ANALYSIS OF AN AC–DC FULL-CONTROLLED CONVERTER SUPPLYING TWO DC-SERIES-MOTOR LOADS

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الخلاصــة :

المُقَوِّمات المحكومة بزاوية الطّور تُستعمل بكثرة، لأنها بسيطة، وتكلفتها قليلة، وسريعة الإستجابة، وعالية الكفاءة، ويعوَّل عليها، ولا تحتاج الى دوائر قطع. كما يشيع استخدام محركات التَوَال في التطبيقات التي تحتاج الى عزوم عالية عند بداية التشغيل، وكذلك التي تحتاج الى قدرات ثابتة.

يُعنى هذا البحث بدراسة مفصلة (بإستخدام الحاسوب) لخصائص وتحليلات المُقوِّم المحكوم بالطور ذي الأربعة ثايرستورات المغذّي لُمحرِّكيْ تَوَال، (حيث إن خصائص وتحليلات المقوِّم المغذي لمحرك واحد لا تنطبق في هذه الحالة). وقد تمَّتْ هذه الدراسة لكلٍّ من نسقيْ تيار المُقـوِّم (نسق التيار المتصل، ونسق التيار المتقطع).

فالبحث يقدِّم معادلة زاوية الإشعال الحرجة التي عندها يتحول تيار المقوِّم من متصل الى منقطع، ويقدِّم أيضا دراسة مستفيضة ومعادلات لكل من: معامل قدرة مدخل المُقـوِّم، ومعامل التَشوّيه في تيار المنبع، وتغير العزوم مع السرعات، ومعامل تموّج تيار كلٍّ محرك. وقد تمَّت هذه الدراسة لكل من حالتي ثبات زاوية الإشعال، وثبات قدرة أحد المحركيْن. وللتوضيح قدَّم البحث نماذج من الأشكال الموجبة للتيارات والفولطيات المختلفة تحت ظروف تشغيل متنوعة.

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ABSTRACT

Phase-controlled converters are widely used because these converters are simple, less expensive, reliable, and do not require any communication circuit. Series motors are extensively used in many applications that require both high starting torque and essentially constant horsepower. This paper is concerned with the detailed study of the performance characteristics of an AC–DC full-controlled converter supplying two DC-series-motor loads. The converter-loads combination is simulated on a digital computer. Different modes of operation (continuous and discontinuous converter currents) are considered. The critical firing angle at which the mode of operation changes from one mode to another is deduced. The performance characteristics such as, input power factor, supply current distortion factor, supply current fundamental power factor, torque-speed, and motor current ripple factor have been derived and studied for both constant firing angle and constant load power of one motor. Waveforms for each load current and converter current are investigated for different modes of operation.

Key words: Controlled rectifiers, AC–DC converters, Full-controlled converters, DC-Series-motor, Parallel loads.

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1. INTRODUCTION

Controlled rectifiers employing power thyristors form the large majority of static power converters. The thyristors are phase controlled to give variable direct voltage from an AC voltage source. The phase controlled converters are simpler and less expensive, and have faster time response, minimal required maintenance, higher efficiency, smaller size, less weight, and more reliable operation, as compared to forced commutated converters. They are extensively used in numerous domestic and industrial applications. In such converters, thyristor commutation (*i.e.* transfer of current from one thyristor to the next) is easily achieved by a process referred to as line commutation. No additional circuitry is required for the commutation process [1, 2].

Some researchers have studied converter performance connected to a passive load. Chung [3] presented a steady-state analysis of a two-branch resistance–inductance parallel circuit controlled by an AC switch. Al-Juhani [4] also made an analysis of an AC–DC converter supplying two-parallel inductive loads. Regarding the active load, Doradla and Sen [5] presented one of the early attempts to study solid state series motor drive. They made an evaluation of control schemes for a thyristor-controlled DC motor [6]. Harmonics presented by a converter supplying a single-motor has also been studied [7]. Most of the work reported in the literature is related to a converter supplying a single DC motor or two parallel passive loads.

Series motors are extensively used in many applications that require high starting torque and that require essentially constant horsepower [8]. The presence of more than one motor connected in parallel is commonly met in practice. The analysis and performance of the phase-controlled converter with single motor load can not be applied in such a situation, unless all loads are of the same parameters, speed, and torque; a situation that is unlikely to be the case in practice.

In this paper, analysis of a phase-controlled (AC–DC full-controlled) converter feeding two DC series motor loads of different parameters, speed, and load torque will be investigated. The steady-state analysis of this system will be obtained for each of the two modes of operation (*i.e.* continuous and discontinuous converter current).

2. METHODOLOGY

In order to evaluate the overall performance of a converter–motors combination, certain performance parameters for the DC motor and the input supply have to be considered. The paper will reach its goal *via* following stages.

1. Study of the behavior of an AC–DC full-controlled converter supplying two DC series-motor loads through the derivation of the expressions for:
   (*a*) Converter current.
   (*b*) Motor current and its ripple factor.
   (*c*) RMS value of the fundamental component of the supply current.
   (*d*) Input power factor.
   (*e*) Supply current distortion factor.
   (*f*) Supply current fundamental power factor.
   (*g*) Critical firing angle at which the mode of operation changes from one mode to the other.
   (*h*) Extinction angle of the discontinuous converter current.
   (*i*) Torque-speed characteristics for each motor.
2. Preparation of a computer program (in Fortran) to compute system performance.
3. Study of the results obtained.

3. STEADY STATE ANALYSIS

The circuit under consideration is shown in Figure 1, where $S1$, $S2$, $S3$, $S4$ are four power thyristors connected as a full converter. The sinusoidal voltage source is considered to have an rms value of V volts, frequency of $\omega/2\pi$ Hz and zero internal impedance. The load consists of two parallel DC series motor loads. The supply and the motor currents are controlled by changing the thyristors-firing angle. The supply current may be continuous or discontinuous depending on the firing angle, the load parameters, and motor speeds. Thyristors $S1$ and $S3$ are fired at an angle α relative to the supply zero voltage whereas $S2$ and $S4$ are gated at $\pi + \alpha$. Each of the two modes of operation will be treated separately.

3.1. Continuous Mode

In this mode, the converter current i_l is assumed to have a positive value at any instant of time. However, as the value of α increases, the value of i_l at $\omega t = \alpha$ decreases until it reaches zero value at the so-called critical firing angle α_c. Beyond this critical firing angle, the converter current will be discontinuous.

Throughout the period $\alpha \le \omega t \le \pi + \alpha$, the voltage across each motor in the continuous mode is equal to the supply voltage, assuming zero drops across thyristors. This means that:

$$\sqrt{2}V \sin \omega t = i_{c1}R_{a1} + L_{a1}\tfrac{d}{dt}i_{c1} + K_{af1}i_{c1}N_1 + K_{res1}N_1 \tag{1}$$

$$\sqrt{2}V \sin \omega t = i_{c2}R_{a2} + L_{a2}\tfrac{d}{dt}i_{c2} + K_{af2}i_{c2}N_2 + K_{res2}N_2 \ , \tag{2}$$

where: i_{c1} and i_{c2} are the armature currents during the continuous mode.

During this mode, the converter current i_{cl} equals the supply current i_{cs}, and is given by:

$$i_{cs} = i_{cl} = i_{c1} + i_{c2}, \ \alpha \le \omega t \le \pi + \alpha \ . \tag{3}$$

If steady-state speed and magnetic linearity are assumed, and the armature circuit resistance R_{a1} and inductance L_{a1} include the resistance and inductance of the series field winding, the solution of Equation (1) is of the form:

$$i_{c1} = \sqrt{2}(V/Z_1)\sin(\omega t - \phi_1) + K_{c1}e^{(-(\omega t-\alpha)/Q_1)} - (K_{res1}N_1/R_1)(1 - e^{(-(\omega t-\alpha)/Q_1)}) \ , \tag{4}$$

where: $R_1 = R_{a1} + K_{af1}N_1$, K_{af1} and K_{res1} are constants of motor one, $Z_1 = \sqrt{R_1^2 + (\omega L_{a1})^2}$; $\phi_1 = \tan^{-1}(\omega L_{a1}/R_1)$; $Q_1 = \omega L_{a1}/R_1$, $K_{c1} = I_{c1} - \sqrt{2}(V/Z_1)\sin(\alpha - \phi_1)$; and I_{c1} is the initial value of i_{c1} at $\omega t = \alpha$.

