ROBUST MODEL-BASED CONTROL OF OPEN-LOOP UNSTABLE PROCESSES

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الخلاصـة :

يتناول هذا البحث تطوير طرق لتقدير الاضطرابات غير المحسوبة وكذلك أخطاء النمذجة لاستخدامها في عمليات التنبؤ ضمن خوارزمية التحكم التنبؤي المبنى على النمذجة الرياضية. وعلى التحديد يتناول البحث تطوير طريقة خاصة بالتحكم المبنى على النماذج الرياضية الخطية وأخرى خاصة بالتحكم المبنى على النماذج الرياضية غير الخطية. والهدف من استخدام تلك الطرق هو تحسين أداء التحكم التنبؤي عند تطبيقه على عمليات كيميائية تصنيعية غير مستقرة وتحت تأثير أخطاء نمذجية أو اضطرابات غير متوقعة. اعتمدت الطريقة في حالة النماذج غير الخطية على تقدير الاضطرابات عن طريق حل مسألة محاكاة متواصلة محدودة الحجم. أما في حالة النماذج الخطية فاعتمدت الطريقة على استخدام نموذج رياضي مفترض للاضطرابات. يتم إعداد وتكوين معاملات النموذج المفترض من المعلومات المستنبطة مباشرة من العمليات التصهيعية. وقد تم اختبار كفاءة الطرق المقترحة بتطبيقها عن طريق المحاكاة على مفاعل المهد المميع ذي ديناميكية غير مستقرة. أبرزت نتائج المحاكاة قدرة الطرق المقترحة على تحقيق استقرارية النظام وبالتالي تحسين أداء التحكم التنبؤي.

ABSTRACT

This paper addresses the development of new formulations for estimating modeling errors or unmeasured disturbances to be used in Model Predictive Control (MPC) algorithms during open-loop prediction. Two different formulations were developed in this paper. One is used in MPC that directly utilizes linear models and the other in MPC that utilizes non-linear models. These estimation techniques were utilized to provide robust performance for MPC algorithms when the plant is open-loop unstable and under the influence of modeling error and/or unmeasured disturbances. For MPC that utilizes a non-linear model, the estimation technique is formulated as a fixed small size on-line optimization problem, while for linear MPC, the unmeasured disturbances are estimated *via* a proposed linear disturbance model. The disturbance model coefficients are identified on-line from historical estimates of plant-model mismatch. The effectiveness of incorporating these proposed estimation techniques into MPC is tested through simulated implementation on non-linear unstable exothermic fluidized bed reactor. Closed-loop simulations proved the capability of the proposed estimation methods to stabilize and, thereby, improve the MPC performance in such cases.

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1. INTRODUCTION

A large family of Model Predictive Control (MPC) algorithms has appeared in the literature during the last fifteen years. Reviews of linear and non-linear MPC algorithms can be found in the literature [1, 2]. MPC has also received a lot of interest due to its successful simulation tests and industrial implementations [3]. To account for model–plant mismatch, the standard MPC algorithms incorporate a constant disturbance estimate to the open-loop prediction, which performs as an integral action, to eliminate steady state offset. However, for open-loop unstable processes or for processes under disturbance of slow drifts, this integral action fails and the feedback performance may deteriorate. Due to the difficulty of obtaining a theoretical analysis for robustness, researchers revert to incorporating process estimation techniques for improving the MPC performance in such situations.

Some researchers have improved their MPC algorithms by using parameters, and/or state estimation, *via* non-linear programming [4–9]. Recently, Robertson *et al.* [10] have modified the above optimization-based estimation methods to a fixed-size constrained least squares problem. One drawback of these methods is the extensive computational demand incurred due to the size of the optimization problem. Moreover, it is sometimes difficult to determine the number and the range of values of the unknown parameters to be estimated.

An alternative way to compensate for the impact of model uncertainty is by augmenting the controller with state estimation by Kalman filter as proposed by Ricker [11]. This approach was further extended by Gattu and Zafiriou [12, 13] to nonlinear quadratic Dynamic Matrix (NLQDMC) control for disturbance rejection of open-loop unstable processes. Similarly, Ali and Zafiriou [14] incorporated extended Kalman filtering in their non-linear MPC algorithm to perform on-line state estimation. Although Kalman filter is easy to use and implement and, thus, computationally undemanding, it requires special assumption of the disturbance structure. Moreover, the solution of the Lyabunov equation, which is required to compute the steady state Kalman filter gain, requires the dynamic matrix of the linearized model to be stable [15], or the pair (C,A) of the traditional linear state space model to be detectable. However, this is not always true as reported by Ali and Elnashaie [16].

The main objective of this paper is to construct new, simple, and easy to implement estimation techniques to improve the performance of LMPC and NLMPC when applied to open-loop unstable processes. Specifically, the objective is two-fold. One purpose is to modify the state estimation by Extended Kalman Filter (EKF) used by Ali and Zafiriou [14] in such a way as to avoid the limitation associated with model characteristics, *i.e.*, loss of detectability, without increasing the computational load. This will be achieved by estimating the EKF gain *via* solving a small size optimization problem. The size of the optimization problem is fixed by setting the number of design parameters equal to the number of measured outputs. No knowledge is required for the end-limits of these design parameters since they are basically constrained between zero and one. The second purpose is to develop a new method for disturbance prediction for linear MPC. The prediction is based on linear model representation of the disturbance over the prediction horizon. The coefficients of this model are identified on-line from the previous estimates of the disturbances. This is achieved by solving a constrained least squares problem of a small fixed size. The size of these coefficient is left as design parameters. However, the accuracy of the prediction model is not very sensitive to the size of the coefficients. Moreover, no knowledge of the end-limits of these coefficients need to be known *a priori*. A similar approach is proposed by Sistu and Bequett [17]. However, their method has tuning parameters of size equal to the number of measured outputs which turned out to affect the performance of the predictor. In addition, their method may lead to unstable prediction since the coefficients of the predictor model are unconstrained.

2. LINEAR MPC ALGORITHM

Our disturbance estimation strategy is developed for the well established linear Model Predictive Controllers (MPC) based on FIR models. A usual MPC formulation (notation follow that in Lee *et al.* [18]), solves the following on-line optimization function:

$$\min_{\Delta u(k_1),\ldots,\Delta u(k+M-1)} \| \Gamma(Y_P(k+1) - r(k+1) \|^2 + \| \Lambda\, U(k) \|^2 \tag{1}$$

subject to:

