A LAMINAR MODEL FOR MIXING IN STRATIFIED FLOW

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الخلاصـة :

تقدّم هذه الدراسة تمثيلا رياضياً للحمل الحراري الانسيابي في طبقات مائع متدرج الكثافة وفي حالة اتزان يخضع لعملية تسخين سفلي وسريان قصيٍّ علويّ. ويتكوّن النظام قيد الدراسة من ثلاث طبقات متدرجة الكثافة، حيث توجد طبقة عالية الكثافة في الأسفل تعلوها طبقة متدرجة الكثافة تنتهي بطبقة ثالثة ذات كثافة منخفضة. وتخضع الطبقة العليا لسريان قصيٍّ بينما تخضع الطبقة السفلي لتدفق حراري ثابت. وقد تم تمثيل حركة المائع في هذا النظام عن طريق حل المعادلات التفاضلية الكاملة لتدفق إنسيابي ثنائي البُعد مع الاستفادة من فرضية (بوسنسك) وتطبيق الشروط الابتدائية والحدية المناسبة. ويهدف التمثيل الرياضي المقدم إلى دراسة توزيع الكثافة والحرارة والسرعة وتغيرها مع الزمن في الموائع المتدرجة الكثافة.

ABSTRACT

The laminar simulation of convection in stably stratified fluid layers subjected to bottom heating and upper shear flow is presented. The system consists of three layers stratified in a stable manner, with a dense layer at the bottom, a light layer at the top, and a stratified layer in the middle. The top layer undergoes shear flow and the system is subjected to a constant heat flux at the bottom. The system is simulated by solving the complete form of the 2-D laminar equations, utilizing the usual Boussinesq approximation, along with the appropriate initial and boundary conditions. The simulation is employed to investigate the density, temperature, and velocity profiles for the three layers as they progress with time.

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1. INTRODUCTION

The mixing process across a density interface of gravitationally-stable stratified fluids is of considerable interest, due to its occurrence in many engineering and natural processes. Of particular interest is the mixing process in: lakes and detention ponds; Salt Gradient Solar Ponds (SGSP); the deepening of the oceanic thermocline; the thickening of the gravity currents flowing under a stratified fluid; the disposal of municipal wastewater and power plants cooling water into water bodies; and quality control in stratified reservoirs.

There have been relatively few numerical models which consider stratified systems when both thermal and concentration stratifications coexist. Cha *et al.* [1] developed a one-dimensional numerical model to predict the diurnal variation of vertical temperature and concentration profiles in solar ponds. In their model, the convection term in the equations was neglected, and the fluid motion was retained through the use of augmented diffusivities. Atkinson and Harleman [2] developed a one-dimensional turbulent kinetic energy (TKE) model to simulate the temporal evolution of vertical temperature and salinity profiles in a salt gradient solar pond. A better version of the model, which accounts for the role of sidewalls, was developed by Schladow [3, 4].

Murty [5] used a numerical model based on finite elements to investigate a laminar and steady double diffusive convection in a cell. Kazmierczak [6] used a stream function–vorticity approach to solve numerically the laminar Navier–Stokes equations for double diffusion in a horizontal fluid layer subjected to a constant bottom heat flux.

Ohtsuka and Yamakawa [7] analyzed a stably stratified shear flow generated by two streams of water at different densities and velocities by solving the Navier–Stokes equations without any turbulence average modeling. Instead they considered the turbulence transport mechanics analytically. Al-Ghamdi [8, 9] developed a one-dimensional turbulent flow model based on algebraic Stress Model (ASM) to examine the mixing in double-diffusive systems.

Under consideration in this contribution is the laminar simulation of unsteady flow in stratified system where a stably stratified salt layer is being eroded from below due to heating and from the top due to heat removal and shearing, as shown schematically in Figure 1. The system has a free surface and a solid bottom boundary and the two side walls are also solid and insulated except at the inlet where the inflow was held at a constant temperature. The system consists initially of three layers; the upper mixed layer, the lower mixed layer, and stably stratified layer in between. Heat is being added to the bottom wall at a constant heat flux. The upper layer is subjected to shear flow. The behavior of the temperature and density profiles as they evolved with time is investigated. The simulation is based on solving the full 2-D laminar Navier–Stokes equations along with the energy and species conservation equations.

