_f}.$$

In the case of laminar flow through a porous medium, the second term on the right-hand side of Equation (9) is usually small compared with the first term. In turbulent flow through a porous medium the second term is much larger than the first term, but we still retain the latter to compare the present results with those of laminar flow.

Equations (9) and (10) involve both mean and fluctuating quantities, and hence we need a suitable closure theory. For this, we introduce the eddy viscosity and eddy diffusion coefficients in the following form:

$$-C_b \overline{|q_i'|q_j'} = \frac{K_m}{\kappa_T}\bar{q}_i \tag{11}$$

$$\frac{\partial}{\partial x_j}(-\overline{q_j' T'}) = \frac{\partial}{\partial x_j}\left(K_h'\frac{\partial \bar{T}}{\partial x_j}\right) \tag{12}$$

where K_m and K_h' are, respectively, the eddy viscosity and eddy diffusion coefficients. Substituting Relations (11) and (12) into Equations (9) and (10) and simplifying further, we get

$$\left(\frac{1}{\varepsilon}\frac{\partial}{\partial t} + \frac{\nu}{\kappa}\right)\bar{q}_i + \frac{\bar{\rho}}{\rho_0} g\delta_{i3} + \frac{1}{\rho_0}\frac{\partial \bar{P}}{\partial x_i} = 0, \tag{13}$$

$$\left(M\frac{\partial}{\partial t} - K_{h_r}\frac{\partial^2}{\partial x_r^2}\right)\bar{T} - \bar{W}\beta = 0, \tag{14}$$

where

$$\frac{1}{\kappa} = \left(\frac{1}{\kappa_L} + \frac{K_m}{\nu\kappa_T}\right), \quad \beta = -\frac{dT_0}{dz}, \quad \text{and} \quad K_{h_r} = K_{h_r}' + K',$$

the extra index, r, on the eddy diffusion coefficient takes care of its anisotropy. This coefficient is taken to be spatially uniform; however, it is still free to depend on the wave parameters. As in the case of ordinary viscous flow (see [9]), although it is reasonable to assume that for horizontally homogeneous porous media the mixing does not depend on position, some restriction must be placed on the density field to extend this to the vertical direction because the stratification greatly influences the vertical component of velocity. Since the vertical intensity of turbulence will depend on the effective resistance and on the degree of stratification of the fluid, to take the eddy coefficients independent of z we assume that the fluid in the porous medium must at least be uniformly stratified. For weak stratification, $\Delta\rho_0 \ll \rho_0$ and hence

$$\frac{1}{\rho_0}\frac{d\rho_0}{dz} = \text{constant}.$$

Under this approximation, we can assume that the eddy diffusion coefficient is constant.

The density stratification is due to temperature only. For simplicity, ρ is assumed to be linear in T for a Boussinesq fluid. For a steady field

$$\rho_0(z) = \rho_0(0) - \alpha T_0(z) \tag{15}$$

and for the wave field

$$\bar{\rho}(\mathbf{r}, t) = -\alpha\bar{T}(\mathbf{r}, t), \tag{16}$$

where the coefficient of volumetric expansion, α, is positive.

Taking the curl of Equation (13) twice, the z-component is

$$\left(\frac{1}{\varepsilon}\frac{\partial}{\partial t} + \frac{\nu}{\kappa}\right)\nabla^2\bar{W} + \nabla_h^2\frac{\bar{\rho}}{\rho_0}g = 0. \tag{17}$$

The vector symbol for the Laplacian and its horizontal component ∇_h^2 have been used for brevity.

The y-averaged continuity equation is:

$$\frac{\partial \bar{q}_i}{\partial x_i}=0. \quad (18)$$

Operating

$$\left(M\frac{\partial}{\partial t}-K_{hr}\frac{\partial^2}{\partial x_r^2}\right)$$

on Equation (17) and then using Equation (14), the following equation in $\bar{W}$ results

$$\left(\frac{1}{\varepsilon}\frac{\partial}{\partial t}+\frac{v}{\kappa}\right)\left(M\frac{\partial}{\partial t}-K_{hr}\frac{\partial^2}{\partial x_r^2}\right)\nabla^2\bar{W}+\frac{g}{\rho_0}\nabla_h^2(\alpha\beta\bar{W})=0. \quad (19)$$

