ORGANIZATION AND OPTIMIZATION TECHNIQUES OF LEGAL SURVEYS IN HOUSING AREAS

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الخلاصة :

لقد نوقشت مشاكل التنظيم والحصول على الحلول المُثلى لمساحة المناطق السكنية والممتلكات . ولقد إدخلنا نظماً مساحية وأسساً لتقييم الدقة وكذلك طرق إختبارات إحصائية جديدة. والنظم الجديدة يمكن اعتبارها مثالية ليس فقط للدول المتطورة بل أيضاً للمناطق السكنية الجديدة في الدول النامية . إن هذه النظم الجديدة قد صممت لتحل كل المشاكل المساحية التفصيلية المعمول بها إلى الآن . .

ABSTRACT

The problem of organization and specifications of block and property survey is discussed. New survey systems, accuracy criteria, and statistical testing are introduced. The new approach is appropriate not only for developed countries but also for housing areas of developing countries. It is designed to solve the problems involved in all conventional cadastral survey systems.

0377–9211/81/010057–07$01.00

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1. INTRODUCTION

Statistical predictions indicate a rapid increase of world population, and that due to the concentration of attractive services and facilities, more than half of this population will be living in cities. Accordingly, present towns and suburban areas are expected to be the cities and metropolises of the near future.

A coordinated growth of urban areas and their required utilities such as electricity, gas, water, and domestic transportation necessitates the availability of a survey system to which all legal and physical details are tied. The coordinates, and their accuracies, of the property boundaries, streets, housing lots and parcels should be readily available to the planners and contractors.

In most developing countries there is no integrated survey system on which all the data coordinates are displayed. Survey work is done by different governmental agencies and different companies and each of these applies its own standards and specifications. On the other hand the price of land is increasingly drastically and disputes among property owners are exponentially increased. Lack of an integrated survey system with unified and updated survey standards and specifications may also cause irrecoverable damage and lost of life and money.

The topic of diversified survey systems and the need for an integrated one, in developing countries in general and Saudi Arabia in particular, requires intensive discussion and treatment and will not be dealt with here. In this paper a unified system of survey standards and specifications to be applied in urban and suburban areas is proposed.

2. DISADVANTAGES OF EXISTING SYSTEMS

There are already several systems of cadastral and legal surveys [1–3], but each one has its disadvantages and drawbacks [2, 4]. The problems involved in the use of these systems can be summarized as:

(1) Nobody other than surveyors, understands or accepts the ratio concept of accuracy. For instance, in the courtroom, judges have not understood the meaning of 1/5000 accuracy. All that interests them is how much error can exist at the corner of a particular building lot or property. As far as the public is concerned the situation is more difficult. The land surveyor's responsibility to his client is a definitive, numerical statement of the accuracy of his property, not only the corners and the lines but also the value. This means that the accuracy of the coordinates will not be sufficient. The property owner cannot understand how valuable it is since it is not transferable into money.

(2) The existing land surveyors are neither qualified to do sophisticated survey adjustment nor are they familiar with statistical testing. At the same time they should inform both, owner and judges about the accuracy of the property they have established [5].

(3) In North America different formulas are used by the Department of Energy, Mines and Resources of Canada and the USA National Academy of Sciences. Although they are based on long experience with field surveys, their use requires the calculation of the standard deviation of the measurement, which depends to a great extent on the surveyor's judgment [6]. Therefore, the formulas are appropriate to compare different solutions in the design stage but they are not enough to evaluate the survey work statistically. In Europe the situation is completely different since cadastral surveys were established quite a long time ago and fast development and urbanization (if any) do not create any serious problems.

The lack of any urban survey standards in developing countries is advantageous to a certain extent. It allows an opportunity to establish a modern system that satisfies everyone.

3. PROPOSED TWO-STEP APPROACH

In the system proposed here a legal survey in housing areas is achieved through two stages.

(1) Block or parcel survey in which the survey is carried out from the national urvey control to the corners of each housing block.

