A HYBRID FUZZY SELF-ORGANIZING VARIABLE STRUCTURE CONTROLLER

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الخلاصـة :

ندرس في هذه الورقة مشكلة القلقلة التي تقترن باستخدام المتحكمات متغيرات البنية . نستخدم دالة أداء طورها المؤلفان في متحكم متغير البنية بدمج مبادئ المتحكم المتكيف والمنطق المتدرج . نهدف من خلال هذا الدمج التقليص من أثر القلقلة وتحسين أداء المتحكم .

لقد قمنا بعدة دراسات معتمدة على المحاكاة لاختبار أداء هذا المتحكم الجديد ولقد توصلنا إلى نتيجة أن المتحكم المقترح يخفض القلقلة في إشارة التحكم كما أنه يخفض قيمتها بدون أن يؤثر سلباً على سرعة استجابة النظام .

ABSTRACT

In this paper the problem of chattering in variable structure controllers is addressed. A new performance index for chattering developed by the authors is used in a Hybrid Scheme employing both adaptive and fuzzy logic schemes to smooth out the chattering phenomenon and simultaneously increase the speed of the system response. Simulations are carried out for single input multi output and multi input multi output systems. The simulation results show that the proposed Hybrid Fuzzy scheme is very effective in reducing both magnitude of the control and chattering phenomenon without compromising speed of response of the systems.

Keywords: Self-organizing, fuzzy logic, variable structure controller, two link manipulator, chattering.

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1. INTRODUCTION

It has been shown in the literature that Variable Structure Control (VSC) may be used to establish almost certain convergence to a neighborhood of the origin for a class of nonlinear systems. One design approach is based on a nominal constant linearization of the system, with the time-varying, nonlinear and uncertain elements of the system grouped into an 'unknown' function [1].

Another approach adopted by Young [2] employs a hierarchical method, the basic idea being to replace the multi-input problem by a sequence of single input problems. The speed of the sliding mode in this approach is close to the designed speed, which is solely determined by the choice of the switching planes, but the amount of chattering in the control signal is very high. Zaremba [3] also uses the hierarchical VSC with on-line iterative tuning of sliding hyperplanes and control function switching terms. The algorithm is based on defining certain local performance indices instead of a global one, and is shown to improve the response of the system considerably.

A recent approach to the design of variable structure controllers has come through the area of fuzzy logic implementation. Sundaramoorthy *et al.* [4] consider the change of fuzzy rules by monitoring the process output. Ghalia [5] implements the variable structure controller using fuzzy logic but no adaptation process is used. Guang [6] introduced fuzzy sliding mode control by proposing a controller based on sliding mode control and fuzzy set theory. Zhang *et al.* [7] developed a fuzzy logic-based adaptive variable structure controller for linear systems only. Zong-Mu [8] proposed a systematic methodology for the design of a decentralized fuzzy logic controller for large scale nonlinear systems. The method is based on using the performance index of a sliding surface to derive fuzzy logic rules.

The literature survey suggests two different approaches to enhance the performance of variable structure control; namely: the Self Organizing approach and the Fuzzy Logic approach. Both approaches are used to achieve the common goal of system performance and reduction in chattering of the control signal.

Lu *et al.* [9] considered the adaptation of the weights of the defuzzifying process to improve on the performance of the variable structure controller. We propose a new Hybrid Scheme employing both a self-organizing and a fuzzy logic scheme to smooth out the chattering phenomenon and simultaneously increase the speed of the system response. The algorithm is based on local performance indices [3, 10]. The error-based evaluation of performance indices are used to calculate self-organizing coefficients of the hyperplane and gains of the control, which maximize the system performance in terms of reduced chattering in the control signal. The fuzzy logic part enhances the system performance in terms of response time and further reduces the chattering in the control system. The two schemes augment each other in the proposed hybrid algorithm in terms of improving system response time and reducing chattering in the control signal.

The organization of this paper is as follows. Section 2 gives some preliminaries for the design of variable structure controllers, Section 3 explains the self-organizing action for variable structure controllers. Section 4 is the main part of this paper discussing in detail the basis and working of the proposed Hybrid scheme. Section 5 is discussion of the results and Section 6 concludes this paper.

2. DESIGN OF VSC

The design of a variable structure controller can be achieved in two phases:

- the design of the switching hypersurface;
- the design of the control law.

The sliding surface is chosen on the basis of ideal sliding dynamics, even if such dynamics only represents an approximation to the actual sliding mode behavior. The aim of the control is to force the motion of the system to be along the intersection of the switching planes $s = 0$. Different choices of switching surface produce radically different system responses. The richness of variable structure control comes from this ability to choose various controller structures at different points in time [11].

To study the stability of sliding modes occurring in nonlinear systems, various state transformations are used to put the differential equations of the nonlinear systems in one of the several possible canonical forms [11].

In the controllability form, the entire system is decomposed into N subsystems and each subsystem is expressed in a companion form:

$$\begin{aligned} \dot{x}_{i1} &= x_{i2} \\ &\vdots \\ \dot{x}_{im_i} &= f_i(X_i) + b(X_i)u_i + d_i(t) \end{aligned} \tag{1}$$

where X_i is a m_i dimensional state vector, $u_i(t) \in \mathcal{R}$ is the control input, and $d_i(t)$ is the external disturbance or uncertainty.