The boundaries are assumed to be isothermal and stress-free, so that the required boundary conditions are:

$$\bar{W}=\frac{\partial^2\bar{W}}{\partial z^2}=\bar{T}=0. \quad (20)$$

WAVE SOLUTION

We look for the solution of Wave Equation (19) in the form:

$$\bar{W}=W(z)\exp(\omega t-ikx). \quad (21)$$

Substituting Equation (21) into Equation (19) and making the equation dimensionless using:

$$z'=z/H;\quad \sigma=\frac{\omega H^2}{K_h};\quad K_h=\frac{1}{3}\sum_{j=1}^{3}K_{hj};\ a^2=k^2H^2,$$

we get

$$[M\sigma-(P_{13}D^2-P_{11}a^2)]\left[\frac{1}{\varepsilon}\sigma+P_0P_t^{-1}\right](D^2-a^2)W-P_0Ra^2W=0, \quad (22)$$

where

$$D=\frac{d}{dz},$$

$$P_0=\frac{v}{K_h},$$

$$R=\frac{\alpha g\beta H^4}{vK_h},$$

$$P_t^{-1}=\frac{H^2}{\kappa}$$

and

$$P_{fr}=K_h^{-1}(K_{hr}\delta_{f1}+K_h\delta_{f0});\quad f=0,1. \quad (23)$$

The solution of Equation (22) satisfying Boundary Conditions (20) is of the form

$$W=A\sin n\pi z, \quad (24)$$

where A is a constant.

Substituting Equation (24) into Equation (22), the dispersion relation

$$\sigma^2+\left[\frac{\eta_1}{M}+P_0P_t^{-1}\varepsilon\right]\sigma+\left[\eta_1\frac{P_0P_t^{-1}}{M}\varepsilon+\frac{\varepsilon P_0Ra^2}{M\eta_0}\right]=0, \quad (25)$$

where

$$\eta_f=P_{f3}n^2\pi^2+P_{f1}a^2,\quad f=0,1, \quad (26)$$

is obtained.

Equation (25) leads to an oscillatory solution only when it has complex roots, viz when

$$\sigma=-\frac{1}{2}\left(\frac{\eta_1}{M}+\varepsilon P_0P_t^{-1}\right)\pm i\left[\frac{a^2P_0R\varepsilon}{M\eta_0}-\frac{1}{4}\left(\frac{\eta_1}{M}-P_0P_t^{-1}\varepsilon\right)^2\right]^{1/2}. \quad (27)$$

The real part of σ is always negative, and hence the waves are always damped. Comparing this with the viscous flow results, we conclude that the effect of the permeability of a porous medium is to dampen the waves much faster by increasing the magnitude of the exponential damping coefficient. In the viscous case, ($P_t^{-1}\to 0$), the magnitude of the exponential damping coefficient is proportional to the square of the wave number multiplied by the appropriate diffusion coefficient. This is not so in the case of a porous medium. However, the heat diffusion and porous parameter have a directly additive influence on the exponential damping. The imaginary part of σ also contains a term arising from the porous parameter in addition to the mixing process, so that the frequency of damped waves is smaller than that of undamped ones. Moreover, if this second term becomes so large that

$$\left[\frac{\eta_1}{M}-\varepsilon P_0P_t^{-1}\right]^2\eta_0\geq 4\left[\frac{a^2P_0R\varepsilon}{M}\right] \quad (28)$$

only critically damped motions (i.e. $\sigma_i=0$) will be found. We can characterize the damping-rate by a time-constant, τ, in which the wave amplitude decreases by a factor e^{-1} where

$$\tau = \frac{2H^2}{K_h\left(\frac{\eta_1}{M} + \varepsilon P_0 P_l^{-1}\right)}. \qquad (29)$$

This shows that the damping-rate is much larger in the case of a porous medium than in the case of viscous flow.

For propagating waves it is meaningful (since then the velocity is affected by the porous parameter in addition to mixing) to compare the wavelength $L(=2\pi/k)$ with the distance x_e traveled by the wave at the phase velocity during the time τ, which is given by

$$\frac{x_e}{L} = \frac{a}{\pi\left(\frac{\eta_1}{M} + \varepsilon P_0 P_l^{-1}\right)} \left(\frac{RP_0\varepsilon}{M\eta_0}\right)^{1/2} \times$$

$$\left[1 - \frac{M\eta_0}{4\varepsilon a^2 RP_0}\left(\frac{\eta_1}{M} - \varepsilon P_0 P_l^{-1}\right)^2\right]^{1/2}. \qquad (30)$$

Physically this represents, for small dissipation rates, the ratio of the average energy content of the wave to the amount of energy lost per cycle through the turbulent interaction. This wave energy is dissipated by internal friction within the porous material.

CONCLUSIONS

The attenuation of internal gravity waves in a stratified fluid in a porous medium with turbulent motion is discussed on the basis of mixing theory. Since the problem is so complicated, in order to understand the physics with a minimum of mathematics some narrowing assumptions are made.

