# AN ESTIMATION ALGORITHM FOR AN INDEPENDENT FREQUENCY LOAD MODELING IN THE PRESENCE OF HARMONICS

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الخلاصـة :

يقدم هذا البحث تطبيقاً جديداً لخوارزم تقليل مربع الخطأ لنمذجة الأحمال الكهربية والتي لا تعتمد على التردد في وجود أو عدم وجود توافقيات. ويفترض الخوارزم المعرفة المسبقة لتوافقيات كلٍّ من الجهد وتيار الحمل. وباستخدام العينات الرقمية لهذه الموجات وكذلك مشتقاتها بالنسبة للزمن يمكن حساب عناصر الحمل (R, L, C). ويتم في هذا البحث أيضاً حساب التيار المتبقي والذي ينتج من عدم التوافق عند الترددات المختلفة بين جهد وتيار الحمل. بالاضافة فإنه يمكن حساب التيار الناتج من التوافقيات غير المشتركة بين موجة الجهد والتيار وكذلك عند عدم توافقها. واخيراً فإنّ النتائج لبيانات حقيقية مسجلة لحمل لاخطي في منظومة قوى قد أعطيت أيضاً في نهاية البحث.

## ABSTRACT

This paper presents a new application to the least errors squares (LES) algorithm for independent frequency load modeling in the presence and absence of harmonics in the time domain. The proposed algorithm assumes that the harmonics content of both load voltage and current waveforms are known in advance, and uses the digitized samples of these wave forms, as well as their derivatives w.r.t. time, to estimate the equivalent load parameters (R, L, C parameters). The residual current source, which is the current resulting from the mismatch between the load voltage and load current at different harmonic frequencies, is calculated. Furthermore, the black-box current source, which is the current resulting from the uncommon harmonics in the voltage and current waveforms, is also determined. Simulated results are presented for matching the voltage and current waveforms, as well as mismatching. Finally, results for actual recorded data for a nonlinear load in a power system are also presented.

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## 1. INTRODUCTION

The widespread use of power electronic applications in power systems increases the level of harmonics in the utility and consumer networks. Nonlinear loads such as electric arc furnaces and fluorescent lamps [1] increase the level of harmonics in the networks, and it has been become necessary to find an accurate model for such nonlinear loads to be presented in the load flow program. Linear loads on a power system may also be affected by the voltage harmonics that exist in the system buses they are connected to.

A method for modeling the distribution load impedances as unbalanced three-phase impedance matrices for detailed harmonics studies was presented in [2]. This method is applied under specific condition and can produce useful estimates of the $Z_{abc}$ matrix. The method [2] did not solve the question of load harmonic sources, whose output is correlated with the applied disturbance.

A method based on parallel processing for modeling the loads and other power system components in the presence of harmonics is also presented in [3]. The algorithm in this method assumes an unbalanced three phase system, also the coupling effects that may exist between harmonics of different orders are neglected. The system is modeled using phase coordinates and frequency dependence of the parameters.

A dynamic system model for dynamic nonlinear loads using the symbolic mathematical program (SMP) is offered in [4]. The Effects of several types of time varying current, including actual recorded waves, have been studied using this dynamic model. The resonance problems in the presence of time varying current injections are also studied. Reference [4] indicated that a sudden variation in the rate of change in the current waveform could result in harmonic voltage transients at frequencies close to the parallel resonant frequencies of the system.

A time domain load modeling technique based on the calculation of the power components is presented in [5]. The developed load model can be used for modeling linear and nonlinear loads in the presence or absence of harmonic distortion. The model developed in [5] is not a unique model and there is no justification whether the developed model is adequate or not. It has been shown in [5] that nonlinear buses that have current injection are conveniently absorbed into the bus admittance matrix, and the proposed method in [5] can be used very effectively in isolating the effects of customers with nonlinear loads.

In this paper the authors present a technique which is an extension to the method in [5], for presentation of an independent frequency load model in the presence and absence of harmonics in the time domain. The proposed technique is based on least error squares parameter estimation algorithm and assumes that the harmonics content of the load voltage and load current waveforms are known in advance. The proposed technique uses the digitized samples of voltage and current waveforms, as well as their derivatives w.r.t the time. Simulation results for matching the harmonics of the voltage and current waveforms at all frequencies, as well as the mismatching, are presented. Finally, the results of an actual record for a nonlinear load in a power system is also presented. In the next section the proposed technique is presented.

