CLOSED FORM SOLUTION OF THE ISOLATED OPEN CIRCUIT OPERATION OF INDUCTION MACHINES

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الخلاصـة :

يقـدّم هذا البحث حـلاً جبريّـاً لحساب الأداء الديناميكي لمحركات الحـثّ في حالة فصلها عـن المصدر، وقد تم ذلك باستخدام المتغيرات المركبة المحوّلة إلى المحور الدوّار وحقيقية كون تيار الملف الثابت في هذه الحالة صفراً، وبذلك تم تحويل معادلات الأداء الديناميكي التفاضلية لمحرك الحث الى معادلات تفاضلية خطية من الدرجة الأولى يمكن حلها جبرياً.

ولقد تم التأكد من دقة نتائج هذه المعادلات الجبرية المقترحة بمقارنتها مع نتائج دقيقة تم الحصول عليها باستخدام النموذج المحوري المفصل لمحركات تتراوح مقنناتها من ٣ إلى ٢٢٥٠ حصـانـاً، وفي جميع الحالات كانت النتائج متطابقة تماماً، وقد كان ذلك متوقعاً لاشتقاق المعادلات الجبرية المقترحة مباشرة من النموذج المحوري المفصل، وبفضل هذه المعادلات الجبريـة يمـكـن حـسـاب الأداء الديناميكي لمحركات الحث في أي لحظة كانت خلال مدة الفصل وبسرعة فائقة على عكس النموذج المحوري المفصل – فيجب باعتباره نموذجاً رقميا – اختيار الدرجة الزمنية بدقة للحصول على نتائج معقولة.

ABSTRACT

This paper presents a closed form solution of induction machines when disconnected from their main supply. Using time complex variables, expressed in the rotor reference frame, and the fact that the stator current is zero during this mode of operation, the differential equations that describe the dynamic behavior of the symmetrical induction machine were transformed into first order linear differential equations which could be easily solved in closed form. The performance of the closed form solution was evaluated using several induction machines with different horse power ratings ranging from 3 hp to 2250 hp. In all simulated cases, the closed form solution and the detailed *d/q* model results were identical. This is expected, since the closed form solution was obtained directly from the detailed *d/q* formulation. However, the closed form solution is faster than the detailed *d/q* model. Furthermore, the closed form solution can directly determine the dynamic response of induction machines at any time during their isolated open circuit mode of operation. However, in the case of the detailed *d/q* model, being a numerical solution, the time step must be properly selected in order to obtain reasonable accuracy.

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LIST OF SYMBOLS

R_s, R_r	=	Stator and rotor resistances
X_{ls}, X_{lr}	=	Stator and rotor leakage reactances
X_m	=	Magnetizing reactance
H	=	Inertia constant(s)
ψ_{ds}, ψ_{qs}	=	d/q stator windings flux linkages
ψ_{dr}, ψ_{qr}	=	d/q rotor windings flux linkages
v_{ds}, v_{qs}	=	d/q stator windings terminal voltages
i_{ds}, i_{qs}	=	d/q stator windings currents
i_{dr}, i_{qr}	=	d/q rotor windings currents
ψ_s, ψ_r	=	Stator and rotor windings time complex flux linkages
v_s	=	Stator windings time complex terminal voltage
i_s, i_r	=	Stator and rotor windings time complex currents
ω_e	=	Electrical angular velocity of the synchronously rotating reference frame
ω_r	=	Rotor speed
ω_b	=	Base electrical angular velocity which is generally selected equal to ω_e
T_e, T_L	=	Electromagnetic and load torques
t_o	=	Time at which machine is isolated
ω_{ro}, s_o	=	Rotor speed and slip at t_o
v_{so}	=	Stator windings time complex terminal voltage at t_o
v_{sa}, i_{sa}	=	Phase a stator voltage and current
i_{ra}	=	Phase a rotor current
X_{ss}	=	$X_{ls} + X_m$
X_{rr}	=	$X_{lr} + X_m$
D	=	$X_{ss}X_{rr} - X_m^2$

All inductive reactances are calculated at base frequency.

1. INTRODUCTION

Considerable attention has been given to the accuracy of reduced order models which predict the dynamic response of induction machines [1–6]. A traditional method of reducing the order of induction machine dynamic equations neglects the time rate of change of the stator flux linkages (stator transients) [2]. This standard model is widely used in power system stability studies. This approach is based on the fact that changes in the stator flux linkages are much faster than those in the rotor flux linkages. The reduced order equations are then linearized using the small displacement approach. Such a model is investigated in [3] where some guidelines were suggested to determine when this model would be accurate. Various modifications of this method were developed in [3–6]. An investigation and comparison of three of these methods are given in [5].