The attenuation rates obtained in this paper apply to a porous layer bounded on both sides by stress-free boundaries and is strongly dependent upon the permeability parameter P_l^{-1}, Prandtl number P_0, and the magnitude of the mixing coefficients K_{h3} and K_{h1}. Since the problem is linearized, it is difficult to take into account any dependence of the strength of the turbulence upon the wave amplitude. We may, however, use the fact that the turbulent mixing process is more important for large scale motion to establish some relationship between the wavelength of the internal waves, the porous parameter, and the magnitude of the eddy coefficients.

The period lengthening is directly related to the ratio

$$\frac{M\eta_0}{4a^2 RP_0\varepsilon}\left(\frac{\eta_1}{M} - \varepsilon P_0 P_l^{-1}\right)^2,$$

the dependence of which on some of the parameters such as depth of the porous layer H, $P_0 P_l^{-1}$, porosity ε, and ratio of heat capacities M is illustrated in Figures 1, 2, 3, and 4, respectively. From these figures it is clear that the effects of these quantities on the frequency of the waves are almost identical because all of them are connected with the structure of the porous medium. The dependence of the relative damping length, $x_{e/L}$, on the depth of the porous layer and on $P_0 P_l^{-1}$ is shown in Figures 5 and 6, respectively. The dependence of the damping rate on the thickness of the porous layer is shown in Figure 7. In these examples, the following values:

$$\frac{\Delta\rho_0}{\rho_0} = 10^{-3}, \qquad K_{h3} = 10\,\text{cm}^2\cdot\text{s}^{-1},$$

and suitable values of $P_0 P_l^{-1}$ have been used to compute the values depicted in these figures. The numerical computation applies to fluid of small Prandtl number (i.e. low kinematic viscosity with large