Under steady state operation, i_{c1} at $\omega t = \alpha$ equals that at $\omega t = \pi + \alpha$ and is equal to:

$$I_{c1} = -\sqrt{2}(V/Z_1)[(1 + e^{(-\pi/Q_1)})/(1 - e^{(-\pi/Q_1)})]\sin(\alpha - \phi_1) - (K_{res1}N_1/R_1) \ . \tag{5}$$

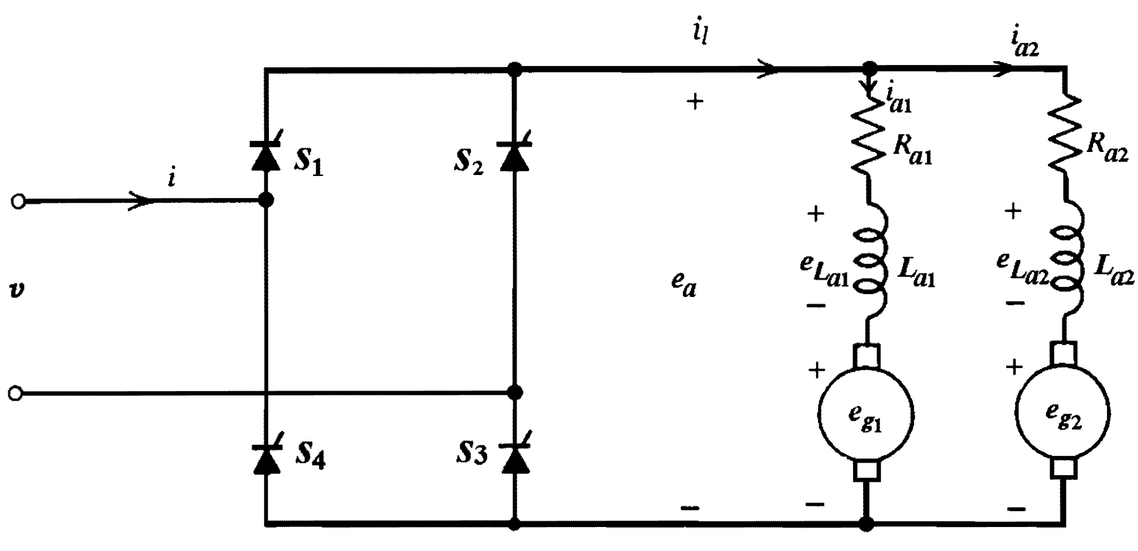

Figure 1. Terminal voltage control of two DC series motors by a single phase full-converter.

The critical firing angle α_c, which is the highest firing angle, for the continuous mode, satisfies the condition:

$$I_{c1} + I_{c2} = 0 \ . \tag{6}$$

Therefore, the critical firing angle α_c is given by:

$$\alpha_c = \sin^{-1}[(A_1 C_1 \pm \sqrt{A_1^2 C_1^2 + (A_1^2 + B_1^2)(B_1^2 - C_1^2)})/(A_1^2 + B_1^2)] \ , \tag{7}$$

where: $A_1 = F_1 \cos\phi_1 + F_2 \cos\phi_2$; $F_1 = -(\sqrt{2}V / Z_1)(1 + e^{(-\pi/Q_1)})/(1 - e^{(-\pi/Q_1)})$;

$F_2 = -(\sqrt{2}V / Z_2)(1 + e^{(-\pi/Q_2)})/(1 - e^{(-\pi/Q_2)})$; $B_1 = F_1 \sin\phi_1 + F_2 \sin\phi_2$; and

$C_1 = (K_{res1} N_1 / R_1) + (K_{res2} N_2 / R_2)$.

For negligible values of K_{res1} and K_{res2}, α_c (as expected) is independent of the supply voltage.

The ripple factor of i_{c1} is given by:

$$K_{cr1} = \sqrt{I_{c1rms}^2 / I_{c1av}^2 - 1} \ , \tag{8}$$

where I_{c1rms} and I_{c1av} are the rms and the average values of the motor current, respectively.

The rms value of the fundamental component of the supply current is:

$$I_{cf} = \sqrt{(a_{c1}^2 + b_{c1}^2)/2} \ , \tag{9}$$

and the phase angle of that component is given by:

$$\theta = \tan^{-1}(a_{c1} / b_{c1}) \ , \tag{10}$$

where a_{c1} and b_{c1} are the amplitudes of the cosine and sine fundamental components of i_{cs} respectively in the continuous mode.

The supply power factor is:

$$Pf = (I_{cf} / I_{csrms}) \cos\theta \ , \tag{11}$$

and the supply current distortion factor is:

$$Df = I_{cf} / I_{csrms} \ , \tag{12}$$

where I_{csrms} is the rms value of the supply current.

Full expressions for I_{c1rms}, I_{c1av}, I_{csrms}, a_{c1}, and b_{c1} are given in the Appendix.

3.2. Discontinuous Mode

The discontinuous mode of operation occurs at firing angles greater than α_c. In this mode, the current i_l increases from zero at $\omega t = \alpha$ to a maximum value and then decreases to become zero again at $\omega t = \beta$ which is referred to as the extinction angle. The angle β is, of course, $< \pi + \alpha$. Thus, the current i_{dl} is zero for $\beta \le \omega t \le \pi + \alpha$.

Throughout the period $\alpha \le \omega t \le \beta$, the load voltage is equal to the supply voltage. This means that:

$$\sqrt{2}V \sin\omega t = i_{d1} R_{a1} + L_{a1} \tfrac{d}{dt} i_{d1} + K_{af1} i_{d1} N_1 + K_{res1} N_1 \text{, and} \tag{13}$$

$$\sqrt{2}V \sin\omega t = i_{d2} R_{a2} + L_{a2} \tfrac{d}{dt} i_{d2} + K_{af2} i_{d2} N_2 + K_{res2} N_2 \ . \tag{14}$$

During $\alpha \le \omega t \le \beta$, the converter current i_{dl} equals the supply current and is given by:

$$i_{ds} = i_{dl} = i_{d1} + i_{d2} \ . \tag{15}$$

The solution of Equation (13) is of the form:

$$i_{d1} = \sqrt{2}(V / Z_1)\sin(\omega t - \phi_1) + K_{d1}e^{(-(\omega t-\alpha)/Q_1)} - (K_{res1}N_1 / R_1)(1 - e^{(-(\omega t-\alpha)/Q_1)}) \ , \tag{16}$$

where: $$K_{d1} = I_{d1} - (\sqrt{2}V / Z_1)\sin(\alpha - \phi_1) \ , \tag{17}$$

and I_{d1} is the initial value of i_{d1} at $\omega t = \alpha$.

During the period $\beta \le \omega t \le \pi + \alpha$, the converter current is equal to zero, and $i_{d1} = -i_{d2}$.

The motors are connected in parallel. Thus:

$$R_1 i_{d1} + L_{a1}\tfrac{d}{dt}i_{d1} + K_{res1}N_1 = R_2 i_{d2} + L_{a2}\tfrac{d}{dt}i_{d2} + K_{res2}N_2 \ . \tag{18}$$

Solving (18), one obtains:

$$i_{d1} = (K_{res2}N_2 - K_{res1}N_1)/(R_1 + R_2) + (I'_{d1} - (K_{res2}N_2 - K_{res1}N_1)/(R_1 + R_2))e^{(-(\omega t-\beta)/Q)} \tag{19}$$

where: $Q = \omega(L_{a1} + L_{a2})/(R_1 + R_2)$, and I'_{d1} is the initial value of i_{d1} at $\omega t = \beta$ and is given by:

$$I'_{d1} = (\sqrt{2}V / Z_1)\sin(\beta - \phi_1) + K_{d1}e^{(-(\beta-\alpha)/Q_1)} - (K_{res1}N_1 / R_1)(1 - e^{(-(\beta-\alpha)/Q_1)}) \ . \tag{20}$$

At $\omega t = \pi + \alpha$, i_{d1} equals I_{d1}. Therefore, from Equations (17), (19), and (20) one gets:

$$\begin{aligned} I_{d1} = [((K_{res2}N_2 - K_{res1}N_1)/(R_1 + R_2))(1 - e^{(-(\pi+\alpha-\beta)/Q)}) + (\sqrt{2}V / Z_1)(\sin(\beta - \phi_1) - \sin(\alpha - \phi_1) \\ e^{(-(\beta-\alpha)/Q_1)})e^{(-(\pi+\alpha-\beta)/Q)} - (K_{res1}N_1 / R_1)(1 - e^{(-(\beta-\alpha)/Q_1)})e^{(-(\pi+\alpha-\beta)/Q)}]/(1 - e^{(-(\beta-\alpha)/Q_1)}e^{(-(\pi+\alpha-\beta)/Q)}) \ . \end{aligned} \tag{21}$$

Similarly:

$$\begin{aligned} I_{d2} = [((K_{res1}N_1 - K_{res2}N_2)/(R_1 + R_2))(1 - e^{(-(\pi+\alpha-\beta)/Q)}) + (\sqrt{2}V / Z_2)(\sin(\beta - \phi_2) - \sin(\alpha - \phi_2) \\ e^{(-(\beta-\alpha)/Q_2)})e^{(-(\pi+\alpha-\beta)/Q)} - (K_{res2}N_2 / R_2)(1 - e^{(-(\beta-\alpha)/Q_2)})e^{(-(\pi+\alpha-\beta)/Q)}]/(1 - e^{(-(\beta-\alpha)/Q_2)}e^{(-(\pi+\alpha-\beta)/Q)}) \ . \end{aligned} \tag{22}$$

The extinction angle β is determined by the solution of the equation:

$$I_{d1} = -I_{d2} \tag{23}$$

And the critical firing angle α_c can also be obtained when β is assigned to the value of $\pi + \alpha$ in (21), (22), and (23). The ripple factor of i_{d1} is given by:

$$K_{dr1} = \sqrt{(I^2_{d1rms} / I^2_{d1av}) - 1} \ , \tag{24}$$

where I_{d1rms} and I_{d1av} are the rms and the average values of the motor current respectively. The rms value of the fundamental component of the supply current is:

$$I_{df} = \sqrt{(a^2_{d1} + b^2_{d1})/2} \ , \tag{25}$$

and the phase angle of that component is given by:

$$\theta = \tan^{-1}(a_{d1} / b_{d1}) \ , \tag{26}$$

where a_{d1} and b_{d1} are the amplitudes of the cosine and sine fundamental components of i_{cs} respectively in the discontinuous mode.