$$Y_P(k+1) = M_P Y(k) + S_P^m \Delta U(k) \tag{2}$$

$$F^T \Delta U(k) \le b\,. \tag{3}$$

The symbol || . || denotes the Euclidean vector norm and k denotes the current sampling point. Γ and Λ are diagonal weight matrices. $R(k+1) = [r(k+1) \ldots r(k+P)]^T$ contains the desired output trajectories over horizon P. $\Delta U(k) = [\Delta u(k) \ldots \Delta u(k+M-1)]^T$ is a vector of M future changes of the manipulated variable vector u that are to be determined by the on-line optimization. $Y_p(k+1) = [y(k+1) \ldots y(k+P)]^T$ includes the predicted outputs over the future horizon P, where y is the output vector, assuming $\Delta u(k+i) = 0;\ i \ge M$. $Y(k) = [y(k) \ldots y(k+n-1)]^T$ includes the predicted outputs over the truncation horizon n (length of FIR) based on $\Delta u(k+i) = 0;\ i \ge 0$. Equation (2) represents the output prediction based on the process model. A disturbance estimate denoted d should also be added in (2), or alternatively it can be absorbed in $R(k+1)$. The latter is assumed for simplicity. In the implementation of standard MPC, the disturbance is assumed constant over the prediction horizon, and set equal to the difference between plant and model outputs at present time k. The matrices M_p and S_P^m are defined as in Lee *et al.* [18]. M_p is a constant matrix consisting of ones and zeros and S_P^m is what is usually referred to as the Dynamic Matrix of step response coefficients. The above objective function (1) is minimized subject to constraints on the manipulated variables and on the change of manipulated variables represented by (3).

2.1. Proposed Estimation Method

The assumption of a constant (step-like) disturbance may not always be a valid one as well be demonstrate later by an example. Thus, to develop a more realistic time-varying disturbance prediction, the following time series model for disturbance prediction:

$$d(k) = \theta_1 d(k-1) + \theta_2 d(k-2) + \ \ldots\ + \theta_{nd} d(k-nd)$$

or in a vector form:

$$d(k) = \bar{\theta}^T \bar{d}(k-1)$$

will be used in this paper. θ''s are the disturbance model coefficients to be identified on-line from actual model–plant mismatch estimates and nd is the size of the disturbance model which is a parameter to be defined by the user; $d(k)$ is, as defined before, the difference between the plant measurement and the model prediction at sampling instant k. The model coefficients can be determined directly by linear regression from historical data for the mismatch d, or by on-line recursive linear regression each time a new estimate of d is made available. However, in these two cases, constraints on θ''s cannot be imposed directly to ensure internal stability of the disturbance predictor. Alternatively, θ's can be determined by solving the following constrained least squares problem:

$$\min_{\bar{\theta}} \left\| d(k) - \bar{\theta}^T \bar{d}(k-1) \right\|^2 \tag{4}$$

subject to:

$$|\theta_{nd}| \le |\theta_{nd-1}| \le \ldots \le |\theta_1| \le 1 \tag{5}$$

every sampling time. At each sampling instant, and after θ's have been updated, the disturbance model can be used to predict the future disturbance estimates *via* the following recursive equation:

$$d(k+l/k) = \sum_{i=1}^{nd} \theta_i d(k+l-i) \tag{6}$$

where the notation $d(k+l/k)$ denotes the prediction at time $k+l$ based on the actual measurement up to time k, and the forecast estimates of the disturbances $d(k+l-i)$ are defined as follows:

$$d(k+l-i) = \begin{cases} \text{actual estimates of disturbance} & \text{if} \quad l-i \le 0 \\ \text{predicted disturbance based on measurement up to } k & \text{if} \quad l-i > 0 \end{cases}.$$

The predicted disturbance estimates are then added to the open-loop output prediction to minimize the model–plant mismatch and consequently enhance the LMPC performance. The above procedure of updating the coefficients of the disturbance model, forecasting the disturbance estimates over the prediction horizon, and correcting the open-loop prediction are repeated each sampling time. These are done before solving the on-line optimization problem given by Equation (1). The benefit is, of course, to promote the LMPC performance when applied to open-loop unstable processes or processes affected by persisting load changes. The LMPC and its proposed estimation technique are developed and tested using MATLAB Software.

3. NON-LINEAR MPC ALGORITHM

In this section the structure of the MPC version developed by Ali and Zafiriou [14], that utilizes directly the nonlinear model for output prediction is presented. Incorporating an optimization technique to estimate the modeling error on-line is also addressed. A usual MPC formulation solves the following on-line optimization:

$$\min_{\Delta u(t_k),\ldots,\Delta u(t_{k+M-1})} \sum_{i=1}^{P} \left\| \Gamma(y(t_{k+i}) - r(t_{k+i}) \right\|^2 + \sum_{i=1}^{M} \left\| \Lambda\, u(t_{k+i-1}) \right\|^2 \tag{7}$$

subject to:

$$F^T \Delta U(t_k) \le b \tag{8}$$

For nonlinear MPC, the predicted output, y, over the prediction horizon, P, is obtained by the numerical integration of:

$$\frac{dx}{dt} = f(x,u,t) \tag{9}$$

$$y = g(x) \tag{10}$$

from t_k up to t_{k+P}, where x and y represent the states and the output of the model respectively. It should be noted that the above objective function is the same as the one used in the LMPC formulation. The notation, however, is slightly different. This is only done for the sake of consistency with the standard formulation presented in the literature. The symbols $\|\,.\,\|$, k, Γ and Λ are defined as before. r is the desired output trajectory. $\Delta U(t_k) = [\Delta u(t_k) \ldots \Delta u(t_{k+M-1})]^T$ is a vector of M future changes of the manipulated variable vector u that are to be determined by the on-line optimization.

A disturbance estimate should also be added to y in Equation (7) or, alternatively, it can be absorbed in $r(t_{k+1})$ as it was assumed in the linear MPC case. In the standard MPC implementation, the disturbance is assumed constant over the prediction horizon, and set equal to the difference between plant and model outputs at present time k. The function of the "additive" constant disturbance in the model prediction is to introduce integral action and thus removes steady state offset in the presence of model uncertainty or unmeasured disturbances. However, for open-loop unstable systems and/or slow dynamic processes, the model and the actual states may diverge away from each other and, thus, degrade the regulatory performance of MPC. For this reason, the NLMPC formulation of Ali and Zafiriou [14] also incorporates Kalman filtering to perform on-line state estimation. The purpose is to reset observer states in the presence of modeling error or unmeasured disturbances. Detailed information on incorporating Kalman filter with the NLMPC algorithm is given by Ali and Zafiriou [14]. However, the solution of the Lyapunov equation, which is required to compute the steady state Kalman filter gain, requires that the dynamic matrix A of the linearized model to be stable [15]. Unfortunately, this is not the case in this specific example.

