LOAD IMPEDANCE REQUIREMENTS FOR A STAND-ALONE INDUCTION GENERATOR

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الخلاصة :

تُعاني المولِّدات التأثيرية ذاتية الإثارة من عدة مشاكل أهمها اعتماد قيمة وتردد الجهد الناتج من المولِّد على سُرعة دوران المولِّد ، وسعة المكثف المتصل بها المولِّد ، وقيمة معاوقة الحمل الكهربائي.

ولهذا يقدِّم البحث دراسة مستفيضة عن تأثير معاوقة الحمل على استمرارية التغذية الذاتية للمولد والتي تضمن أداء المولد ضِمن سُرعات مختلفة ولمعامل قدرة متغير .

ولقد أوضحت النتائج القيمة الصغرى للمعاوقة عند كلِّ سرعة ولمعامل قدرة متغير . وباعتبار هذا الحد الأدنى للمعاوقة أمكن الحصول على أداء المولد أثناء حالة الاستقرار، وكذلك شكل موجة التيار العابر الخارج من المولد .

ABSTRACT

One of the major problems associated with self-excited induction generators is the dependency of the output voltage and frequency on the speed, terminal capacitance, and load impedance. This paper presents a detailed study of the effect of load impedance on the continuity of the self-excitation process over a wide range of speed and power factor. By satisfying the load impedance requirements, both the steady state performance and the transient output current response are computed and presented.

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LIST OF SYMBOLS

a	Per unit frequency (output frequency/rated frequency)
b	Per unit speed (rotor speed/rated synchronous speed)
R_1, R_2	Stator and rotor resistance per phase referred to stator, ohm
X_1, X_2	Stator and rotor reactance per phase at rated frequency referred to stator, ohm
X_m	Magnetizing reactance per phase at rated frequency, ohm
E_g	Air-gap voltage, V
X_{smax}	Maximum unsaturated magnetizing reactance at rated frequency, ohm
C	Terminal self-excitation capacitance per phase, F
C_{min}	Minimum value of capacitance required for self-excitation, F
C_{max}	Maximum value of capacitance required for self-excitation, F
X_c	Self-excitation capacitive reactance per phase at rated frequency, ohm
R	Load resistance per phase, ohm
X	Load reactance per phase at rated frequency, ohm
V_{out}	Terminal voltage across the load per phase, V
l_1	Stator leakage inductance per phase
M	Mutual inductance per phase
ω	$2\pi f$
f	Rated frequency of the machine, Hz
s	Laplace operator
V_{pu}	(V_{out}/rated voltage per phase)
I_{pu}	(load current/rated current).

INTRODUCTION

Considerable interest has recently been devoted to the study of the steady-state performance of the self-excited stand-alone induction generator [1–12] . Salama [3] has examined the role of the magnetizing reactance and the self-excitation capacitance. It has been stated that the load power-factor influences the values of the magnetizing reactance and the capacitance range necessary for self-excitation process.

Al Jabri and Alolah [4] have discussed the speed limits for a fixed terminal capacitance. It has been shown that there is an optimum choice for the speed, terminal capacitance, and load values that gives maximum output power. However, they concluded that in practical situations, such as generators driven by wind power, the driving speed is not controllable.

Since the practical operating conditions involve variable speed at a preferable fixed value for the self-excitation capacitance, the choice of the load impedance and load power-factor usually have an important effect on the steady state performance of the generator. Therefore, this paper is dedicated to a study of the limiting values of load impedance for a wide range of power factors and at different rotor speeds. Also, it presents the transient response of load current when a generator is driven over a wide range of speed and switched to a load of variable power factor.

A MATHEMATICAL MODEL

The analysis pursued hereafter is based on the assumptions that both the initiation and the transient build-up processes of the air gap voltage have been successfully achieved. Thus the air gap induced emf has built up to its rated no-load value (E_g). The steady-state equivalent circuit of an isolated self-excited induction generator which feeds a balanced 3-phase load is shown in Figure 1. The various reactances presented in the circuit and used in the analysis are corresponding to rated frequency. This equivalent circuit is further simplified in Figure 2. The new parameters R_L and X_L, which depend on the type of load connected at the terminals of the generator as well as the value of capacitance necessary for the self-excitation process, are presented in Appendix 1.