(2) Property or lot survey, in which the different lot corners are tied to the previously determined block corners. The positioning is done in two different coordinate systems, the national survey control system (for the use of public authorities), and a

local or block coordinate system (for the use of the property owners).

3.1 Block or Parcel Survey

A block survey is conducted by running traverses from the national survey control to the block corners. The following problems are encountered.

(a) The land surveyor has to choose the traverse route. In this case he has to pre-analyze the different solutions for the optimum design.

(b) The route may be long. The surveyor may go through different measuring conditions which he has to evaluate to estimate the different accuracies and weights of his measurements. This is very difficult for an inexperienced land surveyor.

(c) A rigorous least squares solution is warranted since the measurements have different accuracies and the observations are numerous.

(d) Careful statistical testing is relevant to accept or reject the measurements and the mathematical model. The surveyor should know what to do if disagreements arise because the neighboring block is located using another set of control points.

All the above sophisticated design, adjustment, and statistical testing, require survey specialists perhaps at the M.Sc. level. Also computer hardware and software are required which are not available to most (if not all) of the existing land survey firms. Block survey should, therefore, be restricted to a governmental agency, or, if not possible, should be completely supervised by the department in charge.

Block survey consists of a traverse network fitted between the surrounding survey control (Figure 1). The design of such a fitting should be optimized according to the ratio of the semi-major axis of the error ellipse of the 95% confidence region to the direct unmeasured distance between the control and the block corners.

Taking into consideration the variance–covariance matrix (VCV) together with the estimated variance of the measurements, two least squares adjustments should be carried out. In the first one all the survey control points are held fixed, but all the error sources are taken into consideration for error analysis. The

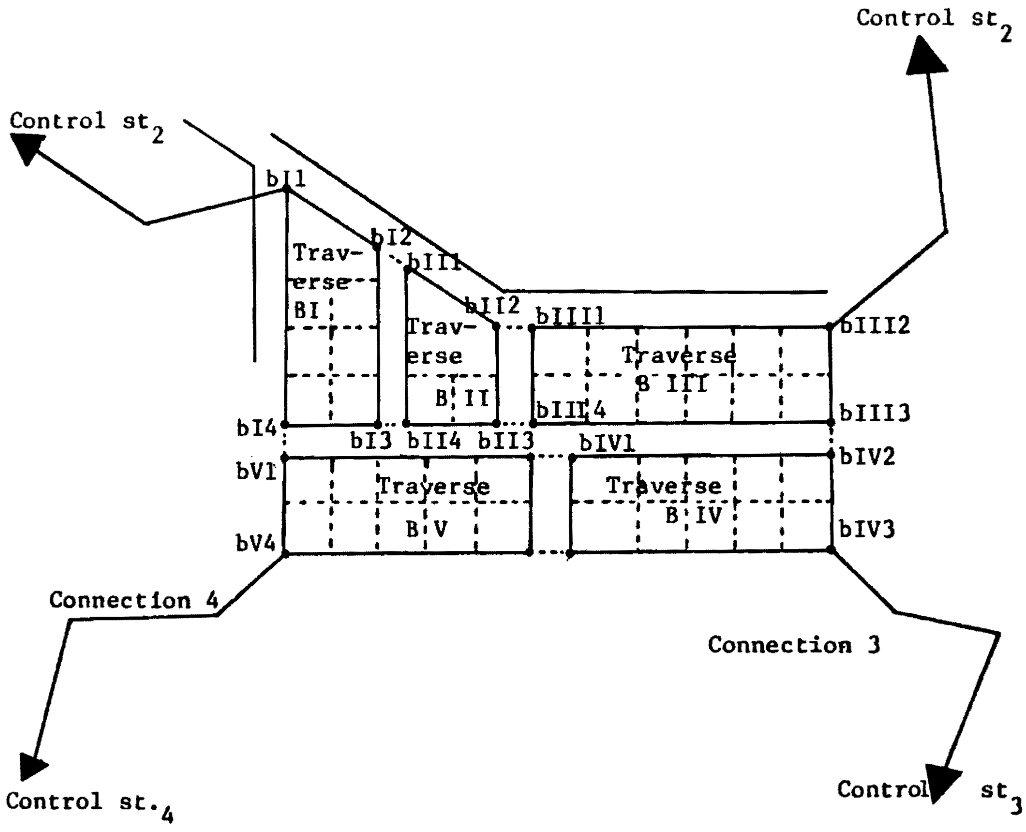

Figure 1

second adjustment is an unconstrained one in which only one point is fixed. The residuals from both adjustments and the misclosures of the different traverses are calculated and statistically tested.