3.1. Modified State Estimation

In the present case, with the characteristics mentioned in the previous section, the state estimation technique is modified in a way that avoids the aforementioned problem. One way to deal with this situation is to introduce completely arbitrary disturbance, say ϕ, in the right hand side of Equation (9) and then try to estimate it on-line. However, since ϕ is arbitrary, it is not easy to determine its end limits. Therefore, alternatively the unmeasured disturbances are estimated in a way which is analogous to the correction term computed by the Kalman filter (Ali and Zafiriou [14]), *i.e.*, the description of the unmeasured disturbance should be written as follows:

$$d = \phi[y_p(t_k) - g(x(t_k))] \tag{11}$$

where the unknown parameter ϕ represents the steady state Kalman gain for continuous system and y_p is the output measurement. ϕ, a diagonal matrix of size $n_y \times n_y$, can be then determined on-line by solving the following objective function:

$$\min_{\phi_1 \cdots \phi_{ny}} \left\| y_p(t_k) - y(t_k) \right\|^2 \tag{12}$$

subject to:

$$0 \le \phi_i \le 1; \qquad i = 1, \ldots, n_y \tag{13}$$

where ϕ_i is the diagonal element of ϕ, n_y the number of measured outputs, and y is obtained by numerically integrating the states Equations (9) and (10) augmented by the estimated disturbance. The augmented states are written as follows:

$$\frac{dx_{um}}{dt} = f_{um}(x_{um}, x_{ms}, u) \tag{14}$$

$$\frac{dx_{ms}}{dt} = f_{ms}(x_{um}, x_{ms}, u) + d \tag{15}$$

$$\frac{dx}{dt} = \left[\frac{dx_{um}}{dt} \quad \frac{dx_{ms}}{dt} \right]^T \tag{16}$$

$$y = g(x) \tag{17}$$

x_{ms} denotes the measured states and x_{um} denotes the unmeasured states. Diagonal ϕ is used to reduce the number of parameters to be estimated. Although the trivial simplification is not optimal, it provides a simple treatment against unmeasured disturbances and modeling errors. The mechanism of solving the MPC control problem with modified state estimation is explained by the following algorithm:

At any sampling time k, given the plant measurement $y_p(t_k)$, the model output $y(t_k)$, and the past input and model states $u(t_{k-1})$ and $x(t_{k-1})$:

Step 1: Obtain the optimum value of ϕ by solving the optimization problem (12) which involves the following steps:

Step 1.1: For a given value of ϕ, which is determined by the optimization software, compute d from the available values of the measured and model outputs at sampling time k using Equation (11).

Step 1.2: Integrate Equation (16) numerically from t_{k-1}, using $u(t_{k-1})$ and $x(t_{k-1})$, up to t_k to obtain a new value of $x(t_k)$.

Step 1.3: Determine the model output $y(t_k)$ using Equation (17).

Step 1.4: If the objective function (12) satisfies the optimality tolerance set by the optimization software, go to Step 2. Otherwise go back to step 1.1.

Step 2: Using the obtained optimum value of ϕ and the current value of d solve the optimization problem (7), which involves the following steps:

Step 2.1: For a specific M future values of manipulated variables set by the optimization software, integrate the state Equation (16) numerically over the prediction horizon P.

Step 2.2: Obtain the P future predicted outputs using Equation (17). Correct the predictions by adding d to them.

Step 2.3: If the objective function (7) does not satisfy the optimality tolerance set by the optimization software then go to step 2.1. Otherwise go to step 3.

Step 3: Implement the obtained optimal manipulated variable for one sampling point. Set $k = k+1$ and go back to step 1.

Correcting the predicted outputs by adding d in step 2.2 follows the internal model control approach to account for the unmeasured disturbances [8]. It should be noted that, in this present work, numerical solution of the differential equations is carried out using DASSL software package [19], and the optimization problems are solved using DNCONF routine provided by the IMSL library.

4. TEST PROCESS FORMULATION

The test process is specifically chosen to present a situation where the standard integral action of LMPC and the EKF of the NLMPC perform poorly. Thus, it serves as a good example to test our proposed estimation methods. The test problem investigated in this paper is that of the catalytic exothermic reaction network $A \rightarrow B \rightarrow C$ taking place in a freely-bubbling fluidized-bed reactor. A schematic diagram of the unit is shown in Figure 1. The desired product is the intermediate component B. In many cases the maximum yield of the product B corresponds to the unstable steady state (saddle type). A number of these situations occur, for example, in the petroleum refining industry where industrial fluid catalytic cracking (FCC) units are to be operated at the middle unstable steady state that gives maximum gasoline yield [20, 21]. Other examples of the occurrence of this situation in petrochemical reactions are partial oxidation of *o*-xylene to

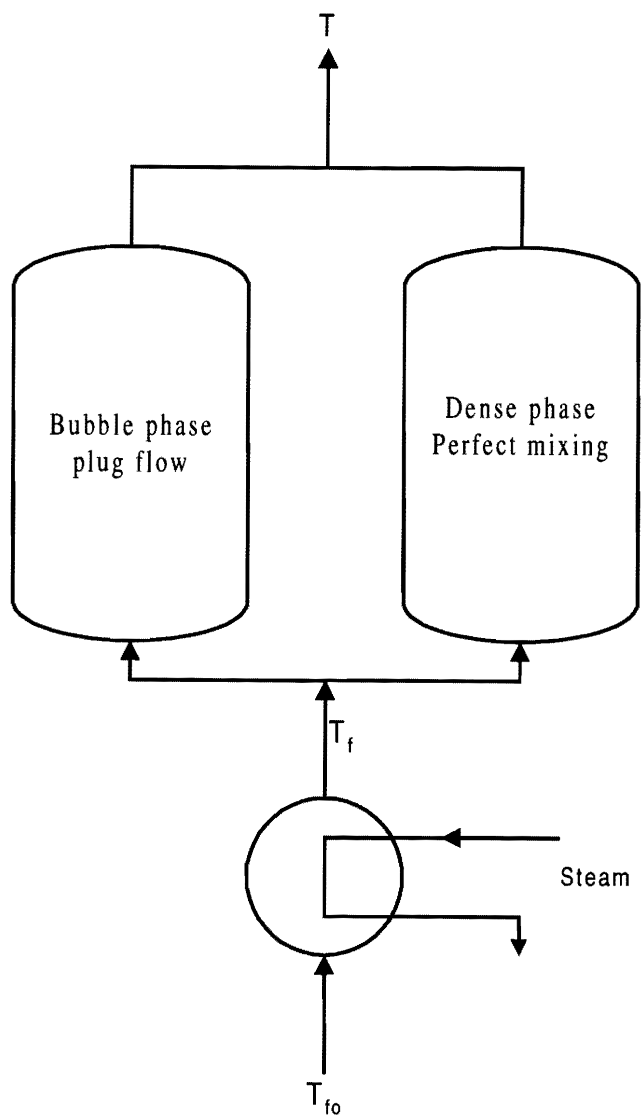

Figure 1. Schematic diagram for the two-phase fluidized bed reactor.

phthalic anhydride [22], and the oxidative dehydrogenation of butane to butadiene [23] and ethylbenzene to styrene [24]. In order to operate the reactor at this unstable steady state a robust feedback control system is needed.