Applying node-voltage analysis for the circuit shown in Figure 2, we get:

$$E_g Y_1 + E_g Y_2 + E_g Y_3 = 0 \tag{1}$$

since $E_g \neq 0$, thus

$$Y_1 + Y_2 + Y_3 = 0. \tag{2}$$

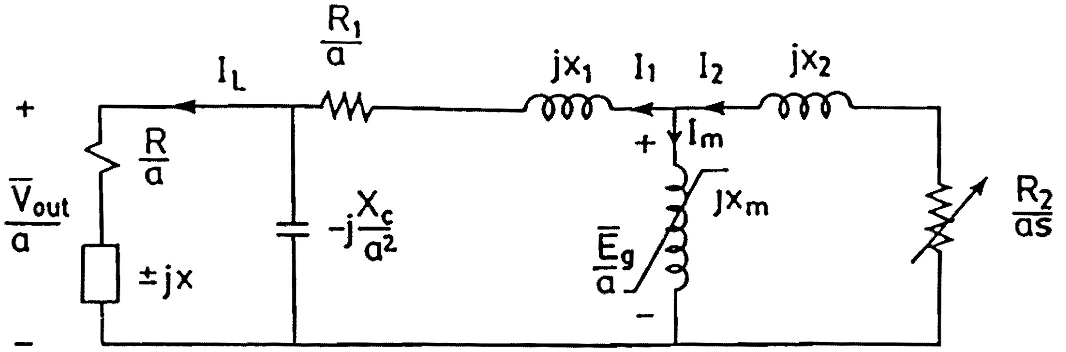

Figure 1. The Steady-State Equivalent Circuit of an Induction Generator.

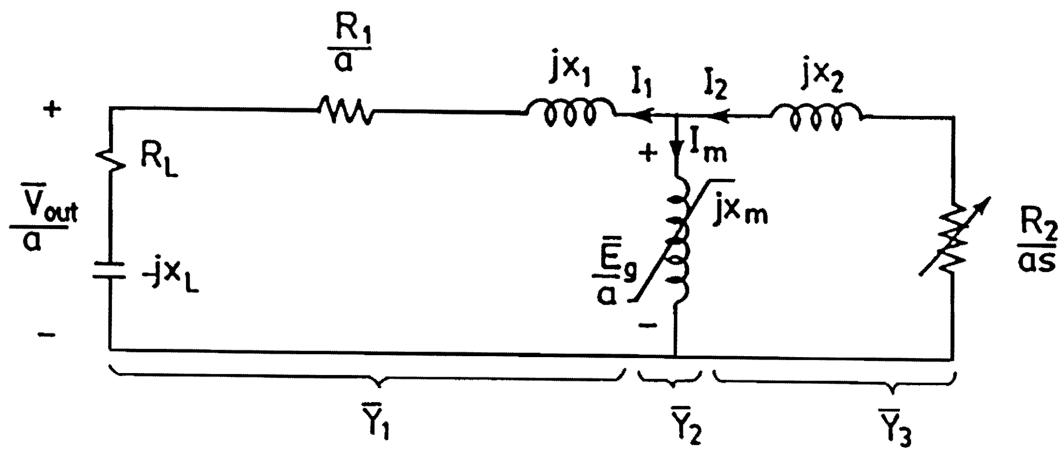

Figure 2. A Simplified Equivalent Circuit of an Induction Generator.

Equating the real and reactive parts of Equation (2) to zero, the following expressions are obtained:

$$\frac{R_L + (R_{1/a})}{[R_L + (R_{1/a})]^2 + (X_1 - X_L)^2} + \frac{[R_2/(a-b)]}{[R_2/(a-b)]^2 + X_2^2} = 0 \tag{3}$$

and

$$\frac{1}{X_m} = \frac{X_2}{[R_2/(a-b)^2] + X_2^2} - \frac{X_1 - X_L}{(R_L + R_{1/a})^2 + (X_1 - X_L)^2} = 0. \tag{4}$$

For a specified load impedance, a given self-excitation capacitance and a known operating speed, Equation (3) is rewritten as a polynomial in terms of the per unit frequency 'a':

$$p_7 a^7 + p_6 a^6 + p_5 a^5 + p_4 a^4 + p_3 a^3 + p_2 a^2 + p_1 a + p_0 = 0. \tag{5}$$

The coefficients of the polynomial (p_0 up to p_7) for no-load, pure resistive, inductive and capacitive loading conditions are presented in Appendix 2. Solution of the polynomial will give the frequency of the output signal. The value of frequency obtained together with an optimization technique are used to search for the minimum value of load impedance using Equations 3 and 4. Such a search technique is carried out for the following constraints:

1. $n_{rotor} \geq n_s$.