If the control problem is to get the state X_i to track a specific time-varying state $X_{di} = [x_{di}, \dot{x}_{di}, ..., x_{di}^{m_i-1}]^T$ for the i-th subsystem, then the error signal vector can be represented as $E_i = X_i - X_{di} = [e_{i1}, e_{i2}, ..., e_{im_i}]^T$. The error dynamics is given by:

$$\begin{aligned} \dot{e}_{i1} &= e_{i2} \\ &\vdots \\ \dot{e}_{im_i} &= -f_i(E_i) - b(E_i)u_i - d_i(t) \qquad i = 1, 2, ...N, \end{aligned} \tag{2}$$

where E_i is an m_i dimensional error vector. The control input u_i is used to drive the subsystem states X_i to the subsystem sliding surface $s_i(E_i, t) = 0$, where $s_i(E_i, t) = 0$ is the i-th component of the sliding hyperplanes s and can be represented as:

$$s_i(E_i) = h_{i1}e_{i1} + h_{i2}e_{i2} + ... + e_{im_i} = 0. \tag{3}$$

Let the Lyapunov function be $V = 0.5s_i^2$. The condition for the sliding mode to exist on the i-th hyperplane is known as the 'reaching condition' and is given by:

$$\lim_{s_i \to 0} s_i \dot{s}_i < 0. \tag{4}$$

Because the system representation is in companion form, the motion on the sliding hyperplane $s_i = 0$ can be considered as the dynamics of a $(m_i - 1)$-th-order subsystem described by:

$$\begin{aligned} \dot{e}_i &= e_{i+1}, \qquad i = 1, 2, 3, ..., (m_i - 2) \\ \dot{e}_{m_i-1} &= -h_{i1}e_{i1} - h_{i2}e_{i2} - ... - h_{m_i-1}e_{m_i-1}. \end{aligned} \tag{5}$$

Since the model is in controllable canonic form with order $(m_i - 1)$ and the characteristic polynomial is specified by the coefficients of the vector h, therefore stability of each sliding mode is guaranteed by choosing the elements of h to match a desired characteristic equation.

The corresponding differential equation is:

$$D^{m_i-1}e + h_{m_i-1}D^{m_i-2}e + ... + h_1 e = 0, \tag{6}$$

where $D = (d/dt)$ and $e = e_1$.

The characteristic values of the differential equation determine the coefficients of the hyperplane *i.e.* h_is. Based on the above discussion the sliding hyperplane with desired dynamics of the corresponding sliding motion can be easily determined [6, 11].

The next step is to design the control input so that the state trajectories are driven and attracted toward the sliding hyperplane and then remain sliding on it for all subsequent time. Based on the above mentioned Lyapunov function the control must satisfy the following condition also called *'reaching condition'* at all times:

$$s\dot{s} < 0. \tag{7}$$

The control law can be designed using a variety of methods [11]. The structure of the control $u(x)$ may be pre-assigned or free. In either case the objective is to satisfy the reaching condition. The controller selected together with the properly chosen sliding hyperplane ensures that the states of the system will be driven towards the origin of the state space along the sliding hyperplane from any given initial state.

3. A SELF-ORGANIZING CONTROLLER

A number of considerations limit the practical use of VSC. A practical problem is that the ideal sliding mode requires an infinitely fast switching mechanism in the controller. The real nonlinear control mechanism generates fast switching called chattering, which may excite unmodelled high frequency plant dynamics. Any deviation from the prescribed dynamics of the error between the state variables of the reference model and the plant is reflected by a decrease in the system performance.

Sliding motion is to a large extent independent of the control and depends mainly on the properties of the plant and on the orientation of the switching surface. The orientation of the switching surface may be adjusted for better performance of the overall system and the performance measures which are critical for the proper operation of the VS may be used in regulating the adaptation process. A combination of error dynamics, robustness or quality of sliding mode, and chattering of the control signal have been recently introduced by Zaremba [3].

Different characteristics of the system affect each of the chosen performance indexes in their own way. The inclination of the hyperplane affects the error dynamics. A greater slope causes a faster descent of the error. The robustness (determined by the sliding mode) is usually easier to ensure on less steep hyperplanes. The level of chattering (switching frequency and amplitude) exhibits a more complex dependence on the hyperplane slope: *e.g.*, if the hyperplane slope is nearly tangent to the natural trajectory of the system, the switching frequency would be small. The amplitude of the switching depends on the working conditions and is often proportional to the slope of the hyperplane. However, the sliding hyperplane can be reached faster if the control signal is larger.

This conflict between the different performance indices (PIs) has to be resolved through a proper choice of the functional relation between individual performance measures, and is the basis of the self-organizing controller, and may be summarized as follows.

If the system trajectories are within the specified limits then the slope of the hyperplanes is increased iteratively so as to cause a faster descent of error; however, if the error is too large then hyperplane slopes are chosen depending upon the error dynamics of the system. The gains of the controllers are similarly adjusted depending upon the chattering level of the control signal. If the chattering is high the gains are decreased else if the chattering level is low the gains are increased taking into consideration the performance of robustness. The performance index of robustness acts as the balancing factor between the self-organizing action of slopes of hyperplanes and the gains.

The adaptive, self organizing (SO) action is located in the general structure of a MRAC at the input of the variable structure controller as shown in Figure 1.

The SO action is performed by varying the slope of the sliding plane and changing the form and/or coefficients of the switching function.

The performance index for error dynamics was defined in [3] as a low pass filter of the inclination $\dot{e}/e$ of the sliding hyperplane. For the jth step of control for the subsystem i:

$$P_i^E = a_0 e_i^j/\dot{e}_i^j + a_1 e_i^{j-1}/\dot{e}_i^{j-1} + ... + a_n e_i^{j-n}/\dot{e}_i^{j-n}, \tag{8}$$

where n is the order of the filter. Now P_i^E is the filtered version of the ratio $e_i/\dot{e}_i$ which can be used as indicator of the quality of the output.