## 2. LOAD MODELING

Assume that the load bus voltage $v(t)$ as well as the load current $i(t)$ are known in advance in a digital form. These two signals may or may not be contaminated with harmonics. These two signals can be written in general forms as

$$v(t) = \sum_{n=1}^{N} V_n \sin\left(n\omega_o t + \phi_n\right) \tag{1}$$

$$i_T(t) = \sum_{i=1}^{K} I_k \sin\left(i\omega_o t + \theta_i\right) \tag{2}$$

where $N$ = number of harmonics contaminating the bus voltage

$K$ = number of harmonics contaminating the load current. $N$ and $K$ may or may not be equal.

$V_n$ = maximum value of the $n$th harmonic of the voltage
$I_i$ = maximum value of the $i$th harmonic of the current
$\phi_n$ = phase angle of the $n$th harmonic of the load voltage
$\phi_i$ = phase angle of the $i$th harmonic of the load current
$\omega_o$ = fundamental frequency ($2\pi f_o$).

The circuit in Figure 1 represents the equivalent network for the load which carries such a current and is connected across such a voltage [5]. At this stage, the loads on each phase are assumed to be symmetrical and there is no mutual coupling between phases.

Referring to Figure 1, the total bus current $i_T(t)$ can be written as:

$$i_T(t) = i_R(t) + i_c(t) + i_L(t) + i_r(t) + i_g(t), \tag{3}$$

where we define:

$i_T(t)$ = total bus current injected to the load, we assume it is given in digitized samples.
$i_R(t)$ = but current component passing in the resistor $R$ of the load, which is in phase with $v(t)$ and equals $v(t)/R$.
$i_L(t)$ = orthogonal component of the bus current which is passing through $L$ and equals $\frac{1}{L}\int v(t)/\mathrm{d}t$.
$i_c(t)$ = bus current component passing in the capacitance $C$ of the load, which is orthogonal to $v(t)$ and equals $C\,\mathrm{d}v(t)/\mathrm{d}t$.
$i_r(t)$ = part of the total current that exists when complete extraction of orthogonal components and inphase component is impossible, as indicated in Figure 1 by a black box (B.B).
$i_g(t)$ = part of the total current that corresponds to uncommon harmonics with the voltage.

Equation (3) can now be written as

$$i_T(t) = \frac{1}{R}v(t) + c\frac{\mathrm{d}v(t)}{\mathrm{d}t} + \frac{1}{L}\int v(t)\mathrm{d}t + \left[i_r(t) + i_g(t)\right] \tag{4}$$

Note that, if there is no uncommon harmonic with the voltage, $i_g(t)$ in (3) and (4) disappears. Furthermore, if the orthogonal components and inphase component are completely extracted from the total current, then $i_r(t)$ will also disappear.