The accuracy of the standard reduced order model in predicting the isolated operation of the induction machine has been investigated in [7]. In [7], the standard reduced order model has been modified in order to correctly predict this mode of operation. The suggested modification involves changing the frame of reference from the synchronously rotating one, of the electrical system (from which the machine was disconnected), to a frame of reference fixed in the rotor of the induction machine and then back to the synchronously rotating reference frame upon reconnecting the machine to the system.

The machine equations can be expressed in either real or complex variables form. Despite the early introduction of complex variables analysis [8, 9], the trend in the U.S.A. has been directed toward real variables except for a few isolated instances [10–12]. However, the complex variable method is extensively used outside of the U.S.A. [13–15]. The complex variables analysis greatly simplifies the mathematical representation of induction machines dynamics [16].

This paper presents a closed form solution of the dynamic response of an induction machine which is disconnected from the system (open circuited). Like the detailed d/q representation [1], the closed form solution accounts for electrical transients in both stator and rotor circuits as well as mechanical transients. The new formulation is obtained using complex time variables with the machine dynamic equations expressed in the rotor frame of reference. The proposed closed form solution accurately predicts the induction machine dynamic response at any time during the open circuit operation. Accurate prediction of the machine terminal voltage during this mode of operation is important in order to correctly represent the influence of protective relaying. Also of practical interest is accurately predicting the magnitude and phase angle of the decaying terminal voltage of the induction machine relative to the system from which it was disconnected so that the average electromagnetic torque immediately following reclosure may be adequately predicted. This is important if reclosing occurs before the terminal voltage of the isolated induction machine has decayed to a relatively small value. The proposed closed form solution and the detailed d/q model gave identical predictions of the machine response during the open circuit operation. However, the closed form solution is faster than the detailed d/q model. Furthermore, the closed form solution can determine the dynamic response of induction machines at any time during their isolated open circuit mode of operation. However, in the case of the detailed d/q model, being a numerical solution, the time step must be properly selected in order to obtain reasonable accuracy.

2. DETAILED MODEL OF INDUCTION MACHINES

Accurate prediction of the induction machines transients can be obtained using the well-established detailed d/q formulation. The full order model of induction machines may be written in terms of either currents or flux linkages as state variables. In this study, flux linkages were selected as state variables because they tend to vary more slowly than currents providing more numerical stability. The equations of a symmetrical induction machine with no neutral connection may be expressed in a synchronously rotating frame of reference by [1]:

$$\frac{1}{\omega_b}\frac{d\psi_{ds}}{dt} = -a_1\psi_{ds} + \frac{\omega_e}{\omega_b}\psi_{qs} + a_2\psi_{dr} + v_{ds} \tag{1}$$

$$\frac{1}{\omega_b}\frac{d\psi_{qs}}{dt} = -a_1\psi_{qs} - \frac{\omega_e}{\omega_b}\psi_{ds} + a_2\psi_{qr} + v_{qs} \tag{2}$$

$$\frac{1}{\omega_b}\frac{d\psi_{dr}}{dt} = a_3\psi_{ds} - a_4\psi_{dr} + \frac{\omega_e - \omega_r}{\omega_b}\psi_{qr} \tag{3}$$

$$\frac{1}{\omega_b}\frac{d\psi_{qr}}{dt} = a_3\psi_{qs} - a_4\psi_{qr} - \frac{\omega_e - \omega_r}{\omega_b}\psi_{dr} \tag{4}$$

where

$$\left.\begin{aligned} a_1 &= R_s X_{rr}/D, \quad a_2 = R_s X_m/D, \\ a_3 &= R_r X_m/D, \quad a_4 = R_r X_{ss}/D \end{aligned}\right\} \tag{5}$$

and the flux linkages are related to the d/q stator and rotor currents by:

$$\psi_{ds} = X_{ss} i_{ds} + X_m i_{dr} \tag{6}$$

$$\psi_{qs} = X_{ss} i_{qs} + X_m i_{qr} \tag{7}$$

$$\psi_{dr} = X_m i_{ds} + X_{rr} i_{dr} \tag{8}$$

$$\psi_{qr} = X_m i_{qs} + X_{rr} i_{qr}. \tag{9}$$

It is important to note that the inductive reactances are calculated by multiplying ω_b by the appropriate inductance. Furthermore, the mechanical equation is given by:

$$\frac{1}{\omega_b}\frac{d\omega_r}{dt} = \frac{1}{2H}(T_e - T_L) \tag{10}$$

where

$$T_e = \frac{X_m}{D}(\psi_{qs}\psi_{dr} - \psi_{ds}\psi_{qr}). \tag{11}$$

Finally, the d/q stator currents can be obtained from the flux linkages as follows:

$$i_{ds} = \frac{1}{D}(X_{rr}\psi_{ds} - X_m\psi_{dr}) \tag{12}$$

$$i_{qs} = \frac{1}{D}(X_{rr}\psi_{qs} - X_m\psi_{qr}). \tag{13}$$

3. TIME DOMAIN COMPLEX MODEL

The machine equations can be expressed in either real or complex variables form. The complex variables analysis greatly simplifies the mathematical representation of induction machine dynamics. To obtain time domain complex equations for the induction machine, define the following variables [10]:

$$\left.\begin{aligned} \mathcal{F}_s &= \mathcal{F}_{ds} + j\mathcal{F}_{qs} \\ \mathcal{F}_r &= \mathcal{F}_{dr} + j\mathcal{F}_{qr} \end{aligned}\right\} \tag{14}$$

where $\mathcal{F}$ denotes current, voltage, or flux linkage. Applying these definitions to (1–4), the four real equations can be reduced to the following two time complex equations:

$$\frac{1}{\omega_b}\frac{d\psi_s}{dt} = -\left(a_1 + j\frac{\omega_e}{\omega_b}\right)\psi_s + a_2\psi_r + v_s \tag{15}$$

$$\frac{1}{\omega_b}\frac{d\psi_r}{dt} = a_3\psi_s - \left\{a_4 + j\frac{\omega_e - \omega_r}{\omega_b}\right\}\psi_r \tag{16}$$

where, from (6–9) with the relations of (14) applied to them, the time complex flux linkages can be related to the time complex currents by:

$$\psi_s = X_{ss}i_s + X_m i_r \tag{17}$$

$$\psi_r = X_m i_s + X_{rr}i_r. \tag{18}$$

The electromagnetic torque is related to the time complex flux linkages by:

$$T_e = \frac{X_m}{D}\Im\{\psi_s\psi_r^*\} \tag{19}$$

where * and $\Im$ denote complex conjugate and imaginary part respectively.

4. CLOSED-FORM SOLUTION OF THE ISOLATED OPERATION MODE

Assume that an induction machine is initially operating at rated conditions. The machine is then disconnected from its main supply (open circuited) with the rated load torque held constant. During the isolated operation of the induction machine, its stator current will be zero and its frequency will be the rotor electrical angular speed, ω_r, and not the supply frequency, ω_e. Therefore, (15) and (16) can be rewritten in the rotor frame of reference by replacing ω_e by ω_r, and hence, $\omega_e - \omega_r = 0$, *i.e.*:

$$\frac{1}{\omega_b}\frac{d\psi_s}{dt} = -(a_1 + j\omega_r/\omega_b)\psi_s + a_2\psi_r + v_s \tag{20}$$

$$\frac{1}{\omega_b}\frac{d\psi_r}{dt} = a_3\psi_s - a_4\psi_r \tag{21}$$

where from (17) and (18) and noting that $i_s = 0$

$$\psi_s = X_m i_r \tag{22}$$

$$\psi_r = X_{rr}i_r. \tag{23}$$

Consequently,

$$\psi_s = \frac{X_m}{X_{rr}}\psi_r. \tag{24}$$

Substituting (24) into (21) and rearranging give:

$$\frac{d\psi_r}{dt} + \frac{1}{\tau}\psi_r = 0 \tag{25}$$

where the time constant, τ is given by

$$\tau = \frac{1}{\omega_b(a_4 - a_3X_m/X_{rr})} = \frac{X_{rr}}{\omega_b R_r}. \tag{26}$$