2. MATHEMATICAL FORMULATION

The distribution of the flow quantities in stratified flow can be described by the conservation laws of mass, momentum, thermal energy, and species concentration along with an equation of state. Presented in tensor notation, for incompressible

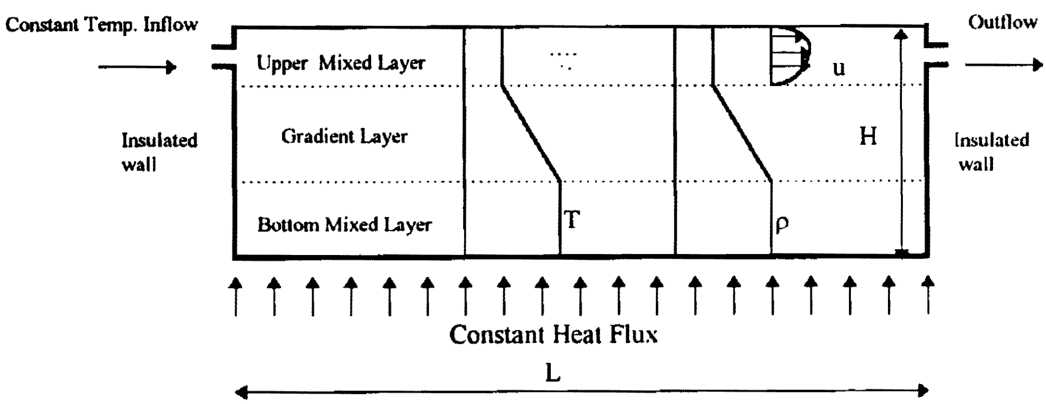


Figure 1. Schematic of the Problem.

flow, constant properties, no chemical reaction, and utilizing the Boussinesq assumption, which states that the density is constant except in the buoyancy terms of the momentum equations, these laws are as described below.

The 2-D unsteady continuity equation is expressed as:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_j)}{\partial x_j} = 0 \tag{1}$$

where x_j is distance in the jth direction, u_j is a fluid velocity component, and ρ is the fluid density.

The x and y-momentum equations are:

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_j} = \rho_0 g_i - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu \frac{\partial u_i}{\partial x_j}\right] \tag{2}$$

where P is the pressure, μ is the dynamic viscosity of the fluid, g_i is the acceleration due to gravity, ρ_0 is the reference density.

The energy equation, when presented in the form of a transport equation for enthalpy, is given by:

$$\frac{\partial(\rho h)}{\partial t} + \frac{\partial(\rho h u_i)}{\partial x_i} = \frac{\partial}{\partial x_i}\left[\frac{k}{c_p}\left(\frac{\partial h}{\partial x_i}\right)\right] \tag{3}$$

where $h = \Sigma X_j h_j$, $h_j = \int C_{p,j} dT$, X_j is the mass fraction, h_j is the enthalpy, and $C_{p,j}$ is the specific heat of species j.

The conservation of species equation can be written as:

$$\frac{\partial(\rho X_i)}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i X_i) = \frac{\partial}{\partial x_i}\left(\rho D_{ij} \frac{\partial X_i}{\partial x_i}\right), \tag{4}$$

where D_{ij} is the diffusion coefficient.

The density of the fluid is defined as a function of the densities of the individual species forming the fluid, thus:

$$\rho = \Sigma X_i \rho_i \,. \tag{5}$$

3. NUMERICAL SOLUTION

The partial differential equations presented in the previous section (Equations (1) through (4)) can be solved by finite difference techniques using the SIMPLE (Semi-Implicit Methods for Pressure-Linked Equations) algorithm, (Patanker [10]). The governing equations when integrated over small but finite volume (Figure 2) yield the following generalized form:

$$\frac{\rho(\phi^{t+1} - \phi^t)V}{\Delta t} + \left[(\rho u \phi)_e - (\rho u \phi)_w\right]\Delta y + \left[(\rho v \phi)_n - (\rho v \phi)_s\right]\Delta x$$

$$= \left[\left(\Gamma \frac{\partial \Phi}{\partial x}\right)_e - \left(\Gamma \frac{\partial \Phi}{\partial x}\right)_w\right]\Delta y + \left[\left(\Gamma \frac{\partial \Phi}{\partial y}\right)_n - \left(\Gamma \frac{\partial \Phi}{\partial y}\right)_s\right]\Delta x + S_\phi \,, \tag{6}$$

where the values of ϕ, Γ, and S_ϕ are as shown in Table 1.