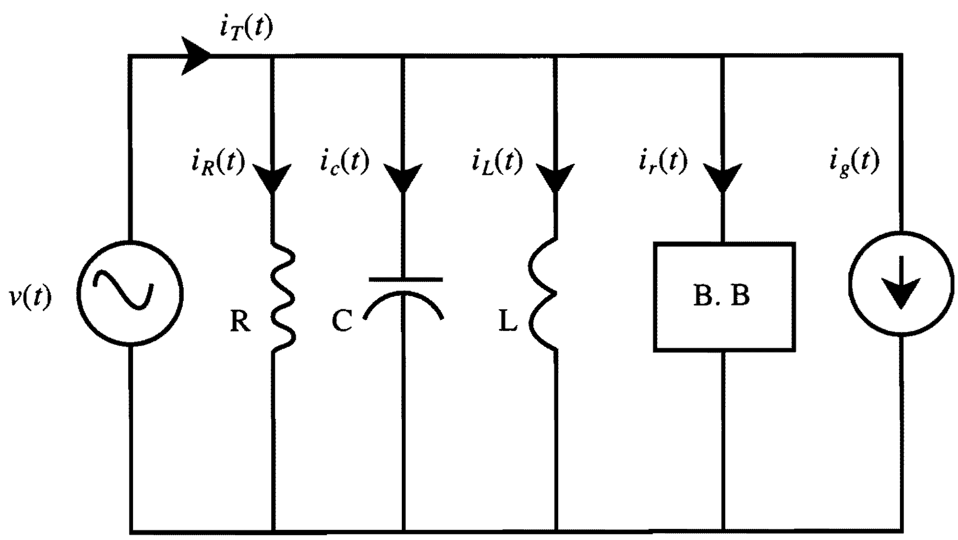


*Figure 1. General Equivalent Network and Current Components.*

Differentiating (4), w.r.t time, one obtains

$$i'_T(t) = \frac{1}{R} v'(t) + C\, v''(t) + \frac{1}{L} v(t) + \left[ i'_r(t) + i'_g(t) \right] \tag{5}$$

The last bracket in (5) can be considered as an error term and could be identified after we identify the parameter $R$, $L$, and $C$. The problem now can be split into two sub-problems. In the first problem, we identify first the parameters $R$, $L$, and $C$ as follows.

We have, from (1) that

$$i'_T(t) = \sum_{i=1}^{K} i\omega_o\, I_i \cos\left(i\omega_o t + \theta_i\right) \tag{6}$$

and

$$v'_T(t) = \sum_{n=1}^{N} n\omega_o\, V_n \cos\left(n\omega_o t + \phi_n\right). \tag{7}$$

Furthermore

$$v''(t) = -\sum_{n=1}^{N} n^2 \omega_o^2\, V_n \sin\left(n\omega_o t + \phi_n\right). \tag{8}$$

To obtain the above derivatives, we need to identify in advance the harmonics content of the total current $i_T(t)$ as well as the load bus voltage $v(t)$. Having identified the harmonics content, we can easily determine the above derivatives as well as the uncommon harmonics if any. Generally speaking, if there is uncommon harmonic current $i_g(t)$, then (5) can be rewritten as:

$$i'_T(t) - i'_g(t) = \frac{1}{R} v'(t) + C\, v''(t) + \frac{1}{L} v(t) + i'_r(t). \tag{9}$$

Define the 3 by 1 vector $\mathbf{X}$ as

$$\mathbf{X} = \begin{vmatrix} 1/R \\ C \\ 1/L \end{vmatrix} \tag{10}$$

and the coefficients vector as

$$\mathbf{h}_{1i} = \begin{vmatrix} v'(t) \\ v''(t) \\ v(t) \end{vmatrix}, \quad i = 1, 2, 3. \tag{11}$$

Furthermore, define the observation vector $z(t)$ as

$$z(t) = i'_T(t) - i'_g(t). \tag{12}$$

Then, (9) can be written as

$$z(t) = h_{11}(t) x_1 + h_{12}(t) x_2 + h_{13}(t) x_3 + i'_r(t). \tag{13}$$

If the bus voltage and load current are sampled at a preselected rate, say $\Delta T$, then $m$ samples would be obtained at $t_1, t_1 + \Delta T, \ldots\ldots, t_1 + (m-1)\,\Delta T$, where $t_1$ is the initial sampling time. Equation (13) can be written as:

$$z(j\Delta T) = h_{11}(j\Delta T)x_1 + h_{12}(j\Delta T)x_2 + h_{13}(j\Delta T)x_3 + i_r'(t);\quad j = 1, 2, \ldots, \ldots, m. \tag{14}$$

In compact form, (14) can be written as

$$\mathbf{Z} = HX + \xi \tag{15}$$

where $\mathbf{Z}$ an $\Re^m$ measurements vector of samples of the current derivatives.
$H$ an $\Re^m \times \Re^3$ measurements matrix whose elements are given by (11).
$\mathbf{X}$ an $\Re^3$ parameters vector to be identified.
$\xi$ an $\Re^m$ errors vector, containing $i'_r(t)$, to be minimized.

At least three samples are required to obtain the parameters vector in (16). Using three samples may produce poor estimates, since we force $\xi$ to be zero. Thus, we assume $m > 3$. The solution to (15) based on the standard least errors squares (LES) is given by (16) [8].

$$\mathbf{X}^* = \left[H^T H\right]^{-1} H^T \mathbf{Z}. \tag{16}$$

Having identified the parameters vector $\mathbf{X}$, then the elements of the general equivalent circuit of Figure 1 can be determined from (10).

