# A LASER METHOD FOR PRECISION ALIGNMENT OF STRUCTURE AXES 

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#### Abstract

الملاصة :

لقد أمكن استحداث طريقة جديدة لتعين وتوقي أي عدد من النقاط على ابجاه ععين بنقطتين وذلك باستخدام أشعة الليزر ذات الموجات المستمرة وذلل بعل تجنب الصعوبة النابجة من ثرير محور الأشعة بالنقطتين الحددتين للْإنجاه . إن شعاع اللينذ قّد إستخدم في هذه الطريقة كحط مساعد تنسب إليه النقطتين اللسابق ذكرهما وعليه فقد أصبح من الممكن تعيين النقط المجيدة على امتداد هاتين النقطتين .  يككن تنفيذ عمليات التخطيط بدقة زاوية تصل إلى Y, • ثانية . وتعتبر الطريقة البلديدة مفيدة للغاية لتخططط عحاود المنثآت الهندسية بدقة بالغة


#### Abstract

A method has been developed, using CW lasers, to determine and locate any number of points on a direction given by two fixed points. The difficulty of bringing the centroid of energy of the laser beam into coincidence with the two fixed points is avoided. The laser beam is used as an auxiliary line and the correlation between the laser and the given direction is determined. It is thus possible to locate the new point on the desired line. A prototype was constructed and a special technique used for this alignment procedure. Several experiments proved that the accuracy attainable, using the presented technique, is in the order of $0.2^{\prime \prime}$. The new method is very useful for precision alignment of the structure axes of different engineering projects.


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## INTRODUCTION

The task of precise alignment has been under extensive investigation during the last few years. The importance of this task is increasing with the increase of construction and highly sophisticated projects in modern and developingocountries. Examples of these are found in the direction definition of linear accelerators and the alignment of the axes of radio telescopes. In civil engineering, the application of precise alignment techniques shows its merit in laying out bearings of cement furnaces, hinged bridges, and crane tracks. In mines, precision alignment is essential for rock deformation measurement and for direction extension in tunnels, shafts, and mine drifts.

In alignment operations, it is usually required to place one or more points on, or along, a line defined by two points. In addition to the required accuracy of $0.2^{\prime \prime}$ or better, simple operation, low cost, and simple equipment are also needed.

There are already different methods for alignment purposes. Each method has its limitations and drawbacks. The simplest, of course, is alignment with theodolites [1]. The accuracy of this method depends chiefly on the experience of the observer in the estimation of averaging the quivering image of the target with the cross-mark of the graticule. The standard deviation of a single pointing is given by $\sigma_{p}=a / M$, where $a=40^{\prime \prime}$ to $60^{\prime \prime}$ and $M$ is the telescope magnification. Therefore, if the magnification is 20 the accuracy is between $2^{\prime \prime}$ and $3^{\prime \prime}$, depending on the observer. The use of the Autocollimator and Sphere Method or the Axicon Method [2] decreases the standard deviation to about $0.3^{\prime \prime}$ but decreases the operational range to a few meters. Using a laser with Koster's prism [3] or zone plates [4] increases the range to tens of meters but it does not solve the main problem of bringing the centroid of energy to the given two points. Also, the zone plate should be changed according to the distance at which the new point is required; accordingly, a certain error is introduced each time, in addition to the cost of the numerous zone plates required for alignment.

## MATHEMATICAL MODEL OF THE TECHNIQUE

The geometry of the technique is illustrated in Figure 1. Points $a$ and $b$ are two given points represent-
ing a fixed direction. Point $n$ is required to be aligned on this direction at a certain distance from $a$. An auxiliary line $O d g$ is assumed so that it is as close as possible to the line $a b$. Oef is the line of intersection of two planes. The first plane is the vertical one through the line $a b$ and the second is the plane through the reference line Odg which is so chosen that the line of its intersection with any plane perpendicular to the reference line is horizontal. The verticals through points $a$ and $b$ meet with the line of intersection of the two planes at points $e$ and $f$, respectively. The perpendiculars from $e$ and $f$ to the auxiliary line are horizontal and lie in the second plane.