The necessary statistical tests of the field data, for the mathematical model and for the adjustment outcome are explained below.

3.1.1 *Statistical Tests for Block Surveys*

The block traverse net has a small number of degrees-of-freedom (df). It also serves as a reference datum for lot and property surveys. On the other hand the simultaneous adjustment of the block traverses is final and the resulting coordinates of the block corners should not be updated after the completion of lot surveys. Accordingly, the adjustment should be as rigorous as possible.

The fact that the traverse net always has a small number of degrees-of-freedom should be taken into consideration. It implies that the estimated standard deviation of unit weight is unreliable, and accordingly the *a priori* standard deviation of unit weight should be used. Hence very good estimates of standard deviations of observations should be available. This is an important condition for obtaining a sound least squares solution. To reach this goal the following statistical tests are necessary [7].

I. The residual of each observation has to be tested for acceptance. An observation should be rejected if its residual v_i exceeds the 95% confidence interval.

$$-t\sigma_i < v_i < +t\sigma_i$$

The value of t is given in the Student's Distribution table. When the *a priori* variance factor (σ_0^2) is used the number of degrees-of-freedom is infinity and the value of t for a 95% confidence interval is 1.96. Hence the rejection criteria is

$$-1.96\sigma_i < v < 1.96\sigma_i$$

Thus, the probability that a residual from a given set of observations will not be within 1.96σ of the mean, is 0.95 [7].

If good estimates of the standard deviations of observations are not available the calculated standard deviation of unit weight should be used in that test and the factor corresponding to the actual number of degrees-of-freedom should be used from tables [7, p. 206].

For instance if $\sigma = 5''$ when 4 excess observations are available for specific measurement, the corresponding $t_{4,0.05} = 2.776$ and the observation is rejected if its residual v_i is greater than 13.9″.

In many cases most of the measurements, especially the distances, are conducted twice. Any paired observationns can be tested for acceptance. The procedure is to calculate

$$t = \frac{\Delta_i}{S\Delta_i}$$

where Δ_i is the difference between the paired observations to be tested (in the case of distances Δ_i is the difference in ppm between the two observations of the distance $i = l_{i1} - l_{i2}$),

$$S^2 = \frac{\Sigma v_i^2}{n-1}$$

where v_i is the deviation of Δ_i from their mean and n is the number of differences. The hypothesis $l_{i1} = l_{i2}$ is rejected at the 95% confidence interval if $t \geqq t_{n-1,0.025}$ or if $t \leqq t_{n-1,0.975}$.

II. Essentially a test has to be applied to determine whether the estimate of variance obtained from the adjustment σ^2, where $\sigma^2 = \mathbf{V}^{\mathrm{T}}\mathbf{P}\mathbf{V}/\mathrm{df}$ ($\mathbf{P}$ is the weight matrix and df is the degrees-of-freedom), is compatible with the *a priori* value σ_0^2. The probability statement used is the Chi-Square test on the results of the adjustment and for the 5% significance level is:

$$P_r\left(\frac{\mathrm{df}\,\sigma^2}{\chi^2_{\mathrm{df},0.025}} < \sigma_0^2 < \frac{\mathrm{df}\,\sigma^2}{\chi^2_{\mathrm{df},0.975}}\right) = 0.95$$

If the probability is too small or too large, different reasons should be investigated. One is that the mathematical model is poor (for instance incomplete removal of systematic effect or poor network design), another is that the weights are not in proper proportion to each other. Therefore there is no definite answer for the strength of the adjustment. In addition to this is the unreliability of σ^2 due to the fact that any traverse net has a small number of degrees-of-freedom.