The chattering index introduced in [10] is used and reads as follows:

$$P_i^C = 1/m \sum_{l=j-m}^{j} \triangle(sign(u_i^l)), \tag{9}$$

where m is the window size, u_i^l is the control input at instant l to the ith subsystem, and $\triangle(x)$ is defined as:

$$\triangle(x) = 1 \qquad if \qquad sign(x_j) \neq sign(x_{j-1}).$$

The 0 to 1 scaling of the performance index of chattering gives direct control in the self-organizing process of the gains of the switching function. Depending upon the allowable physical limitation of the process, the change in the gains of the switching function can be forced to limit the chattering within the specified limits. Thus if the desired or allowable chattering in the control signal is 0.5, then the current index for chattering can be compared with this desired 0.5, and the gains of the switching function can be appropriately adjusted. It was shown in [10] that a self-organizing scheme with this performance index of chattering performs better in terms of chattering suppression and response quality of the system. The scheme in [10] is the self-organizing variable structure controller scheme described in this paper without the fuzzy part.

In order to keep the VS system invariant to parameter variations and disturbances, the control should maintain the system trajectory on the sliding plane. To achieve this, the performance measure for robustness was chosen as in [3]:

$$P_i^R = 1/m\left| \sum_{l=j-m}^{j} (1(sign(s_i^l)) - 1(-sign(s_i^l)))\right| , \tag{10}$$

where $1(x)$ is the Heaviside function and s_i is the sliding surface.

Let h_i^j be the coefficient of slope of the plane and k_i^j be the switching function gain for the control i in step j of the action. The adaptation is done in iterative form using a weighted function of the performance indices according to the following equations:

$$h_i^{j+1} = h_i^j[1 + \phi_i(P_i^E, P_i^C, P_i^R)] \tag{11}$$

$$k_i^{j+1} = k_i^j[1 + \eta_i(P_i^E, P_i^C, P_i^R)] , \tag{12}$$

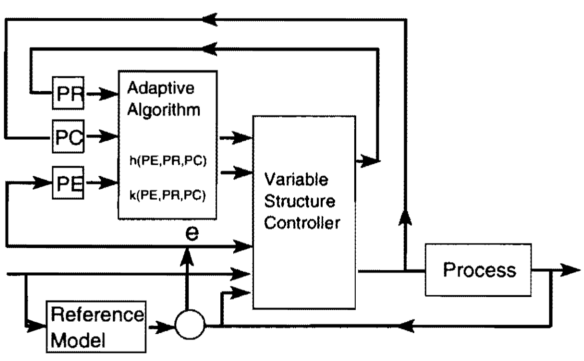

Figure 1. Self organizing model reference VSC.

where ϕ_i and η_i are weighted numerical values for determining the increment or decrement in h_i and k_i respectively depending on the performance indices. A possible set of equations describing the effect is given as:

$$\phi_i = \frac{c_i^{\phi E}[P_i^E - H_i]}{|P_i^E - H_i| + \delta_i^E} + c_i^{\phi R}[1(P_i^R) + 1(P_i^R - R_i) - 1] \tag{13}$$

$$\eta_i = \frac{c_i^{\eta C}[P_i^C - K_i]}{|P_i^C - K_i| + \delta_i^C} + c_i^{\eta R}[1 - 1(P_i^R) - 1(P_i^R - R_i)] \ , \tag{14}$$

where H_i, K_i, R_i are design parameters providing comparison points for the performance indices; $c_i^{\phi E}$, $c_i^{\phi R}$, $c_i^{\eta C}$, and $c_i^{\eta R}$ specify the weights of change in hyperplane slope, gain of switching function, and robustness respectively. In our scheme, K_i has a value in the range of [0,1] and may be chosen according to the allowable limits of chattering in the system. The adaptive process is oriented towards adjusting the coefficients of the hyperplanes and the gains of the controller in the direction of minimizing the error and maintaining the system trajectory on the sliding plane. The stability of the sliding plane is guaranteed as long as the reaching conditions of Equation (4) are satisfied. This is taken care in the controller design and the specification of fuzzy rules as explained in the next section. The adaptation process can be switched off when the states of the system have reached a desired error level.

4. HYBRID SCHEME

Sliding mode control uses discontinuous control action to drive the state trajectories toward a specific hyperplane in the state space, and to maintain the state trajectories sliding on the specific hyperplane until the origin of the state space is reached. This principle provides the basic guidance towards the design of a fuzzy logic controller based on the sliding mode theory. The existence of the sliding mode and asymptotic stability of the system is guaranteed as long as the reaching condition of Equation (4) is satisfied. Therefore the control input to the system has to be designed so that the condition on the sliding mode is always satisfied. To achieve this objective consider the sliding surface 's' for any nonlinear system expressed in companion form as given in Equation (2). For the sake of clarity we drop the subscript 'i' which stands for the i-th subsystem; then the sliding surface may be given as:

$$s(e) = h_1 e_1 + h_2 e_2 + ... + e_m = 0. \tag{15}$$

By taking the time derivative of both sides and substituting the value of $\dot{e}_m$ from Equation (2) we obtain:

$$\dot{s} = \sum_{i=1}^{n-1} h_i e_{i+1} - f(e) - b(e)u - d(t). \tag{16}$$

Then multiplying both sides of the above equation by 's' gives:

$$s\dot{s} = \sum_{i=1}^{n-1} h_i e_{i+1} s - f(e)s - b(e)us - d(t)s; \tag{17}$$

here if $b(e) > 0$ for any 'e' then in Equation (16) $\dot{s}$ increases as u decreases and vice versa. This implies that in Equation (17) if $s > 0$ then increasing u will result in decreasing $s\dot{s}$ and that if $s < 0$ then decreasing u will result in decreasing $s\dot{s}$. This appropriate action on 'u' will guarantee the system stability as it will satisfy the reaching condition (Equation (4)).

Based on the above qualitative analysis, design of u dependent on the sliding mode conditions can be achieved [6]. A sample of control rules is shown in Table 1.

For illustration the (1,7) entry in Table 1 is read as:

IF s is PB AND $\dot{s}$ is PB, THEN Δu is PB.

The rules in Table 1 are designed to guarantee that the reaching condition is always satisfied: that is, $s\dot{s}$ is always decreasing.

