REAL-TIME FAILURE DETECTION OF AIRCRAFT ENGINE OUTPUT SENSORS

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الخلاصة :

لقد تم وضع منهاج رياضي للحاسب الآلي لاستخدامه في البحث عن حالات الأعطال في مؤشرات المخارج لمحركات الطائرات . ويعتمد هذا المنهاج على طريقة باياسيان لاختبارات الفروض للبحث عن الأعطال . ويتم إستخدام مجموعة من مصفيات كالماء الملاغة للاقرتاحات لتحديد المصفوفات اللازمة ذات الخطأ المعايير. ويتم إستخدام مقياس زمني حقيقي — يشتمل على قراءات المؤشرات الحالية في تطبيق المنهاج الرياض للبحث عن أعطال المؤشرات . ويتم اقتراح منطق رياضي لإتخاذ القرارات للتغلب على الصعوبات التحليلية الناتجة من الخواص اللاخطية لمحركات الطائرات . ولقد تم تطبيق الطريقة المذكورة على مثال عددي يمثل نموذج حقيقي لمحرك تربيني أحادي الملف يفترض تعرضه لأعطال في مؤشرات المخارج .

ABSTRACT

A digital computer algorithm is developed for real-time failure state detection in aircraft engine output sensors. The technique employs the Bayesian approach for hypothesis testing in failure detection. A set of hypothesis-conditioned Kalman filters is used to estimate the outputs corresponding to the failed sensors and to determine the associated error covariance matrices. Real-time processing which incorporates the current sensor readings is employed in the implementation of the algorithm for sensor failure detection. A decision logic that overcomes the analytical difficulties resulting from inherent nonlinearities of aircraft engines is proposed. As a numerical example, the technique developed is applied to a realistic single-spool turbojet engine model undergoing simulated output sensor failures.

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INTRODUCTION

High reliability of engine output sensors is essential for proper operation of automated landing and take-off systems in future generations of aircraft. Not only is the possibility of instrument failure inevitable, it is often true that by observing instrument readings alone one cannot judge whether failure has actually occurred. Recent work in the area of aircraft failure detection during automatic landing [1] had addressed the problem of modeling the human pilot as a monitor of instrument failures. The implementation of that procedure is dependent to a large extent on the presence of an on-board digital computer for full authority control. An alternative procedure for real-time detection of the failure state of aircraft engine output sensors is to use the actual instrument readings and engine inputs in a digital computer-based decision logic that incorporates dynamic models for the engine and output sensors. An algorithm based on the second approach is developed in the present paper.

This method is not a scheme merely to detect whether a sensor failure has occurred. The failure detection will be done in real time, and an estimate for the correct reading will be determined. This information can be employed in the flight control until corrective action is taken. The method developed can be used in manual as well as automated flight control as a backup system for emergency flight control.

ENGINE MODEL

In conventional turbojet analysis and control, fuel flow and altitude are considered as the independent input variables. The intake air temperature and pressure are assumed to be fixed by the altitude. These inputs determine the engine speed and thrust, and the aircraft cruising speed. When a simplified model incorporating these assumptions is used, the turbojet control principally consists of a 'chart look up' procedure and corresponding open-loop scheduling of the fuel flow.

Under unsteady atmospheric conditions, however, altitude alone may not determine the intake air temperature and pressure. In addition to burner fuel flow, after burner fuel flow, and exhaust nozzle area can be used as independent input variables. This necessitates a more complex multivariable feedback control strategy. State space techniques are particularly suitable for multiinput-multioutput systems. Using these techniques, superior control systems can be implemented for modern aircraft engines. This potential is discussed in the literature [2, 3]. Output variables that can be utilized in such control systems include engine rotor speed, compressor discharge pressure, nozzle inlet temperature, nozzle inlet pressure, turbine inlet temperature, turbine inlet pressure, and engine thrust.

The turbojet engine dynamics can be modeled as a set of first-order non linear ordinary differential equations given by

$$\dot{\mathbf{z}} = \mathbf{f}(\mathbf{z}, \mathbf{p}) \tag{1}$$

and

$$\boldsymbol{\xi} = \boldsymbol{\xi}(\mathbf{z}), \tag{2}$$

in which $\mathbf{z}$ is the state vector, $\mathbf{p}$ is the input vector, and $\boldsymbol{\xi}$ is the output sensor reading vector. Linearization about the steady state values $\mathbf{z}_s$, $\mathbf{p}_s$, and $\boldsymbol{\xi}_s$, results in