   This inequality constraint is to ensure generator action for the induction machine.

2. $C_{min} < C < C_{max}$.

   This is to ensure that the value of capacitance C used will achieve the initiation of the self-excitation process.

3. $X_m \leq X_{smax}$.

   This constraint is to ensure that the value of the magnetizing reactance X_m computed from Equation 4 will not exceed the maximum unsaturated magnetizing reactance X_{smax} [3].

THE STEADY-STATE PERFORMANCE

To compute the steady-state performance of a stand-alone induction generator, the value of the air gap voltage E_g is required. The E_g–X_m curve obtained from the open circuit test is expressed by an n^{th} order polynomial. For $n = 5$, the accuracy obtained in comparison to the experimental curve is about 98%. Both X_m and the E_g–X_m polynomial are used to compute E_g at the given loading conditions.

The terminal voltage will have a value that depends on the type of loading at the terminals of the machine.

(*a*) For an Inductive Impedance Load

$$\begin{aligned} V_{out} = [a\,E_g]\,[(a^2(XX_c)^2 + (R\,X_c)^2]/[a^6(XX_1)^2 \\ + a^4(R^2X_1^2 + R_1^2X_1^2 - 2X_1X^2X_c - 2X_1^2XX_c)) \\ + a^2(X^2X_c^2 + R^2R_1^2 + 2XX_1X_c^2 + X_1^2X_c^2 - 2X_1X_c^2R \\ + 2XX_cR_1^2) + (R^2X_c^2 + 2RR_1X_c^2 + R_1^2X_1^2)]. \end{aligned} \tag{6}$$

(*b*) For a Pure Resistive Load

$$\begin{aligned} V_{out} = [a\,E_g]\,R^2X_c^2/[a^4(R^2X_1^2) + R_1^2X_1^2 + a^2(R^2R_1^2 + X_1^2X_c^2 - 2X_1X_c^2R) \\ + (R^2X_c^2 + 2RR_1X_c^2 + R_1^2X_c^2)]. \end{aligned} \tag{7}$$

The output current from the generator is expressed as:

$$I = \frac{V_{\mathrm{out}/a}}{\sqrt{(R/a)^2 + X^2}}. \tag{8}$$

THE INTERNAL IMPEDANCE OF THE GENERATOR

Looking back across the load terminals of the equivalent circuit of the generator, its internal impedance is given as:

$$Z_{\mathrm{int}} = K - jN, \tag{9}$$

where

$$K = \frac{X_c^2 \, R_1/a}{R_1^2 + a^2\left(X_1 + X_m - \dfrac{X_c}{a}\right)^2}$$

$$N = \frac{2R_1 \, X_c(X_1 + X_m) - X_c^2 \, R_{1/a}}{R_1^2 + a^2\left(X_1 + X_m - \dfrac{X_c}{a}\right)^2}.$$

Equation (9) reveals that K is always positive, while N is either negative or positive. Hence, the internal impedance of the stand-alone induction generator will have:

an inductive reactance if $C < \dfrac{0.5}{a\omega^2(l_1+M)}$

or

a capacitive reactance if $C > \dfrac{0.5}{a\omega^2(l_1+M)}$

or

a pure resistance $C = \dfrac{0.5}{a\omega^2(l_1+M)}$.