III. Since the tested surveys consist of several traverses with different accuracies, each misclosure should be tested against its variance–covariance matrix. If the misclosures of a traverse in northing and easting is ΔN and ΔE respectively, and the variances and covariances of the estimated coordinates of the last point with respect to the fixed first point are $\sigma^2_{\Delta N}$, $\sigma^2_{\Delta E}$, $\sigma_{\Delta N \Delta E}$ and $\sigma_{\Delta E \Delta N}$ then the χ^2 test can be used in which

$$[\Delta N \quad \Delta E]\begin{bmatrix}\sigma^2_{\Delta N} & \sigma_{\Delta N\Delta E} \\ \sigma_{\Delta E\Delta N} & \sigma^2_{\Delta E}\end{bmatrix}\begin{bmatrix}\Delta N \\ \Delta E\end{bmatrix} = \chi^2$$

χ^2 should be less than the theoretical $\chi^2_{2,0.05} = 5.99$, if so the misclosure is acceptable at the 5% significance level.

IV. The traverse nets under discussion are usually established by different private or governmental agencies. Before the simultaneous adjustment of all these nets is carried out each net should be adjusted separately and the estimated variance factor σ_i^2 is calculated. The different variance factors should be tested for homogeneity. The test proposed by Bartlett [7] is used in this respect. In that test the pooled variance estimate S^2 is given by

$$S^2 = \sum^n r_i \sigma_i^2 / \Sigma r_i$$

where r_i are the degrees-of-freedom in each net and n is the number of nets. The statistic to be calculated is

$$M = (\Sigma_i r_i) \log S^2 - \Sigma_i r_i \log \sigma_i^2$$

M is approximately distributed as χ^2_{n-1}. Therefore the calculated value of M should be compared with $\chi^2_{n-1,0.05}$; if $M < \chi^2_{n-1,0.05}$ the hypothesis of homogeneity of variances of the traverse networks should be accpeted at the 5% significance level.

V. The linear relative misclosure is defined as $M = (\Delta N^2 + \Delta E^2)^{1/2} / \Sigma l$. For a set of traverses the values of their relative misclosures should be comparable. Their average value $\bar{M}$ together with the discrepancies $d_i = M_i - \bar{M}$ can be calculated. The estimated standard deviation for n traverses is then calculated as $\sigma_i = [\Sigma d_i / (n-1)]^{1/2}$ and the statistic $t = d_i / \sigma_i$ should be compared with the theoretical value from the t-distribution. The hypothesis of equal relative misclosures should be rejected at the 5% significance level if $t \geqq t_{0.025,n-1}$ or if $t \leqq -t_{0.975,n-1}$.

VI. The above statistical tests are good for an unconstrained adjustment of the traverse net. In the other adjustment when this net is to fit a set of fixed coordinates, the effect of such a constraint should be investigated and tested for consistency. In this case the agreement factor ratio [7, pp. 159]

$$R' = \frac{R_1}{R_0}$$

is calculated where $R_1^2 = \mathbf{v}_1^{\mathrm{T}} P v_1$ (for the constrained adjustment) and $R_0^2 = \mathbf{v}_0^{\mathrm{T}} \mathbf{P} \mathbf{v}_0$ (for the unconstrained adjustment). But

$$\frac{R_1}{R_0} = \left[\frac{R_1^2 - R_0^2}{R_0^2} + 1 \right]^{1/2}$$

Therefore

$$R' = \left[\frac{\mathbf{v}_1^{\mathrm{T}} \mathbf{P} \mathbf{v}_1 - \mathbf{v}_0^{\mathrm{T}} \mathbf{P} \mathbf{v}_0}{\mathbf{v}_0^{\mathrm{T}} \mathbf{P} \mathbf{v}_0} + 1 \right]^{1/2}$$

But $(R_1^2 - R_0^2)/R_0^2$ is distributed as $b/r\, F_{b,r,\alpha}$, where b is the number of fixed parameters (higher order surveys), r is the degrees-of-freedom $= n - u$ (u is the total number of parameters), and α is the significance level. Therefore

$$R'_{b,r,\alpha} = \left[\frac{b}{r} F_{b,r,\alpha} + 1 \right]^{1/2}$$

If R' is bigger than $R'_{b,r,0.05}$ the traverse survey is not consistent with higher order control network at the 95% confidence interval and there should be more nonacceptable approximation in the mathematical model such as a reduction of EDM or angular measurements.