The differential equations for the dense phase material and energy balances are written in dimensionless form as given by the following three non-linear equations [25]:

$$\frac{1}{Le_A}\frac{dX_A}{dt} = \bar{\beta}(X_{Af} - X_A) - \alpha_1 \exp\left(\frac{-\gamma_1}{T}\right)X_A \tag{18}$$

$$\frac{1}{Le_B}\frac{dX_B}{dt} = \bar{\beta}(X_{Bf} - X_B) + \alpha_1 \exp\left(\frac{-\gamma_1}{T}\right)X_A - \alpha_2 \exp\left(\frac{-\gamma_2}{T}\right)X_B \tag{19}$$

$$\frac{dT}{dt} = \bar{\beta}(T_f - T) + \alpha_1\beta_1 \exp\left(\frac{-\gamma_1}{T}\right)X_A + \alpha_2\beta_2 \exp\left(\frac{-\gamma_2}{T}\right)X_B\ . \tag{20}$$

The first two equations represent the mass balance of the components A and B in the dense phase of the reactor, while the third equation represents the energy balance. Here X_A and X_B are the dimensionless concentrations of the reactant A and product B, T is the dimensionless dense phase temperature and T_f is the dimensionless feed temperature.

The model parameters along the data used in this model are given in Table 1. The fluidized bed two-phase model parameters can be computed from many procedures available in the literature. The simplest procedure due to Patridge and Rowe [26] has been used after introducing into it the modifications suggested by Chavarie and Grace [27] regarding the gas flow in the dense phase. However any two-phase model other than the Patridge and Row can be used instead. Details of these calculations are given elsewhere [28]. The parameters Le_A and Le_B are lewis numbers. They represent the ratio between the heat capacity of the system and the mass capacities for components A and B respectively. The model equations may then be written in the continuous-time state space form [Equations (9) and (10)].

5. SIMULATION RESULTS

The control objective in this paper is to startup the reactor from an initial steady state of no reaction to a desirable steady state of maximum yield. The desirable steady state corresponds to an open-loop unstable operating point as mentioned in Section 4 and was found by steady state analysis. This will make the control problem even more challenging and more suitable for testing the proposed estimation techniques. Since the composition of the product

Table 1. Data Used for the Fluidized Bed Catalytic Reactor.

Normalized pre-exponent factor for the reaction $A \rightarrow B$, α_1	10^8
Normalized pre-exponent factor for the reaction $B \rightarrow C$, α_2	10^{11}
Dimensionless overall exothermicity factor for the reaction $A \rightarrow B$, β_1	0.4
Dimensionless overall exothermicity factor for the reaction $B \rightarrow C$, β_2	0.6
Dimensionless activation energy for the reaction $A \rightarrow B$, γ_1	18.0
Dimensionless activation energy for the reaction $B \rightarrow C$, γ_2	27.0
Lewis number of component A, Le_A	1.0
Lewis number of component B, Le_B	0.4545
Feed concentration of Component A, X_{Af}	1.0
Feed concentration of Component B, X_{Bf}	0.0
Dimensionless feed temperature to the reactor, (base value) T_{fo}	0.55342
Reciprocal of the effective residence time of the bed, $\bar{\beta}$	0.12543

cannot be measured easily in practise, the reactor temperature has been used as the controlled variable instead. The set point of the feed temperature is used as the manipulated variable assuming that the feed temperature is controlled by a lower-level fast loop where the steam flow rate is the manipulated variable. The feed temperature is constrained between 0.4 and 1.4 in dimensionless units. Table 2 lists the values of the state variables at the initial and the desired steady states points.

In order not to diverge from our aim, no different control objectives will be tested. Rather we will adhere to a specific situation where the MPC algorithms fail to perform well and investigate their improvement by the proposed estimation methods.

5.1. Using Nonlinear MPC

Perfect Model Case

Simulation of NLMPC for a set point change of 0.376 in the reactor temperature using sampling time, $T_k = 0.2$, and prediction horizon, $P = 1$ is shown in Figure 2. The other NLMPC tuning parameters values are $M = 1$, $\Gamma = [1]$, and

Table 2. Steady State Values of the States.

State	Initial	Desired
x_1	1.0	0.243
x_2	0.0	0.663
x_3	0.5543	0.995

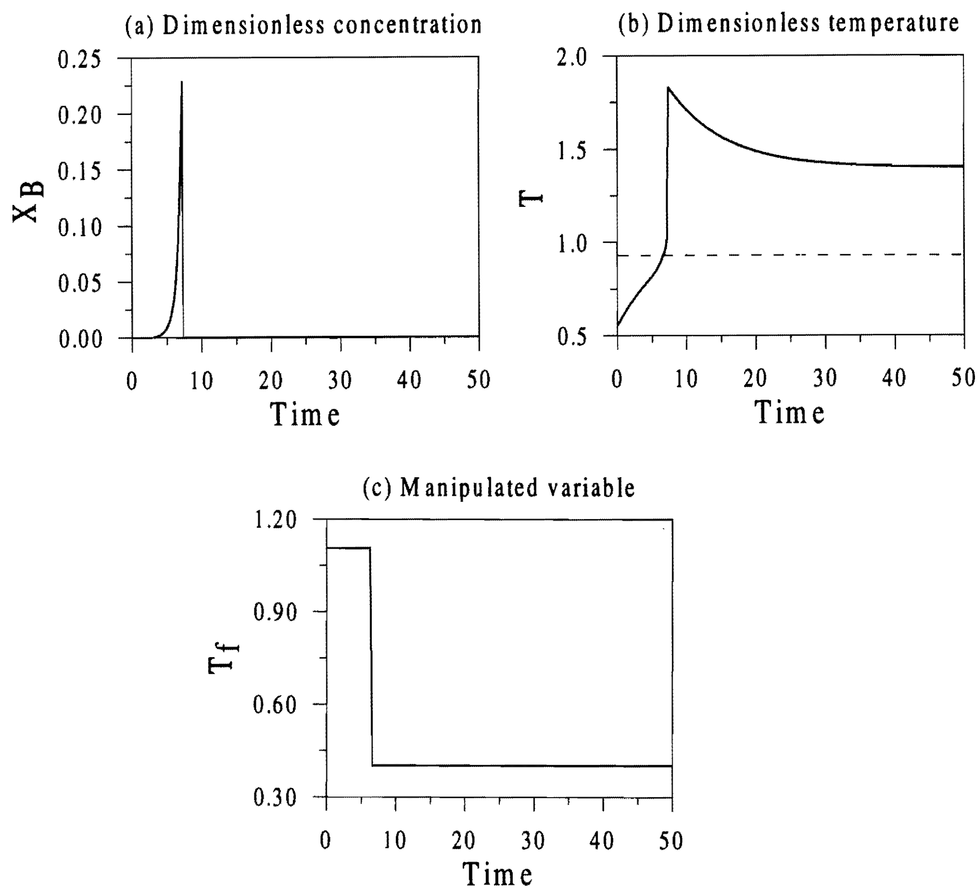

Figure 2. Set point change response using nonlinear MPC with perfect model, $M = 1$, $P = 1$, $\Gamma = 1$, $\Lambda = 0$.