The supply power factor is:

$$Pf = (I_{df} / I_{dsrms})\cos\theta \ , \quad (27)$$

and the supply current distortion factor is:

$$Df = I_{df} / I_{dsrms} \ , \quad (28)$$

where I_{dsrms} is the rms value of the supply current.

Full expressions for I_{d1rms}, I_{d1av}, I_{dsrms}, a_{d1}, and b_{d1} are given in the Appendix.

The developed power is given by:

$$P_{motor} = K_{af} N I^2_{motor_{rms}} \ , \quad (29)$$

And the torque is obtained from:

$$T_{motor} = K_{af} I^2_{motor_{rms}} \ . \quad (30)$$

4. PERFORMANCE CHARACTERISTICS

In order to study the performance characteristics of the system under consideration (full-converter supplying two series-motor loads), a computer program has been developed based on the equations derived in section 3.

The following data of the load parameters are used:

Motor one:	$R_{a1} = 1.0\ \Omega$.	Motor two:	$R_{a2} = 0.15\ \Omega$.
	$L_{a1} = 0.012$ H.		$L_{a2} = 0.02$ H.
	$K_{af1} = 0.027$ H.		$K_{af2} = 0.03$ H.
	$K_{res1} = 0.0273$ V/rad/s.		$K_{res2} = 0.075$ V/rad/s.

And the rms value of the supply voltage is $V = 120.0$ Volt.

4.1. Computer Results

4.1.1. Critical Firing Angle

According to Equation (7), the variation of the critical firing angle α_c with respect to the speed of motor two, $N2$, for different values of that of motor one, $N1$, is shown in Figure 2. It is noted that the critical firing angle α_c decreases as the speed of any motor or both is increased.

4.1.2. Current Waveforms

The waveforms of the converter current and motor current are obtained using Equations (3) and (4) in the continuous mode and using Equations (15), (16), and (19) in the discontinuous mode. These waveforms are shown in Figure 3*a* and 3*b* for continuous mode and in Figure 4 for discontinuous mode.

4.1.3. Torque-Speed Characteristics

The torque speed characteristics ($N2$ *versus* $T2$) at different firing angles are shown in Figure 5 keeping the speed $N1$ unchanged. The regions of discontinuous converter current are obtained by the dotted lines. As expected, the converter current is discontinuous at high values of the firing angle α, high speeds and low values of torque.

4.1.4. Constant-HP Operation

Series motors are normally used for constant-horsepower applications. However, the torque speed curves for a particular firing angle do not conform to constant power characteristics. If the motor is required to operate at a constant power level, the firing angle has to be adjusted accordingly [1].

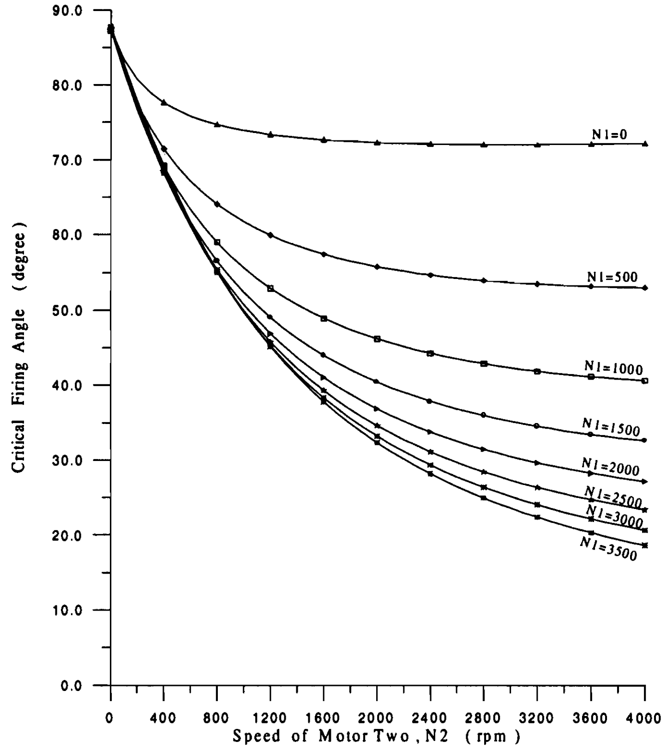

Figure 2. The critical firing angle versus *speed* N2 *for different values of speed* N*1.*

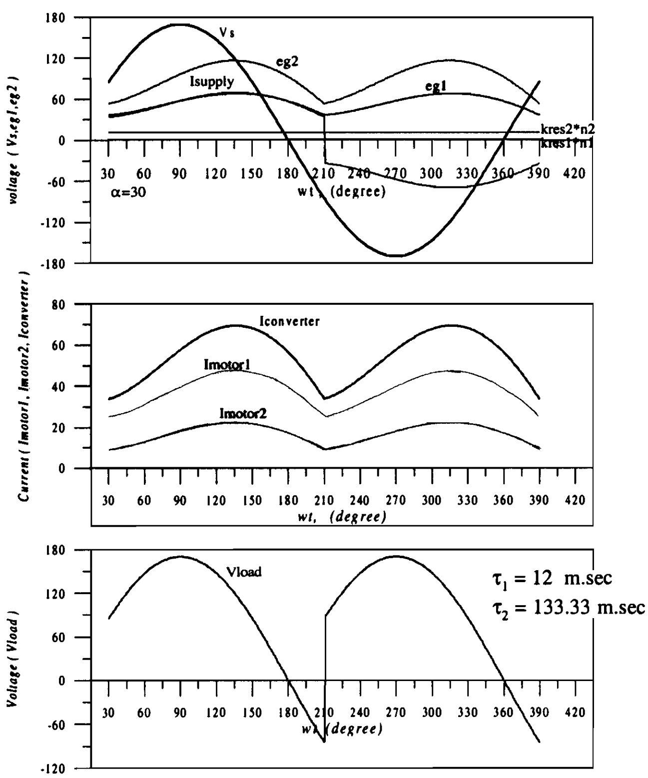

Figure 3a. Current and voltage waveforms of the system operation in continuous converter current mode at N*1=500 rpm,* N*2=1500 rpm, and* α *=30deg.*

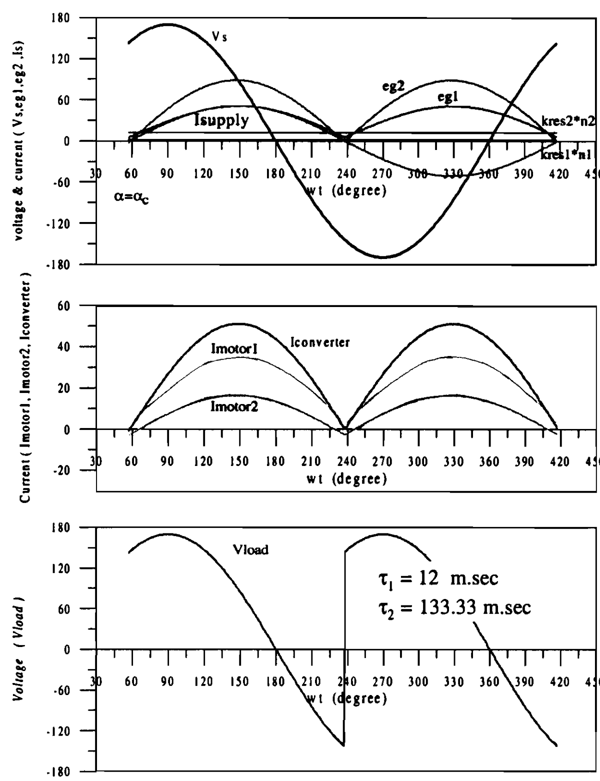

Figure 3b. Current and voltage waveforms of the system operation in continuous converter current mode at N*1=500 rpm,* N*2 =1500 rpm, and* $\alpha = \alpha_c$*=57.949 deg.*

Figure 6 shows variation of firing angle α with respect to the speed *N*2 for different values of horsepower of motor two keeping the speed *N*1 unchanged. The critical firing angle is also indicated. Again the dotted lines indicate discontinuous converter current. Torque-speed characteristics of motor two for different horsepower levels of the same motor are that shown in Figure 7 keeping the speed *N*1 unchanged. These characteristics are drooping in nature. The system produces high torque at low speed and low torque at high speed.