This control rule states that if s and $\dot{s}$ are both positive big, which is equivalent to that $s\dot{s}$ is largely positive; then a large positive change of the control input is required to decrease $s\dot{s}$ quickly.

For the step j, the control is then designed to be of a learning type and is given as:

$$u^j = u^{j-1} + G\phi(p(y), q(z)), \tag{18}$$

where:

$y = GS{*}s$ and $z = GCS{*}\dot{s}$; s is the switching surface; G, GS, and GCS are scaling factors; and $\phi(p(y), q(z))$ is the defuzzified output.

For implementation purposes the rule table may be converted into a lookup table of predefined quantization level to save computer processing time [11].

The basic idea behind the hybrid scheme emerged from the following observations:

The self-organizing scheme with the new performance index results in improvement over other schemes reported in the literature. But the chattering problem is still persistent.

The pure fuzzy logic scheme is shown to reduce chattering in comparison to pure variable structure controller scheme as reported by Zong-Mu [8] but the limitation imposed on it through fixed or predefined coefficients of the hyperplanes, h_is, makes the response slower while nearing the steady state.

A possible solution to the above problems is to combine the two schemes, namely Self-Organizing and the Fuzzy Logic together to get the best response possible from the system.

The Hybrid scheme is best represented in the block diagrams of Figures 2 and 3 and can be stated in words as: retain the self-organizing action both on slopes (h_i) and gains (k_i), the slopes being used to calculate the change in control through the fuzzy logic based lookup table, and the gains being used in the general structure of the control law, which can now be referred to as Hybrid fuzzy control law given as:

$$u_i^j = u_i^{j-1} + k_i\phi(p(y_i), q(z_i)), \tag{19}$$

where:

$y_i = GS_i * s_i$ and $z_i = GCS_i * \dot{s}_i$; $\phi(p(y_i), q(z_i))$ is the output obtained from the lookup or decision matrix.

The Hybrid controller of Equation (19) is different from the fuzzy controller of Equation (18) in two aspects. Firstly the constant gain G in fuzzy controller has been replaced by adaptive gain k_i in the Hybrid controller and

Table 1. Rule Table for the FL Controller.

	$\dot{s}$						
s	NB	NM	NS	ZE	PS	PM	PB
PB	ZE	PS	PM	PB	PB	PB	PB
PM	NS	ZE	PS	PM	PB	PB	PB
PS	NM	NS	ZE	PS	PM	PB	PB
ZE	NB	NM	NS	ZE	PS	PM	PB
NS	NB	NB	NM	NS	ZE	PS	PM
NM	NB	NB	NB	NM	NS	ZE	PS
NB	NB	NB	NB	NB	NM	NS	ZE

secondly the state variable x in the fuzzy controller which is replaced by s_i in the Hybrid controller is constantly being updated through adaptation on h_i.

This approach was adopted here to get the combined effect of both schemes. The advantage here is to get the fastest response of the system through adaptation on h_is and get the minimal chattering of states through adaptation on k_i, and improve the system time constant through fuzzy logic. Hence the fuzzy logic part improves the system response initially and the self-organizing part takes care of the response towards the steady state. The chattering reduction effects of fuzzy logic and self organizing schemes complement each other in the Hybrid scheme and the level of chattering is brought down to a minimum.

4.1. Hybrid Controller for the Inverted Pendulum

The inverted pendulum is a single input multivariable output problem as shown in Figure 2.

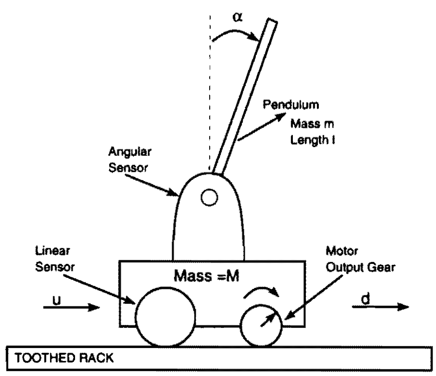

Figure 2. Model of an inverted pendulum.

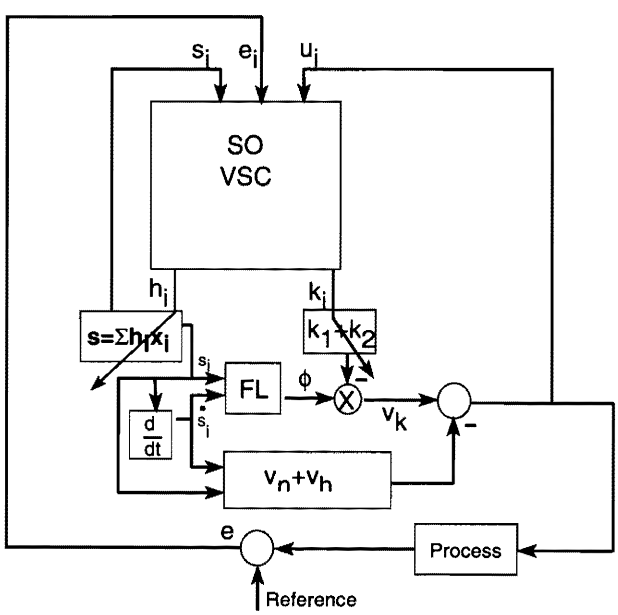

Figure 3. Hybrid scheme based controller for the inverted pendulum.

For the complete dynamics of the system we have to consider the system to be constrained not only by the angle of the inverted pendulum but also the distance traveled by the cart. For complete system equations see [12]. The system parameters are defined as:

mass of the cart $M = 0.455$ kg, mass of the pendulum rod $m = 0.210$ kg, acceleration due to gravity $g = 9.8$ m/sec^2, length of the rod $L = 30.5$ cm, moment of inertia of the pendulum $I = mL^2/3$. The sliding surface for the inverted pendulum is chosen to be:

$$s = h_1x_1 + h_{11}x_2 + h_2x_3 + h_{22}x_4 - h_1x_d, \tag{20}$$

where h_is represent the slopes and x_d is the desired cart position, which is set to zero for the regulation problem.