Equation (25) can be easily solved for $\psi_r(t)$ *i.e.*

$$\psi_r(t) = \psi_{ro}\exp[-(t-t_o)/\tau] \tag{27}$$

where ψ_{ro} is the value of ψ_r at the instant of isolation from the main supply, t_o. Its value can be obtained from (15) and (16) by equating both equations to zero and solving for the rotor flux linkage:

$$\psi_{ro} = \frac{v_{so}\exp(js_o\omega_b t_o)}{(a_1+j)(a_4+js_o)/a_3 - a_2} \tag{28}$$

where v_{so}, s_o, and $\omega_{ro}(s_o = (\omega_e - \omega_{ro})/\omega_b)$, are respectively the machine complex terminal voltage, machine slip, and rotor speed at the time of disconnecting the induction machine from the main supply. Note that ω_b is chosen to be equal to ω_e. It is important to note that the rotor flux linkage at t_o is multiplied by $\exp(js_o\omega_b t_o)$ in order to transform it from the synchronously rotating frame of reference to the rotor frame of reference. Substituting for ψ_r into (23) and (24), i_r and ψ_s can be easily obtained

$$i_r(t) = \frac{1}{X_{rr}}\psi_r(t) \tag{29}$$

$$\psi_s(t) = \frac{X_m}{X_{rr}}\psi_r(t). \tag{30}$$

It can be shown that the developed electromagnetic torque of the disconnected machine is zero. This can be shown by substituting (24) into (19) *i.e.*:

$$T_e(t) = \frac{X_m}{D}\Im\left\{\frac{X_m}{X_{rr}}|\psi_r|^2\right\} = 0 \tag{31}$$

because $|\psi_r|$ is the magnitude of ψ_r and hence $(X_m/X_{rr})|\psi_r|^2$ is real. Setting T_e equal to zero into (10) and integrating, the rotor speed can also be obtained in closed form *i.e.*:

$$\frac{\omega_r(t)}{\omega_b} = \frac{\omega_{ro}}{\omega_b} - \frac{T_L}{2H}(t-t_o) \tag{32}$$

where t is the time from the instant of isolation. Finally, the induction machine terminal voltage can be obtained by substituting into (20) for ψ_r from (24) and noting that $\frac{d\psi_s}{dt} = -\frac{1}{\tau}\psi_s$,

$$v_s(t) = \omega_b\left\{-\frac{1}{\tau} + j\omega_r(t)\right\}\psi_s(t) + \left(a_1 - a_2\frac{X_{rr}}{X_m}\right)\psi_s(t). \tag{33}$$

Since $(a_1 - a_2X_{rr}/X_m) = 0$, hence

$$v_s(t) = \omega_b\left\{-\frac{1}{\tau} + j\omega_r(t)\right\}\psi_s(t). \tag{34}$$

The machine phase a stator and rotor quantities may be obtained from the following equation:

$$f_{na} = |f_n|\cos(\theta + \angle f_n) \tag{35}$$

where f denotes current, voltage or flux linkage; $n = s$ for the stator variables and $n = r$ for the rotor quantities; $|f_n|$ is the magnitude and $\angle f_n$ is the angle. Note that $\theta = 0$ for the rotor variables and that for the stator variables it is given by:

$$\theta = \omega_{ro} t_o + \int_{t_o}^{t} \omega_r dt = \omega_{ro} t - \frac{\omega_b T_L}{4H}(t - t_o)^2. \tag{36}$$

It is important to note that Equations (32) and (36) are only valid for a constant load torque during the open-circuited operation. However, if the load torque is assumed to be dropping off linearly with the rotor speed, *i.e.* $T_L = k\omega_r(t)$ (k is a constant), then the rotor speed will be dropping off exponentially, *i.e.*

$$\omega_r(t) = \omega_{ro} \exp\left(-\frac{k}{2H}(t - t_o)\right) \tag{37}$$

and consequently,

$$\theta = \omega_{ro}\left(t_o + \frac{2H}{k}\right) - \frac{2H}{k}\omega_r(t). \tag{38}$$

It is worth mentioning that it is also possible to obtain a closed form solution in the synchronously rotating frame of reference, however the solution which was derived in the rotor frame of reference is simpler.

5. RESULTS AND DISCUSSION

The stator voltages of phase a and phase a of the rotor currents as well as rotor speeds, are shown for two, 3 hp and 2250 hp, induction machines when the load torque was held constant during the open-circuit operation. The two machines are three-phase, four-pole, 60 Hz and their parameters are given in Table 1.

Table 1. Machines Parameters.

hp	volts	rpm	R_s Ω	R_r Ω	X_{ls} Ω	X_{lr} Ω	X_m Ω	J $kg - m^2$
3	220	1710	0.435	0.816	0.75	0.75	26.13	0.089
2250	2300	1786	0.029	0.022	0.226	0.226	13.04	63.87

Figures 1 and 2 show phase a of the stator voltages of the large and small induction machines respectively. Figure 3 shows phase a of the rotor currents of both the large and small machines. From this figure, the rotor current of the small machine jumped to a higher value, at the instant of isolation from the main supply, than that of the large machine. This is due to the rather large slip value of the small machine (over 6 times that of the large machine).