The values of $\phi_{e,w,n,s,}$

$$\left(\frac{\partial \phi}{\partial x}\right)_{e,w}, \left(\frac{\partial \phi}{\partial y}\right)_{n,s}$$

are evaluated in terms of $\phi_{E,W,N,S}$ using the power law.

Equation (6) is solved for the flow variables, the sequence of the solution is: (*i*) solving x-momentum equation for u-velocity, (*ii*) solving y-momentum equation for v-velocity, (*iii*) solving mass conservation equation and updating velocities and pressure, (*iv*) solving the energy equation for h and T, (*v*) solving species equation for concentration, and (*vi*) repeating the steps for the next time step.

4. RESULTS AND DISCUSSIONS

As presented in Figure 1, the system under consideration consists initially of three layers: the upper mixed layer, the bottom mixed layer, and a gradient layer in between. The side walls are thermally insulated except at the inlet which has a constant temperature and the outlet at which the temperature gradient is zero. Heat is being added through the bottom wall at a constant heat flux. No flux of species is allowed through the walls except through the outlet. The upper layer undergoes a shear velocity generated by a slug flow entering through the inlet at a uniform velocity of 0.03 m/s. This shear flow simulates the effect of wind on the surface and represents also the flow of the fluid usually added to solar ponds upper layer to compensate for the evaporated water or for flushing the salt transported to the upper layer. The upper boundary of the system is usually in equilibrium with the ambient temperature, hence its temperature was fixed at 295 K. The physical dimensions of the system are: length = 5.0 m, height = 1.50 m, grid spacing in x-direction is 10 cm and grid spacing in y-direction is 3.33 cm. Finer grid spacing required a prohibitively large computational time.

As a sample result, the results for a constant heat flux of 40 W/m^2 being supplied to the bottom boundary are presented here. The velocity vectors presented in Figure 3 for t=10 and 20 hours, show the circulation process taking place in the bottom layer as a result of heating. As the time progresses, the circulation increases and the erosion of the gradient layer continues. The erosion of the gradient layer is a result of the bombardment of the interface between this layer and the convecting layer below by the thermals which plunge into the stable region before spreading along the interface [11]. The shear velocity in the upper layer does not seem to have a considerable effect on the entrainment between the upper mixed layer and the gradient layer. The contour lines for temperature and density at t=10 and 20 hours are presented in Figures 4 and 5, respectively. These two figures indicate that even though the flow pattern is strongly two-dimensional, especially in

Table 1. Values of φ, Γ, and S_φ for Different Equations.

Equation	ϕ	Γ	S_ϕ
Conservation of mass	1	0	0
Conservation of momentum	u, v	μ	$-\partial p/\partial x, -\partial p/\partial y - \rho g$
Conservation of energy	h	κ/Cp	0
Conservation of species	X_j	ρD_{ij}	0

