OPTIMUM DESIGN OF I-COLUMNS AND BEAM-COLUMNS

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INTRODUCTION

In designing steel columns and beam-columns (members subjected to simultaneous actions of axial compression and bending moment), builtup I-sections are used whenever a suitable rolled section either is not available or would not yield an economical solution. Most building codes, for example the AISC specification [1], prescribe guidelines for design of columns and beam-columns. In recent years, considerable attention has been devoted to the optimum design of structural members. This has resulted in extensive research work on the optimization of I-shaped members subjected to different types of loading. References [2–7] can be cited as a representative sample of such work.

The objective of this paper is to present design-oriented procedures for optimum design of built-up homogeneous I-shaped columns and beam-columns. For columns, the general case of different effective lengths corresponding to the two principal axes is considered so that the failure axis is not known *a priori*. For beam-columns, the bending is assumed to be about the axis perpendicular to the web. Though the AISC specification has been used and referred to throughout the text, the prescribed approach can be modified to adopt any other specification.

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METHOD

For a typical symmetrical I-section as shown in Figure 1, the cross-sectional properties can be expressed as

$$A = 2A_f + A_w \tag{1}$$

$$A_f = C_1 A \tag{2}$$

$$A_w = C_2 A \tag{3}$$

$$I_x = (6C_1 + C_2)Ah^2/12 \tag{4}$$

$$I_y = C_1 A b^2/6 \tag{5}$$

$$r_x = h\sqrt{(6C_1 + C_2)/12} \tag{6}$$

$$r_y = b\sqrt{C_1/6}, \tag{7}$$

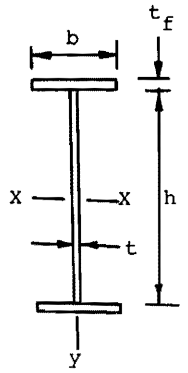

Figure 1. Typical I-Section

in which A is the total area of the cross section, I is the moment of intertia, r is the radius of gyration, and C_1 and C_2 (C_1, $C_2>0$) are related by

$$2C_1+C_2=1. \tag{8}$$

Setting $b/2t_f=\xi_1$, ξ_1 being the permissible width–thickness ratio of the flange, Equations (2) and (7) lead to

$$b=\sqrt{2\xi_1 C_1 A} \tag{9}$$

$$A=3r_y^2/(\xi_1 C_1^2). \tag{10}$$

Column Section

It is assumed as a general case that the column has different effective lengths K_xL_x and K_yL_y about the x and y axes. Defining the ratios $b/h=\alpha$, $r_x/r_y=\beta$ and $K_yL_y/K_xL_x=\gamma$, it can be shown from Equations (6)–(8) that

$$C_1=\frac{1}{2(\alpha^2\beta^2-2)} \tag{11}$$

$$C_2=\frac{a^2\beta^2-3}{a^2\beta^2-2}. \tag{12}$$

For C_1, $C_2>0$, $\alpha^2\beta^2$ must be greater than three i.e. $\alpha\beta>1.732$. The objective is then to find the minimum value of A subject to the following constraints

$$A=\frac{P}{\phi_1 F_y} \tag{13a}$$

$$\alpha\beta>1.731 \tag{13b}$$

$$\alpha>\alpha_0 \tag{13c}$$

$$h/t\leq\xi_2 \tag{13d}$$

in which $\phi_1 F_y$ ($\phi_1<1$ and F_y=yield stress of steel) is the allowable compressive stress for the column, α_0 is an upper limit on the value of b/h decided from practical consideration and ξ_2 is the allowable depth–thickness ratio of the web.

To derive a condition for the minimum area [8], a design curve for the allowable stress coefficient ϕ_1 is plotted against the slenderness ratio parameter λ in Figure 2 using [1].

$$\lambda=\frac{KL}{rC_c}=\frac{K_yL_y}{r_y\delta C_c} \tag{14}$$

where

$$C_c=\sqrt{2\pi^2E/F_y}\ [1];$$

$$\delta=\begin{cases}1, & \text{if } \gamma\beta\geq 1\\ \gamma\beta, & \text{if } \gamma\beta<1.0.\end{cases}$$