The control chosen has three factors:

$$u = -v_n - v_k - v_h \tag{21}$$

where v_n is the input to cancel the nonlinearity in the system, v_k is the switching input whose magnitude depends on the k_i which are updated through the fuzzy logic based lookup Table 1 and v_h is calculated as the pure sliding mode control with the coefficients of the slope h_i being updated through self-organizing VSC. The system initial values are given as $d = x_1 = 2$ cm, $\alpha = x_3 =$ -1 degrees and:

$h_1 = 4$, $h_2 = 5$, $k_1 = 0.1$ and $k_2 = 0.1$;
the constants are $h_{11} = 8$ and $h_{22} = 1$.

4.2. Hybrid Controller for the Two link Manipulator

The second model considered is that of a two link two degree of freedom robotic manipulator and is shown in Figure 4. Let θ_1 and θ_2 be the angles of the two links and u_1 and u_2 be the two inputs to the system. For complete system representation see [2].

The system parameters are defined as:

Length of link 1 $r_1 = 1$m, length of link 2 $r_2 = 0.8$m, mass of link 1 $m_1 = 0.5$kg, mass of link 2 $m_2 = 0.5$kg, acceleration due to gravity $g = 9.81$ m/sec^2, inertia with respect to axis of rotation of link 1 $J_1 =5$ kgm^2, inertia with respect to axis of rotation of link 2 $J_2 =5$ kgm^2.

The two link manipulator is a two-input, two-output problem. Using the hierarchical approach, the problem can be subdivided into i subsystems (here i=2). The hyperplane for the i-th subsystem may be chosen as

$$s_i = h_i(\theta_i - \theta_{id}) + \dot{\theta}_i \tag{22}$$

where θ_{id} represents the desired angle of link i, for regulation problem it is set to zero.

For the i-th subsystem the controller is given by:

$$u_i^j = u_i^{j-1} + k_i\phi(p(s_i), q(z\dot{s}_i)). \tag{23}$$

The scaling factors for all the subsystems were chosen to be the same and are given by:

g_1=10; g_2=5; and g_3=1.

The controller block diagram is shown in Figure 5. The controller works by updating h_is through the self-organizing VSC and then using s_i and $\dot{s}_i$ to lookup the corresponding change in k_i from the fuzzy logic based lookup Table 1.

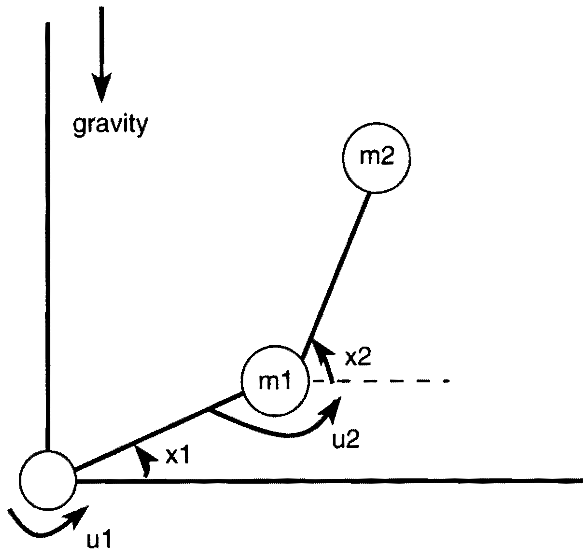

Figure 4. Model of a two link manipulator.

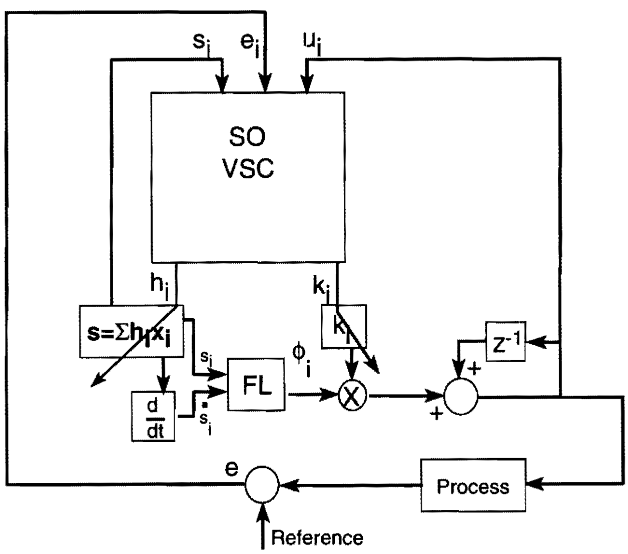

Figure 5. Hybrid scheme based controller for the two link manipulator.

5. RESULTS AND ANALYSIS

To facilitate comparison the state response and the control signal are shown for the inverted pendulum with restricted travel and the two link manipulator for the following three cases:

(1) Fuzzy Logic Scheme;
(2) Self Organizing [10]; and
(3) the Proposed Hybrid Scheme.

The response of the states of the inverted pendulum with restricted travel is shown in Figure 6 and the control signal is shown in Figure 7. The Fuzzy Logic scheme fails to drive the states of the system to steady state in a reasonable amount of time also the chattering in the control signal is persistent. Comparison with [10] and the Hybrid scheme shows that the Hybrid scheme results in marked improvement of the response of the system and the magnitude of the control required has also been reduced. Also, as the system approaches steady state the chattering in the control has effectively been reduced to minimum.