THE TRANSIENT ANALYSIS

Assuming no-load conditions, the internal generated voltage of a stand-alone induction generator is being build-up. When steady-state is reached, the generator terminals are then switched on to the load. The transient response of the load current can be computed if the internal impedance of the source presented by V_{out}, is obtained. The internal impedance expressed in Laplace transform is given as:

$$Z_{\mathrm{th}}(s) = A\frac{s + \alpha}{a^2 + \alpha s + \omega_n^2} \tag{10}$$

where

$$A = \frac{1}{aC}$$

$$\alpha = \frac{R_1}{a}\left(\frac{1}{l_1 + M}\right)$$

$$\omega_n^2 = \frac{A}{l_1 + M} .$$

Thus the load current $i(t)$ will be given as:

$$i(t) = \left\{ \frac{\frac{V_{\text{out}}}{a}(s)}{Z_{\text{th}}(s) + Z_{\text{L}}(s)} \right\} . \tag{11}$$

Since the no load terminal voltage has a sinusoidal wave form, Equation (11) will be used to compute the transient response of the load current at different operating conditions.

Machines Data

Data for the three induction machines used in the investigation is presented in Appendix 3.

RESULTS AND DISCUSSIONS

Assuming all other requirements has been fulfilled an induction machine will continue to operate as a generator if the load impedance connected to the machine terminals is higher than a specific minimum value [3]. The effect of varying either the operating speed or the self-excitation capacitance on that minimum value for the load impedance is thoroughly investigated and the results obtained are presented.

For a pure resistive load, the computed results indicate that the minimum value exhibits two different characteristics when either the operating speed or the self-excitation capacitance is increased.

The first characteristic for low hp machine, Figure 3, shows that the value of minimum resistance tends to increase with higher values corresponding to both high capacitance and high speed.

The second characteristic for high hp machine, Figure 4 shows that the trend is reversed where the minimum value tends to decrease as either the capacitance or the speed increases. The study was extended to include the case of inductive reactive loads. The computed results for 10 hp machine, Figure 5 indicate that the amplitude of minimum load impedance is almost five times that of a pure resistive load when the generator operates at speeds close to its synchronous speed. However, that difference is diminished as the operating speed shifted away from the synchronous speed.

The results also indicate that as the load power factor is decreased the higher must be the impedance amplitude necessary to ensure continuous operation as a generator.

The steady state performance of three induction generators are presented in Figures 6–8. For a specified speed and a given capacitance (Figure 6) the output frequency shows little change with variations of load power factor. However, the output voltage shows a rise of 10% for a corresponding power factor drop of 40%. The results presented in Figure 7 show the effect of both the operating speed and the excitation capacitance on the output voltage and frequency, for a pure resistive load. Figure 8 shows that the output frequency exhibits the same performance irrespective of the load power factor, although the output voltage shows a drop of 20% at a speed of twice the synchronous value and when the load power factor changes from unity to 0.8 lagging.

To conclude the previous discussion, the output voltage tends to increase with either operating speed or excitation capacitance, however, the output frequency tends to increase with the operating speed but decreases with increasing the self excitation capacitance.

Since the no-load terminal voltage has a sinusoidal wave form, Equation (11) can be used to compute the transient response of the load current at different load impedances and power factors. The transient load current obtained is of the form:

$$i(t) = k_1 \cos(\omega_1 t - \theta_1) e_1^{-\alpha} t + k_2 e_2^{-\alpha} t + k_3 \cos(\omega_2 t + \theta_2) \qquad \text{for } t > 0. \qquad (12)$$

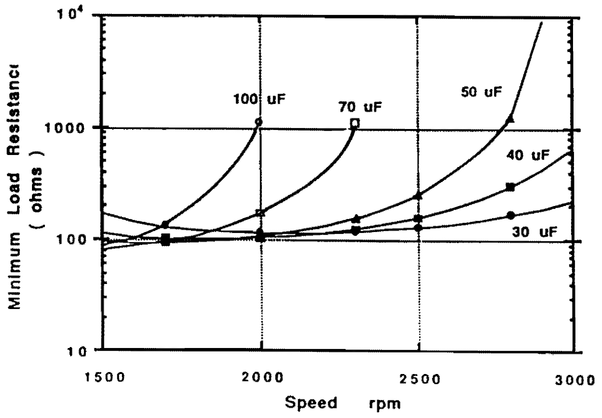

Figure 3. Minimum Load Resistance for 1 kW Machine.