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{3}$$

and

$$\mathbf{y} = \mathbf{C}\mathbf{x} \tag{4}$$

in which

$$\mathbf{x} = \mathbf{z} - \mathbf{z}_s, \quad \mathbf{u} = \mathbf{p} - \mathbf{p}_s, \ \mathbf{y} = \boldsymbol{\xi} - \boldsymbol{\xi}_s,$$
$$\mathbf{A} = \partial \mathbf{f}/\partial \mathbf{z}(\mathbf{z}_s, \mathbf{p}_s),$$
$$\mathbf{B} = \partial \mathbf{f}/\partial \mathbf{p}(\mathbf{z}_s, \mathbf{p}_s),$$

and

$$\mathbf{C} = \partial \boldsymbol{\xi}/\partial \mathbf{z}\,(\mathbf{z}_s).$$

For digital computer simulation it is desirable to represent the continuous-time system model (3)-(4) by the corresponding discrete model

$$\mathbf{x}(k+1) = \ \Phi\mathbf{x}(k) + \Gamma\mathbf{u}\ (k) + \Gamma\mathbf{w}(k) \tag{5}$$

and

$$\mathbf{y}(k+1) = \ \mathbf{C}\mathbf{x}\ (k+1) + \mathbf{v}\ (k+1), \tag{6}$$

which includes random sensor noise $\mathbf{v}$ and random input disturbances $\mathbf{w}$. The state transition matrix is given by

$$\Phi = \Phi(k+1,\ k) = \exp\ (\mathbf{A}T)$$

and the input transition matrix by

$$\Gamma=\Gamma(k+1, k)=\int_0^T \exp(\mathbf{A}\tau)\mathbf{B}d\tau$$

in which T is the sampling interval.

FAILURE DETECTION ALGORITHM

The general statistical decision problem constitutes making a decision which is best in a certain sense, dependent on some past observations in a stochastic environment. It is convenient to restate this problem in terms of hypothesis testing [4]. It is assumed that the system can takd M disjoint states $H_1, H_2, \ldots, H_M$ having *a priori* probabilities $P_{H_1}, P_{H_2}, \ldots, P_{H_M}$ respectively. H_i are termed hypotheses. The hypothesis testing decision problem now becomes that of selecting the most likely hypothesis, depending on the sensor readings $\mathbf{y}$ and based upon a suitable test. There are a number of tests that can be used for this purpose. The most common two are Bayes test (of which the likelihood ratio test is a special case) and the minimax test. These two approaches to the multiple hypothesis decision problem have been proven successful in various studies including failure detection both in the shuttle orbiter reaction control system [5] and in gyro-accelerometer systems [6]. For the purpose of the present development, the Bayes test is chosen and described in the following.

Definition 1: Cost Function

The cost function $C(\mathbf{H}, \mathbf{H})$ specifies the penalty or loss or cost of making an incorrect decision. $\mathbf{H}$ denotes the hypothesis set $(H_1, H_2, \ldots, H_M]$. In particular, C_{ij}=cost of accepting H_i when H_j is true.

Definition 2: Simple Cost Function

If the cost of a particular event does not depend on the probability of that event, the corresponding cost function is a simple cost function. In the present work only the simple cost functions are considered.

Definition 3: Risk Function

The risk function B is the expected value of the cost function. Therefore

$$B=\sum_{i=1}^{M}\sum_{j=1}^{M} C_{ij}\, P(H_i, H_j), \tag{7}$$

in which $P(H_i, H_j)$ is the joint probability that H_i is accepted and H_j is true.

Multiple (M-ary) hypothesis testing requires the partitioning of the entire observation space Z into M disjoint subspaces

$$Z_i, \quad i=1, 2, \ldots, M,$$

where

$$Z=\bigcup_{i=1}^{M} Z_i,$$

so that if the observation (sensor reading) vector

$$\mathbf{y}\in Z_l$$

the hypothesis H_l is accepted. The appropriate Z_i are determined by minimizing B, the risk of incorrect decisions.

Using the standard results for joint probability,

$$P(H_i, H_j)=P(H_i/H_j)P_{H_j}$$

and the conditional probability of accepting H_i given H_j is true,

$$P(H_i/H_j)=\int_{Z_i} f_{\mathbf{Y}/\mathbf{H}}(\mathbf{y}/H_j)\, d\mathbf{y},$$

in which

$f_{\mathbf{Y}/\mathbf{H}}(\mathbf{y}/H_j)$=conditional probability density of the sensor reading random vector, given that H_j is true, the risk function (7) may be expressed as

$$B=\sum_{i=1}^{M}\sum_{j=1}^{M} C_{ij}\, P_{H_j}\int_{Z_i} f_{\mathbf{Y},\mathbf{H}}(\mathbf{y}, H_j)\, d\mathbf{y} \tag{8}$$

It should be noted that for $i\neq j$, the conditional probability $P(H_i/H_j)$ is not necessarily zero, for the decision of selecting H_i, which is based on testing whether $\mathbf{y}$ lies in Z_i, may have a nonzero probability even when it is given that H_j is true.