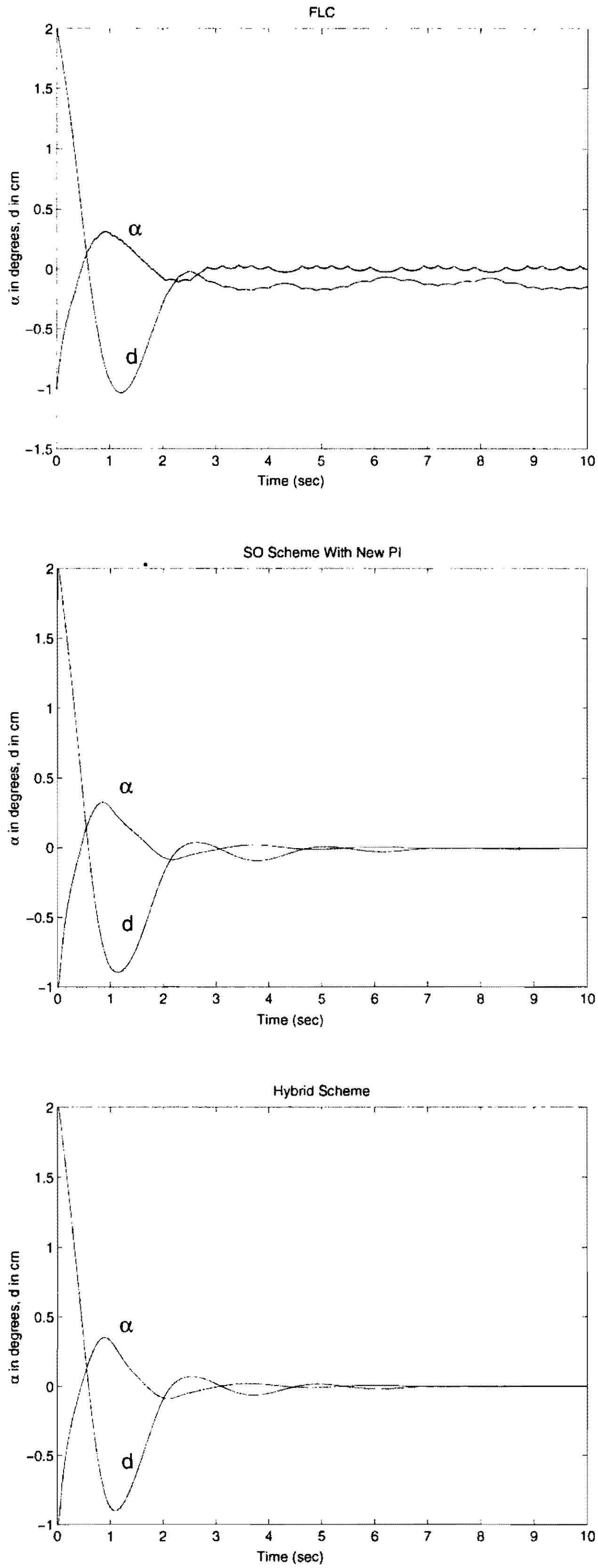

Figure 6. States for the inverted pendulum.

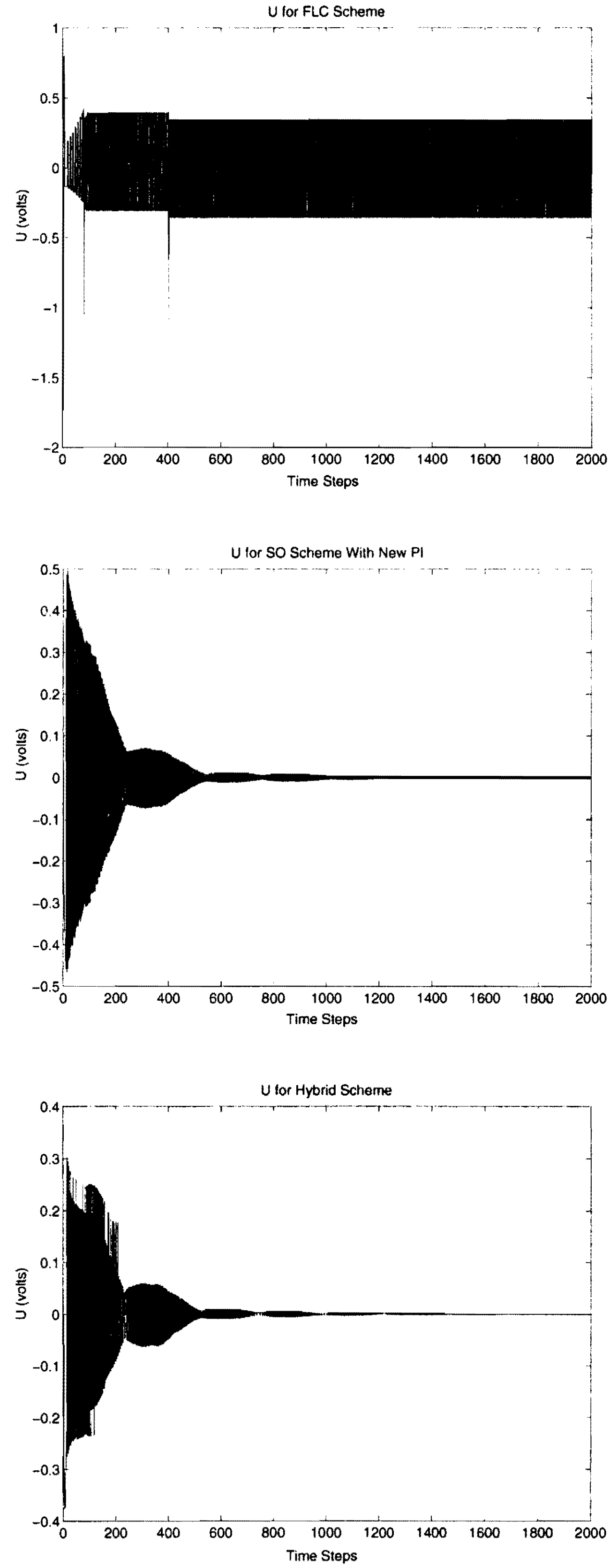

Figure 7. Control for the inverted pendulum.

