OPTIMUM COLUMN FOOTING UNDER BIAXIAL BENDING

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INTRODUCTION

Most designers design a single column spread footing following the guidelines of a building code such as, for example, ACI [1] and the procedure similar to that prescribed in Reference [2]. For the case of a concentrically loaded column footing, the optimum footing size is a square. For such footings, Furlong [3] has proposed design aids that include the thickness of the footing slab necessary to avoid shear reinforcement. For the design of rectangular footings, Henye [4] has produced nomographs. A computer program for the optimum design of a concrete spread footing has been presented by Kohli [5].

For a column footing loaded eccentrically about an axis or a column footing subjected to an axial load and a moment, the determination of an optimum footing is relatively difficult, and most designers use trial and error procedures. Davis and Mayfield [6] have prepared design charts to aid designers to select readily the plan dimensions of a footing subjected to an axial load and a moment. It should be noted that such plan dimensions may not lead to an optimum footing size from the viewpoint of minimum concrete volume.

In this technical note, a procedure is outlined to determine the optimum footing size for a single column subjected to an axial load and biaxial bending moments. The problem has been dealt with in two parts. In the first part, only the optimum plan dimensions on the basis of the minimum plan area are considered. In the second case, the optimum footing size is determined considering the minimum concrete volume as the optimality criterion and satisfying the assumed constraint that no shear reinforcement would be necessary in accordance with the ACI [1]. In the absence of accurate cost data, it is simpler and more appropriate to consider the minimum concrete volume rather than the total cost of the footing as the objective function, as the latter case would include the cost of the reinforcing steel and the form work. Linear distribution of soil pressure is assumed throughout with no uplift of the footing.

THEORETICAL CONSIDERATIONS

Optimum Plan Dimensions

Figure 1 shows a footing of size $B \times H$ $(B > H)$ subjected to an axial load P and moments M_x and M_y $(M_x \geq M_y)$. Assuming linear distribution of soil pressure, the following conditions can be stipulated to satisfy the upper and lower bounds of soil pressure:

$$\frac{P}{A}\left(1+\frac{6e_x}{B}+\frac{6e_y}{H}\right) \leq q_0, \tag{1}$$

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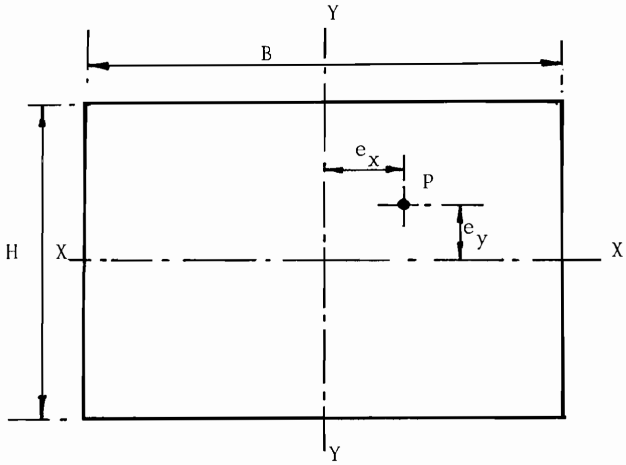

Figure 1. A Typical Footing under Eccentric Load

and

$$\frac{P}{A}\left(1-\frac{6e_x}{B}-\frac{6e_y}{H}\right)\geq 0, \tag{2}$$

in which $A=BH$; q_0 = permissible soil pressure under service loads, $e_x=M_x/P$, and $e_y=M_y/P$. Setting $\delta=B/H(\delta\geq 1)$, the aspect ratio of the footing, Equations (1) and (2) can be rearranged as

$$A\geq\frac{P}{q_0}\left[1+\frac{6(e_x+\delta e_y)}{\delta H}\right], \tag{3}$$

and

$$H\geq\frac{6(e_x+\delta e_y)}{\delta}. \tag{4}$$

Substituting $\delta=A/H^2$ in Equation (3), a quadratic equation in A and a cubic equation in H can be obtained for the minimum values of A and H:

$$A^2-\frac{P}{q_0}\left(1+\frac{6e_y}{H}\right)A-6e_x H\frac{P}{q_0}=0, \tag{5}$$

and

$$\delta H^3-\frac{P}{q_0}H-\frac{6(e_x+\delta e_y)}{\delta}\frac{P}{q_0}=0. \tag{6}$$