In the second problem, the errors vector $\omega$ is determined from (15) and (16).

$$\xi = \left[I - H\left[H^T H\right]^{-1} H^T\right]\mathbf{Z}. \tag{17}$$

This residual vector contains the derivative of the residual current $i_r(t)$, as well as the noise from the $A/D$ conversion. The residual vector can be written as

$$\xi(t) = i_r'(t) + \varepsilon(t). \tag{18}$$

The residual current $i_r(t)$ has a form similar to the form given by (2) as

$$i_r(t) = \sum_{k=1}^{K} I_{rk} \sin\left(k\omega_o t + \Psi_k\right) \tag{19}$$

*i.e.* the residual current contains the same order of harmonics as those for the total bus load current. The derivative of (27) w.r.t the time is given by:

$$i_r'(t) = \sum_{k=1}^{K} k\, I_{rk}\omega_o \cos\left(k\omega_o t + \Psi_k\right). \tag{20}$$

Now, $\xi(t)$ in (18) is available in a form of samples. Without loss of generality, we assume that $k = 1, 2$, in this study. Hence (18) can be written as

$$\xi(t) = I_{r1}\omega_o \cos\left(\omega_o t + \Psi_1\right) + 2\omega_o I_{r2} \cos\left(2\omega_o t + \Psi_2\right) + \varepsilon(t). \tag{21}$$

Using the trigonometric identities;

$$\begin{aligned}\xi(t) = &\left(\omega_o \cos\omega_o t\right)\left(I_{r1}\cos\Psi_1\right) - \left(\omega_o \sin\omega_o t\right)\left(I_{r1}\sin\Psi_1\right) \\ &+ \left(2\,\omega_o \cos 2\omega_o t\right)\left(I_{r2}\cos\Psi_2\right) - \left(2\omega_o \sin 2\omega_o t\right)\left(I_{r2}\sin\Psi_2\right) + \varepsilon(t).\end{aligned} \tag{22}$$

Define the states vector **Y** as

$$\mathbf{Y} = \begin{vmatrix} I_{r1}\cos\Psi_1 \\ I_{r1}\sin\Psi_1 \\ I_{r2}\cos\Psi_2 \\ I_{r2}\sin\Psi_2 \end{vmatrix} \tag{23}$$

and the parameters vector $a_{1i}$ as

$$a_{1i}(t) = \begin{vmatrix} \omega_o \cos\omega_o t \\ \omega_o \sin\omega_o t \\ 2\omega_o \cos 2\omega_o t \\ 2\omega_o \sin 2\omega_o t \end{vmatrix}, \quad i = 1,2,3,4. \tag{24}$$

Then, (22) can be written as

$$\xi(t) = a_{11}(t)y_1 + a_{12}(t)y_2 + a_{13}(t)y_3 + a_{14}(t)y_4 + \varepsilon(t). \tag{25}$$

As we said earlier the $\xi(t)$ is available in the form of samples calculated at $t_1$, $t_2$, $(=t_1 + \Delta t)$, $t_3$, $(=t_1 + 2\Delta t)$,..., $t_m$ $(t_1 + (m-1)\,\Delta t)$. Then (25) can be written as:

$$\begin{vmatrix} \xi(t_1) \\ \xi(t_2) \\ ,, \\ ,, \\ ,, \\ \xi(t_m) \end{vmatrix} = \begin{vmatrix} a_{11}(t_1) & a_{12}(t_1) & a_{13}(t_1) & a_{14}(t_1) \\ a_{11}(t_2) & a_{12}(t_2) & a_{13}(t_2) & a_{14}(t_2) \\ ,, & ,, & ,, & ,, \\ ,, & ,, & ,, & ,, \\ ,, & ,, & ,, & ,, \\ a_{11}(t_m) & a_{12}(t_m) & a_{13}(t_m) & a_{14}(t_m) \end{vmatrix} \begin{vmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ \\ \\ \end{vmatrix} + \begin{vmatrix} \varepsilon(t_1) \\ \varepsilon(t_2) \\ ,, \\ ,, \\ ,, \\ \varepsilon(t_m) \end{vmatrix}. \tag{26}$$

or in vector-matrix notation:

$$\xi = A\mathbf{Y} + \in \tag{27}$$

where $A$ is an $\Re^m \times \Re^4$ measurements matrix and can be calculated off-line. The order of this matrix depends on the number of harmonics in the total current $i_T(t)$.