VII. In the case of relocation survey, when one of the block traverse nets is destroyed and relocated, a test for the significance of difference of coordinates of the same station is to be applied. The new coordinates of the relocation survey have to agree to a certain tolerance with the old ones according to its variance–covariance matrix. In this test if ΔX and ΔY are the differences in coordinates of the same point then:

$$[\Delta X \quad \Delta Y] \begin{bmatrix} \sigma_{X_1}^2 & \sigma_{X_1 Y_1} \\ \sigma_{X_1 Y_1} & \sigma_{Y_1}^2 \end{bmatrix}^{-1} \begin{bmatrix} \Delta X \\ \Delta Y \end{bmatrix} = \chi^2$$

The value of χ^2 should be less than the theoretical value at the 5% significance level $\chi^2_{\mathrm{df},0.05}$ where df = 2 (testing two-dimensional coordinates).

In case of a newly developed housing area and suburban area the final adjustment will not be possible until all the blocks and parcels are established. Then an overall simultaneous adjustment is carried out by the governmental department in charge. In the meantime any new block to be surveyed should follow the following steps. (Figure 2).

The sides and angles of the new block are measured. Also ties to the control P and to all surrounding corners of the old blocks b should be established. Temporary adjustment is carried out considering that the control points P and old corner points b are fixed. The coordinates of the corners of the new block together with the VCV matrix are calculated taking into consideration the VCV matrix of P and b. Statistical tests I, II, III, and V may be applied for checking. The resulting coordinates are only temporary ones, they are used for lot survey. They are updated when the area between the control stations is completed. A simultaneous least squares adjustment is then carried out.

3.2. *Property and Lot Survey*

In Figure 3 the corners l of any lto L are to be

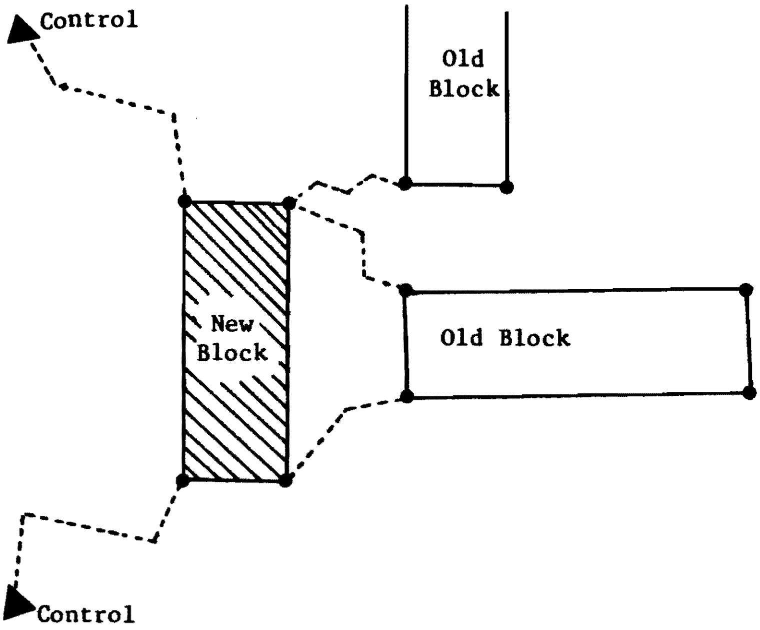

Figure 2. Survey of a New Block

surveyed from the block corners b and not from the control P. If b is destroyed a relocation survey has to be carried out first to relocate b then l can be established.

Traverses are run from b_1 to l_1, b_2 to l_2, b_3 to l_3, and from b_4 to l_4 in addition to the closed traverse of the property $l_1\ l_2\ l_3\ l_4\ l_1$. A simultaneous least squares adjustment is carried out to determine the coordinates of l_1 to l_4 and their VCV matrix with eight degrees-of-freedom. The b's are assumed fixed but their VCV matrix is taken into consideration to propagate variances.