$\Lambda = [1]$. The figure also shows the feedback response for the intermediate product concentration X_B and the manipulated variable T_f (feed temperature). Clearly, as the temperature increases towards the optimal point, the concentration of B also increases as expected. However, as the reactor temperature reaches the unstable region, it drifts away to another higher steady state value, a situation which favors the second reaction resulting in complete consumption (*i.e.*, burn out) of the intermediate product. Nevertheless, the feedback performance can be improved by proper tuning. Specifically, it has been found that a value of $P = 10$ in this case, was able to stabilize the feedback response as shown in Figure 3. It is clear from the figure that, as expected, the response of X_B does reach its maximum value by maintaining the reactor temperature at its corresponding desired set point.

Imperfect Model Case

Simulation of the same control objective with the same NLMPC tuning parameter used in Figure 3, but using an imperfect model in the NLMPC algorithm is shown in Figure 4. The model is assumed to have 20% error in two of its parameters namely, the pre-exponential term α_1 and the activation energy γ_1. These two parameters were selected because their values are poorly known in general. As demonstrated by the figures, NLMPC failed to stabilize the closed-

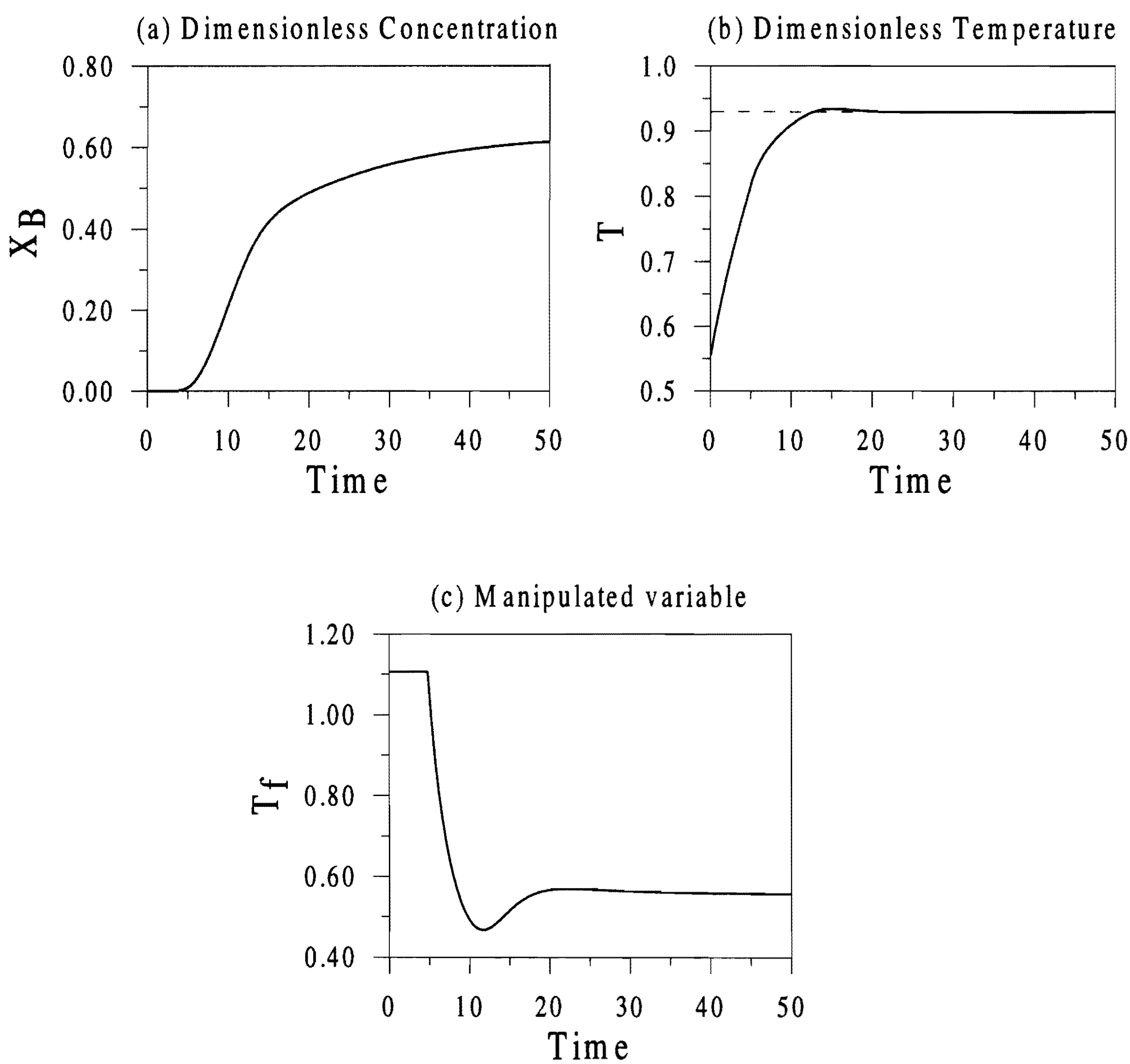

Figure 3. Set point change response using nonlinear MPC with perfect model, $M = 1$, $P = 10$, $\Gamma = 1$, $\Lambda = 0$.

loop response of the reactor temperature although the same values of tuning parameters used in Figure 3 were used. Even for values of P larger than 10 and different values for Λ, the closed-loop response was still poor. This, of course, is due to the presence of modeling error which made the model states diverge away from those of the plant. As discussed in Section 3, estimation of the model uncertainty by a constant additive term on the output does not help in this case. Therefore, it is necessary to reset the model states by either state estimation or parameter estimation in order to improve the performance of NLMPC.

As mentioned earlier, employing Kalman filtering is useless since the model states are unobservable. In fact, different values for σ (the tuning parameter for Kalman filter [14]) were indeed attempted for these cases, but no significant improvement was observed. Hence, the above simulation is repeated using the proposed modified estimation technique. The results are demonstrated in Figures 5(*a–c*) for the same values of the tuning parameters except only P = 10 is used. The modified estimation method is found useful in this case to improve the performance of NLMPC in the face of parametric modeling errors. Some spikes were observed in the time response of T_f due to the fluctuating values of ϕ. The first spike occurred during start up and lasted for one sample, *i.e.*, 0.2, and, thus had negligible effect on the concentration response. The second spike occurred when process started reaching steady state and lasted for about 3 samples. This spike caused a minor effect on the concentration response as illustrated by Figure 5.