4.1.5. Supply Power Factor and Fundamental Power Factor

The supply power factor and the supply fundamental power factor *versus* speed *N*2, for different values of firing angle α keeping the speed *N*1 unchanged are shown in Figures 8 and 9, respectively. These figures show that these factors decrease as the firing angle is increased. The variations of the supply power factor and fundamental power factor with respect to the speed *N*2 for different horsepower levels of motor two are shown in Figures 10 and 11, respectively keeping the speed *N*1 unchanged. When the motor is used for constant-horsepower applications, these figures show that the supply power factor and the fundamental power factor deteriorate with a decrease in speed. Also these figures show that these characteristics decrease as the load horsepower is decreased.

4.1.6. Distortion Factor

Figures 12 and 13 show the relation between the distortion factor and the speed *N*2 for different values of horsepower levels of motor two and for different values of firing angle respectively keeping the speed *N*1 unchanged. These figures show that the distortion factor increases as the load horsepower is decreased and as the firing angle is increased for continuous converter current mode. The opposite conclusions have been obtained for the discontinuous converter current mode. It is noted that the best value of the distortion factor is obtained when the firing angle equals the critical firing angle.

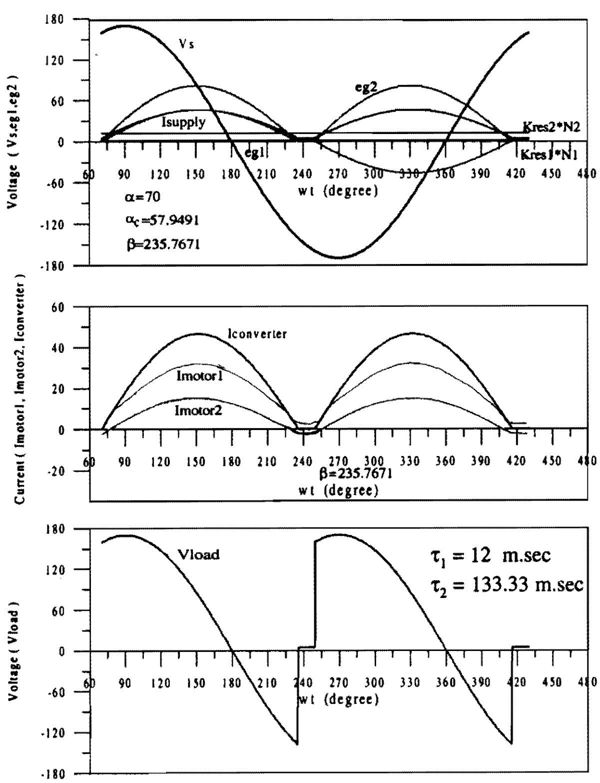

Figure 4. Current and voltage waveforms of the system operation in discontinuous converter current mode at N1=500 rpm, N2=*1500 rpm, and* α *=70 deg.*

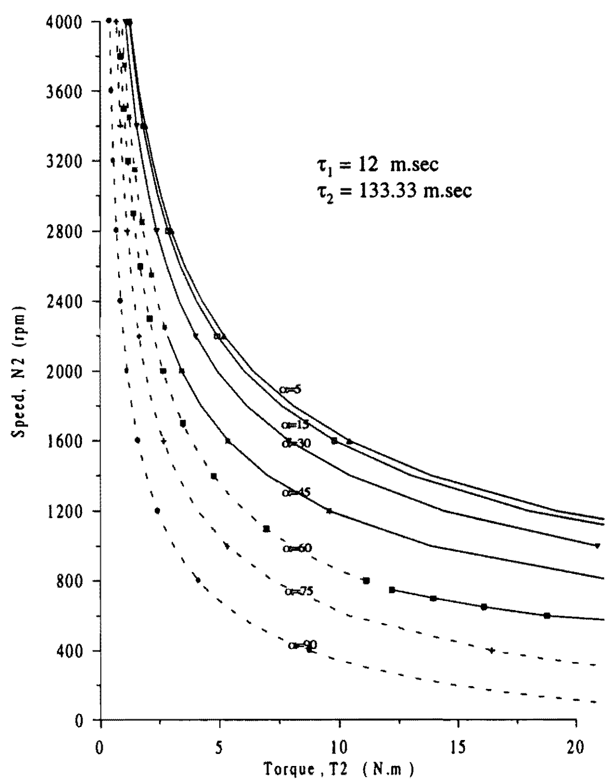

*Figure 5. Torque-speed characteristics (*N2 versus T2*) of the system operation at different values of firing angles at N1= 1000 rpm.*

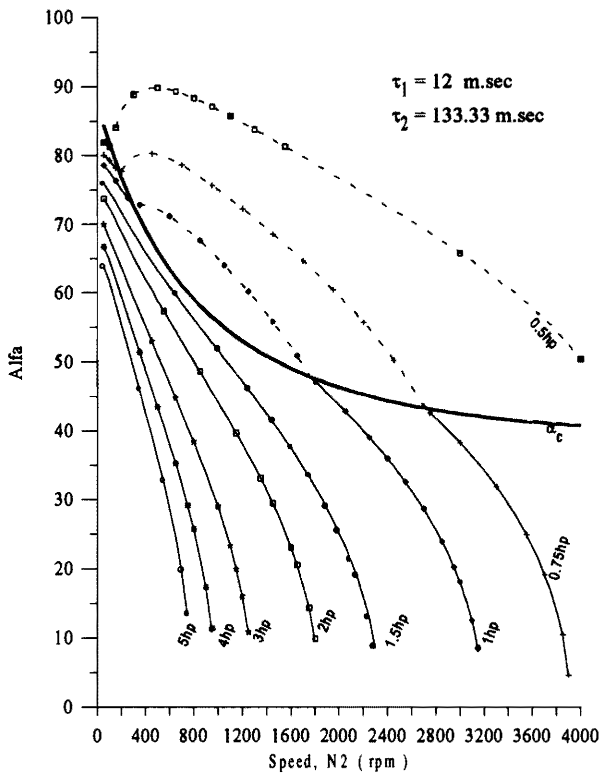

Figure 6. Firing angle versus *speed* N2 *of the system operation at different values of horsepower levels of motor 2, at* N*1=1000 rpm.*

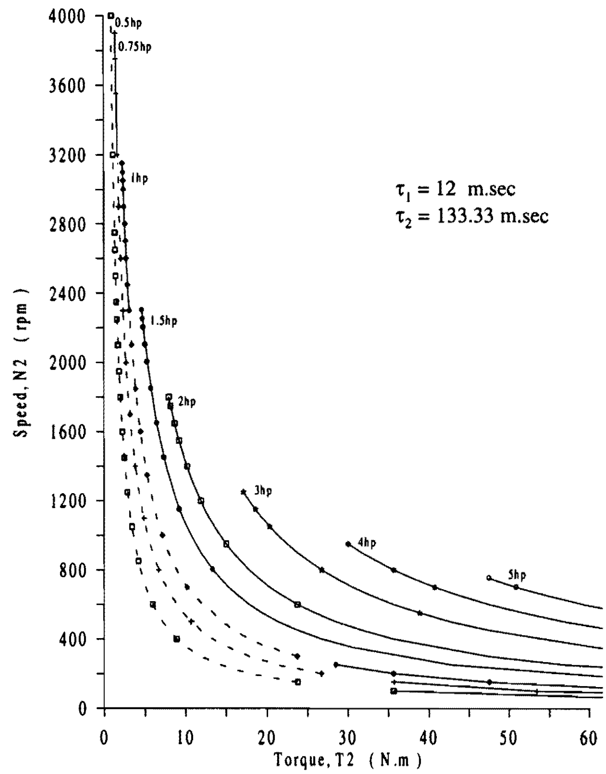

Figure 7. Torque-speed characteristics (N2 versus T2 *) of the system operation at different values of horsepower levels of motor 2, at* N*1 = 1500 rpm.*