This survey is typical for any lot. The solution can be computerized for the use of any land surveyor since this will be the main sort of survey he will face.

3.2.1 *Accuracy Criteria of Property Survey*

In a property survey the situation is different from a block survey. Several landowners were asked about the accuracy they would accept on the corners of their properties. The figures they gave were transferred into errors in the area of the properties. In other words the accuracy was translated into a possible loss of money. Immediately no owner accepted the figures he was given for accuracy or tolerance in the relative location of the corners of his property. It was found, after some

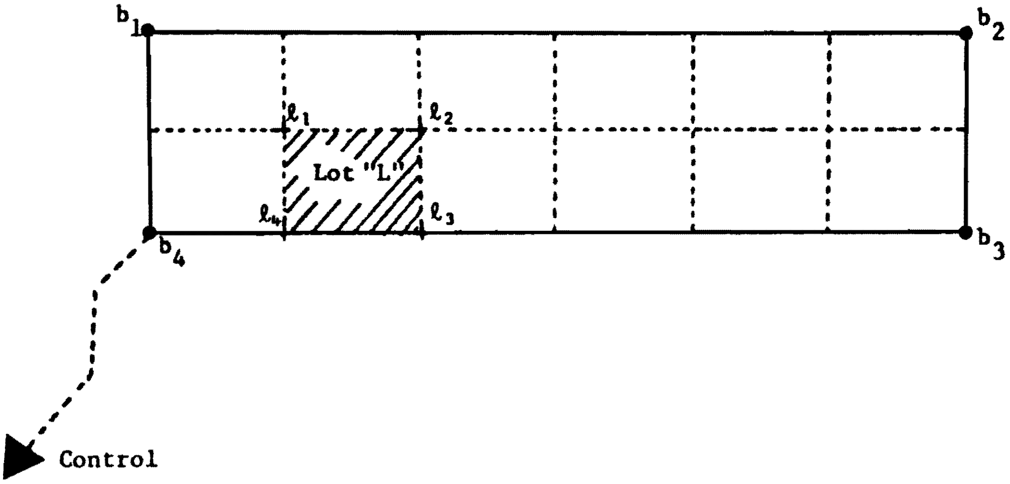

Figure 3. Property Survey

elaboration, that most of the owners can accept error ratio in the area of not more than 1/5000.

It is therefore more suitable in a property survey, where area really counts, to adopt the Area Accuracy Ratio $\sigma_A/A = \text{constant } K$ as accuracy criteria for the design and for testing the standard deviation of the observations. The AAR criteria is a physical quantity for the property and it is understood by the public, clients, and judges. The owner of a 10,000 m^2 housing lot, that is worth $200,000 should be informed that the accuracy in his property is ± 1 m^2. To him this can be transferred into money terms ($20.00) and he can decide whether he should tolerate this or not. Also for the judge and the surveyor the ratio should be constant for all properties.

The rest of the traverse net (connections between block traverses and ties to the main control points) should be designed according to the ratio of the semi-major axis of the error ellipse to the direct unmeasured distance. The semi-major axis of the standard relative error ellipses are calculated along unmeasured lines of the property (e.g. $l_1 \rightarrow l_4$ and $l_2 \rightarrow l_3$) and the results are given to the public as a positive quantity. Eventually the misclosure of any property traverse should not be more than twice its standard deviation.

It should be emphasized that the specifications for a property survey in commercial or residential areas should be the same as for urban and suburban areas. Otherwise the accuracy of the survey in suburban areas will not be sufficient after 10–15 years when such areas become as dense and expensive as today's urban areas.

ACKNOWLEDGMENT

This work originated in the summer of 1977 and was granted by Dr. A. Chrzanowski, Professor, S.E. Dept. U. of New Brunswick, Canada. The author was a visiting scientist to UNB in this period of time.

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Paper Received 28 May 1979; Revised, 3 June 1980.