By straightforward mathematical manipulation, (8) can be reduced to

$$B=\sum_{j=1}^{M} P_{H_j}\, C_{jj}+\sum_{i=1}^{M}\int_{Z_i}\left[\sum_{\substack{j=1\\ j\neq i}}^{M} \beta_{ij}\right] d\mathbf{v}, \tag{9}$$

in which

$$\beta_{ij}=P_{H_j}\,(C_{ij}-C_{jj})f_{\mathbf{Y}/\mathbf{H}}(\mathbf{y}/H_j). \tag{10}$$

For a given problem, the first term in (9) is a constant. Consequently, an equivalent formulation is to select Z_i

such that the risk given by

$$B^{*} = \sum_{i=1}^{M} \int_{Z_i} \left(\sum_{\substack{j=1\\ j\neq i}}^{M} \beta_{ij}\right) d\mathbf{y} \quad (11)$$

is a minimum. Then, for a particular observation sample $\mathbf{y}$, if the minimum of

$$\sum_{\substack{j=1\\ j\neq i}}^{M} \beta_{ij}$$

corresponds to $i=l$, the hypothesis H_l is accepted. Then and only then will each integral in the summation over i be at its minimum, resulting in the minimum of B^*. This results in the following algorithm for the general hypothesis testing problem:

(i) Using (10) compute the $M(M-1)$ unknown terms in the matrix $\{\beta_{ij}\}$ where $\{\beta_{ij}\}$ is known to have zero diagonal terms.

(ii) Compute the M sums

$$S_i = \sum_{\substack{j=1\\ j\neq i}}^{M} \beta_{ij}.$$

(iii) Identify the minimum S_i corresponding to the most likely hypothesis. In other words:

accept H_l if $S_l \leq S_i$ for all $i \neq l$

GAUSSIAN PROBLEM

A common assumption is that the noise vectors $\mathbf{w}(k)$ and $\mathbf{v}(k)$ in the state space model (5) and (6) are independent, zero-mean, Gaussian white noise with covariance matrices given by [7]

$$E[\mathbf{w}(j)\,\mathbf{w}^{T}(k)] = \delta_{jk}\mathbf{Q}(k) \quad (12)$$

$$E[\mathbf{v}(j)\,\mathbf{v}^{T}(k)] = \delta_{jk}\mathbf{R}(k). \quad (13)$$

In addition, the system initial states are assumed Gaussian. Consequently, the conditional probability density function of the random processes $\mathbf{Y}$, given H_i, is normal, i.e.

$$f_{\mathbf{Y}/\mathbf{H}}(\mathbf{y}/H_i) = \frac{1}{(2\pi)^{r/2}|\mathbf{Q}_i|} \exp \times \exp[-1/2(\mathbf{y}-\hat{\mathbf{y}}_i)^{T}\mathbf{Q}_i^{-1}(\mathbf{y}-\hat{\mathbf{y}}_i)] \quad (14)$$

in which $\hat{\mathbf{y}}_i$ is the expected value of the measurement vector conditioned on H_i and r is the dimension of $\mathbf{y}$. The hypothesis-conditioned measurement error $\mathbf{y}-\hat{\mathbf{y}}_i$ and its covariance matrix $\mathbf{Q}_i$ that are required for the decision process may be conveniently obtained by using a set of M Kalman filters [7] each conditioned on one of the hypotheses. This concept of using a bank of Kalman filters in the parameter estimation of dynamic systems is fairly old [8, 9]. The corresponding recursive relations are