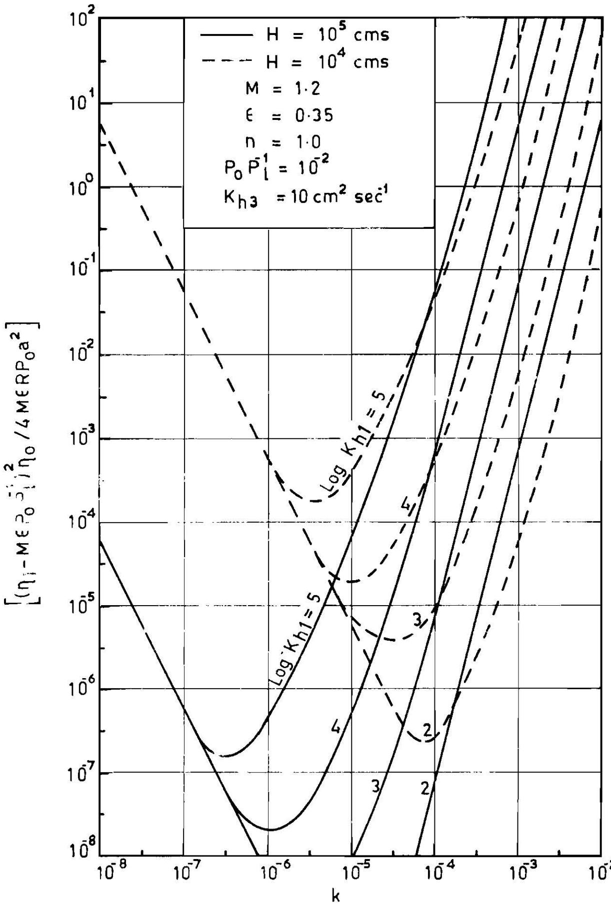

Figure 1. Influence of Thickness of Porous Layer on Frequency of Internal Waves

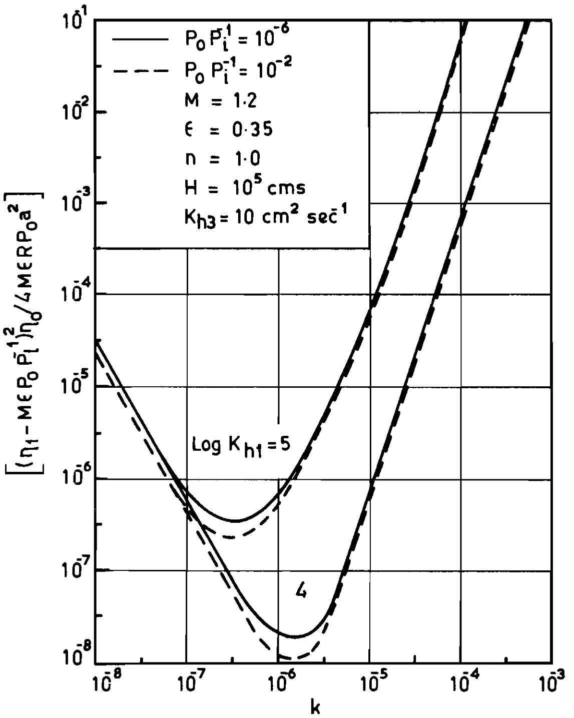

Figure 2. Influence of $P_0P_l^{-1}$ *on Frequency of Internal Waves*

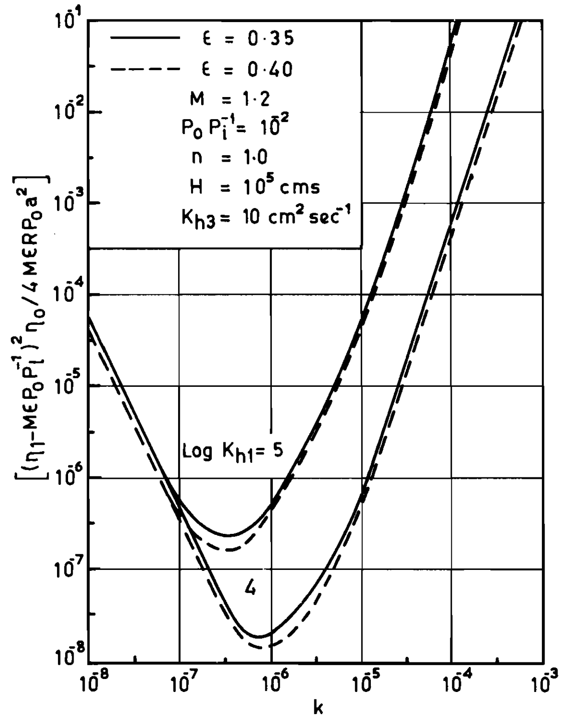

Figure 3. Influence of Porosity on Frequency of Internal Waves

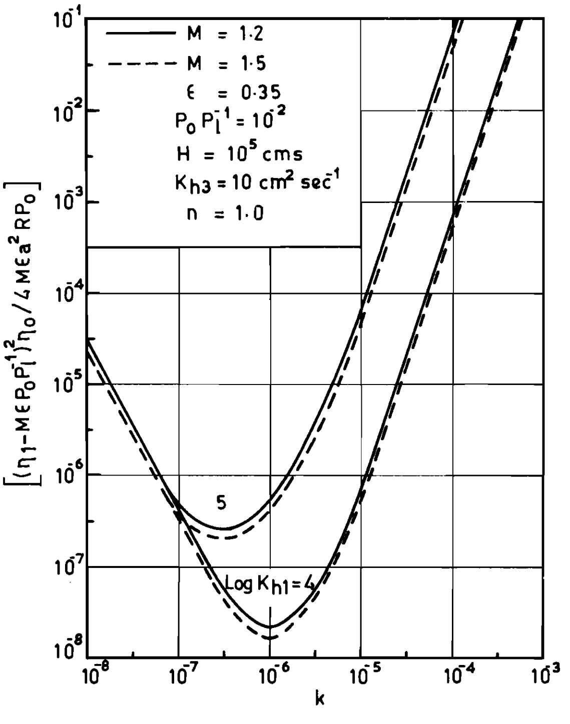

Figure 4. Influence of Ratio, M, of Heat Capacities of Pores and Fluid on Frequency of Internal Waves