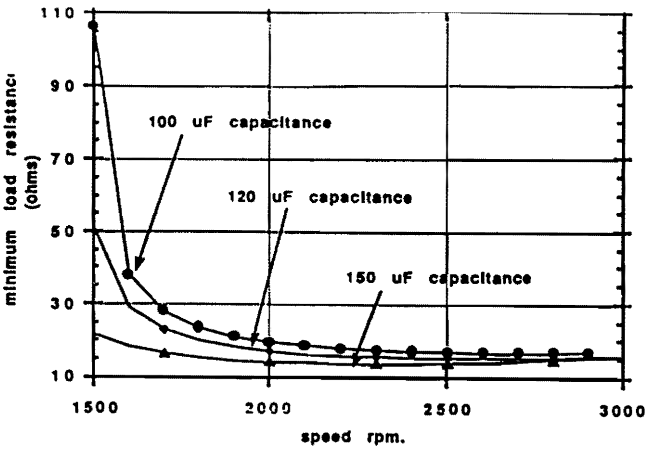

Figure 4. Minimum Load Resistance for 10 hp Machine for Pure Resistive Load.

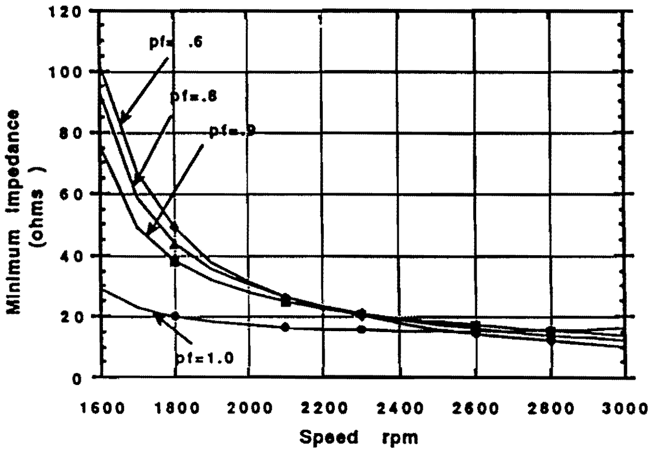

Figure 5. Variation of Minimum Load Impedance with Speed at Different Lagging Power Factors. For 10 hp machine with capacitance of 120 μF.

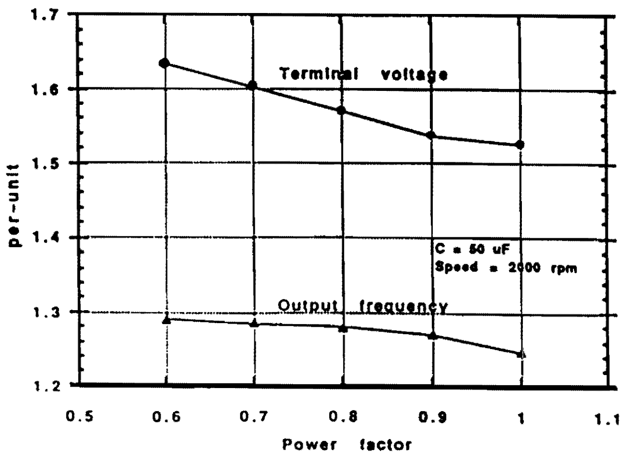

Figure 6. Steady State Performance of 1 kW Generator at Different Power Factors for Load of 200 Ω.

where k_1, k_2, and k_3 are coefficients which depend on the operating speed, the no-load voltage frequency and the rating of the machine under test.

The transient-time response of the load current $i(t)$ which is represented by Equation (12), consists of three components:

(*a*) A time decaying component, Figure 9(*a*).

(*b*) An exponentially decaying sinusoidal component, Figure 9(*b*).

(*c*) A sinusoidal component with a time delay, Figure 9(*c*).

However, components *a* and *b* have little effect on the wave form of the transient response which is found to be of the form:

$$i(t) = k_3 \cos(\omega_3 t + \theta_3) e^{-\alpha_3 t} + k_4 \sin(\omega t + \theta_4)$$

$$\text{for } t > 0. \tag{13}$$

The results obtained show that the instant at which the generator output voltage is applied to the load impedance, will have little effect on both the amplitude and shape of the load current response.