The $y$-axis of the coordinate system coincides with the auxiliary line. It intersects the vertical plane through $a b$ in the origin of the system $O$. The $x$-axis is horizontal and directed towards $a b$.

From the geometry of the figure, distances $x_{n}$ and $h_{n}$ can be calculated from

$$
x_{n}=x_{b}+\left(x_{b}-x_{a}\right) \frac{Y_{n}-Y_{b}}{Y_{b}-Y_{a}}
$$

and

$$
h_{n}=h_{b}+\left(h_{b}-h_{a}\right) \frac{Y_{n}-Y_{b}}{Y_{b}-Y_{a}}
$$

where the variables $h$ are the vertical distances to the plane through the auxiliary line. The first equation is used for horizontal alignment. For vertical alignment the second equation can be used. But since the latter is always possible using precise leveling, only horizontal alignment is dealt with in this paper. With this understanding, a system of points is aligned horizontally when they lie in the same vertical plane. In these two equations, the measurables are $x_{a}, h_{a}, x_{b}$, and $h_{b}$. $\left(Y_{b}-Y_{a}\right)$ is the distance between the two given points and $\left(Y_{n}-Y_{b}\right)$ is the distance from $b$ at which the new point is determined.
The two lines $d g$ and ef may be either divergent, convergent, parallel, or intersecting within their lengths. It can be seen that the solution is applicable for any of these cases.

## TECHNICAL MODEL AND OPERATION

In the technical model, the auxiliary line is represented by the pass of the centroid of energy of a CW, $\mathrm{He}-\mathrm{Ne}, \mathrm{TEM}_{00}$ laser (see [1] and [5]). A prototype of


Figure 1. Different Arrangements of Alignment
an alignment instrument was designed and built (Figures 2 and 3).

It consists of two parts: the first part is a vertical column connected to a tribrach with a built-in optical plummet; the second part is an arm connected perpendicularly to the column by means of a sleeve. The vertical movement of the arm along the vertical
column is maintained by means of a screw-and-nut mechanism. The arm is adjusted horizontally by the use of three leveling screws connected to the tribrach and by the aid of a spirit level fixed on the arm. An optical pentaprism is mounted so that one of its perpendicular surfaces is facing the column. The pentaprism can be moved along the arm with the help of a rack and pinion mechanism and the amount of movement


Figure 2. Field Work Procedure
is read on a millimeter scale and a vernier with a resolution of 0.05 mm . A metallic plate with a pinhole in the middle is mounted on the first surface of the pentaprism. The pinhole is surrounded by four photocells connected in pairs to a null detector (see p. 138 of
[1] for a circuit diagram). A cross-mark is fixed on the sleeve in front of the other surface of the pentaprism. The instrument is calibrated so that the readings on the scale represent the perpendicular distances from the center of the pinhole to the axis of the vertical column. The instrument can be mounted on a tripod by a screw through the tribrach.

## ALIGNMENT OPERATION

The optical cavity of the laser reaches a stable condition some time after switching on the laser. Accordingly, it is recommended to start the alignment operation at least half an hour after switching on the laser.

The laser is placed close to the first given point and directed along the given line so that perpendicular distances from any point on the line to the laser pass are not more than the length of the horizontal arm of the instrument (about 40 cm ). Then the instrument is centered and leveled above the first point (point a). The front surface of the pentaprism is directed to face the laser beam which passes through the pinhole,


Figure 3. Plan View of the Prototype and Photocells Arrangement
changes its direction by $90^{\circ}$, and travels along the horizontal arm toward the cross-mark. With the pinhole as close as possible to the center of the laser spot, the reflected narrow laser beam is made to coincide with the cross-marks by rotating the arm in the horizontal plane around the vertical column. The slow motion screw is used to move the pentaprism along the arm until the differential signal indicated on the null meter and generated by the pairs of photocells along the two orthogonal axes vanishes. The distance from the centroid of energy of the laser spot to the axis of the column, $x_{a}$, is read directly on the scale fixed on the arm.