Differentiating the value of A from Equation (5) with respect to H, it can be shown that for the minimum value of A

$$\delta=e_x/e_y. \tag{7}$$

Thus, for given values of e_x and e_y, the minimum area corresponds to the case when the aspect ratio δ is equal to the ratio of the eccentricities, e_x/e_y. This shows that for small e_y, the long and narrow footing would yield theoretical minimum area, a solution that would obviously be unacceptable in most cases from a practical and economic viewpoint.

Using the relationship in Equation (7), the optimum value of H can then be obtained from Equation (6) by taking the lowest real root of H. However, the chosen value of H must not be less than $12e_y (H\geq 12e_y)$ to satisfy the constraint in Equation (4).

It should be recognized that, for a design, a maximum value of $\delta(\delta\geq 1)$ should be established to avoid a long length which would increase the cost of reinforcing steel. To indicate the variation in the footing area and its sensitivity with δ, Figure 2 is plotted with $P=100$ kips (445 kN), $q_0=3\text{k/ft}^2$ (143.6 kN/m²), $e_x=0.5$ ft (0.152 m), and $e_y=0.3$ ft (0.091 m). The minimum area corresponds to the value of δ as being the ratio of e_x/e_y, which is 1.67. As the area is not very sensitive, and a lesser value of δ would be more practical and desirable, an acceptable range of δ in this case can be taken as 1.2 to 1.3. It should be noted that the optimum plan dimensions may not lead to the optimum footing on the basis of minimum concrete volume.

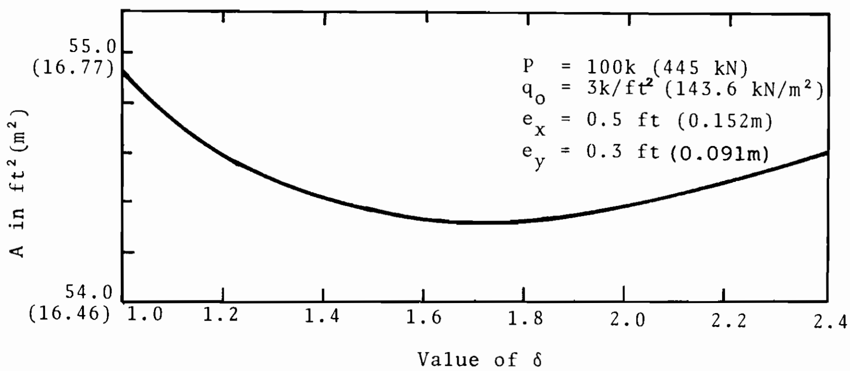

Figure 2. Area of Footing versus δ

OPTIMUM FOOTING

The optimum footing is determined by minimizing the concrete volume and assuming that no shear reinforcement will be required for the footing. The ultimate shear strength of a single footing as prescribed in Reference [1] is based on the two criteria: (a) two-way slab action and (b) beam action. Considering the former case and referring to Figure 3, it can be shown that to avoid shear reinforcement, the effective depth of the footing slab, d, for a column of $a \times b$ can be