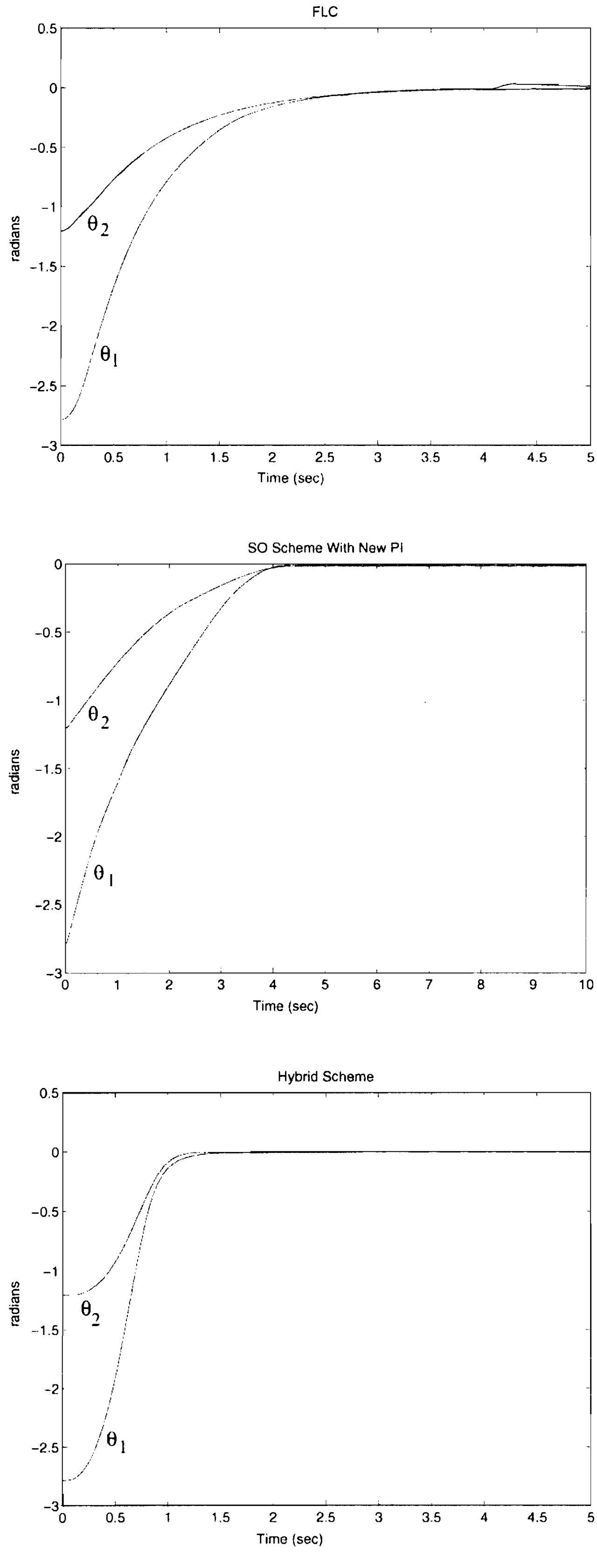

Figure 8. State for the two link manipulator.

The state response for the two link manipulator is given in Figure 8 and the corresponding control signals U_1 and U_2 are shown in Figure 9 and Figure 10 respectively.

The results for the two link manipulator show that time constant of the two links have been improved over the rest of the schemes and simultaneously the maximum torques required for the two links have been reduced. Also the amount of chattering in the control signal have been effectively removed.

6. CONCLUSIONS

In this work we have effectively demonstrated the use of a combination of both the self-organizing and the fuzzy logic in the Hybrid Scheme for the first time. A number of simulations have been reported and shown to achieve the target of minimum chattering and improved response of the system. Moreover, the use of the Hybrid scheme results in minimizing the power of the control signal required in all the systems considered.

For quick reference all the simulation results for the inverted pendulum with restricted travel and the two link manipulator have been summarized in Tables 2 and 3, respectively.

Table 2. Results Comparison for the Inverted Pendulum.

	Self Organizing Scheme [10]	Fuzzy Logic Scheme	Hybrid Scheme
Max. Mag. of U	0.5	0.81	0.3
Power of U	0.101	1.234	0.049
d ISE	1.532	1.887	1.465
d IAE	1.68	2.67	1.57
α ISE	0.18	0.178	0.187
α IAE	0.525	0.591	0.522
Avg. PC	0.75	0.3	0.6
Settling Time	7.2 sec.	Very high	6.8 sec.

Table 3. Results Comparison for the Two Link Manipulator.

	Zaremba Scheme	Self Organizing Scheme [10]	Fuzzy Logic Scheme	Hybrid Scheme
Link 1				
Rise time (sec)	4	4.2	3.75	1.5
Max. of U_1	232	226	80.91	124
Power of U_1	43684	48132	12726	5287
Link 1 ISE	6.432	6.695	3.739	3.763
Link 1 IAE	4.038	4.205	2.274	1.721
Avg. PC	32.3	0.293	0.84	0.003
Link 2				
Rise time (sec)	6.3	4.5	4.1	1.25
Max. of U_2	77	69	63.6	70
Power of U_2	17316	4935	4365	653
Link 2 ISE	1.351	1.320	0.771	0.793
Link 2 IAE	2.03	1.89	1.16	0.823
Avg. PC	37.7	0.331	0.823	0.0038