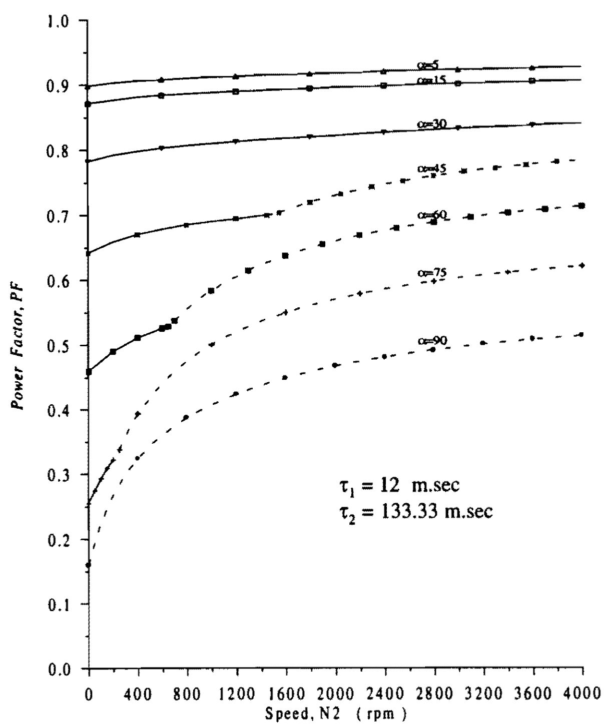

Figure 8. Supply power factor versus *speed* N2 *of the system operation at different values of firing angles, at* N*1=1500 rpm.*

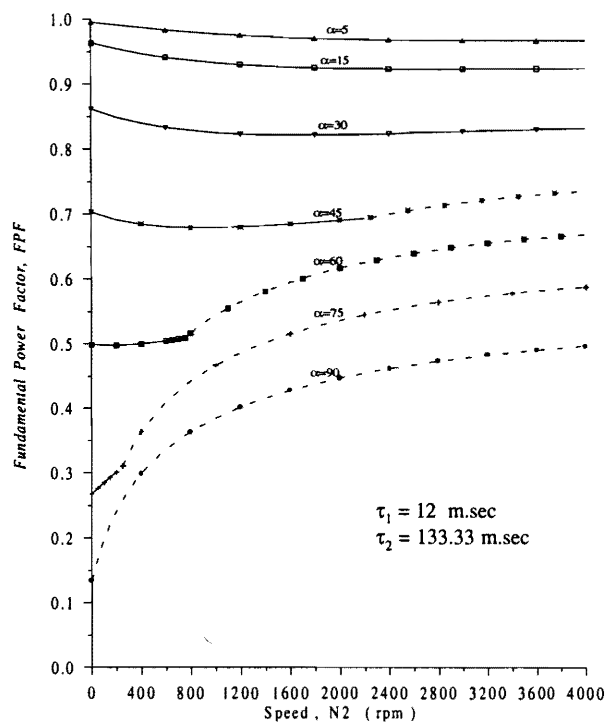

Figure 9. Supply fundamental power factor versus *speed* N2 *of the system operation at different values of firing angles, at* N*1 =1000 rpm.*

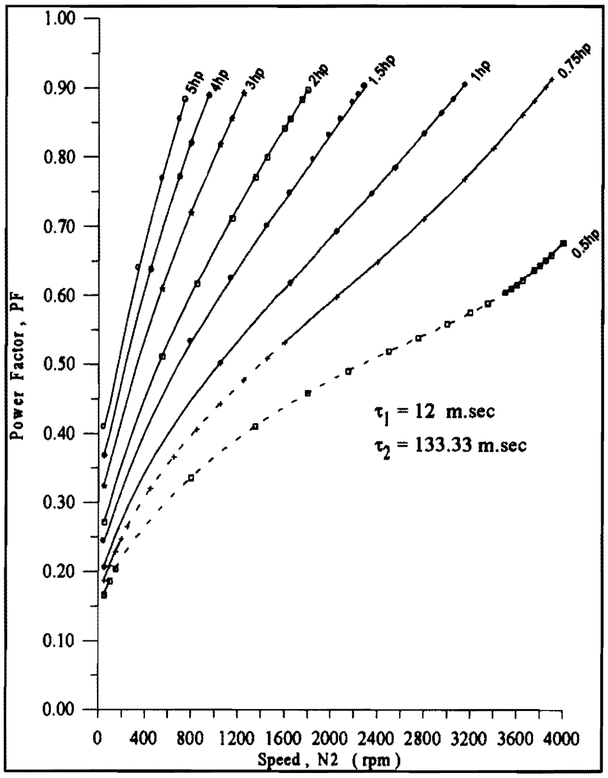

Figure 10. Supply power factor versus *speed* N2 *of the system operation at different values of horsepower levels of motor 2, at* N*1=500 rpm.*

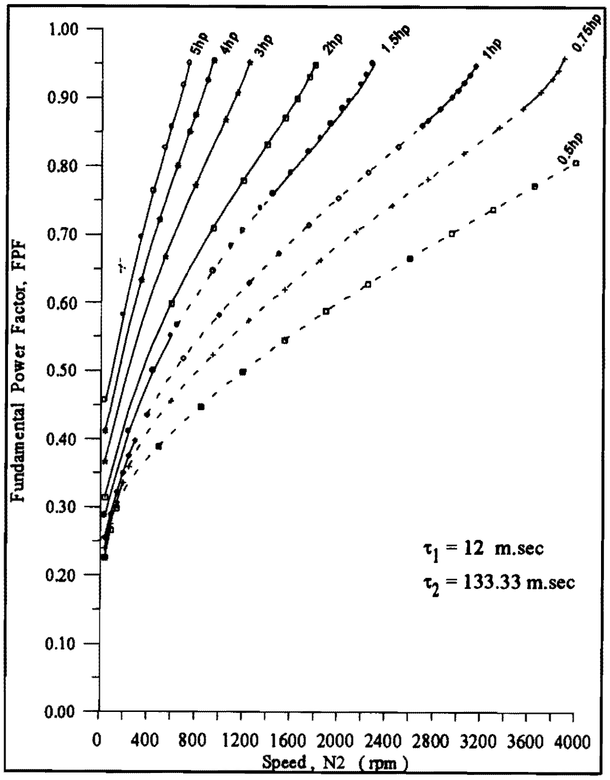

Figure 11. Supply fundamental power factor versus *speed* N2 *of the system operation at different values of horsepower levels of motor 2, at* N*1=2500 rpm.*

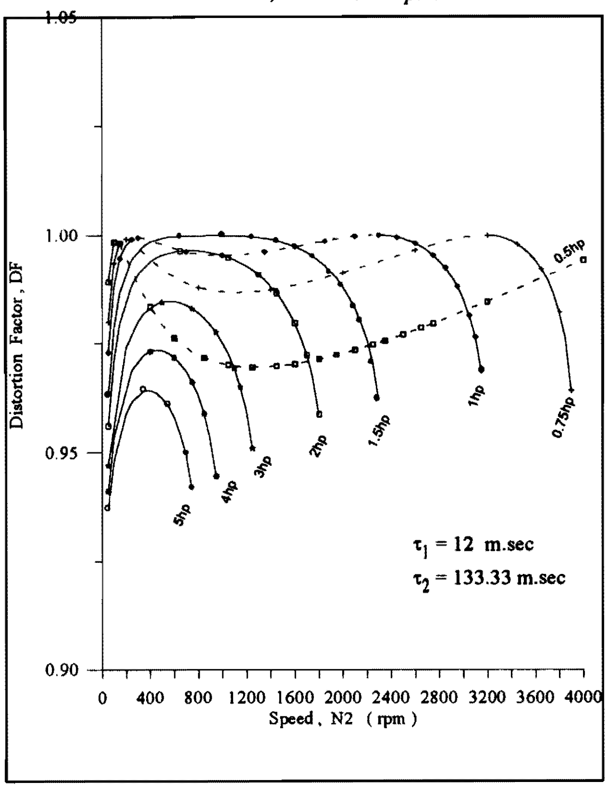

Figure 12. Distortion factor versus *speed* N2 *of the system operation at different values of horsepower levels of motor 2, at* N*1=1500 rpm.*

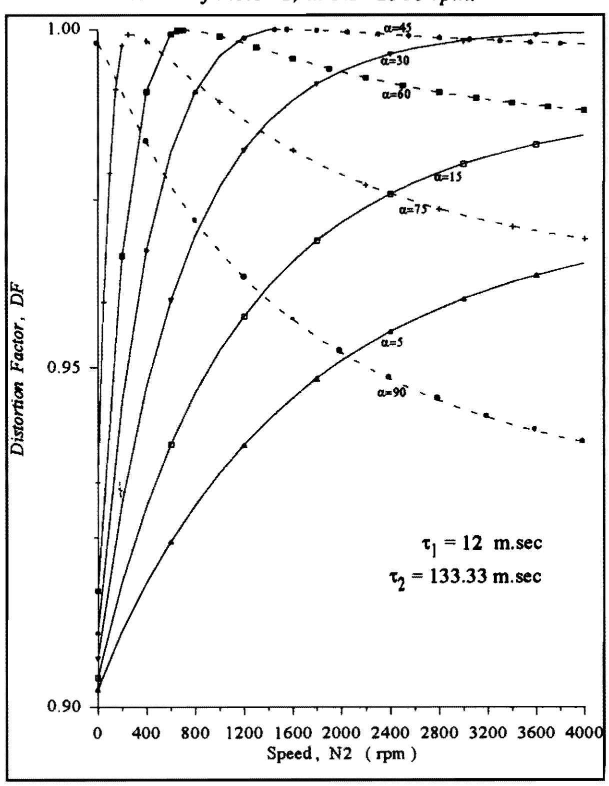

Figure 13. Distortion factor versus *speed* N2 *of the system operation at different values of firing angles, at* N*1=1500 rpm.*