The distance $X_{b}$ is obtained by following the same procedure, and distance $x_{n}$ can be calculated to be set at a distance $Y_{n}-Y_{b}$ from point $b$. To locate point $n$, the instrument is adjusted at the required distance close to the laser beam, say at $n^{\prime}$, and the corresponding distance $x_{n}$, is measured as explained previously. The direction of the perpendicular from point $k^{\prime}$ to the laser beam, which is the direction of the arm, is projected to the same level as $n^{\prime}$ by using either a plumb line or an additional optical plummet. Point $n$ is obtained by setting the distance $x_{n}^{\prime}-x_{n}$ from point $n^{\prime}$ along the line $n^{\prime} m^{\prime}$.

## ACCURACY EVALUATION

The function for the $x$-determination is given by

$$
x_{n}=x_{b}-\left(x_{b}-x_{a}\right) \frac{Y_{n}-Y_{b}}{Y_{b}-Y_{a}}
$$

and the distance $x_{n}^{\prime}-x_{n}$ is given by

$$
\left(x_{n^{\prime}}-x_{n}\right)=x_{n^{\prime}}-\left[x_{b}-\left(x_{b}-x_{a}\right) \frac{D_{n}}{D_{b}}\right]
$$

where

$$
\begin{aligned}
& D_{n}=Y_{n}-Y_{b} ; \text { and } \\
& D_{b}=Y_{b}-Y_{a} .
\end{aligned}
$$

Hence

$$
x_{n}=x_{b}-\left(x_{b}-x_{a}\right) \frac{D_{n}}{D_{b}} .
$$

The measurements are $x_{a}, x_{b}, D_{b}$, and $D_{n}$. Their variance can be estimated from experiments and from error source analysis of each quantity. From the Gaussian law of error propagation for independent measurements, the variance of the distance ( $x_{n^{\prime}}-x_{n}$ ),
$\sigma_{x_{n} n_{n}}^{2}$, can be derived as

$$
\begin{aligned}
\sigma_{x_{n} n_{n}}^{2}=\left(\frac{D_{n}}{D_{b}}\right)^{2} & \sigma_{x_{a}}^{2}+\left(1-\frac{D_{n}}{D_{b}}\right)^{2} \sigma_{x_{b}}^{2} \\
& +\left[\frac{D_{n}\left(x_{b}-x_{a}\right)}{D_{b}}\right]^{2} \sigma_{D_{b}}^{2}+\left(\frac{x_{b}-x_{a}}{D_{b}}\right)^{2} \sigma_{D_{n}}^{2}+\sigma_{x_{n}}^{2} .
\end{aligned}
$$

Assuming that the values of $x$ were measured with the same accuracy, $\sigma_{x}$, the same goes for the values of $D$; therefore, the variance of alignment is

$$
\sigma_{x_{n^{\prime} n}}^{2}=\left[\left(\frac{D_{n}}{D_{b}}\right)^{2}-\frac{D_{n}}{D_{b}}+1\right] 2 \sigma_{x}^{2}+\left(\frac{D_{n}^{2}+D_{b}^{2}}{D_{b}^{4}}\right)\left(x_{b}-x_{a}\right)^{2} \sigma_{D}^{2}
$$

## Accuracy of Measuring the Distances

It is seen from the above relation that the accuracy increases with the decrease of the ratio $D_{n} / D_{b}$ and the difference between $x_{b}$ and $x_{a}$. It is therefore recommended to increase the accuracy of the measurement of $D$ when $D_{n}$ is larger than $D_{b}$. Another way to decrease the second portion of the equation when $D_{n} / D_{b}$ is too large is to decrease $x_{a}$ and $x_{b}$. This can be attained in the field by placing reflectors at points $a$ and $b$ (e.g. luminous tapes) for approximate pointing of the laser beam. Experiments showed that these distances can be reduced easily to less than 10 cm .