CONCLUSION

The minimum load patterns computed at different operating speeds for various self-excited capacitances will help in deciding the possibility of applying a stand-alone induction generator. By driving the generator at higher speeds, close to twice the value of the synchronous speed, variations of the power factor will have a minor effect on the minimum value of impedance necessary to ensure the continuous operation of the generator. Also, load power factor variations show little effect on the steady-state performance of the generator. However, the performance of the generator tends to improve as the speed is increased. The current transient components of a stand-alone generator when feeding various loads at different operating speed die out quickly.

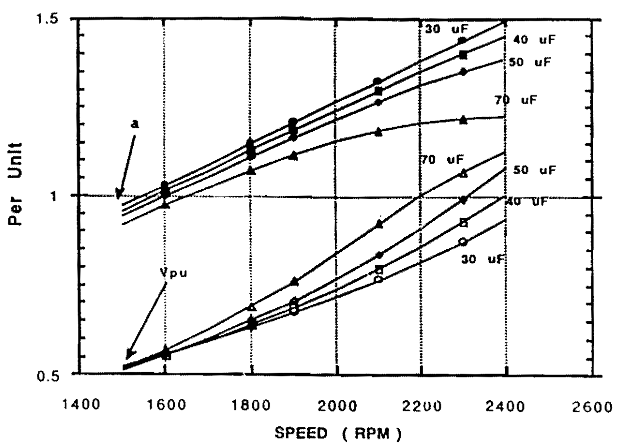

Figure 7. Steady State Performance of 5 hp Induction Generator for a 200 Ω Resistive Load.

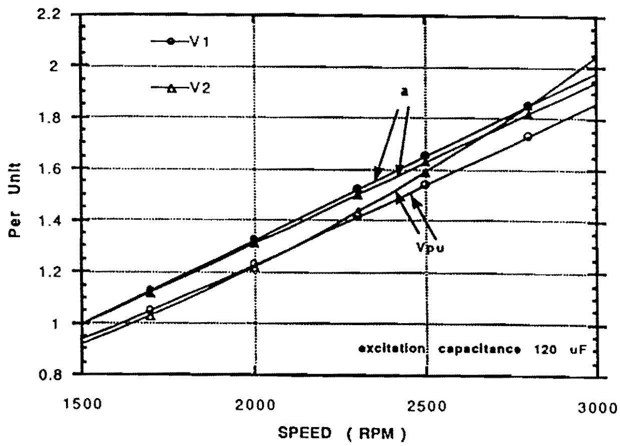

Figure 8. Steady State Performance of 10 hp Induction Generator for a Pure Resistive Load of 200 Ω.