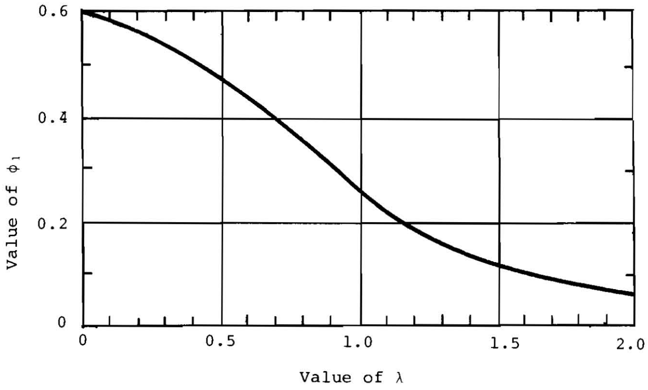

Figure 2. Values of ϕ_1 versus λ Based on [1]

Eliminating A and r_y from Equations (10), (13a), and (14), the following relationship between λ and ϕ_1 prescribes the condition for a safe design.

$$\lambda\leq B\sqrt{\phi_1} \tag{15}$$

where B, a non-dimensional parameter, is given by

$$B=\frac{K_yL_y}{\delta C_c}\sqrt{\frac{3F_y}{\xi_1 C_1^2 P}}. \tag{16}$$

The value of λ required for the column to carry the load P safely is given by Equation (15). When $\lambda=B\sqrt{\phi_1}$, the column will have the minimum area for the chosen values of α and β. For a known value of B, the value of λ to satisfy Equation (15) can be determined easily from the plot of Figure 2 and hence the cross section can be finalized.

The procedure thus requires, at the outset, appropriate values for α and β. This is, however, not difficult to choose considering that there is a practical limit on α; $\alpha\beta>1.731$, but not too much larger for an economic design; and $\gamma\beta$ should be as close as practicable to 1.0 ($\gamma\beta=1.0$ represents the case of identical slenderness about both axes). The value of $\alpha\beta$ will normally be in the range of 1.80 – 1.90. Since the resulting proportioning must satisfy the constraint (13d), it can be shown that

$$\alpha^2\geq\frac{2C_1\xi_1}{C_2\xi_2}. \tag{17}$$

The additional condition makes it easier to select an appropriate value of α.

Beam-Column Section

Beam-columns are designed using simplified interaction equations which include the additional secondary moment due to deflection. In accordance with AISC specification, the latter effect can be ignored if the ratio of the computed axial stress and the allowable compressive stress, f_a/F_a, is less than or equal to 0.15. Dealing with the general case of $f_a/F_a > 0.15$, the AISC strength and stability criteria stipulate

$$\frac{f_a}{F_a} + \frac{C_m f_b}{\left(1 - \frac{f_a}{F'_e}\right) F_b} \leq 1.0 \tag{18a}$$

and

$$\frac{f_a}{0.6F_y} + \frac{f_b}{F_b} \leq 1.0. \tag{18b}$$

In Equation (18), f_b and F_b are the calculated and permissible bending stresses in compression, C_m is a reduction coefficient and F'_e is the Euler buckling stress about the axis of bending with a factor of safety of 23/12.

Unlike a column, the cross section of a beam-column cannot be determined directly due to difficulty in separation of variables. A simple iterative search procedure is therefore formulated to find systematically the minimum value of A. For this purpose, the web thickness t is assumed to be known. The additional properties of an I-section required in the design of a beam-column are

$$S_x = C_2 A^2 (C_1 + C_2/6)/t \tag{19}$$

$$r_T = \sqrt{\frac{\xi_1 C_1^2 A}{C_2 + 6C_1}} \tag{20}$$

$$F'_e = \frac{\pi^2 E C_2^2 (6C_1 + C_2) A^2}{23 (K_x L_x)^2 t^2} \tag{21}$$

where

S_x is the section modulus about the x-axis;

r_T is the radius of gyration of the T-section comprised of the compression flange and one-sixth of the web, taken about the plane of the web.

Substitution of the values of stresses in Equation (18) and noting that $f_b = M/S_x$, M being the design moment, the area of an I-section required must satisfy the following equations

$$A \geq \frac{P}{\phi_1 F_y} + \frac{C_m M t F'_e}{\phi_2 F_y C_2 (C_1 + C_2/6)(AF'_e - P)} \tag{22a}$$

$$A \geq \frac{P}{0.6F_y} + \frac{Mt}{\phi_2 F_y C_2 (C_1 + C_2/6) A} \tag{22b}$$

where

ϕ_2 is a factor for the allowable bending stress $(= f(L/r_y, Lh/A_f))$;

$F_b = \phi_2 F_y$.

Reference [1] gives the equation for computing the value of F_b for a given section. For a chosen t and an assumed value of C_1 (hence C_2), the required area A can be determined from Equation (22) by successive approximation.