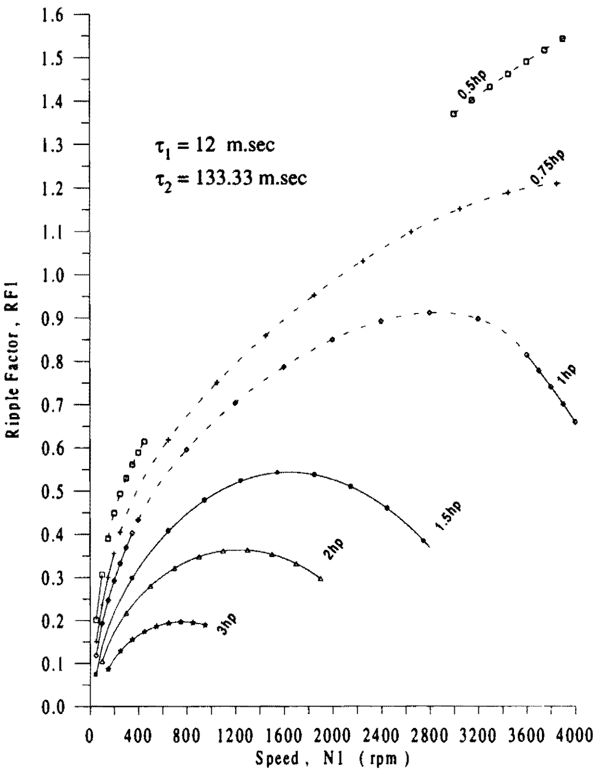

Figure 14. Current ripple factor of motor one versus *speed* N1 *of the system operation at different values of horsepower levels of motor 1, at* N2=*1500 rpm.*

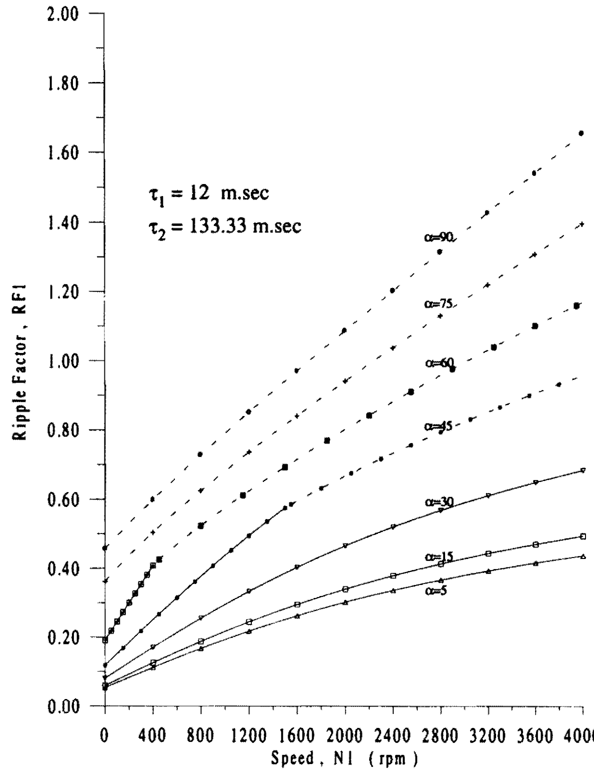

Figure 15. Current ripple factor of motor one versus *speed* N1 *of the system operation at different values of firing angles, at* N2=*1500 rpm.*

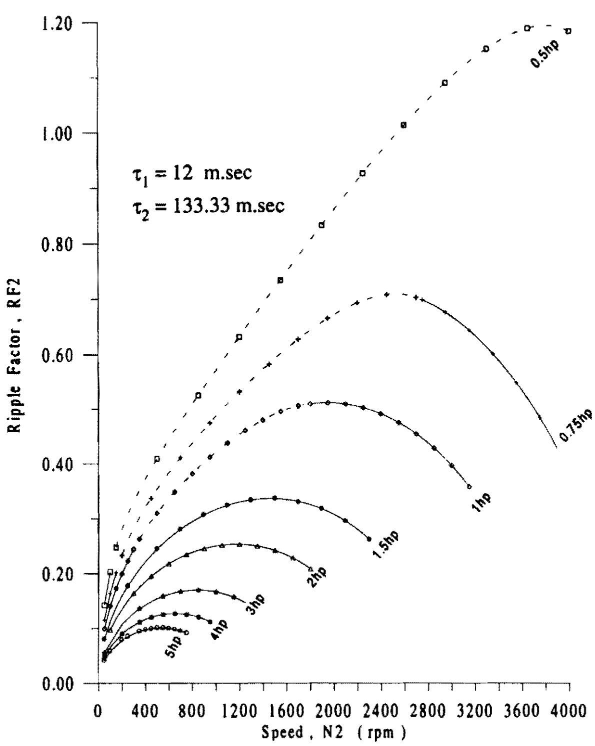

Figure 16. Current ripple factor of motor two versus *speed* N2 *of the system operation at different values of horsepower levels of motor 2, at* N1=*1000 rpm.*

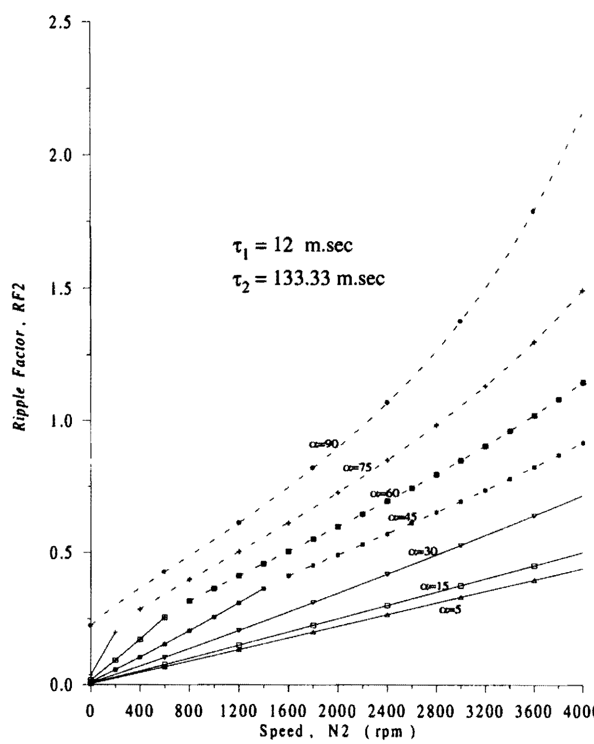

Figure 17. Current ripple factor of motor two versus *speed* N2 *of the system operation at different values of firing angles, at* N1=*1500 rpm.*

4.1.7. Ripple Factor

The variation of the ripple factor of the current of motor one *versus* speed of the same motor for different values of horsepower levels and for different values of firing angle keeping the speed of the other motor unchanged are shown in Figures 14 and 15 respectively. The same relations for the other motor are shown in Figures 16 and 17. It is evident that, the ripple factor increases with the increase of either the firing angle or the speed of any motor. The ripple factor also increases as the load horsepower is decreased.Comparison of Figures 14 and 16 shows that the ripple factor increases as the motor armature circuit time constant, τ, is decreased. ($\tau_1 = L_{a1}/R_{a1}$ = 12m.sec and $\tau_2 = L_{a2}/R_{a2}$ = 133.33m.sec).

5. CONCLUSIONS

In this paper, an AC–DC full-controlled converter supplying two DC series motor loads has been investigated. The steady-state analysis of the system has been derived analytically for each of the two modes of continuous and discontinuous converter current. In each mode, expressions for the converter current and its ripple factor, the input power factor, the motor currents and their ripple factors, the supply current distortion factor, the supply current fundamental power factor, and the torque-speed characteristic for each motor have been derived and studied. Also the critical firing angle at which the mode of operation changes from one mode to the other has been derived and investigated. In view of the analysis and results presented in this paper, the following conclusions have been inferred:

- The critical firing angle decreases as the speed of any motor (or both) is increased.
- The converter current is discontinuous at high values of the firing angle α, high speeds, and low values of torque.
- The supply power factor and the fundamental power factor decreases as the firing angle is increased and as the motor horsepower is decreased provided that the motor speeds are unchanged. These factors increase as the motor speeds are increased provided that the motor horsepower is kept constant.
- For the continuous converter current mode, the distortion factor has a high values at high values of firing angles and low values of the motor horsepower provided that the motor speeds are unchanged. For the same mode, the distortion factor also has high values at high values of the motor speeds provided that the firing angle is kept constant. The opposite conclusions have been obtained for the discontinuous converter current mode. The best values of the distortion factor have been obtained when the firing angles equal the critical firing angles.
- The motor current ripple factor has a low values at low values of firing angles and high values of the motor horsepower provided that the motor speeds are unchanged. It also has low values at low values of motor speeds provided that the firing angle is kept constant. The motor current ripple factor increases as its armature circuit time constant is decreased.