$$\begin{aligned}
\mathbf{V}_i(k+1/k) &= \mathbf{\Phi}\mathbf{V}_i(k)\,\mathbf{\Phi}^{T} + \mathbf{\Gamma}\mathbf{Q}\mathbf{\Gamma}^{T}\\
\mathbf{Q}_i(k+1) &= \mathbf{C}_i\mathbf{V}_i(k+1/k)\,\mathbf{C}_i^{T} + \mathbf{R}_i\\
\mathbf{K}_i(k+1) &= \mathbf{V}_i(k+1/k)\,\mathbf{C}_i^{T}\,\mathbf{Q}_i^{-1}(k+1)\\
\tilde{\mathbf{y}}_i(k+1) &= \mathbf{y}(k+1) - \mathbf{C}_i[\mathbf{\Phi}\hat{\mathbf{x}}_i(k) + \mathbf{\Gamma}\mathbf{u}(k)]\\
\hat{\mathbf{x}}_i(k+1) &= \mathbf{\Phi}\hat{\mathbf{x}}_i(k) + \mathbf{\Gamma}\mathbf{u}(k) + \mathbf{K}_i(k+1)\,\tilde{\mathbf{y}}_i(k+1)\\
\hat{\mathbf{y}}_i(k+1) &= C_i\hat{\mathbf{x}}_i(k+1)\\
\mathbf{V}_i(k+1) &= [\mathbf{I} - \mathbf{K}_i(k+1)\mathbf{C}_i]\mathbf{V}_i(k+1/k)
\end{aligned} \quad (15)$$

Matrix $\mathbf{C}_i$ is obtained from $\mathbf{C}$ by setting to zero the rows corresponding to the failed sensors in hypothesis H_i. The assumption made here is that a failed sensor produces zero output 'in the mean'. However, in the general case, random measurement noise $\mathbf{v}_i$ having nonzero mean values corresponding to the failed sensors have to be incorporated in the filter equations. *A priori* sensor noise covariances $\mathbf{R}_i$ conditioned on H_i, are assumed to be known.

In order to initialize the algorithm (15), the estimated initial state vector $\mathbf{x}_i(0)$ and the estimated error covariance matrix $\mathbf{V}_i(0)$ have to be known. If the initial state is known with probability 1, these parameters become

$$\hat{\mathbf{x}}_i(0) = \mathbf{x}(0) \quad \text{and} \quad \mathbf{V}_i(0) = \mathbf{0},$$

Once the most likely hypothesis H_l is determined using the decision logic developed in the previous section, an optimal estimate for the true output variables is obtained using

$$\hat{\mathbf{y}}(k+1) = \mathbf{C}\hat{\mathbf{x}}_l(k+1). \quad (16)$$

The complete algorithm may be implemented in real time using a digital computer. A block diagram for the principal steps involved is shown in Figure 1. It is noted that real-time processing is necessary for the implementation of the algorithm, because both the instantaneous sensor readings and the engine input variables are used in making the decision at a particular instant. This real-time failure detection and real-time output estimation feature makes the present method quite superior to a simple off-line failure detection scheme.

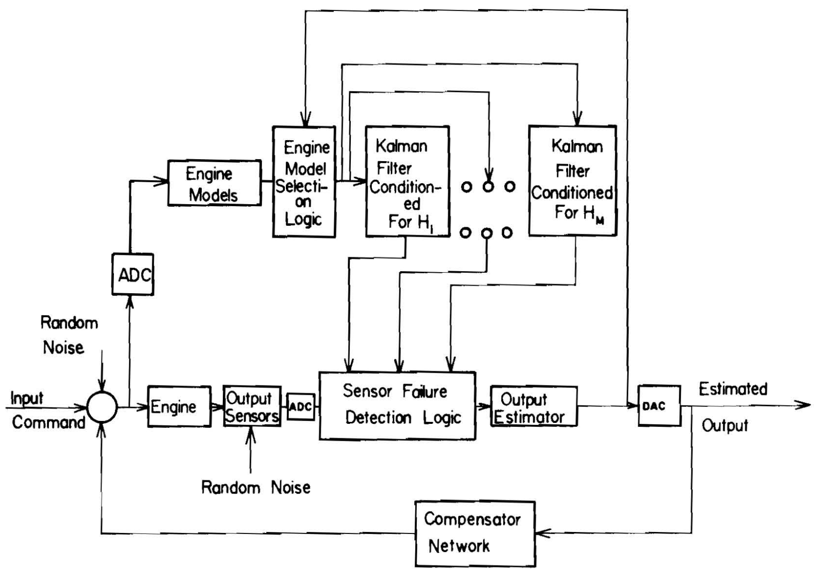

Figure 1. Block Diagram for the Real-Time Sensor Failure Detection Process

SIMPLIFIED ALGORITHM

A simple decision strategy may be obtained using the assumptions

$$P_{H_j} = P_H \quad \text{for all} \quad j$$

and

$$C_{ij} \begin{cases} = 0 & \text{for } i = j \\ = C > 0 & \text{for } i \neq j. \end{cases}$$

Consequently,

$$S_i = CP_H \sum_{\substack{j=1 \\ j \neq i}}^{M} f_{\mathbf{Y/H}} (\mathbf{y}/H_j).$$