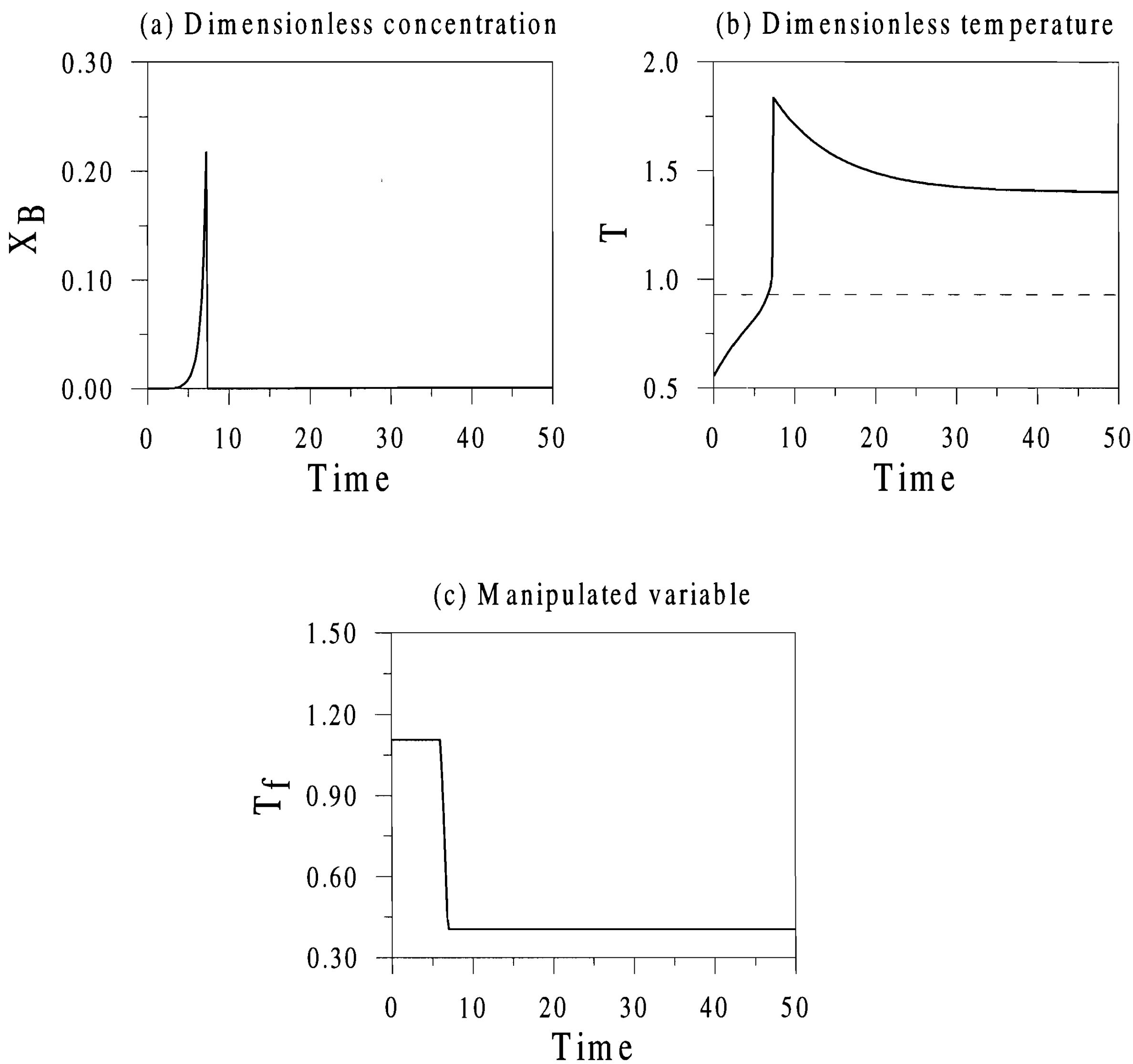

Figure 4. Set point change response using nonlinear MPC with imperfect model, M = *1,* P = *10,* Γ = *1,* Λ = *0.*

5.2. Using Linear MPC

To implement linear MPC, the dynamic model of the reactor developed in Section 4 was linearized at the stable operation point (*i.e.*, initial steady state point in Table 2) and then converted into a step response model. Results from testing the linear MPC for the same above set point change and values for the tuning parameters are shown in Figure 6. The simulation plots indicate that LMPC was not able to stabilize the process. Different values for M, P, and Λ were tried but without success. This poor performance is similar to that of a non-linear MPC with parametric modeling error. This is because linearization itself presents model–plant mismatch. This situation clearly demonstrates how "additive" constant disturbance (*i.e.*, step-like disturbance), as used by the standard MPC, precluded the MPC from stabilizing the reactor temperature around its desired value. In fact, the model–plant mismatch may not be always represented by a step-like function, especially for open-loop unstable processes. Typical transient model–plant mismatch for the above simulation is shown by the solid curve in Figure 7. The dotted curve is the estimated mismatch obtained by the proposed disturbance estimation discussed in Section 2.1, using disturbance model size, *nd*, equal 5. The estimated mismatch is computed without actually implementing it in the MPC algorithm. The figure illustrates the effectiveness of the proposed disturbance model to perfectly track the actual mismatch.