Search Procedure

As the web thickness should be small for an economic design, a minimum acceptable web thickness is chosen first for the beam-column. A starting value of C_1 is taken as 0.2 and an initial value is arbitrarily assigned to A. Using successive approximation, the correct value of A for chosen values of t and C_1 is determined from Equation (22). The convergence is very rapid when the trial value of A is taken as the average of the old and the corresponding new value of A.

The value of C_1 is then increased by a small step and the corresponding A is calculated. If this value of A is smaller than the previous value with lower value of C_1, the search for A is continued with progressively increasing values of C_1. At a stage when the value of A starts to increase, the search is terminated, recording the lowest value of A and the corresponding value of C_1. The resulting optimum proportioning is acceptable if the ratio $h/t \leq \xi_2$. If not, the web thickness is changed to the next higher value and the procedure is repeated. This value of A is compared with the previous value obtained with smaller t that satisfies $h/t \leq \xi_2$ to record the correct optimum solution. While increasing C_1 in small steps, the following condition should be satisfied to ensure that $b/h \leq \alpha_0$.

$$\frac{C_1}{(1 - 2C_1)^2} \leq \frac{\alpha_0^2 A}{2\xi_1 t^2}. \tag{23}$$

The proposed method can be programmed in a computer to find an optimum proportioning readily. It has been observed that the value of A is not very sensitive to the value of C_1 in the neighborhood of the optimum value. Thus, for hand computation, a value of C_1 within the range of 0.30 to 0.35 can be taken in most cases to proportion an economical I-section.

Design Example 1

Proportion an I-section for a compression member of length 7 m to carry an axial load of 2000 kN. Effective length factors are $K_x = 1.0$ and $K_y = 0.5$. Use $F_y = 248$ MPa and the AISC specification. $\gamma = 0.5/1.0 = 0.5$; choose $\beta = 2.0$ (hence $\gamma\beta = 1.0$) and $\alpha = 0.93$ ($\alpha\beta = 1.86$); from Equation (11), $C_1 = 0.3426$ and hence $C_2 = 0.3148$. With $\xi_1 = 16$ and $\xi_2 = 42$ (AISC specification), α satisfies Equation (17). With $C_c = 126.1$ and $\delta = 1.0$, the value of B is 0.335.

From Figure 2, the value of γ to satisfy Equation (15) is 0.29 and $\phi_1 = 0.541$. Thus the required A from Equation (13a) is 14910 mm^2 which leads to $A_f = 5110$ mm^2 and $A_w = 4690$ mm^2. From Equation (9), $b = 404$ mm and hence $h = b/\alpha = 434$ mm. The theoretical flange plate size is 12.65 mm × 404 mm and the web plate is 10.8 mm × 434 mm. Use flange plate 14 mm × 400 mm and web plate 10 mm × 410 mm. $A = 15\,300$ mm^2 and the capacity of the column is 2053 kN.

Design Example 2

Proportion an I-section for a beam-column of unsupported length 6 m which is subjected to an axial compressive load of 1000 kN and a moment of 125 kN·m at one end. Assume simple end conditions with $C_b = 1.0$. For steel, $F_y = 248$ MPa.

Choose a 10 mm web thickness. Assume a value of $C_1 = 0.33$ and an initial value of $A = 11\,000$ mm^2 due to high moment. With $r_y = 79.9$ mm from Equation (10), $C_c = 126.1$ and $\lambda = 0.595$ (y-axis critical), $\phi_1 = 0.438$ from Figure 2. With $r_T = 90.0$ mm, the value of ϕ_2 evaluated from [1] is 0.564. From Equation (21), $F'_e = 774$ MPa. Taking $C_m = 0.6$, the new value of A from Equation (22) is 13 400 mm^2. Repeat the calculation with a new trial value of $A = (11\,000 + 13\,400)/2 = 12\,200$ mm^2.

The correct value of A is 12 420 mm^2 for $t = 10$ mm and $C_1 = 0.33$. For this design, $A_f = 4100$ mm and $A_w = 4220$ mm^2 giving the theoretical flange plate size as 11.3 mm × 362 mm and the web plate as 10 mm × 422 mm. Use flange plate 12 mm × 360 mm and web plate 10 mm × 420 mm.

CONCLUSIONS

Design procedures based on the AISC specification are presented to find the optimum proportioning of built-up homogeneous steel I-sections for columns and beam-columns. While the cross section of a column can be determined directly from the prescribed method, which is relatively simple and straightforward, that of a beam-column is based on a search procedure. The simple iterative scheme can be computerized easily to obtain an optimum I-section of a beam column readily. For a rapid economical design of a beam-column by hand calculation, the suggested procedure as illustrated by an example can be used.

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