It is noted that the minimum S_i is obtained if and only if the maximum $f_{\mathbf{Y/H}}(\mathbf{y}/H_j)$ term is eliminated from the sum. This results in the decision strategy:

accept H_l if

$$f_{\mathbf{Y/H}}(\mathbf{y}/H_l) \geqq f_{\mathbf{Y/H}}(\mathbf{y}/H_i), \quad \text{for all} \quad i \neq l. \tag{17}$$

In view of the Gaussian assumption (14) and noting that the logarithm is a monotonically increasing function, the following equivalent decision strategy is obtained:

accept H_l if

$$\alpha_l \leqq \alpha_i \quad \text{for all} \quad i \neq l$$

in which

$$\alpha_i = \ln|\mathbf{Q}_i| + \tfrac{1}{2}(\mathbf{y} - \hat{\mathbf{y}}_i)^{\mathrm{T}} \mathbf{Q}_i^{-1} (\mathbf{y} - \hat{\mathbf{y}}_i). \tag{18}$$

SYSTEM NONLINEARITIES

Aircraft engines are highly nonlinear systems. The mathematical model linearized about an operating point therefore cannot be used over a large range of output values. During engine accelerations and decelerations, the variations in outputs can be quite large. This will impose serious restrictions on the use of the proposed technique, which may be overcome as follows.

A set of linear engine models is identified with respect to an output parameter, for example the engine speed. This information can be interpolated and stored in the computer data base as a set of curves. During the decision process the correct engine model parameters are chosen depending on the value of the current output parameter. This adaptive model selection technique is possible because of the presence of the on-board digital computer. A block diagram of the complete decision process is given in Figure 1. It is the estimated output that is used in the model selection logic. The engine models will be identical if the nonlinearities are neglected. In the presence of nonlinearities, different models will result in different output estimates. The level of deviation will depend on the degree of nonlinearity. Except for the right model for a given range of engine outputs, the estimated outputs from a particular engine model will be outside that range of outputs.

NUMERICAL EXAMPLE

In order to apply the present technique to the turbojet engine sensor failure detection problem, engine dynamics have to be expressed in state space form. Merrill [3] has derived a linear state model using the Tse–Weinert identification technique, and realistic single-spool turbojet engine output data generated by a digital computer dynamic simulation. The model is second order and has been shown to be quite adequate. The engine rotor speed has been employed as the model linearization parameter. The output variables are:

$$\mathbf{y}=\begin{bmatrix}\text{engine rotational speed (rev/min)}\\ \text{engine thrust (lbf)}\end{bmatrix}$$

and the input is:

$$\mathbf{u}=[\text{fuel flow (lb/s)}].$$

Four linear models given in Table 1 have been determined for the speed ranges indicated therein. The sampling interval $T=0.1$ s.

In the present simulation the following three failure states are considered:

H_1: speed sensor fails,

H_2: thrust sensor fails,

H_3: no sensor failure.

The $\mathbf{Q}$ matrix which is a scalar in the present example, has to be selected so as to reflect the variance of the disturbances in the fuel flow. $\mathbf{R}_i$ matrices give the sensor noise covariances under various failure states H_i. They are selected such that there is a higher noise covariance associated with a failed sensor. Actual values of $\mathbf{Q}$ and $\mathbf{R}_i$ depend on the type of sensors used and will vary from system to system. Besides, a single hypothesis is often associated with a group of sensors. For instance, the thrust sensor in the present example represents more than one sensor output employed to compute the engine thrust. The failure in any one of these sensors is equivalent to the thrust sensor failure.

The variables are scaled to improve the computational accuracy. The scaled-to-unity noise covariance matrices used in the present example are

$$\mathbf{R}_1=\begin{bmatrix}0.10 & 0.00\\ 0.00 & 0.09\end{bmatrix}$$

$$\mathbf{R}_2=\begin{bmatrix}0.10 & 0.00\\ 0.00 & 0.11\end{bmatrix}$$

$$\mathbf{R}_3=\begin{bmatrix}0.10 & 0.00\\ 0.00 & 0.10\end{bmatrix}$$

$$\mathbf{\Gamma Q \Gamma}^{\mathrm{T}}=\begin{bmatrix}0.100 & 0.005\\ 0.005 & 0.200\end{bmatrix}$$

A set of simulated sensor readings may be obtained by first computing the engine outputs using the mathematical model in the absence of noise for certain time periods, taking the outputs to be zero for the remaining time periods (these correspond to the failed sensors) and finally adding Gaussian noise to the resulting data samples for the entire duration. The following failure modes are simulated using this procedure:

(i) During 0.0–0.5 s there is no sensor failure (H_3).
(ii) During 5.1–10.0 s only the speed sensor failed (H_1).
(iii) During 10.1–15.0 s only the thrust sensor failed (H_2).
(iv) During 15.1–20.0 s there is no sensor failure (H_3).
(v) During 20.1–30.0 s only the speed sensor failed (H_1).
(vi) During 30.1–40.0 s there is no sensor failure (H_3).