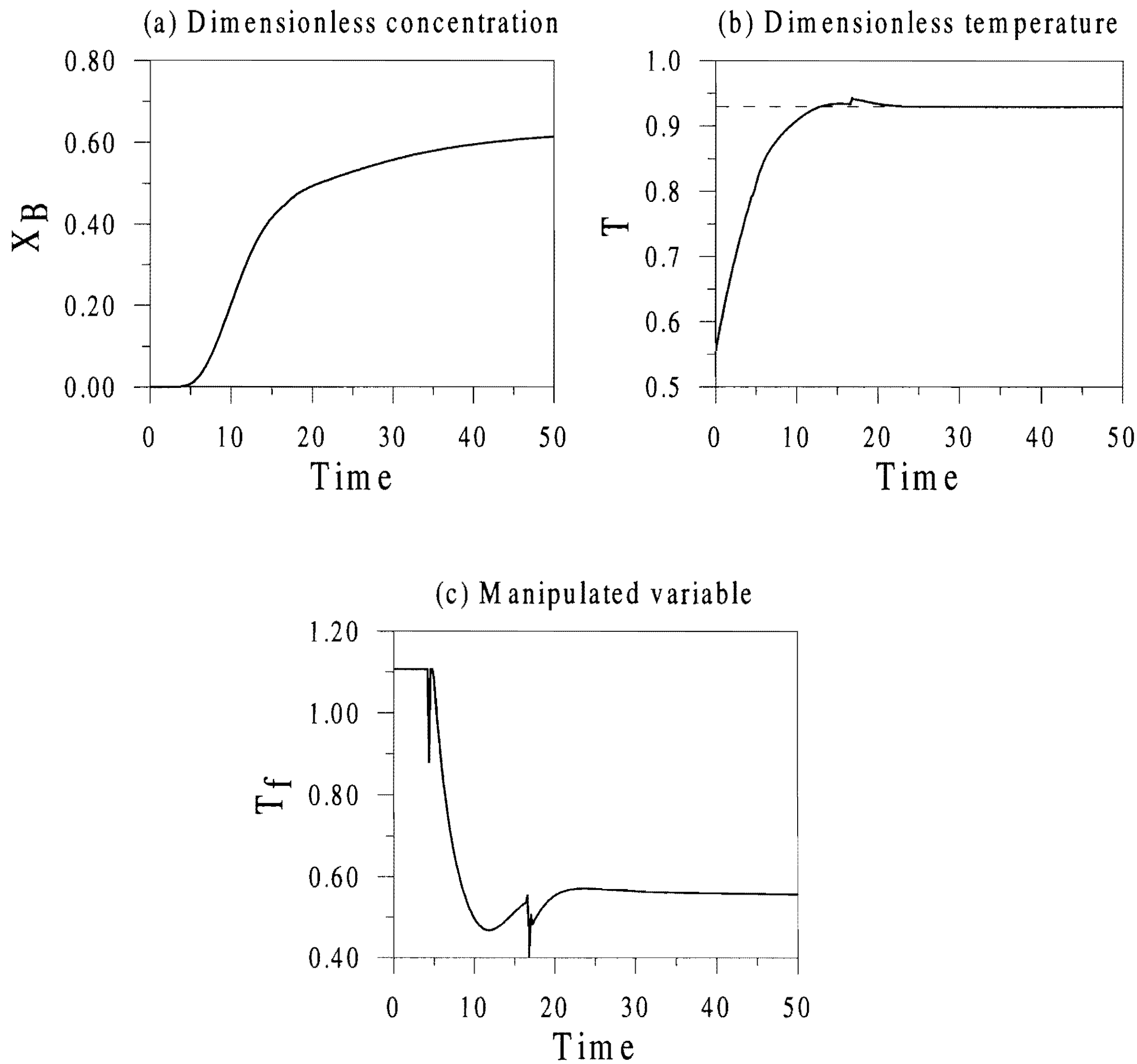

Figure 5. Set point change response using nonlinear MPC with imperfect model and disturbance estimation, $M = 1$, $P = 10$, $\Gamma = 1$, $\Lambda = 0$.

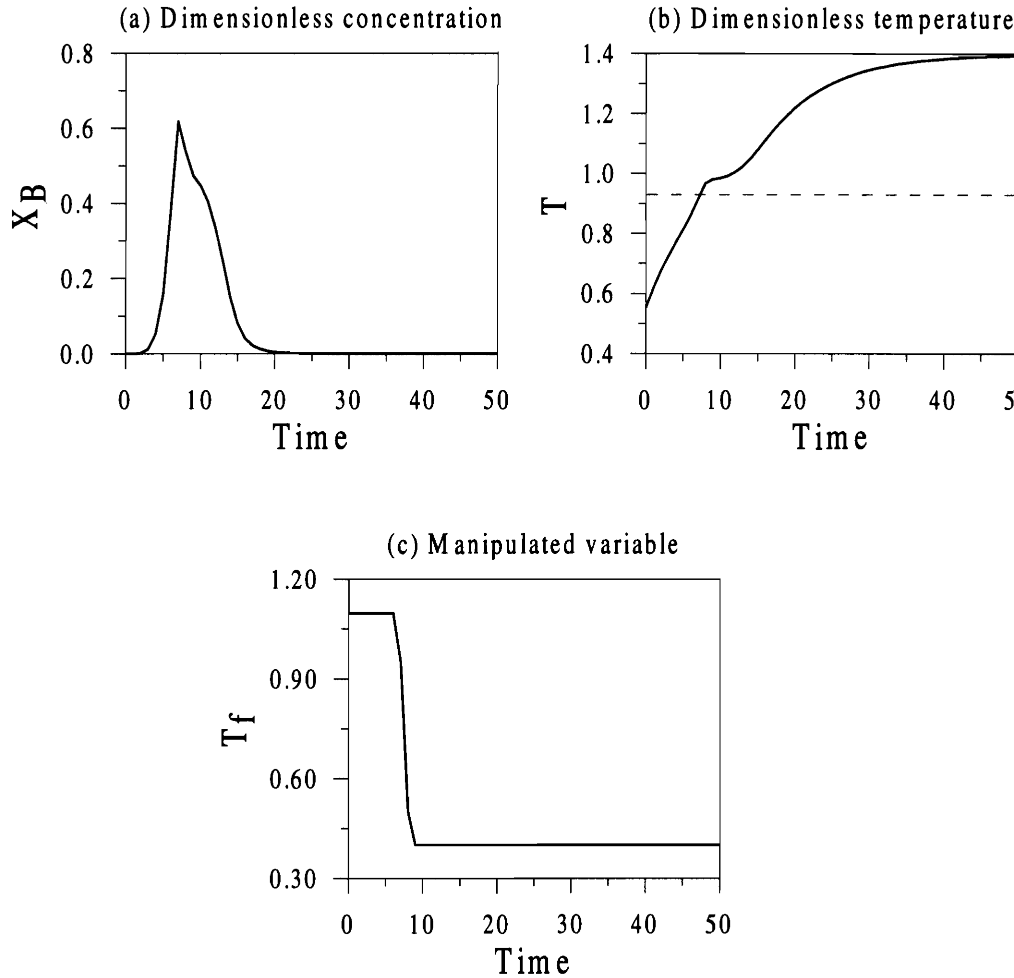

Figure 6. Set point change response using linear MPC.

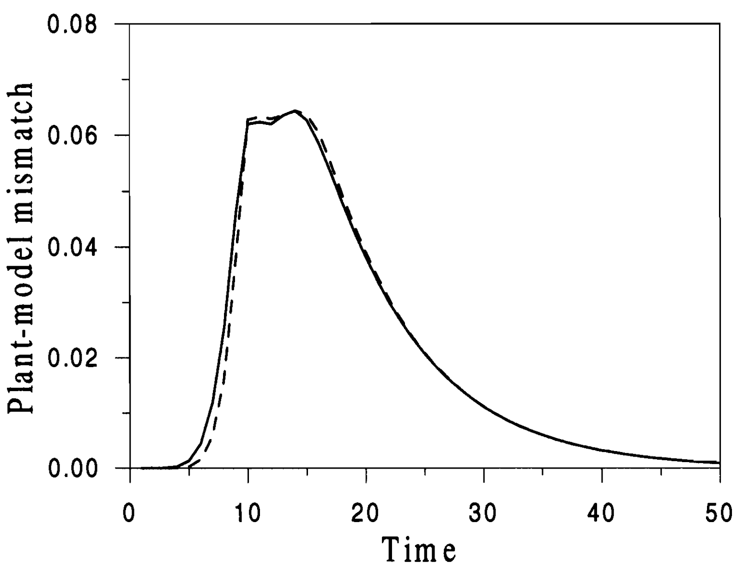

Figure 7. Actual and predicted mismatch for set point change simulation using linear MPC, Solid line: Actual; Dashed line: Predicted.