From Figures 1, 2, and 3, it is clear that the small machine stator voltage and rotor current decay faster than those of the large machine. This is expected because of the rather shorter time constant of the small machine (almost one tenth that of the large machine).

Finally the rotor speeds of the two machines are shown in Figure 4. The rotor speeds decrease linearly with time as clearly indicated by the two figures. This is expected due to the constant load torque during the open circuit operation. However, if the load torque is assumed to be dropping linearly with rotor speed, then the rotor speed will decrease exponentially according to Equation (37). All figures show their respective responses before and during disconnecting the machines from their systems. The two machines were disconnected from their systems (open circuited) at $t_o = 0.1$s. It is worth mentioning that the frequency of the stator phase voltage is ω_e before disconnecting the machine and ω_r during the isolation operation. However, the frequency of the rotor phase current is $s\omega_e$ before disconnecting the machine and 0 during the isolation operation.

It is important to note that both the detailed d/q model and the closed form solutions gave identical results for all tested machines which ranged in size from 3 hp to 2250 hp. This is not surprising, since the closed form solution was directly derived from the detailed d/q formulation as outlined in the previous sections. However, the closed form solution is faster than the detailed d/q model. Furthermore, the closed form solution can determine the dynamic response of induction machines at any time during their isolated open circuit mode of operation. However, in the case of the detailed d/q model, being a numerical solution, the time step must be properly selected in order to obtain reasonable accuracy.

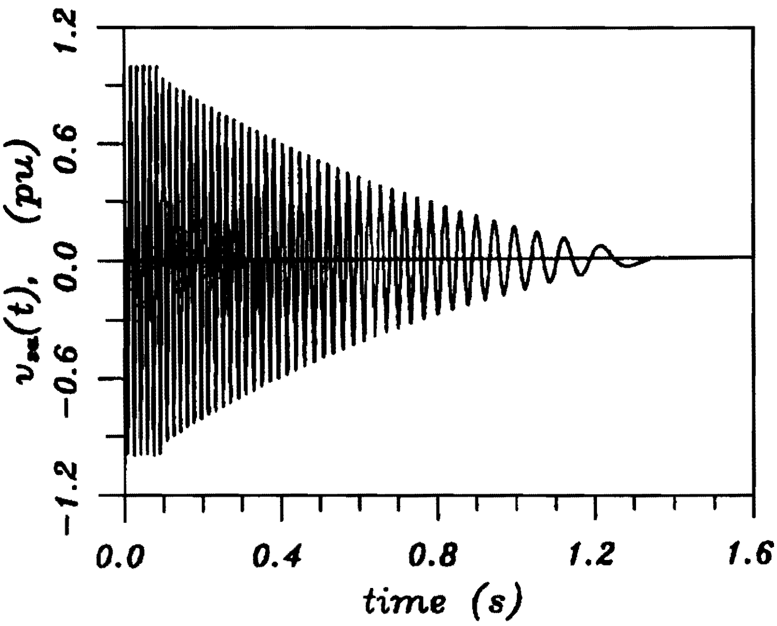

Figure 1. Phase a *Stator Voltage of the 2250 hp Machine Before and After it was Disconnected from the System.*

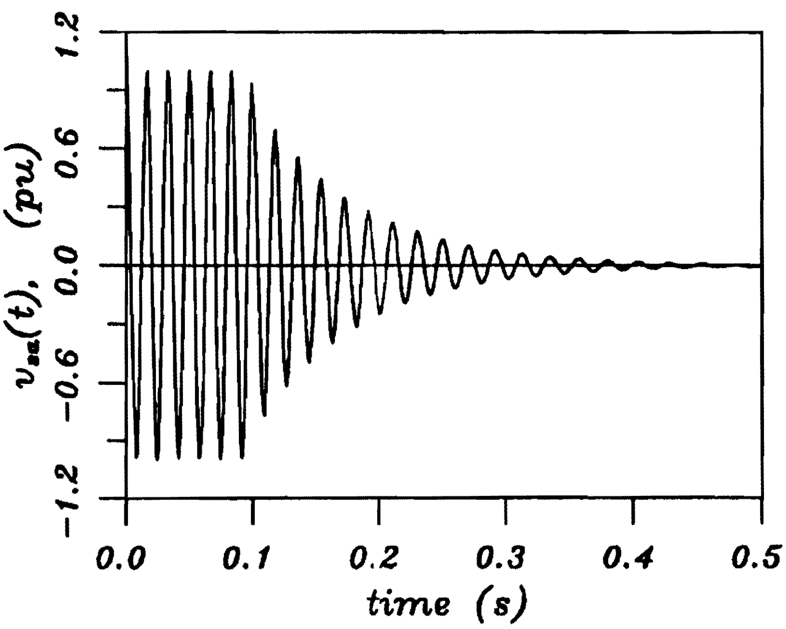

Figure 2. Phase a *Stator Voltage of the 3 hp Machine Before and After it was Disconnected from the System.*

