

BASIC STATISTICS OF STRENGTH LIMIT STATES OF REINFORCED CONCRETE MEMBERS IN SAUDI ARABIA

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الخلاصة :

نستخدم في هذا البحث أسس الموثوقية في تطوير مواصفات تصميم المنشآت الخرسانية المسلحة والمنشآت الفولاذية، حيث تُستخدم كوسيلة لحساب معاملات السلامة للأحمال والمقاومة التي تأخذ في الاعتبار التباين الطبيعي في قيم الأحمال والمقاومة. إن من أهم الخطوات نحو دراسة موثوقية المنشآت هي تقدير الخصائص الإحصائية لمقاومة الأجزاء الإنشائية. وفي هذه الدراسة تم تحديد القيم الوسطى، ومعاملات التباين، ونوع التوزيع الإحصائي لمقاومة العوارض والأعمدة القصيرة مستخدمين إحصائيات عناصر المقاومة الأساسية التي تم الحصول عليها من واقع ممارسات تنفيذ المباني الخرسانية في المملكة العربية السعودية. وقد تم استخدام طريقة مونتني كارلو للمحاكاة. كما تم دراسة حساسية الخصائص الإحصائية للمقاومة عند تغيير الخصائص الإحصائية لعناصر المقاومة الأساسية. كما تم تحديد قيم للخصائص الإحصائية للمقاومة بحيث يمكن الاستفادة منها في حساب معاملات السلامة لأغراض تصميم المنشآت الخرسانية المسلحة في المملكة العربية السعودية .

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ABSTRACT

Reliability principles have been employed successfully in the development of design codes of reinforced concrete and steel structures. These methods were employed to guide the selection of load and resistance factors which account for the variabilities in the individual load and resistance parameters. One of the major steps towards the reliability analysis is to estimate the statistical characteristics of the strength limit states of structural members. In this study, the mean-to-nominal ratios, coefficient of variations, and types of distribution functions of the strength of reinforced concrete beams and short columns were investigated employing the statistics of basic parameters obtained under the prevailing construction practices in Saudi Arabia. The Monte-Carlo simulation technique is employed in the study. The sensitivity of these characteristics to the variations in the basic parameters is investigated. Representative values of these characteristics are selected. Results obtained in this study are essential for reliability analysis and determination of appropriate safety factors for the Saudi Design Code of Reinforced Concrete Structures.

Keywords: beams, columns, reinforced concrete, bending, probability theory, reinforcing steel, compressive strength, yield strength, ductility, reliability, and building code.

BASIC STATISTICS OF STRENGTH LIMIT STATES OF REINFORCED CONCRETE MEMBERS IN SAUDI ARABIA

INTRODUCTION

Reliability-based methods have been employed successfully in the development of design codes of reinforced concrete and steel structures [1–3]. In the reliability based limit state design, probabilistic methods are used to guide the selection of load and resistance factors, which account for the variabilities in the individual load and resistance parameters. The advantages of probabilistic limit states design include the possible achievement of a more consistent reliability for different design situations as the variabilities of the related strength and load parameters are considered explicitly, and the possible selection of suitable reliability level for a structure which reflects the consequences of failure [4].

Reliability Index

Structural reliability is measured by reliability index, β , which, for a particular structure, is defined as the inverse of the standard normal distribution function at the probability of failure. Cornell [5] established a relation between β and the statistics of design parameters as follows:

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \quad (1)$$

in which μ_R , σ_R are the mean and standard deviation for R , and μ_Q , σ_Q are the mean and standard deviation for Q . The load (Q) and resistance (R) variables are mutually independent random variables which have normal distributions. Several expressions for β are available, depending on the nature of limit state and distribution functions of related parameters [4, 6].

Flexural and compression of reinforced concrete members tend to fall within the range of β of 2.5 to 3.0 for the load combinations of dead, live, and wind loads [4, 7]. The recently developed ASCE 7-95 [2] criteria were based on target reliability index of 3.0 for ductile failures such as would occur in under reinforced beams and spiral columns, and of 3.5 for brittle failures expected in shear and tied columns [4, 7].

Load and Resistance Factors

The most simple design criterion, based on partial safety factors, is given as follows:

$$\phi R_n \geq \gamma Q_n \quad (2)$$

where ϕ and γ are the resistance and load factors which can be derived, for lognormal distributions of load and resistance parameters, as follows:

$$\gamma = \lambda_Q \exp(0.75 \beta_T V_Q) \quad (3)$$

$$\phi = \lambda_R \exp(-0.75 \beta_T V_R) \quad (4)$$

where λ_Q and λ_R are the mean-to-nominal ratios of load and resistance parameters, respectively, whereas V_Q and V_R are the coefficient of variations of load and resistance parameters and β_T is the target reliability index.