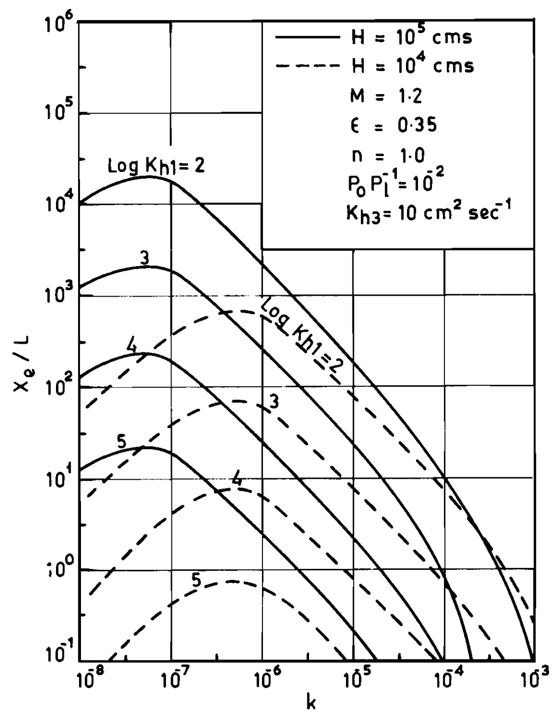

Figure 5. Influence of Thickness of Porous Layer on Relative Damping-Length

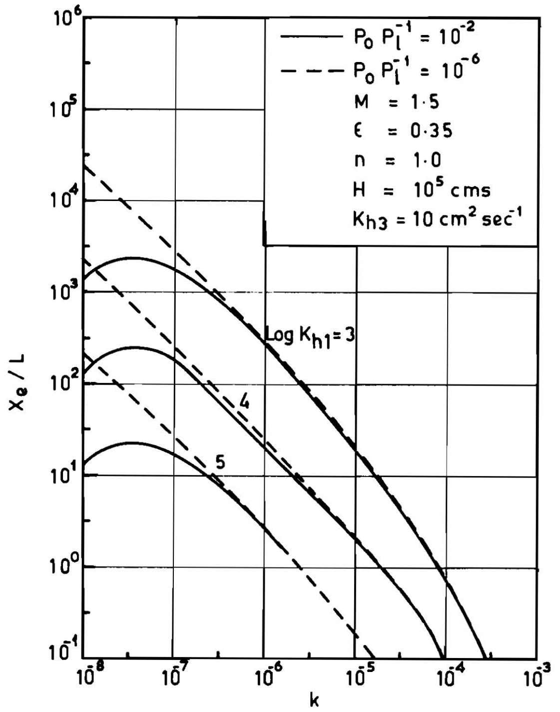

Figure 6. Influence of Porous Parameter $P_0P_l^{-1}$ *on Relative Damping-Length*

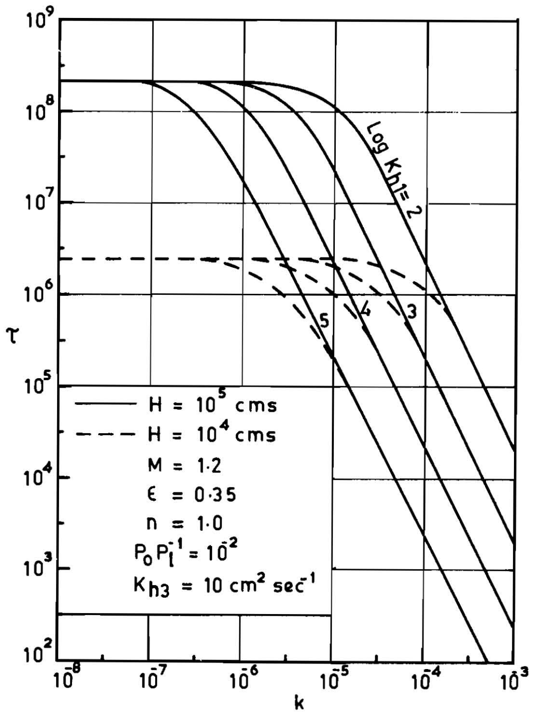

Figure 7. Influence of Thickness of Porous Layer on Damping-Rate

eddy diffusive coefficient) and large permeability parameter (i.e. $\sim 10^2$ to 10^3).

Figures 1 to 6 show that the damping and period-increase are, in general, dependent on the porous parameter, Prandtl number, thickness of the porous layer, and eddy diffusivity parameter K_{h1}, and the following conclusions are drawn.

(1) The damping is more rapid when $P_0P_l^{-1}$, H, and K_{h1} are larger in the case of short waves, but their influence becomes smaller in the case of long waves.

(2) An increase in the value of $P_0P_l^{-1}$ decreases the attenuation of the frequency of internal waves and increases their damping-length, because the smaller the thermal diffusivity the smaller will be the frequency attenuation.

(3) The dependence of the relative damping-length on the depth of the porous layer is in opposite directions for long and short waves.

(4) The period-increase and relative damping-length present a minimum and a maximum, respectively, at intermediate wavelengths; the position of these extrema depends on H, K_{h1}, and $P_0P_l^{-1}$.

(5) The influence of the porosity, ε, and the ratio of the heat capacities, M, is similar in attenuating internal waves, because both of them are intrinsic properties of the medium. We see that as the voids become larger the attenuation becomes smaller owing to reduction in the resistance offered by solid particles to flow. Further, an increase in the heat capacity of the pores, compared with that of the fluid, decreases the eddy heat diffusivity, which consequently causes the augmentation of the Prandtl number and thereby decreases the frequency attenuation.

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