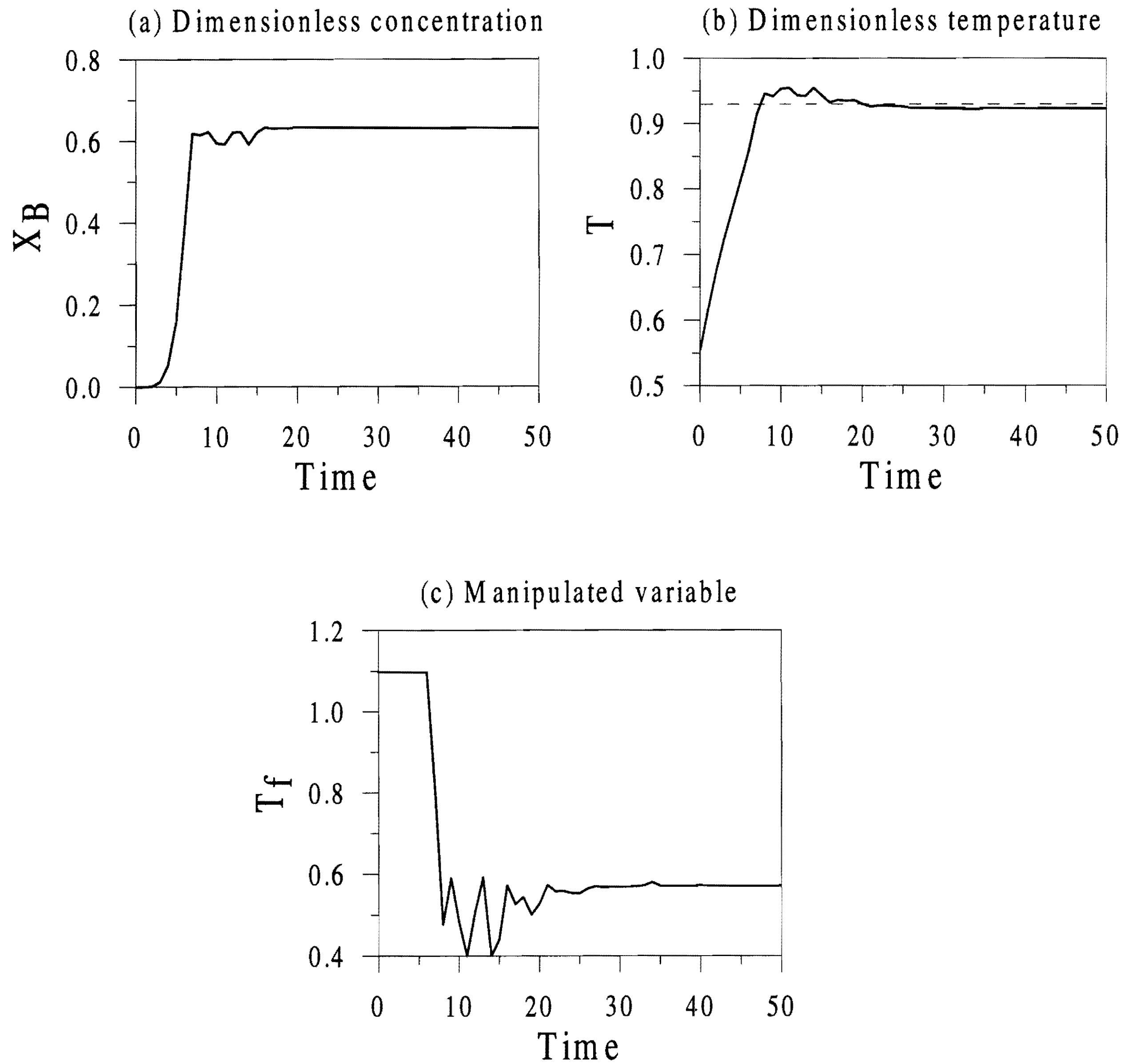

Figure 8. Set point change response using linear MPC and proposed estimation method.

Substantial improvement of the LMPC performance is obtained when the proposed disturbance estimation with *nd* = 5 is implemented. The resulting feedback response is depicted by Figure 8. The reactor temperature increases rapidly and then levels off at the desired optimal value leading to an increase in the production of the desired intermediate product up to the maximum possible value. Several values for the disturbance model size (*nd*) were attempted and found not to have critical effect on the disturbance predictor performance. The non-smooth transient feedback behavior may be attributed to the fluctuation of the coefficients of the disturbance model which are kept updated on-line each sampling time. A smoother response might be obtained if the coefficient identification is relaxed.

6. CONCLUSIONS

In this paper two new formulations for unmeasured disturbances estimation are presented. One formulation is developed for non-linear MPC algorithm and the other for linear MPC algorithm. The idea is to improve their performances in the presence of modeling errors applied to an open-loop unstable process. For NLMPC the new scheme is designed such that it overcomes the limitation associated with non-linear MPC with Kalman filtering developed by Ali and Zafiriou [14]. In the latter, the procedure used for computation of the Kalman filter gain may fail to converge for unstable process, thus, in the new scheme, the gain is computed *via* solving optimization problem. Simulation of the two schemes for the stabilization of open-loop unstable fluidized bed reactor during set point change in the presence of modeling error revealed the superiority of the new scheme. Superiority is based on the fact that computation of the filter gain using the Kalman procedure failed. In addition, the enhancement was achieved without requiring excessive computations since the optimization problem is made small by confining its size to the size of measured outputs.

For linear MPC, the new formulation is designed such that it circumvents the limitation associated with the traditional MPC due to the assumption of using constant projection of the unmeasured disturbance during open-loop prediction. The new scheme uses a linear model to predict the future values of the unmeasured disturbances. The model parameters are identified on-line *via* solving small size least square problem. Successful simulated implementation of the proposed algorithm to stabilize the same unstable fluidized bed reactor during set point change was observed. It was found, despite the non-smooth response, that the new scheme was able to bring the process to its desired set point where the traditional scheme failed.

NOMENCLATURE

A	Reactant in the chemical reaction $A \rightarrow B \rightarrow C$
B	Intermediate product in the chemical reaction $A \rightarrow B \rightarrow C$
b	Vector of the lower and upper bounds of the linear constraints
C	Product in the chemical reaction $A \rightarrow B \rightarrow C$
d	Disturbance estimates
F	Linear constraint matrix
k	Sampling time index
Le_i	Lewis number of component i
M	Control horizon
M_p	Constant matrix of dimension $(P.n_y)x(P.n_y)$
n_y	Number of controlled outputs
nd	Size of the disturbance model in the linear MPC
P	Prediction horizon
r	Set point for the controlled outputs
R	Vector of set points
S_P^m	Step response coefficient matrix
T	Dimensionless reactor temperature
T_k	Controller sampling time
T_f	Dimensionless feed temperature
t	Time
t_k	Time at sampling instant k
u	Manipulated variable vector
x	State vector
x_{ms}	Measured state vector
x_{um}	Unmeasured state vector

X_A	Dimensionless dense phase concentration of the reactant *A*
X_B	Dimensionless dense phase concentration of the intermediate product *B*
X_{Af}	Dimensionless feed concentration of *A*
X_{Bf}	Dimensionless feed concentration of *B*
y	Controlled output vector

Greek Symbols

α_i	Dimensionless pre-exponential factor for reaction *i*
β_i	Dimensionless overall exothermicity factor for reaction *i*
$\bar{\beta}$	Reciprocal of the effective residence time of the bed
Δu	Vector of manipulated variable change
ΔU	Vector of *M* future manipulated variable change
γ_i	Dimensionless activation energy
Γ	Diagonal weight matrix on the predicted error
Λ	Diagonal weight matrix Ion the manipulated variable change
θ	Disturbance model coefficients
ϕ	Static gain for the disturbance estimator

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