Using the $m$ available measurements, $m>4$, the solution to (27) in the least error squares (LES) sense is given by

$$\mathbf{Y} = \left[A^T A\right]^{-1} A^T \zeta. \tag{28}$$

Having identified the states vector **Y**, then by using (23), we can calculate the harmonics content of the residual current, and hence the residual current of (19) can be determined.

Now, the noise vector contaminated the current waveform due to the $A/D$ conversion and others can be determined from (27) as

$$\varepsilon = \left[\zeta - A\left[A^T A\right]^{-1} A^T \zeta\right]. \tag{29}$$

## 3. SIMULATED NUMERICAL EXAMPLES

### 3.1. Complete Extraction of the Total Current Components

**Example 1.**

Consider the following load bus voltage and current [5]

$$v(t) = \sqrt{2}\cos\omega t$$
$$i_T(t) = \sqrt{2}\cos(\omega t - 30°), \quad \omega = 2\pi f, \quad f = 60\,\text{c/s}.$$

In this simple example, no harmonics contaminate either the load bus voltage or the load current. Following the principles of the technique developed earlier, the parameters of the load are; $R = 1.1547$ p.u. and $L = 0.00531$ p.u. These results are the same as those obtained in [5]. Note that, the residual current $i_r(t)$, in this particular example equals zero, and the model of the load in this case will be $R$ and $L$ in parallel.

**Example 2.**

Consider the bus voltage and the load current are contaminated with harmonics as

$$v(t) = \sqrt{2}\left[\cos\omega t + 0.6\cos 3\omega t\right],$$
$$i_T(t) = \sqrt{2}\left[\cos(\omega t - 30°) + 0.5292\cos(3\omega t - 11°)\right].$$

In this example the fundamental component of both the voltage and current produces the same load impedance as the third harmonic component of the voltage and current.

Application of the developed technique yields the same equivalent circuit as Example 1, with the load parameters $R = 1.1547$ p.u. and $L = 0.00531$ p.u., and it has been found that the residual current in this case does not exist.

### 3.2. Incomplete Extraction of the Total Current Components

**Example 3.**

Consider that the harmonics content of both the bus voltage and the current of the load connected to this bus are [5].

$$v(t) = \sqrt{2}\left[\cos\omega t + 0.6\cos 3\omega t\right]$$
$$i_T(t) = \sqrt{2}\left[\cos(\omega t - 30°) + 0.25\cos(3\omega t + 60°)\right].$$

Note that, in this example, the load impedance calculated from the fundamental component of the voltage and current is not the same as that calculated from the third harmonic components, where the load impedance from the fundamental is $Z_1 = 1\ \angle 30°$, which gives $R_1 = 1.1547$ p.u., $L_1 = 0.00531$ p.u., and the load impedance at the third harmonic is $Z_3 = 2.4\ \angle{-60°}$ which gives $R_3 = 4.8$ p.u, and a capacitance of $C_3 = 9.572 \times 10^{-4}$ p.u. In conclusion the load changes its mode of operation from one frequency to the other.

The application of the proposed technique yields the parameters:

$R = 2.71515$ p.u. and $L = 0.0327$ p.u. The residual current in this case exists and is extracted to be:

$$i_r(t) = \sqrt{2}\left[0.6542\cos(\omega t + 50°) + 0.251\cos(3\omega t - 158°)\right].$$

Note that theoretically $i_r(t)$ can be represented by two impedances as $Z_1 = 1.5286\ \angle 50°$, and $Z_3 = 2.39\ \angle 158°$, that can be connected in parallel with $R$, and $L$ calculated above, where the subscript represents the harmonic order.