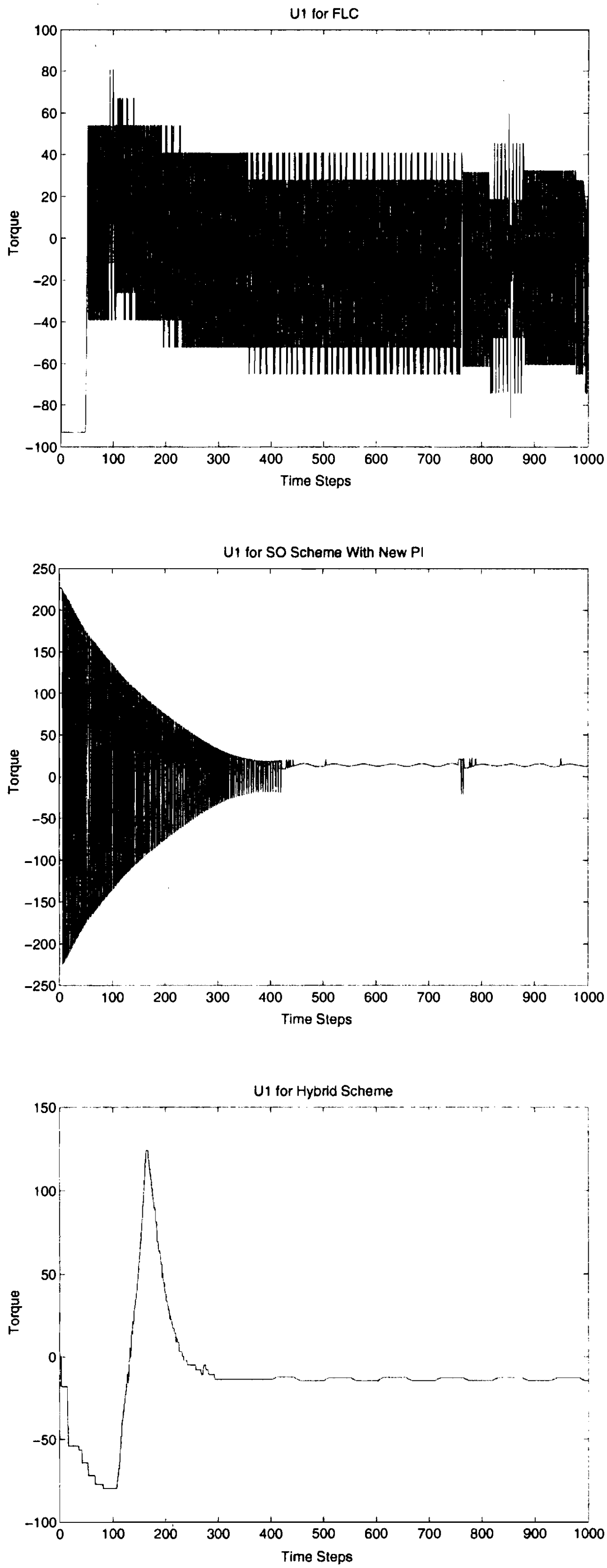

Figure 9. U_1 *for the two link manipulator.*

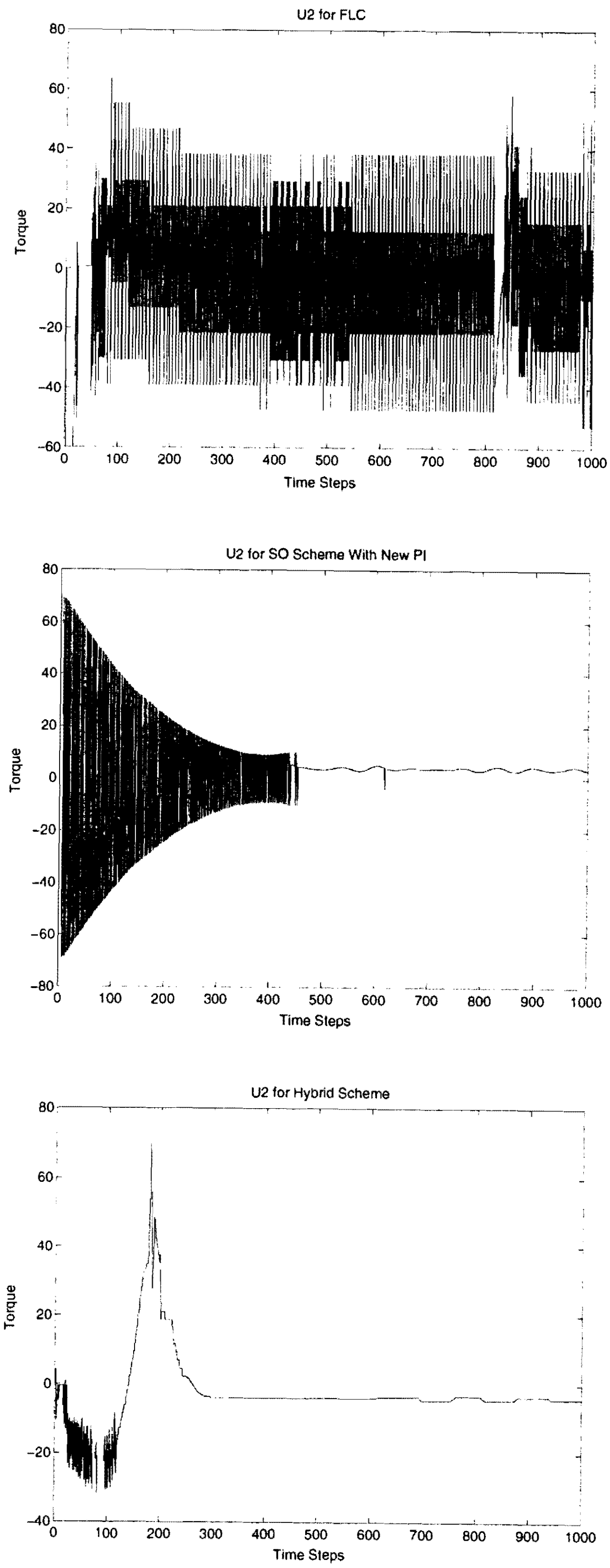

Figure 10. U_2 *for the two link manipulator.*

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