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Paper Received 6 September 1998; Revised 28 June 1999; Accepted 11 October 1999.

APPENDIX

Continuous Mode

The average value of the motor current is obtained from: $I_{c1av} = (1/\pi)\int_{\alpha}^{\pi+\alpha} i_{c1} d\omega t$ and is given by:

$$I_{c1av} = 2\sqrt{2}(V/\pi Z_1)\cos(\alpha-\phi_1) + (K_{c1}Q_1/\pi)(1-e^{(-\pi/Q_1)}) - (K_{res1}N_1/R_1)[1-(Q_1/\pi)(1-e^{(-\pi/Q_1)})] \; ; \qquad \text{(A-1)}$$

and the rms value of the motor current is obtained from: $I_{c1rms} = \sqrt{(1/\pi)\int_{\alpha}^{\pi+\alpha} i_{c1}^2 d\omega t}$ and is given by:

$$I_{c1rms} = [(V^2/Z_1^2) + (K_{c1} + (K_{res1}N_1/R_1))(2\sqrt{2}VQ_1^2/(\pi Z_1(1+Q_1^2))[(1/Q_1)\sin(\alpha-\phi_1) + \cos(\alpha-\phi_1)]$$

$$(1+e^{(-\pi/Q_1)}) - (4\sqrt{2}VK_{res1}N_1/\pi Z_1 R_1)\cos(\alpha-\phi_1) + (Q_1/2\pi)(K_{c1} + (K_{res1}N_1/R_1))^2(1-e^{(-2\pi/Q_1)}) -$$

$$(2Q_1K_{res1}N_1/\pi R_1)(K_{c1} + (K_{res1}N_1/R_1))(1-e^{(-\pi/Q_1)}) + (K_{res1}N_1/R_1)^2]^{1/2}. \qquad \text{(A-2)}$$

Similarly, the instantaneous, average, rms value, and the ripple factor of i_{c2} are given by (4), (A-1), (A-2), and (8), respectively after replacing the subscript 1 by 2.

The rms value of the supply current is obtained as:

$$I_{csrms} = \sqrt{(1/\pi)\int_{\alpha}^{\pi+\alpha} i_{cs}^2 d\omega t} = \sqrt{I_{c1rms}^2 + I_{c2rms}^2 + (1/\pi)\int_{\alpha}^{\pi+\alpha} 2 i_{c1} i_{c2} d\omega t}\,. \qquad \text{(A-3)}$$

It could be shown that rms value of supply current is given by:

$$I_{csrms} = \sqrt{I_{c1rms}^2 + I_{c2rms}^2 + A + B + C + D + E + F + G + H + J}\;, \qquad \text{(A-4)}$$

where:

$A = (2V^2/Z_1Z_2)\cos(\phi_2 - \phi_1)$,

$B = (2Q_1Q_2/\pi(Q_1+Q_2))(K_{c1} + (K_{res1}N_1/R_1))(K_{c2} + (K_{res2}N_2/R_2))(1-e^{(-\pi(Q_1+Q_2)/Q_1Q_2)})$,

$C = (2\sqrt{2}VQ_2^2/\pi Z_1(1+Q_2^2))(K_{c2} + (K_{res2}N_2/R_2))((1/Q_2)\sin(\alpha-\phi_1) + \cos(\alpha-\phi_1))(1+e^{(-\pi/Q_2)})$

$D = (2\sqrt{2}VQ_1^2/\pi Z_2(1+Q_1^2))(K_{c1} + (K_{res1}N_1/R_1))((1/Q_1)\sin(\alpha-\phi_2) + \cos(\alpha-\phi_2))(1+e^{(-\pi/Q_1)})$

$E = (-4\sqrt{2}V/\pi Z_2)(K_{res1}N_1/R_1)\cos(\alpha-\phi_2)$,

$F = (-4\sqrt{2}V/\pi Z_1)(K_{res2}N_2/R_2)\cos(\alpha-\phi_1)$,

$G = (-2Q_1/\pi)(K_{c1}(K_{res2}N_2/R_2) + (K_{res1}N_1/R_1)(K_{res2}N_2/R_2))(1-e^{(-\pi/Q_1)})$,

$H = (-2Q_2/\pi)(K_{c2}(K_{res1}N_1/R_1) + (K_{res1}N_1/R_1)(K_{res2}N_2/R_2))(1-e^{(-\pi/Q_2)})$, and

$J = 2(K_{res1}N_1/R_1)(K_{res2}N_2/R_2)$,

and a_{c1} and b_{c1} are the amplitudes of the cosine and sine fundamental components of i_{cs} respectively and are given by:

$$a_{c1} = (1/\pi)\int_{\alpha}^{2\pi+\alpha} i_{cs}\cos\omega t d\omega t$$

$$-(\sqrt{2}V/Z_1)\sin\phi_1 + (2Q_1^2/(1+Q_1^2)\pi)(K_{c1}+(K_{res1}N/R_1))((1/Q_1)\cos\alpha - \sin\alpha)(1+e^{(-\pi/Q_1)})$$
$$+(4K_{res1}N_1/\pi R_1)\sin\alpha - (\sqrt{2}V/Z_2)\sin\phi_2 + (2Q_2^2/(1+Q_2^2)\pi)(K_{c2}+(K_{res2}N_2/R_2))((1/Q_2)$$
$$\cos\alpha - \sin\alpha)(1+e^{(-\pi/Q_2)}) + (4K_{res2}N_2/\pi R_2)\sin\alpha, \tag{A-5}$$

and by $b_{c1} = (1/\pi)\int_{\alpha}^{2\pi+\alpha} i_{cs}\sin\omega t d\omega t$,

$$(\sqrt{2}V/Z_1)\cos\phi_1 + (2Q_1^2/(1+Q_1^2)\pi)(K_{c1}+(K_{res1}N/R_1))((1/Q_1)\sin\alpha + \cos\alpha)$$
$$(1+e^{(-\pi/Q_1)}) - (4K_{res1}N_1/\pi R_1)\cos\alpha + (\sqrt{2}V/Z_2)\cos\phi_2 + (2Q_2^2/(1+Q_2^2)\pi)(K_{c2}+$$
$$(K_{res2}N_2/R_2))((1/Q_2)\sin\alpha + \cos\alpha)(1+e^{(-\pi/Q_2)}) - (4K_{res2}N_2/\pi R_2)\cos\alpha. \tag{A-6}$$

Discontinuous Mode

The average value of the motor current is given by: $I_{d1av} = (1/\pi)\int_{a}^{\pi+a} i_{dl}\, d\omega t$. Therefore:

$$I_{d1av} = (\sqrt{2}V/Z_1\pi)[\cos(\alpha-\phi_1) - \cos(\beta-\phi_1)] + (Q_1/\pi)(K_{d1}+(K_{res1}N_1/R_1))(1-e^{(-(\beta-\alpha)/Q_1)})+$$
$$(K_{res1}N_1/R_1)((\alpha-\beta)/\pi) + ((K_{res2}N_2 - K_{res1}N_1)/(R_1+R_2))((\pi+\alpha-\beta)/\pi) + (Q/\pi)$$
$$(((K_{res2}N_2 - K_{res1}N_1)/(R_1+R_2)) - I'_{d1})(e^{(-(\pi+\alpha-\beta)/Q)} - 1). \tag{A-7}$$

The rms value of the motor current is found to be: $I_{d1rms} = \sqrt{(1/\pi)\int_{\alpha}^{\pi+\alpha} i_{d1}^2 d\omega t}$. Therefore:

$$I_{d1rms} = \{(2V^2/\pi Z_1^2)[(\beta-\alpha)/2 + [\sin 2(\alpha-\phi_1) - \sin 2(\beta-\phi_1)]/4] + (2\sqrt{2}V/\pi Z_1)(K_{res1}N_1/R_1)$$
$$[\cos(\beta-\phi_1) - \cos(\alpha-\phi_1)] + (2\sqrt{2}V/\pi Z_1)(Q_1^2/(1+Q_1^2))(K_{d1}+(K_{res1}N_1/R_1))[e^{(-(\beta-\alpha)/Q_1)}(-(1/Q_1)$$
$$\sin(\beta-\phi_1) - \cos(\beta-\phi_1)) + ((1/Q_1)\sin(\alpha-\phi_1) + \cos(\alpha-\phi_1))] - (Q_1/2\pi)(K_{d1}+(K_{res1}N_1/R_1))^2$$
$$(e^{(-2(\beta-\alpha)/Q_1)} - 1) + (Q_1/\pi)(2K_{d1}(K_{res1}N_1/R_1) + 2(K_{res1}N_1/R_1)^2)(e^{(-(\beta-\alpha)/Q_1)} - 1) + (K_{res1}N_1/R_1)^2 \tag{A-8}$$
$$(\beta-\alpha)/\pi + ((K_{res2}N_2 - K_{res1}N_1)/(R_1+R_2))^2((\pi+\alpha-\beta)/\pi) + (2Q/\pi)[((K_{res2}N_2 - K_{res1}N_1)$$
$$/(R_1+R_2))^2 - I'_{d1}((K_{res2}N_2 - K_{res1}N_1)/(R_1+R_2))](e^{(-(\pi+\alpha-\beta)/Q)} - 1) - (Q/2\pi)(((K_{res2}N_2 - K_{res1}N_1)$$
$$/(R_1+R_2)) - I'_{d1})^2(e^{(-2(\pi+\alpha-\beta)/Q)} - 1)\}^{1/2}.$$

Similarly, the instantaneous, average, rms values, and the ripple factor of i_{d2} are given by (16), (19), (A-7), (A-8), and (24), respectively with the exception that the subscripts 1 and 2 are interchanged.