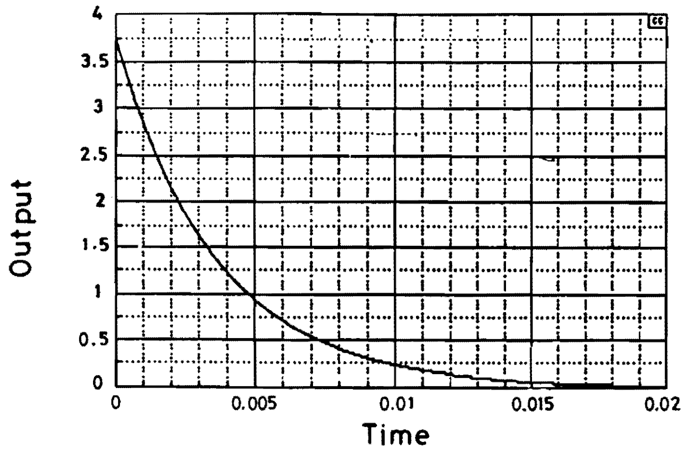

Fig.(9a) Component (a) of the transient current

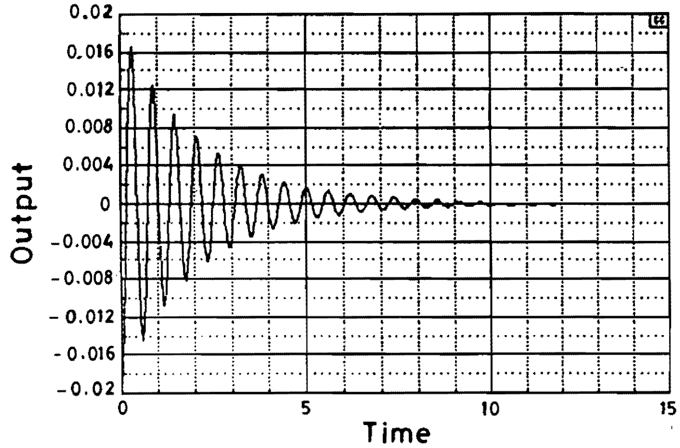

Fig.(9b) Component (b) of the transient current

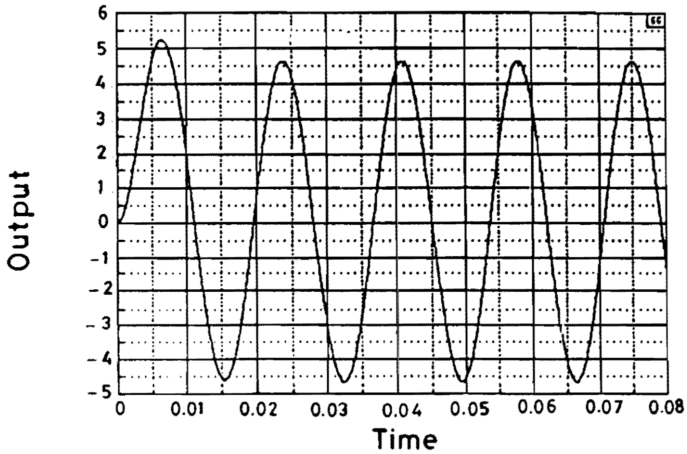

Fig.(9c) Component (c) of the transient current

Figure 9. Transient Response of 10 hp Machine Connected to a Load Impedance of 100 Ω, 0.6 pf and Operating at 2000 rpm with Capacitance 50 μF.

The current transient components obtained when the generator is switched on to a load, tend to die out quickly leaving a sinusoidal wave of current to feed the load.

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APPENDIX 1

The Expressions For R_L and X_L

(a) For Inductive Load

$$R_L = \frac{RX_c^2}{aD}, \qquad X_L = \frac{X_c(XX_c + R^2 + a^2X^2)}{D}$$

(b) For Pure Resistive Load

$$R_L = \frac{RX_c^2}{a(a^2R^2 + X_c^2)}, \qquad X_L = \frac{X_c R_2}{a^2R^2 + X_c^2}$$

(c) For Capacitive Load

$$R_L = \frac{RX_c^2}{aD^*}, \qquad X_L = \frac{X_c(-XX_c + R_2 + a^2 X^2)}{D^*}$$

(d) For No-Load

$$R_L = 0 \qquad X_L = X_c / a^2$$

where $D^* = a^4 X^2 + a^2 (R_2 - 2XX_c) + X_c^2$.

APPENDIX 2

The Coefficients of the Polynomial

1. For Inductive Load

$$p_0 = -bR_2 (R^2 X_c^2 + 2RR_1 X_c^2 + R_1^2 X_c^2)$$

$$p_1 = R_2 (R^2 X_c^2 + 2RR_1 X_c^2 + R_1^2 X_c^2) + (R_2^2 + b^2 X_2^2)(R X_c^2 + R_1 X_c^2)$$

$$p_2 = -bR_2 (X_c^2 X_2 + R_1^2 R_2 - 2XX_c R_1^2 + X_1^2 X_c^2 + 2X_1 X_c^2 X - 2X_1 X_c R_2) - 2b X_2^2 (R X_c^2 + R_1 X_c^2)$$

$$p_3 = R_2 (X_c^2 X_2 + R_1^2 R_2 - 2X X_c R_1^2 + X_1^2 X_c^2 + 2X_1 X_c^2 X - 2X_1 X_c R_2) + X_2^2 (R X_c^2 + R_1 X_c^2) + (R_2^2 + b^2 X_2^2)(R_1 R^2 - 2XX_c R_1)$$

$$p_4 = -bR_2 (R_1^2 X_2 + X_1^2 R_2 - 2X X_c X_1^2 - 2X_1 X_c X^2) - 2b X_2^2 (R_1 R^2 - 2XX_c R_1)$$