**Example 4.**

Consider the following bus voltage and load current:

$$v(t) = \sqrt{2}\,\left[\cos\omega t + 0.6\cos 3\omega t\right]$$
$$i_T(t) = \sqrt{2}\,\left[\cos(\omega t - 30°) + 0.6\cos(3\omega t + 30°) + 0.05\cos(5\omega t + 30°)\right].$$

In this example, there is an uncommon harmonic between the total current and the bus voltage, which is

$$i_g(t) = \sqrt{2}\left[0.05\cos(5\omega t + 30°)\right].$$

The rest of the total current is used together with the bus voltage to find the entire compete circuit for the load given in Figure 1. As we can see, at the fundamental frequency the load is an inductive load ($R_1$, $L$) while at the third harmonic the load is a capacitive load ($R_3$, $C$). The load impedance at the fundamental is $Z_1 = 1\ \angle 30°$, which gives as before, $R_1 = 1.1547$ p.u. and $L = 0.00531$ p.u., while at the third harmonic gives $R_3 = 1.1547$ and $C = 0.00133$ p.u. In this example, the value of $R$ is the same at both the fundamental and the third harmonics.

The application of the proposed algorithm yields the equivalent network shown in Figure 1 with the parameters $R = 1.1547$ p.u., $C = 0.006632$ p.u., and $L = 0.0035368$ p.u. and the residual current in this case does exist and can be represented by a current source $i_r(t)$ given by

$$i_r(t) = \sqrt{2}\,\left[2.6483 \times 10^{-7}\cos\omega t + 1.7555 \times 10^{-7}\cos 3\omega t\right]\ \text{p.u.}$$

**Example 5.**

In this example the harmonic content is determined for a certain bus in a 44 kV power system [5]. 6-pulse converter is connected to this bus and it is the source of harmonic current injection into the system. This converter drives a large motor of 1250 HP. The voltage as well as the injected current can be written as,

$$v(t) = \sqrt{2}\ [0.2822\cos(\omega t - 28.648°) + 0.0016\cos(3\omega t - 22.491°)$$
$$+ 0.01152\cos(5\omega t - 162.36°) + 0.01274\cos(7\omega t - 145.22°)]$$

and the injected current is

$$i_T(t) = -\sqrt{2}\ [\cos(\omega t + 100) + 0.027\cos(3\omega t) + 0.151\cos(5\omega t) + 0.1335\cos(7\omega t)].$$

Table 1 gives the impedance of the load, using the basic circuit, calculated at each harmonic frequency. In this table, we use $\omega = 2\pi f_0 = 2\pi \times 60 = 377$ rad/s. From this table, we can see that the load changes its parameters from one frequency to the other; also, at the seventh harmonic the load is a capacitive load.

**Table 1. Load Parameters at Different Harmonics Frequencies.**

| Order of Harmonics | $V$ | $\phi_v$ | $I$ | $\phi_i$ | $R$ p.u | $L$ p.u | $C$ p.u |
|---|---|---|---|---|---|---|---|
| 1 | 0.2822 | −28.648° | −1.0000 | 100 | 0.3613 | $9.584 \times 10^{-4}$ | 0 |
| 3 | 0.0016 | −22.491 | −0.027 | 0 | −0.06414 | $4.11 \times 10^{-4}$ | 0 |
| 5 | 0.01152 | −162.36 | −0.151 | 0 | 0.0801 | $6.678 \times 10^{-4}$ | 0 |
| 7 | 0.01274 | 145.22 | −0.1335 | 0 | 0.1162 | 0 | 0.0159 |

Applying the proposed technique, we can obtain the parameters $R = 0.318187$, $L = 9.803093 \times 10^{-4}$ p.u. connected in parallel and the residual current in this case exists and is given by

$$\begin{aligned} i_r(t) = \sqrt{2}\, [&0.26294 \cos(\omega t + 155.15°) \\ &+ 0.03126 \cos(3\omega t + 174.02°) \\ &+ 0.011472 \cos(5\omega t + 177.5°) \\ &+ 0.010686 \cos(7\omega t - 165.43°)]. \end{aligned}$$

Figure 2 shows the total current $i_T(t)$, the residual current $i_r(t)$, and the final error. It can be noticed from the figure that the final error in the samples is almost zero.