The value of the supply rms current is obtained from: $I_{dsrms} = \sqrt{(1/\pi)\int_{\alpha}^{\beta} i_{ds}^2 d\omega t}$. (A-9)

Thus: $I_{dsrms} = \sqrt{(1/\pi)\int_{\alpha}^{\beta} i_{d1}^2 d\omega t + (1/\pi)\int_{\alpha}^{\beta} i_{d2}^2 d\omega t + (1/\pi)\int_{\alpha}^{\beta} 2 i_{d1} i_{d2} d\omega t}$. (A-10)

Then rms value of the supply current is found to be:

$$I_{dsrms}=\{(2V^2/\pi Z_1^2)[(\beta-\alpha)/2+[\sin 2(\alpha-\phi_1)-\sin 2(\beta-\phi_1)]/4]+(2\sqrt{2}V/\pi Z_1)(K_{res1}N_1/R_1)$$
$$[\cos(\beta-\phi_1)-\cos(\alpha-\phi_1)]+(2\sqrt{2}V/\pi Z_1)(Q_1^2/(1+Q_1^2))(K_{d1}+(K_{res1}N_1/R_1))[e^{(-(\beta-\alpha)/Q_1)}(-(1/Q_1)$$
$$\sin(\beta-\phi_1)-\cos(\beta-\phi_1))+((1/Q_1)\sin(\alpha-\phi_1)+\cos(\alpha-\phi_1))]-(Q_1/2\pi)(K_{d1}+(K_{res1}N_1/R_1))^2$$
$$(e^{(-2(\beta-\alpha)/Q_1)}-1)+(Q_1/\pi)(2K_{d1}(K_{res1}N_1/R_1)+2(K_{res1}N_1/R_1)^2)(e^{(-(\beta-\alpha)/Q_1)}-1)+(K_{res1}N_1/R_1)^2$$
$$(\beta-\alpha)/\pi+(2V^2/\pi Z_2^2)[(\beta-\alpha)/2+[\sin 2(\alpha-\phi_2)-\sin 2(\beta-\phi_2)]/4]+(2\sqrt{2}V/\pi Z_2)(K_{res2}N_2$$
$$/R_2)[\cos(\beta-\phi_2)-\cos(\alpha-\phi_2)]+(2\sqrt{2}V/\pi Z_2)(Q_2^2/(1+Q_2^2))(K_{d2}+(K_{res2}N_2/R_2))[e^{(-(\beta-\alpha)/Q_2)}$$
$$(-(1/Q_2)\sin(\beta-\phi_2)-\cos(\beta-\phi_2))+((1/Q_2)\sin(\alpha-\phi_2)+\cos(\alpha-\phi_2))]-(Q_2/2\pi)$$
$$(K_{d2}+(K_{res2}N_2/R_2))^2(e^{(-2(\beta-\alpha)/Q_2)}-1)+(Q_2/\pi)(2K_{d2}(K_{res2}N_2/R_2)+2(K_{res2}N_2/R_2)^2)$$
$$(e^{(-(\beta-\alpha)/Q_2)}-1)+(K_{res2}N_2/R_2)^2(\beta-\alpha)/\pi+(2V^2/\pi Z_1Z_2)\cos(\phi_2-\phi_1)(\beta-\alpha)-(V^2/\pi Z_1Z_2)$$
$$[\sin(2\beta-\phi_2-\phi_1)-\sin(2\alpha-\phi_2-\phi_1)]+(2\sqrt{2}V/\pi Z_2)(K_{res1}N_1/R_1)[\cos(\beta-\phi_2)-\cos(\alpha-\phi_2)]$$
$$+(2\sqrt{2}V/\pi Z_1)(K_{res2}N_2/R_2)[\cos(\beta-\phi_1)-\cos(\alpha-\phi_1)]+(2\sqrt{2}V/\pi Z_2)(Q_1^2/(1+Q_1^2))$$
$$(K_{d1}+(K_{res1}N_1/R_1))[e^{(-(\beta-\alpha)/Q_1)}(-(1/Q_1)\sin(\beta-\phi_2)-\cos(\beta-\phi_2))+((1/Q_1)\sin(\alpha-\phi_2)+$$
$$\cos(\alpha-\phi_2))]+(2\sqrt{2}V/\pi Z_1)(Q_2^2/(1+Q_2^2))(K_{d2}+(K_{res2}N_2/R_2))[e^{(-(\beta-\alpha)/Q_2)}(-(1/Q_2)\sin(\beta-\phi_1)$$
$$-\cos(\beta-\phi_1))+((1/Q_2)\sin(\alpha-\phi_1)+\cos(\alpha-\phi_1))]+(2Q_1Q_2/\pi(Q_1+Q_2))(K_{d1}+(K_{res1}N_1/R_1))$$
$$(K_{d2}+(K_{res2}N_2/R_2))(1-e^{(-(Q_1+Q_2)(\beta-\alpha)/Q_1Q_2)})+(2Q_2/\pi)(K_{res1}N_1/R_1)(K_{d2}+(K_{res2}N_2/R_2)) \quad \text{(A-11)}$$
$$(e^{(-(\beta-\alpha)/Q_2)}-1)+(2Q_1/\pi)(K_{res2}N_2/R_2)(K_{d1}+(K_{res1}N_1/R_1))(e^{(-(\beta-\alpha)/Q_1)}-1)+(2/\pi)$$
$$(K_{res1}N_1/R_1)(K_{res2}N_2/R_2)(\beta-\alpha)\}^{1/2},$$

and a_{d1} and b_{d1} are the amplitudes of the cosine and sine fundamental components of i_{ds} respectively and are given by:

$$a_{d1}=(\sqrt{2}V/2\pi Z_1)[\cos(2\alpha-\phi_1)-\cos(2\beta-\phi_1)-2(\beta-\alpha)\sin\phi_1]+2(Q_1^2/(\pi(1+Q_1^2)))(K_{d1}+$$
$$(K_{res1}N_1/R_1))[e^{(-(\beta-\alpha)/Q_1)}(-(1/Q_1)\cos\beta+\sin\beta)+((1/Q_1)\cos\alpha-\sin\alpha)]-(2K_{res1}N_1/\pi R_1))$$
$$(\sin\beta-\sin\alpha)+(\sqrt{2}V/2\pi Z_2)[\cos(2\alpha-\phi_2)-\cos(2\beta-\phi_2)-2(\beta-\alpha)\sin\phi_2]+2(Q_2^2/(\pi(1+ \quad \text{(A-12)}$$
$$Q_2^2)))(K_{d2}+(K_{res2}N_2/R_2))[e^{(-(\beta-\alpha)/Q_2)}(-(1/Q_2)\cos\beta+\sin\beta)+((1/Q_2)\cos\alpha-\sin\alpha)]-$$
$$(2K_{res2}N_2/\pi R_2))(\sin\beta-\sin\alpha), \text{ and}$$

$$b_{d1}=(\sqrt{2}V/2\pi Z_1)[\sin(2\alpha-\phi_1)-\sin(2\beta-\phi_1)+2(\beta-\alpha)\cos\phi_1]+2(Q_1^2/(\pi(1+Q_1^2)))(K_{d1}+$$
$$(K_{res1}N_1/R_1))[e^{(-(\beta-\alpha)/Q_1)}(-(1/Q_1)\sin\beta-\cos\beta)+((1/Q_1)\sin\alpha+\cos\alpha)]+(2K_{res1}N_1/\pi R_1))$$
$$(\cos\beta-\cos\alpha)+(\sqrt{2}V/2\pi Z_2)[\sin(2\alpha-\phi_2)-\sin(2\beta-\phi_2)+2(\beta-\alpha)\cos\phi_2]+2(Q_2^2/(\pi(1+ \quad \text{(A-13)}$$
$$Q_2^2)))(K_{d2}+(K_{res2}N_2/R_2))[e^{(-(\beta-\alpha)/Q_2)}(-(1/Q_2)\sin\beta-\cos\beta)+((1/Q_2)\sin\alpha+\cos\alpha)]+$$
$$(2K_{res2}N_2/\pi R_2))(\cos\beta-\cos\alpha).$$