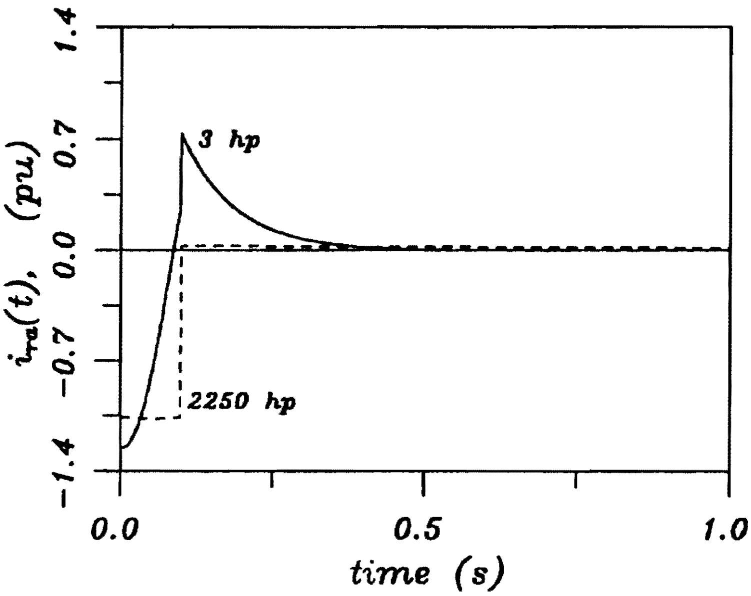

Figure 3. Phase a *rotor Currents of the Two Machines Before and After they were Disconnected from the System.*

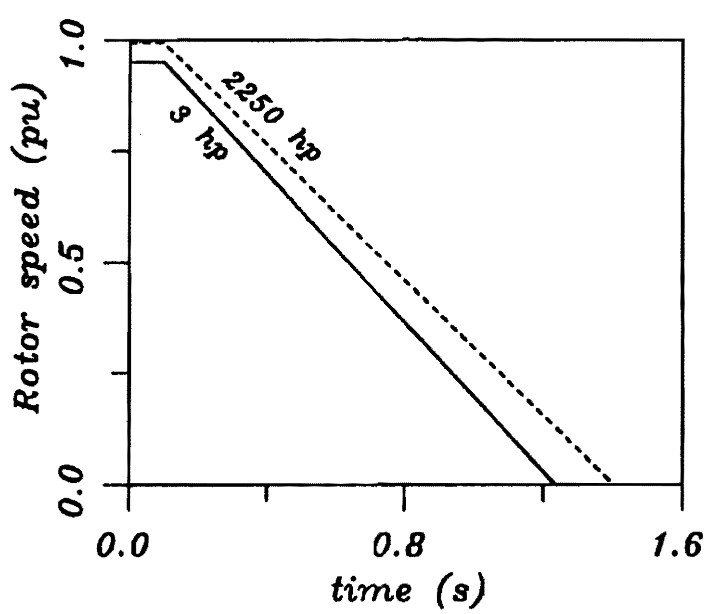

Figure 4. Rotor Speeds of the Two Machines Before and After they were Disconnected from the System.

6. CONCLUSION

This paper presents a closed form solution for the dynamic behavior of an induction machine when disconnected from its system. Using time complex variables and the fact that, during the isolated mode of operation, the machine is open circuited, the differential equations that describe the dynamic behavior of the symmetrical induction machine were transformed into two first order linear differential equations which could be easily solved in closed form. The performance of the proposed model was evaluated using several induction machines. In all simulated cases, both the closed form solution and the detailed d/q model gave identical results. However, the closed form solution is faster than the detailed d/q model. Furthermore, the closed form solution can directly determine the dynamic response of induction machines at any time during their isolated open circuit model of operation. However, in the case of the detailed d/q model, being a numerical solution, the time step must be properly selected in order to obtain reasonable accuracy.

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