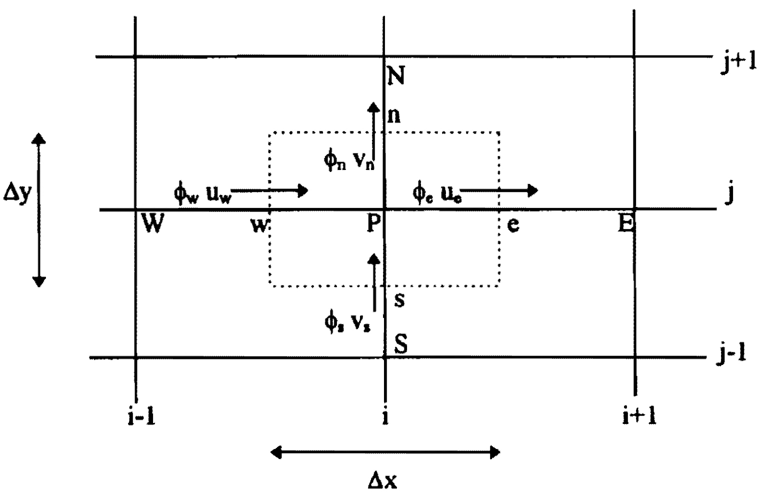
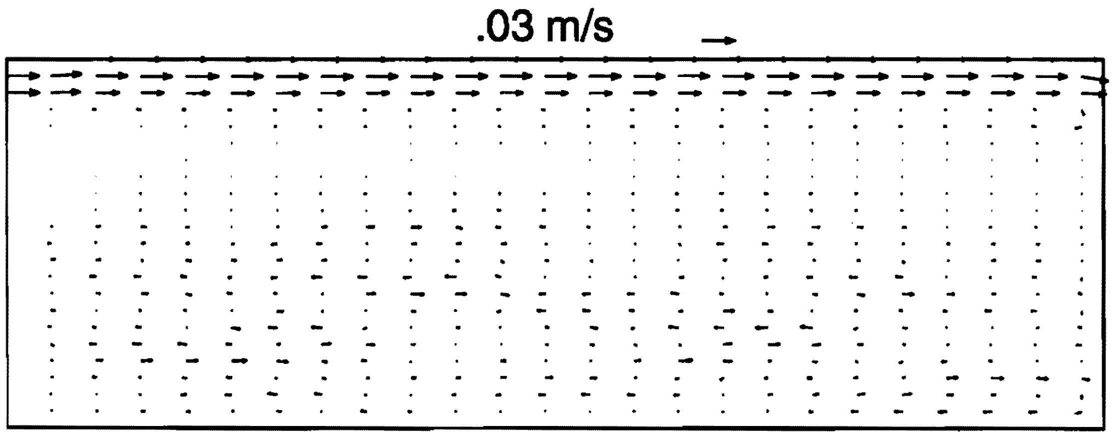
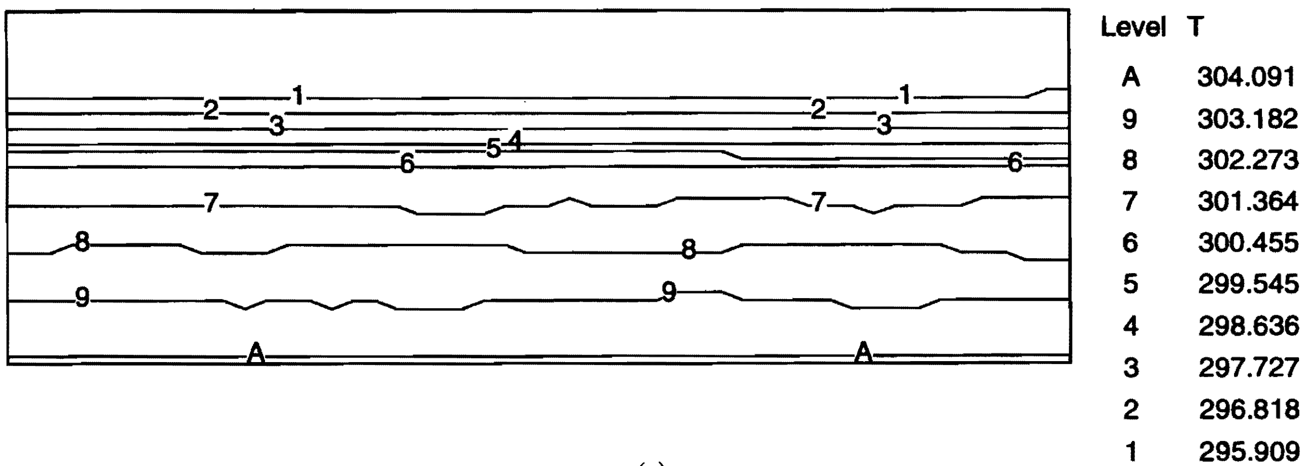


Figure 2. Control Volume for the Staggered Grid.