The accuracy of $x_{a}$ and $x_{b}$ determinations depends on centering of the laser spot with the pinhole. Accuracy of centering depends in turn on the method of centering and on the weather conditions. Extensive work is this field [5] produced several sorts of centering detector (space integrating, time, and self-aligning). An accuracy of $0.5 \times 10^{-6}$ rad over 1 km in all bad weather conditions was obtained with the second and the third detectors. With these kinds of detector, if the alignment is carried out at an average distance of 1 km and the accuracy ( $1 \sigma$ ) of $D$ is 0.1 m , an average value of $\sigma_{x_{n, n}}$ is calculated from the formula previously derived and found to be 1 mm . The corresponding angular standard deviation of the new location at 1 km distance is therefore $0.2^{\prime \prime}$ (i.e. $10^{-6} \mathrm{rad}$ ).

## Effect of Side Refraction

Side refraction affects the direction of the laser pass. Hence, if the laser beam passes through vertical layers of atmosphere that have a certain temperature
gradient the direction of the beam will change in the horizontal plane and the alignment will be affected. Such a condition may exist when the laser beam passes close to an object which is of a different temperature from that of the atmosphere. Hence, the effect of refraction on the laser beam decreases with increase of the distance from that object. Several experiments were carried out in different field conditions and at extreme temperature gradients. It was found, in all cases, that the effect of the object temperature diminished to nil at a distance of 1 m . Accordingly, the influence of side refraction can be neglected if the laser beam passes at a distance more than 1 m from the object. Such a condition can easily be fulfilled in the
field since, in the present technique, the laser beam direction is expressly chosen to be away from the alignment direction itself.

## ANALYSIS OF INSTRUMENTAL ERRORS

These errors result from maladjustment of the instrument. Also, the analysis of such errors determines the necessary resolving power of each part of the instrument. In this respect, three sources of error are investigated, as follows.


Figure 4. Effect of Error of the Optical Plummet

## Errors of the Optical Plummet

When the spirit level is leveled, the line of sight of the optical plummet mounted in the tribrach should coincide with the axis of the vertical shaft. The angular deviation will introduce changes to the values of $x$, depending on the height of the plummet above the points to be aligned. The effect of this systematic error can be cancelled by rotating the tribrach through $180^{\circ}$, releveling the instrument, and taking another value of $x$. The average, $x_{n}^{\prime}$, will be free from this error (Figures 4(a) and 4(b)). The distance $x_{n}^{\prime}-x_{n}$ is set from point $n_{1}^{\prime}$ toward the laser to locate point $n$.

## Errors Due to Misleveling of the Horizontal Arm

If the arm has a small inclination $\delta_{1}$ the distance $x_{1}^{\prime}$ will be read on the scale instead of $x_{1}$ (Figure 5). If the corresponding vertical distance from the pinhole is $k_{1}$ the resulting error $\Delta x_{1}$ in the measured distance can be calculated from

$$
\Delta x_{1} \cdot 2 x_{1}=k_{1}^{2}=x_{1}^{2} \delta_{1}^{2}
$$

hence

$$
\Delta x=0.5 x_{1} \delta_{1}^{2} .
$$

Therefore, if $\delta_{1}=10^{\prime}$ and $x_{1}=20 \mathrm{~cm}$, then $\Delta x_{1}=0,001 \mathrm{~mm}$. This means that even with such misleveling, the expected error in the measured distance is not serious.


Figure 5. Effect of Misleveling of the Horizontal Arm

## Effect of Collimation Errors of the Reflected Laser

If the reflected laser spot from the pentaprism was not made to coincide with the cross-mark by a distance $k_{2}$, an error $\Delta x_{2}$ will be introduced. $\Delta x_{2}$ is the difference between $x_{2}^{\prime}$ and $x_{2}$ (Figure 6) and can be calculated from

$$
\Delta x_{2}=k_{2}^{2} / 2 x_{2} .
$$

So if $k_{2}=2 \mathrm{~mm}$, and $x_{2}=10 \mathrm{~cm}$, then $\Delta x_{2}=0.2 \mathrm{~mm}$.
It is clear from the above analysis that the resulting


Figure 6. Effect of Displacement of the Laser Spot
errors $\Delta x_{1}$ and $\Delta x_{2}$ are not serious if some care is taken in leveling the horizontal arm and in bringing the reflected laser spot into coincidence with the crossmark.