An indicator function I_f is defined as follows:

$$\begin{aligned} I_f &= 1.0 \quad \text{when } H_3 \text{ is true}\\ &= 0.5 \quad \text{when } H_2 \text{ is true}\\ &= 0.0 \quad \text{when } H_1 \text{ is true.}\end{aligned}$$

Table 1. Numerical Values for Engine Model Parameters

Matrix	Percentage of nominal speed: 80%		90%		100% 36,960 rev/min		104.5%	
Φ	0.000	1.0000	0.000	1.0000	0.000	1.0000	0.000	1.0000
	−0.354	1.2330	−0.340	1.1830	−0.258	1.0600	−0.318	1.1190
Γ	48004.70		43653.50		45947.60		40617.60	
	27815.90		24165.80		19977.63		19662.80	
C	1.000	0.0000	1.000	0.0000	1.000	0.0000	1.000	0.0000
	0.018	−0.0012	0.023	−0.0023	0.026	−0.0034	0.0028	−0.0044

Table 2. Engine Model Selection Logic

For Speed Variations (rev/min) About the 100% speed:			Switch to the model
2494.8	to	831.6	104.5%
831.6	to	−1848.0	100%
−1848.0	to	−5544.0	90%
−5544.0	to	−9240.0	80%

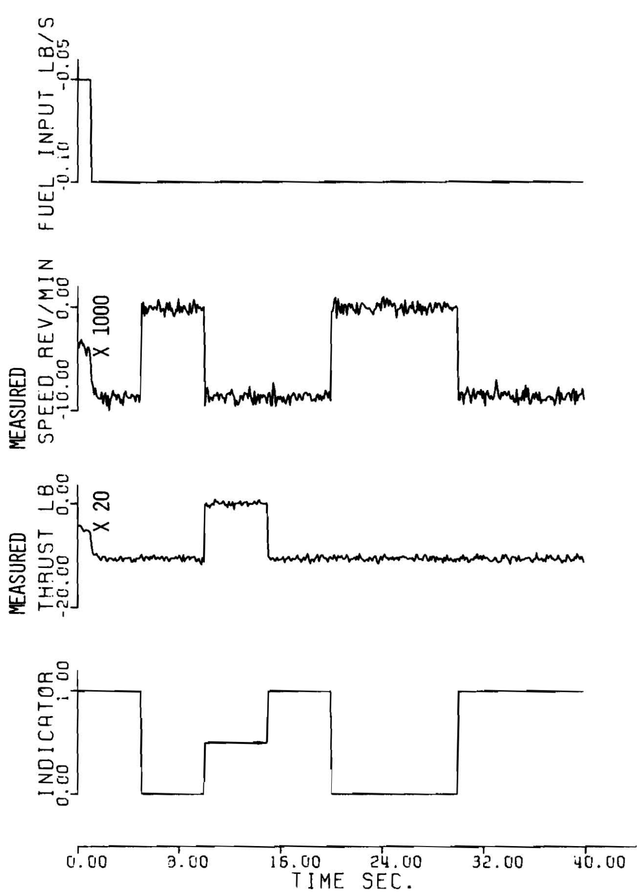

Figure 2. Failure Detection for a Step Input

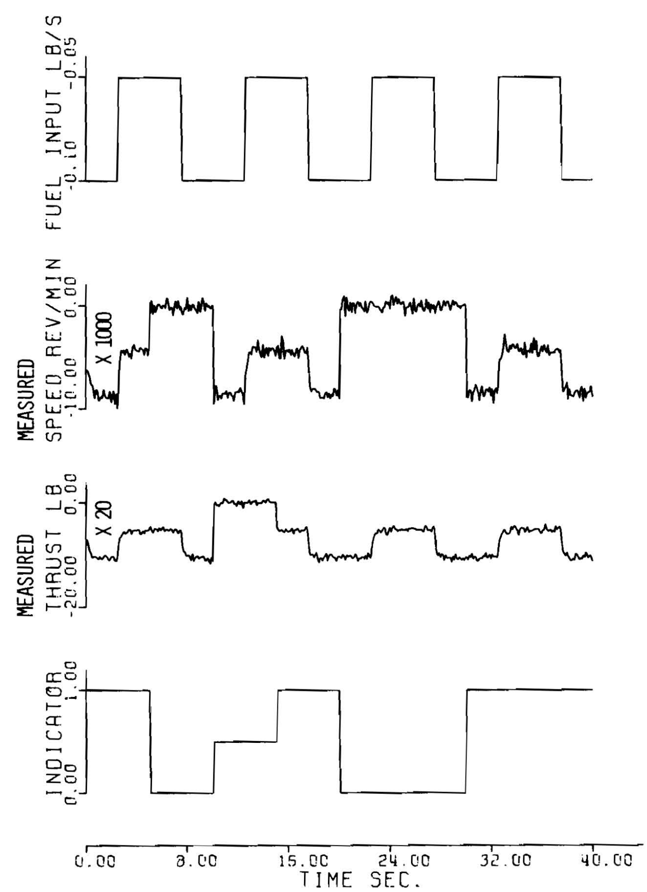

Figure 3. Failure Detection for a Bang-Bang Input

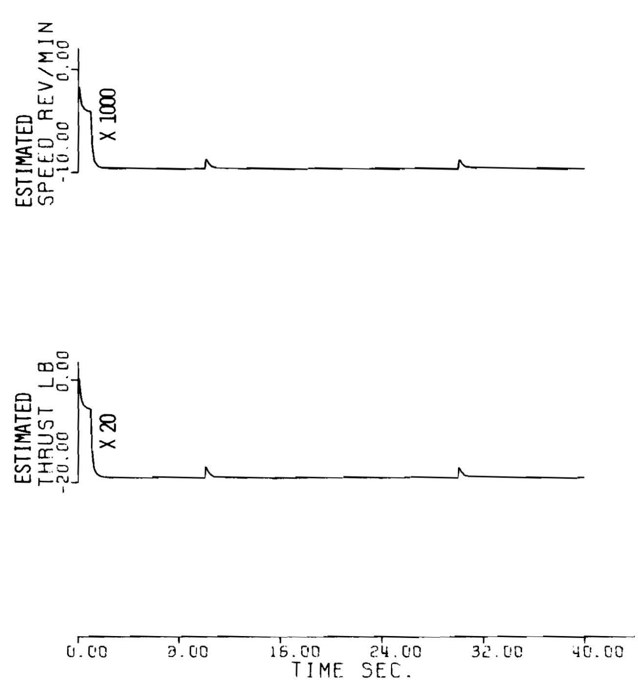

Figure 4. Estimated Outputs for a Step Input

The model selection logic given in Table 2 is incorporated within the original process. Variations about a nominal 100% speed are considered. Estimated speed is used in the engine model selection logic.