Probabilistic Behavior of RC Beams

Flexural Strength

The statistical characteristics of beam strength in flexure depends on statistics of basic strength parameters: compressive strength of concrete, yield strength of reinforcement, sectional dimensions, and effective depth of reinforcement. Results are also depend on the accuracy of the selected analytical model. Allen [8] investigated the probabilistic behavior of RC beams in bending. The statistics of basic parameters employed in the study are listed in Table 1. He concluded that λ_R is between 1.06 and 1.25 depending on the rate of loading and reinforcement ratio and V_R is between 9 and 21 percent. The higher values of V_R being for shallow members and poor workmanship. He also found that the distribution of the ratio of

flexural strength to the nominal strength is normally distributed while the lower tails of the distribution functions at high reinforcement ratios are affected by the occurrence of the compression failure.

MacGregor, *et al.* [9] established the statistical descriptions of the variability of flexural strength of RC beams. The statistics of basic parameters employed in the study are listed in Table 1. Results, as listed in Table 2, indicated that for beams with grade 60 and f'_c of 35 MPa the λ_R was between 1.01 and 1.09 and V_R was between 8 and 12 percent. They selected λ_R of 1.05 and V_R of 11 percent as representative values for calculating the resistance factor. They concluded that as the reinforcement ratio increases toward the nominal balanced steel ratio ρ_b , the strength λ_R decreases. This was attributed to the increased probability of compression failures.

Table 1. Statistics of Basic Parameters [4].

Basic Variable	λ	$V\%$	CDF
<u>Concrete Compressive Strength</u>			
$f'_c = 3000$ psi (21 MPa)	0.92	18	Normal
$f'_c = 5000$ psi (34 MPa)	0.81	15	Normal
<u>Steel Yield Strength</u>			
Grade 40, static yield	1.07	9.0	Normal
	1.19	9.0	lognormal
	1.13	11.6	Beta
Grade 60, static yield	1.13	9.8	Beta
<u>Geometry</u>			
Beam depth		40/h*	Normal
Beam width		40/b*	Normal
Effective depth		68/h*	Normal
	1.0	25+(20/d)**	Normal

* b and h are the sectional dimensions (inch)

** d is the effective depth of the tension reinforcement.

Table 2. Resistance Statistics [4].

Limit State	Type of member	Reinforcement ρ/ρ_b	λ	$V\%$
Flexure	Beams, grade 60, $f'_c = 5$ ksi	0.14	1.04	8
		0.31	1.09	11
		0.57	1.05	11
		0.73	1.01	12
Axial Load	Short Columns Compression failure			
		$f'_c = 3$ ksi	1.05	16
		$f'_c = 5$ ksi	0.95	14
Shear	Beams with $a/d > 2.5$	No stirrups	0.93	21
		Min stirrups	1.00	19
		$r_v f_y = 150$ psi	1.09	17

Shear Strength

The variability of shear strength depends on the statistics of basic strength parameters and the accuracy of selected analytical model. The main problem in the study is the complexity of the mechanical models for predicting shear strength. Approximate models are usually associated with high prediction errors.

Variability of shear strength employing shear strength equation developed by Zsutty [10] was investigated [4]. The study is limited to beams with a/d between 2.3 and 4.9. Results indicated that the mean-to-nominal ratio is between 1.08 and 1.13 where the strength coefficient of variation is between 8.2 and 13.7 percent.

MacGregor, *et al.* [9] studied the variability of shear strength employing the statistics listed in Table 1. The study was limited to beams with a/d greater than or equal to 2.5. The longitudinal steel ratio was 0.008. Results indicate that the mean-to-nominal ratio is between 0.93 and 1.09 where the strength coefficient of variation is between 17 and 21 percent depending on web reinforcement as shown in Table 2.

Compressive Strength of Columns

The variability in strength of reinforced concrete tied columns has been studied by Ellingwood [11] and by Grant *et al.* [12]. Results indicated that for short columns under compression failure for concrete strength of 3000 psi (21 MPa) the mean-to-nominal ratio and coefficient of variation of the strength were 1.05 and 16 percent, respectively, and for concrete strength of 5000 psi (34 MPa), these values were 0.95 and 14 percent, respectively.