## FIELD EXPERIMENTS

Two stations $A$ and $B$ were established 500 m apart. The prototype was used to determine the location of three new points $C, D$, and $E$ so that $A C=C B=$ $B D=D E=250 \mathrm{~m}$. Accordingly, point $C$ will fall between $A$ and $B, D$ and $E$ will fall along the line $A B$, and all the points should fall on the same vertical plane as $A$ and $B$. The horizontal distances between the points were measured with a steel tape giving an accuracy ( $1 \sigma$ ) of 5 cm . To conduct the alignment, a collimated $\mathrm{He}-\mathrm{Ne}, 2 \mathrm{~mW}, \mathrm{TEM}_{00}$ laser was set about 10 m from station $A$ at five different locations. At each location the laser was projected at a different angle to the line $A B$ (Figure 7). Hence five different alignment operations were carried out. In each alignment operation the perpendicular distance, $x$, from any of the points to the laser beam did not exceed 25 cm . Each alignment operation was repeated ten times by changing the laser direction by a few are seconds. The corresponding discrepancies in the $x$-determinations were measured with the help of a millimeter scale having a 0.05 mm vernier. Hence it was possible to compute the standard deviation of the point location on the basis of the discrepancies in the values of $x$ obtained at each alignment operation. Table 1 shows the results. Table 1 also shows the standard deviation of the alignment angle that is included between $A B$ and $B C, B D$ and $B E$. A high precision theodolite (Wild T3) was used to check the gross errors. The theodolite was located at station $B$ and points $C, D$, and $E$ were determined after observing station $A$. The instrumental errors were cancelled by conducting the observations in each of the two theodolite positions. A complete coincidence was obtained at the three stations $C, D$, and $E$ between


Figure 7. Experimental Arrangement

Table 1. Alignment Accuracy for the Points $C, D$, and $E$

| Point | Standard deviation (mm) |  |  |
| :---: | :---: | :---: | :---: |
|  | $C$ | $D$ | $E$ |
| 1 | 0.15 | 0.2 | 0.50 |
| 2 | 0.15 | 0.15 | 0.40 |
| 3 | 0.25 | 0.25 | 0.65 |
| 4 | 0.20 | 0.25 | 0.70 |
| 5 | 0.20 | 0.30 | 0.70 |
|  | 0.19 | 0.23 | 0.59 |
|  |  |  |  |
| Average accuracy (mm) <br> Accuracy of angular <br> alignment (arc seconds) | 0.16 | 0.19 | 0.24 |

the points determined by the theodolite and those obtained from the laser arrangement.

The discrepancies were recorded, and hence the standard deviation was calculated at the three stations to be $0.05 \mathrm{~mm}, 0.20 \mathrm{~mm}$, and 0.32 mm respectively. Accordingly, the angular standard deviation did not exceed $0.2^{\prime \prime}\left(10^{-6} \mathrm{rad}\right)$ in any of the three cases.
Several other alignment operations were carried out using the same arrangement and similar results were obtained.

## CONCLUSION

The results obtained from the present technique and prototype have shown that an accuracy ( $1 \sigma$ ) in the
order of $0.2^{\prime \prime}$ or better is achievable in establishing any new point concurrent with two given points. The equipment is inexpensive and simple to operate. The average time required for an alignment operation is 30 min .

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## REFERENCES

[1] F. A. Ahmed, Ph.D. Dissertation, UNB, Canada, June 1970, unpublished.
[2] R. O. Naess, 'Methods for Precise Alignment', Applied Optics, 7 (1968).
[3] R. A. Woodson, 'Using Laser and Koster's Prism for Alignment', Engineering and Instrumentation, 67C (1953).
[4] A. C. Van Heel, 'Precise Alignment Using Zone Plates', Transactions in Instrument Technology, 2 (1962).
[5] A. Chrzanowski and F. Ahmed, 'Alignment Surveys in a Turbulent Atmosphere Using a Laser', Proceedings of the American Congress of Surveying and Mapping, Washington DC, 12 (1971), pp. 494-513.

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