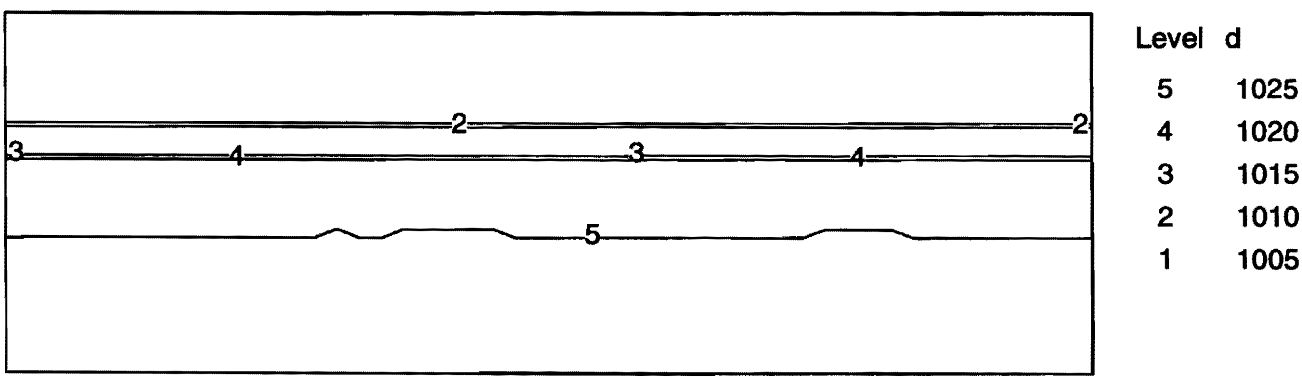
(*a*)

Figure 3(a). *Velocity Vectors.* t = *10 hours.*



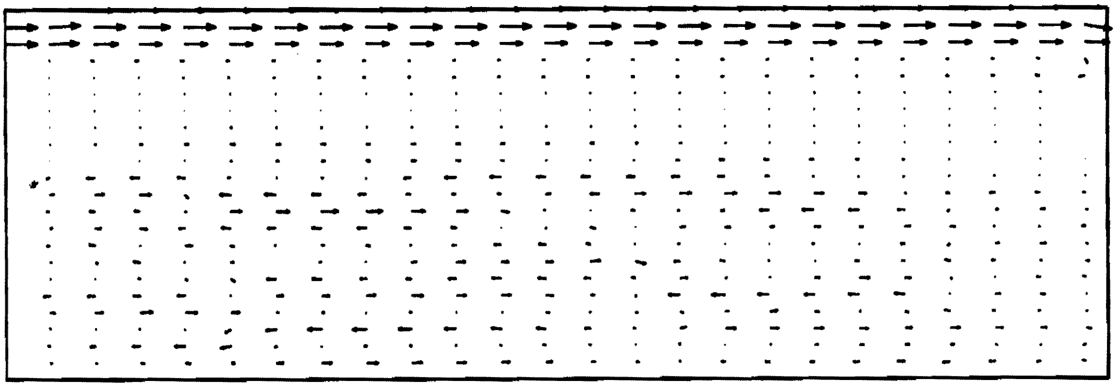
(*a*)

Figure 4(a). *Temperature Contours.* t = *10 hours.*



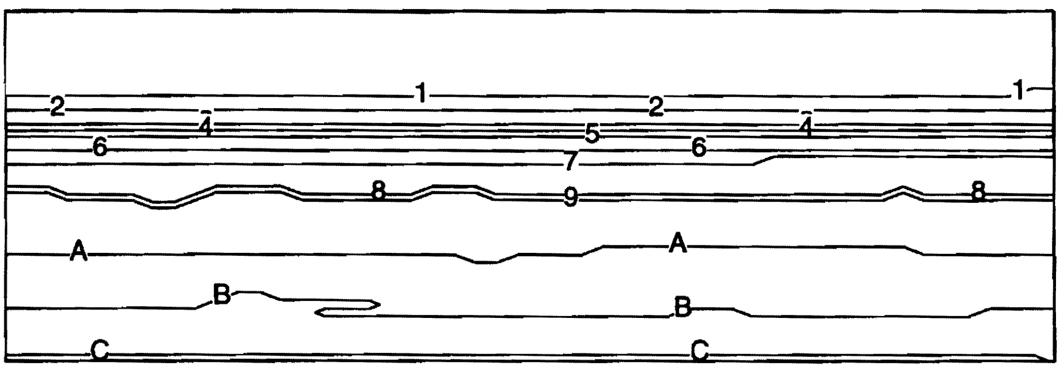
(*a*)