Figure 2 gives the results obtained using the present algorithm for a step input. The results for a bang-bang input are shown in Figure 3. It is seen that in both cases the algorithm has correctly determined the failure states of the sensors. Figures 4 and 5 show the optimal estimates for the output variables. These are adequate to control the engine during emergency maneuvering in the event of a sensor failure. They are also used in selecting the proper engine model.

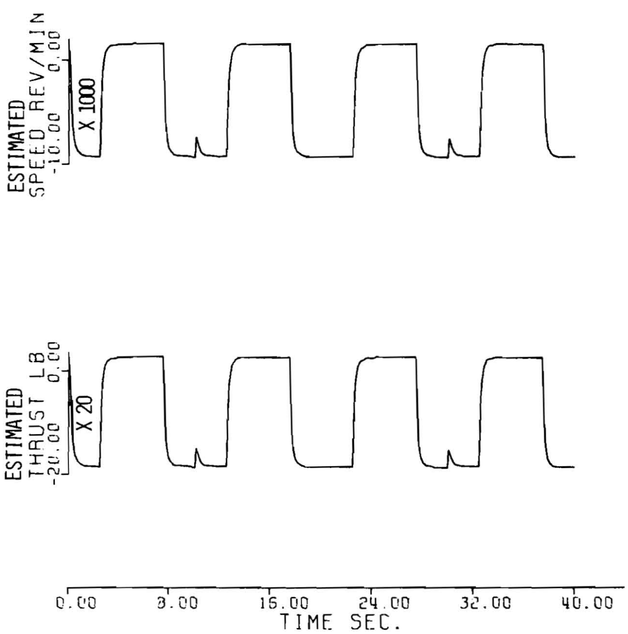

Figure 5. Estimated Outputs for a Bang-Bang Input

DISCUSSION AND CONCLUSIONS

A digital computer algorithm was developed for the detection of failure states in real time for aircraft engine output sensors. The technique employed the Bayesian approach for the hypothesis testing associated with the failure detection. A set of hypothesis-conditioned Kalman filters was used to estimate the outputs corresponding to the failed sensors and to determine the associated error covariance matrices. Real-time processing which incorporates the instantaneous sensor readings and the engine input values was necessary in the implementation of the procedure for sensor failure detection. The algorithm was applied to a realistic single-spool turbojet engine model. To account for the engine nonlinearities a model selection logic was used. In this example, the present technique accurately determined the sensor failure states and provided good estimates for the engine outputs.