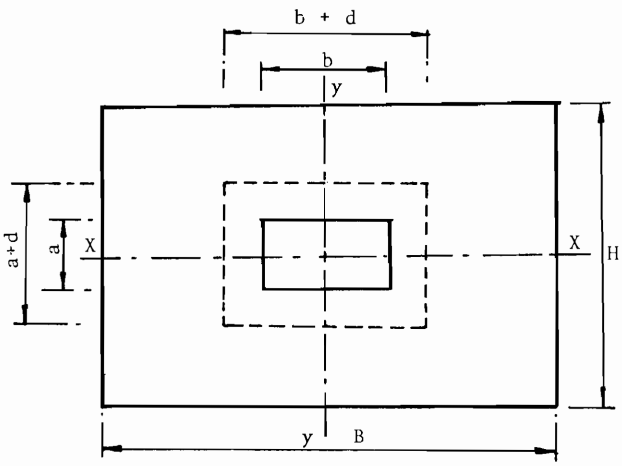

Figure 3. Critical Sections for Two-way Slab Action

determined from

$$\left(\frac{P_u}{\delta H^2}+16\phi\sqrt{f_c'}\right)d^2+(a+b)\left(\frac{P_u}{\delta H^2}+8\phi\sqrt{f_c'}\right)d$$

$$+P_u\frac{ab}{\delta H^2}-P_u=0, \qquad (8)$$

where P_u = ultimate axial load on column, f_c' = ultimate compressive strength of concrete in pounds per square inch, and ϕ = strength reduction factor. Equation (8) is formulated by equating the total shear on the area of $[BH-(a+d)(b+d)]$ to the shear strength of the slab, computed using a permissible stress of $4\sqrt{f_c'}$ for $B/H \leq 2$ as prescribed in the ACI and an effective area of $2(a+b+2d)d$.

For the beam action, two critical sections must be considered, as both the footing and the column are assumed to be rectangular. Considering a distance d from the shorter face of the column (Figure 4), it can be shown that the value of d should satisfy the following equation to avoid any shear reinforcement:

$$d=\frac{-K_3+\sqrt{(K_3^2+4K_1K_2L)}}{2K_2}, \qquad (9)$$

where

$$K_1=\frac{3bM_{ux}}{\delta^3H^4}+\frac{3M_{ux}}{\delta^2H^3}+\frac{P_u}{\delta H^2}, \qquad (10a)$$

$$K_2=\frac{6M_{ux}}{\delta^3H^4}, \qquad (10b)$$

and

$$K_3=2\phi\sqrt{f_c'}-K_2L+K_1. \qquad (10c)$$

For the beam action, the permissible shear stress in the footing without shear reinforcement is taken as $2\sqrt{f_c'}$. Likewise, for the critical section at a distance d from the longer face of the column (Figure 4), the value of d can be calculated from

$$d=\frac{-C_3+\sqrt{(C_3^2+4C_1C_2N)}}{2C_2}, \qquad (11)$$

in which

$$C_1=\frac{3aM_{uy}}{\delta^3H^4}+\frac{3M_{uy}}{\delta^2H^3}+\frac{P_u}{\delta H^2}, \qquad (12a)$$

$$C_2=\frac{6M_{uy}}{\delta^3H^4}, \qquad (12b)$$

and

$$C_3=2\phi\sqrt{f_c'}-C_2N+C_1. \qquad (12c)$$

Thus, the required effective depth is the maximum value of d obtained from Equations (9)–(11). The total depth of the footing, D, is the effective depth plus the effective cover to the reinforcement.