Figure 5(a). *Density Contours.* t = *10 hours.*



(*b*)

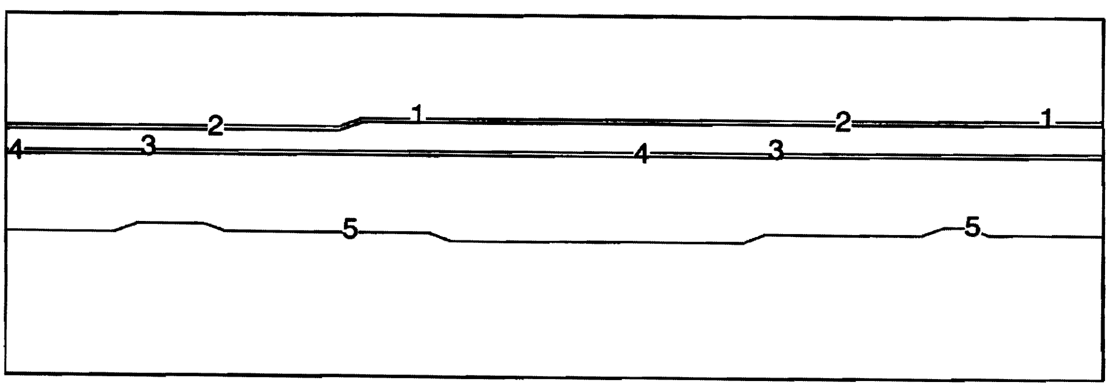
Figure 3(b)*. Velocity Vectors.* t = *20 hours.*



Level	T
C	304.231
B	303.462
A	302.692
9	301.923
8	301.154
7	300.385
6	299.615
5	298.846
4	298.077
3	297.308
2	296.538
1	295.769

(*b*)

Figure 4(b)*. Temperature Contours.* t = *20 hours.*



Level	d
5	1025
4	1020
3	1015
2	1010
1	1005

(*b*)

Figure 5(b)*. Density Contours.* t = *20 hours.*

the lower layer, the temperature and the density fields depend mainly on the vertical direction and no considerable variation is noticed along the horizontal direction. The transient temperature and density profiles for this case at the centerline (*i.e.* at $x = 0.5\ L$, where L is the length of the system in x-direction) are shown in Figures 6 and 7, respectively. As one can see from these two figures, there is a continuous growth of the bottom layer due to the mixing generated by heating process, and the temperature distribution in this layer is uniform except at the very bottom where a thin thermal boundary layer develops as a result of the constant heat flux condition. In the middle layer the convection process is suppressed by salinity gradient and the heat transfer in this layer is dominated by conduction. This agree, in principle, with the flow visualization results reported by Behnia and Viskanta [11], which indicate no convection in the gradient layer. Obviously, as the time proceeds, one expects the convecting layer to grow continuously on the expenses of the gradient layer and after a sufficiently long time the whole system will turn into a large single circulation cell. The growth rate of the bottom convecting layer depends on the heating rate, the temperature and salinity gradients across the gradient layer and the mixing processes which occurs at the interface between the convecting and stable layers [8, 11]. From field measurements, it was found [12] that if an instability occurs in the system or a long period of time is elapsed, the system stratification is destructed and a restratification of the system becomes necessary.