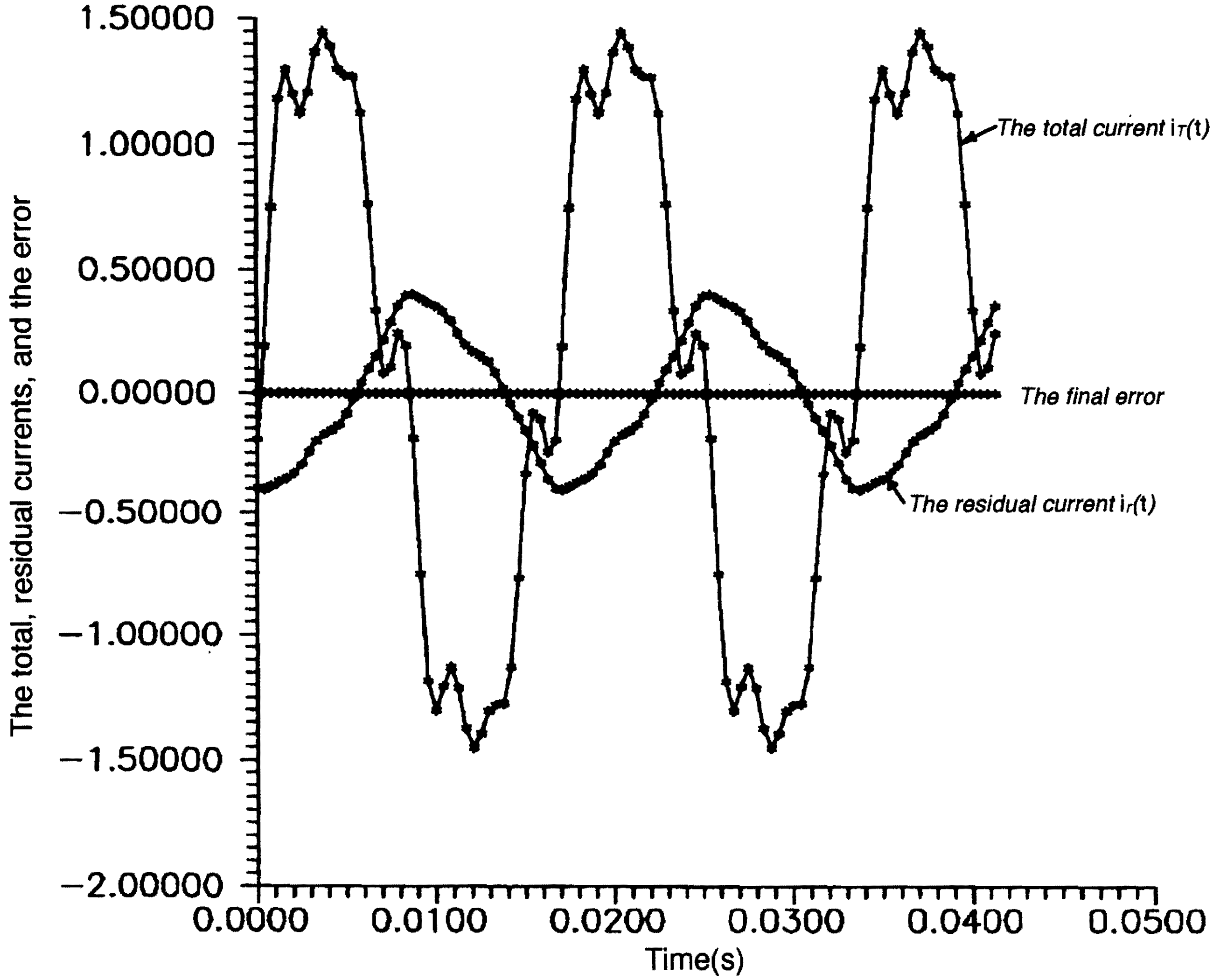


*Figure 2. The Total Current, the Residual Current, and the Final Error.*

## CONCLUSIONS

The algorithm proposed in this paper has successfully estimated the parameters of the load in the absence and presence of harmonics in the load voltage and load current waveforms. The estimated load parameters are unique in the sense of the least error squares criteria. It has been shown through extensive runs that the principle of conservation of power is valid within the proposed algorithm. The simulated examples, as well as the actual recorded data example, indicate that the proposed algorithm can generally be applied for any order of harmonics whether the load is linear or nonlinear, and no restrictions are imposed on the algorithm.

In this paper we assume that the parameters of the load are constant. It is worth while to estimate the parameters of the load in the presence of harmonics, when the voltage and current waveforms are time-varying. In this case a dynamic estimation algorithm is required, which is the current research of the authors.

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