$$p_5 = R_2 (R_1^2 X_2 + X_1^2 R_2 - 2XX_c X_1^2 - 2X_1 X_c X^2) + X_2^2 (R_1 R^2 - 2\, XX_2 R_1) + R_1 X^2 (R_2^2 + b^2 X_2^2)$$

$$p_6 = -bR_2 X_1^2 X_2 - 2\, b X_2^2 R_1 X^2$$

$$p_7 = X_2^2 R_1 X^2 + R_2 X_1^2 X_2 .$$

2. For Pure Resistive Load

The coefficients of the polynomial can be obtained from case (1) by substituting $X = 0$.

3. For the No-Load Condition

$$p_0 = -\, bR_2X_c^2 \; p_1 = R_2X_c^2$$

$$p_2 = 2bR_2X_1X_c - bR_2R_1^2$$

$$p_3 = -2X_1X_cR_2 + R_2^2 R_1 + b^2X_2^2 R_1 + R_2R_1^2$$

$$p_4 = -2bX_2^2 R_1 - bR_2X_1^2$$

$$p_5 = X_2^2 R_1 + R_2X_1^2$$

$$p_6 = 0$$

$$p_7 = 0.$$

4. For Capacitive Load

$$p_0 = -\, bR_2(R^2X_c^2 + 2R_1RX_c^2 + R_1^2 X_c^2)$$

$$p_1 = R_2(R^2X_c^2 + 2R_1RX_c^2 + R_1^2 X_c^2) + (R_2^2 + bX_2^2)(R_1X_c^2 + RX_c^2)$$

$$p_2 = -\, bR_2(X_c^2 X_2 + R_1^2 R_2 + 2XX_cR_1^2 - 2X_1X_c^2 X - 2X_1X_cR^2 + X_1^2 X_c^2)$$
$$- 2bX_2^2(R_1X_c^2 + RX_c^2)$$

$$p_3 = R_2(X_c^2 X_2 + R_1^2 R_2 + 2XX_cR_1^2 - 2X_1X_c^2 X - 2X_1X_cR^2 + X_1^2 X_c^2)$$
$$+ X_2^2(R_1X_c^2 + RX_c^2) + (R_2^2 + b^2X_2^2)(R_1R^2 + 2XX_cR_1)$$

$$p_4 = -\, bR_2(R_1^2 X_2 - 2X_1X_cX^2 + X_1^2 R_2 + 2XX_cX_1^2)$$
$$- 2bX_2^2 (R_1R^2 + 2XX_cR_1)$$

$$p_5 = R_2(R_1^2 X_2 - 2X_1X_cX^2 + X_1^2 R_2 + 2XX_cX_c^2) + X_2^2(R_1R^2 + 2XX_cR_1)$$
$$+ (R_2^2 + b^2X_2^2)(R_1X^2)$$

$$p_6 = -\, bR_2(X_1^2 X_2) - 2bX_2^2 (R_1X^2)$$

$$p_7 = X_2^2(R_1X^2) + R_2(X_1^2 X^2)\,.$$

APPENDIX 3

The Data of the Three Induction Machines

First Machine

1 kW, 4 pole, 3-phase squirrel cage, 50 Hz, 420 V

$R_1 = 8.5\ \Omega$
$X_1 = 15.715\ \Omega$
$R_2 = 3.95\ \Omega$
$X_2 = 15.715\ \Omega$
$X_m = 133.7\ \Omega$

Second Machine

5 hp, 4 pole, 3 phase squirrel cage, 50 Hz, 420 V.

$R_1 = 2.788\ \Omega$
$X_1 = 7.1\ \Omega$
$R_2 = 3.464\ \Omega$
$X_2 = 7.1\ \Omega$
$X_m = 139.86\ \Omega$

Third Machine

10 hp, 420 V, 50 Hz, 4 pole, squirrel cage, 3 phase machine.

$R_1 = 0.743\ \Omega$
$X_1 = 1.8\ \Omega$
$R_2 = 0.246\ \Omega$
$X_2 = 1.8\ \Omega$
$X_m = 27.13\ \Omega$