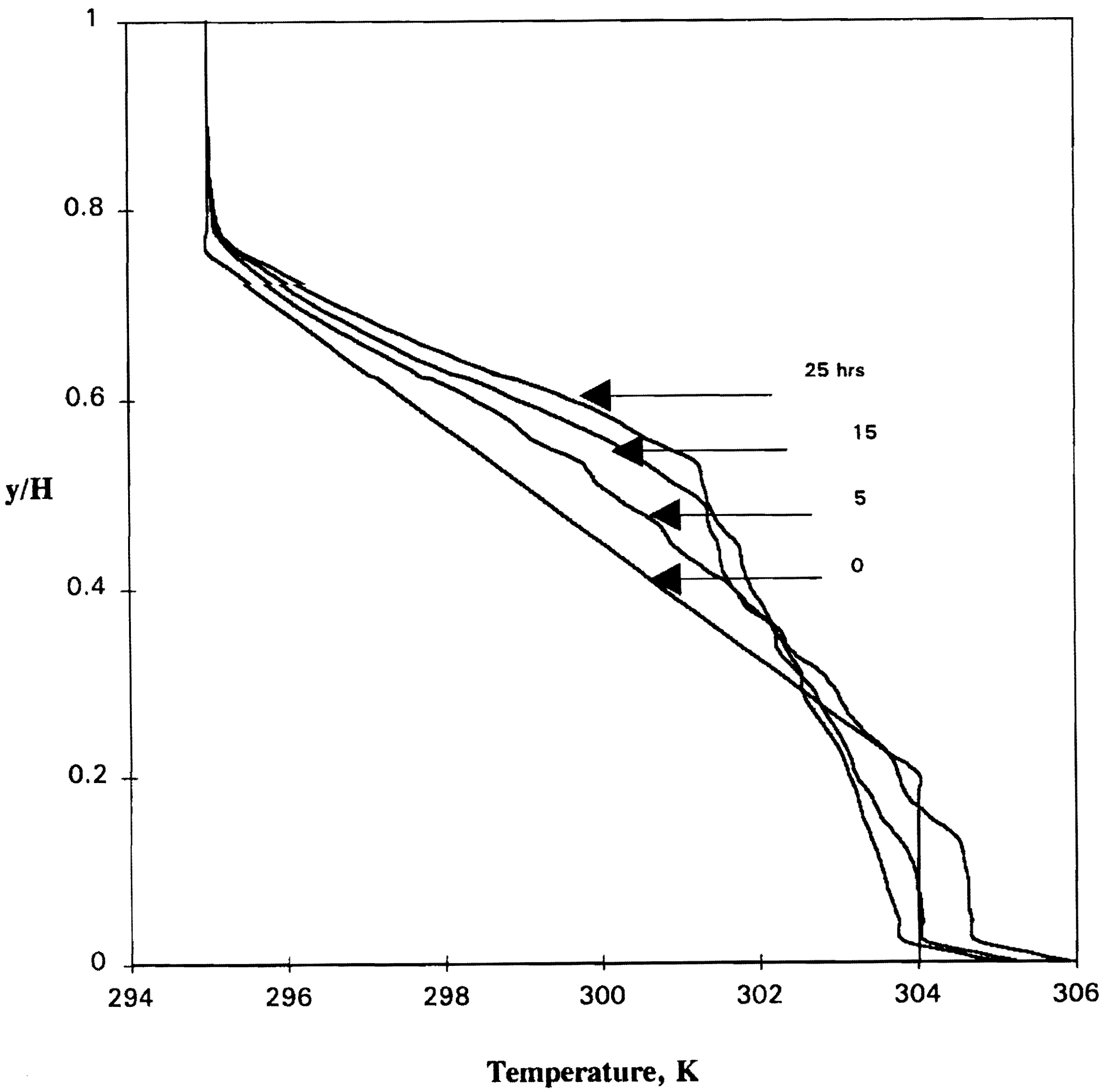


Figure 6. Centerline Temperature Profiles (x/L = *0.5).*

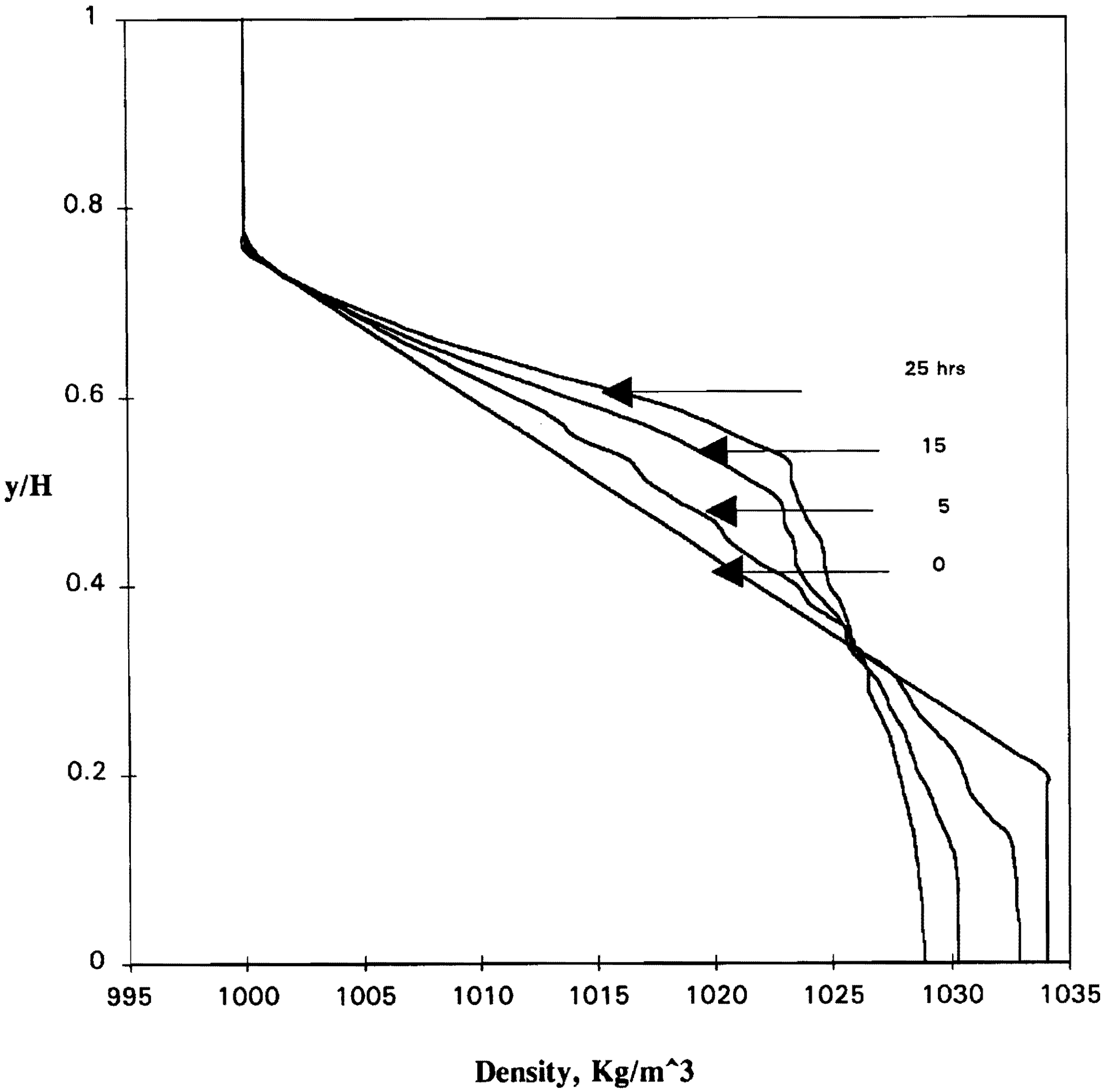


Figure 7. Centerline Density Profiles (x/L = 0.5).

The density is almost uniform in the bottom layer and it decreases with time as the layer grows due to the entrainment of lighter fluid from the layer above. The shear flow in the upper layer does not seem to influence significantly the thickness and density of the upper layer, this suggests that mixing due to heating is more appreciable than mixing resulted from shear flow.

The results presented agree qualitatively with previous numerical studies (Kazmierczak [6]) and experimental investigation (Al-Ghamdi [8]), but quantitative comparison with experimental findings is not possible because a purely laminar flow data is not available.

5. CONCLUSION

The laminar simulation of convection process in a stably stratified fluid layer subjected to shear flow and bottom heating is presented in this contribution. The simulation is based on solving the complete form of the 2-D laminar Navier–Stokes equations along with the conservation of heat and species equations. The simulation agrees qualitatively with previous numerical simulations and gives a similar trend to experimental results.

The current study could be extended in the future to include the effects of meteorological conditions over the surface, penetration of radiation, and fluid instability on the mixing process. Also, taking into account turbulent fluctuations in the system may provide a more realistic simulation.

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