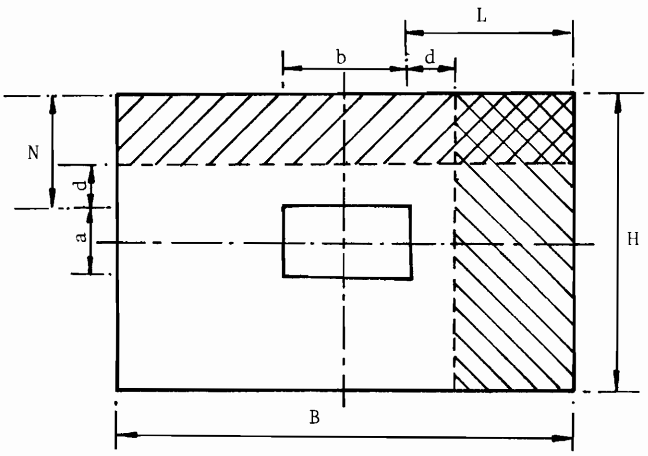

Figure 4. Critical Sections for Beam Actions

SEARCH PROCEDURE

For given input data, the optimum column footing can be determined by a simple search technique which can easily be programmed in a small computer. The procedure starts with the initial value of $\delta = 1.0$ (square footing). The value of H for this assumed value of δ is calculated from Equation (6) and satisfying the constraint of Equation (4). Using Equations (9)–(11), the maximum value of d is determined to avoid shear reinforcement. Adding effective cover to d, the depth of the slab, D, and hence the volume of the footing are evaluated.

The value of δ is then increased by a small step and the new volume is calculated and compared with the previous value. The procedure is repeated with new increased values of δ until and unless the optimum value of δ is reached at which the volume is a minimum. The dimensions of the footing, B, H, and D, represent the optimum footing on the basis of the minimum concrete volume.

Example

Determine the optimum footing for a column of 12 in × 18 in (305 × 457 mm) using the following data:

$$P = 100 \text{ kips } (445 \text{ kN}),\ P_u = 170 \text{ kips } (756 \text{ kN}),$$
$$M_x = 50 \text{ ft-K } (67.8 \text{ kN.m}),\ M_y = 30 \text{ ft-K } (40.7 \text{ kN.m}),$$
$$M_{ux} = 85 \text{ ft-K } (115.3 \text{ kN.m}),\ M_{uy} = 50 \text{ ft-K } (67.8 \text{ kN.m}),$$
$$q_0 = 3k/\text{ft}^2 \ (143.6 \text{ kn/m}^2),\ f'_c = 3000 \text{ psi } (20.7 \text{ MPa}),$$

and effective cover = 3 in (75 mm).

The variation in the volume of the footing with various values of δ up to 2.0 is depicted in Figure 5. The minimum volume is 53.1 ft^3 (1.504 m^3) and the corresponding optimum footing dimensions are $B = 7.73$ ft (2.356 m), $H = 7.02$ ft (2.140 m), and $D = 0.98$ ft (0.299 m). The value of δ at which the optimum volume is obtained is 1.1.

CONCLUSIONS

A simple procedure has been prescribed to aid designers in determining the optimum footing for a single column subjected to biaxial bending. It has been shown that the optimum plan dimensions on the basis of minimum footing area are obtained when the ratio of the length to width of the footing equals the ratio of the eccentricities of the applied load with respect to the two principal axes. However, for all practical purposes, the value of δ should not be too large for an economical design. The prescribed procedure for seeking the optimum footing dimensions that do not require any shear reinforcement can be programmed easily in a small computer to obtain the footing size conveniently.

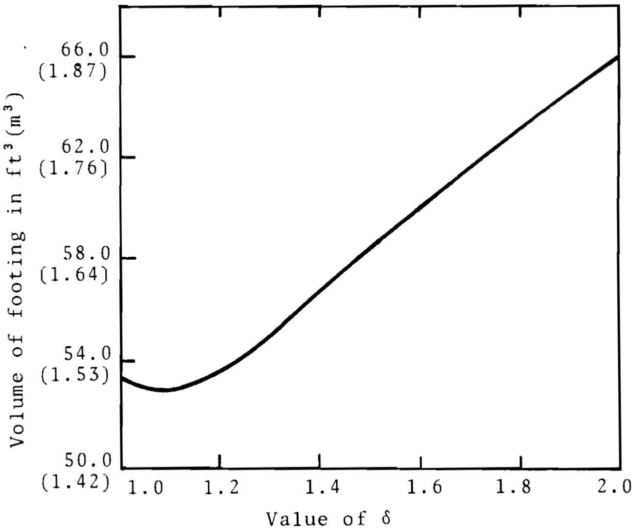

Figure 5. Volume of Footing versus δ for Example Footing

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