In general, the efficiency of the algorithm depends on factors such as the accuracy of the mathematical model used, proper selection of *a priori* covariance matrices $\mathbf{Q}$ and $\mathbf{R}_i$ for the input and sensor noise, and proper scaling of the system variables. Since the hypothesis-conditioned measurement error covariances depend to a large extent on the sensor failure itself (in addition to the *a priori* sensor noise covariances), they are time-dependent quantities in general. Consequently, time-dependent Kalman filters must be employed. The hypothesis-conditioned measurement error covariances computed in the present example were found to deviate considerably from the *a priori* sensor noise covariances, justifying the use of unsteady Kalman filters.

Various inputs including ramp, step, sine and bang-bang were used to test the algorithm developed and in each case the failure states were predicted accurately. Most of these results were omitted from the paper for brevity.

A desirable feature of the technique is that optimal estimates of the outputs corresponding to failed sensors are a by-product of the algorithm. However, the technique requires as many Kalman filters as there are failure states. The real-time processing assumes the availability of a digital computer on-board for the implementation of the present algorithm.

Depending on the aircraft features and its mission, the method developed could be quite cost-effective in comparison to the classical method of adding hardware redundancy. It should be noted that weight, space requirements, and cost should be taken into account when employing hardware redundancy. In any case, hardware redundancy does not result in a 100% fail-safe system. On the other hand, if an on-board digital computer is available, the addition of the software feature developed in this paper involves negligible increase in weight, space requirements, and effort. In this sense the software-oriented technique developed in this paper is cost-effective. However, the economic aspects depend on specific applications considered. This could be the subject of a different research project where comparative evaluations could be made.

APPENDIX

Notation

$\mathbf{A}$ = System matrix in the linear turbojet state model
B = Risk function
B^* = Modified risk function
$\mathbf{B}$ = Input system matrix
$C(\mathbf{H},\mathbf{H})$ = Cost function
C_{ij} = Cost of accepting H_i when H_j is true
$\mathbf{C}$ = Measurement system matrix
$\mathbf{C}_i$ = Measurement system matrix conditioned on H_i
$f_{\mathbf{Y}/\mathbf{H}}(\mathbf{y}/H_j)$ = Conditional probability density of the observation vector given that H_j is true
H_i = Hypothesis i

$\mathbf{H}$ = Set of hypotheses
M = Total number of possible hypotheses
$\mathbf{p}$ = Absolute input vector
P_{H_i} = Probability of occurrence of H_i
$P(H_i, H_j)$ = Joint probability that H_i is accepted and H_j is true
$P(H_i/H_j)$ = Conditional probability of accepting H_i given H_j is true
$\mathbf{Q}$ = Input noise covariance matrix
$\mathbf{Q}_i$ = Hypothesis conditioned sensor error covariance matrix for H_i
r = Size of the engine output vector
$\mathbf{R}$ = Sensor noise covariance matrix
$\mathbf{R}_i$ = Sensor noise covariance matrix conditioned on H_i
T = Sample interval for the discrete system model
$\mathbf{u}$ = Incremental input vector
$\mathbf{v}$ = Sensor noise vector
$\mathbf{w}$ = Input noise vector
$\mathbf{x}$ = Incremental state vector
$\hat{\mathbf{x}}$ = Estimated state vector
$\mathbf{y}$ = Incremental output vector
$\hat{\mathbf{y}}$ = Estimated output vector
$\ddot{\mathbf{y}}$ = Output estimation error vector
$\mathbf{z}$ = Absolute state vector
Z = Observation space
Z_i = Observation subspace corresponding to H_i
δ_{ij} = Kronecker delta
Φ = State transition matrix
Γ = Input transition matrix
τ = Dummy time variable
ξ = Absolute output vector
$(\,)_s$ = Steady state values
$|\cdot|$ = Determinant
$[\,]^T$ = Matrix transposition
$E[\,]$ = Expected value
$(\dot{\,})$ = Time derivative

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