RESEARCH SIGNIFICANCE

In this study, the statistical characteristics of strength limit states of reinforced concrete beams in flexure and shear force have been estimated. Short columns under compression failure were also included. The investigated parameters included the mean-to-nominal ratios, coefficient of variations and types of distribution functions. The sensitivity of these characteristics to the variations in the basic parameters is investigated. The study is essential for the evaluation of reliability and risk analysis of structural members and systems. It is also one of the major steps towards the calculation of resistance factors in the limit state design for the Saudi Design Code for reinforced concrete structures.

SIMULATION OF FLEXURAL, SHEAR, AND COMPRESSIVE STRENGTH

Statistical Characteristics of Basic Parameters

Arafah, *et al.* [13] estimated the statistics of ready-mixed (RM) and at-site mechanically-mixed (SM) concrete under the prevailing concreting practices in Saudi Arabia. The results of 636 strength test on RM indicated that λ_c and V_c are about 1.0 and 20 percent respectively, and the strength is well represented by the normal distribution. The results of 45 strength tests on SM concrete indicated that λ_c and V_c are about 0.85 and 40 percent respectively, and concrete strength is well represented by the log-normal distribution. These results were listed in Table 3 and employed in this study.

Al-Behairi [14] investigated the probabilistic characteristics of steel bars produced through Bar Quenching Process. He concluded that λ_s and V_s are 1.34 and 4.3 percent respectively, and the yield strength is well represented by the normal distribution function. These statistics were listed in Table 3 and employed in this study.

The deviation of sectional dimension parameters from their nominal values affects the behavior of beam sections. Based on the results obtained in [13] the coefficient of variation of the depth of reinforcement in the tension and compression are found as about 2.0 and 20.0 percent, respectively.

Constitutive Models of Concrete and Steel

The stress-strain curve for concrete suggested by Hognestad *et al.* [15] is employed in the procedure. The curve is presented by a second degree parabola for the ascending part of the relation, as shown in Figure 1, can be expressed by:

$$f_{ci} = f_c \left[2 \left(\frac{\epsilon_{ci}}{\epsilon_o} \right) - \left(\frac{\epsilon_{ci}}{\epsilon_{co}} \right)^2 \right] \quad (5)$$

and a straight line over the descending part which can be expressed by:

$$f_{ci} = f_c [1 - z(\epsilon_{ci} - \epsilon_{co})], \tag{6}$$

where f_{ci} is the compressive stress, ϵ_{ci} is concrete strain, f_c is ultimate concrete compressive strength, ϵ_{co} is the concrete strain at the ultimate concrete compressive strength which is assumed to be 0.002. The ultimate strain of the concrete, ϵ_{cu} , is function of compressive strength and can be calculated as follows [16]:

$$\epsilon_{cu} = 0.004 - 2.23 \times 10^{-5} f_c, \tag{7}$$

and z is the slope of the linear descending part of the relation which represents the level of concrete confinement. Linear brittle stress–strain relation for concrete in tension with a rupture tensile strain equal to f_t/E_c is employed.

The model expresses the constitutive behavior over the three strain-ranges, as shown in Figure 1 is as follows:

$$f_s = E_s \epsilon_s \quad \text{for} \quad 0 < \epsilon_s \leq \epsilon_y \tag{8a}$$

$$f_s = f_y \quad \text{for} \quad \epsilon_y < \epsilon_s \leq \epsilon_{sh} \tag{8b}$$

$$f_s = f_y + E_{sh} (\epsilon_s - \epsilon_{sh}) \quad \text{for} \quad \epsilon_s > \epsilon_{sh} \tag{8c}$$

in which f_s and ϵ_s are the steel stress and strain respectively, f_y is the yield strength, ϵ_y is yield strain, ϵ_{sh} is the strain at the initiation of strain hardening and E_s and E_{sh} are the steel moduli of elasticity in the elastic and strain hardening ranges, respectively.

Model Error

Model error is a random variable with mean equal to the ratio of the mean test strength to the nominal strength, λ_m , and coefficient of variation, V_m , equal to

$$V_m = \sqrt{V_{t/n}^2 - V_t^2 - V_{spec}^2} \tag{9}$$

where $V_{t/n}$ is the coefficient of variation obtained directly from comparison of measured and calculated strengths, V_t represents uncertainties in the testing procedure, and V_{spec} represents errors introduced by test specimens. The λ_m and V_m for flexural and axial strength were estimated as 1.01 and 4.6 percent, respectively [4]. For shear strength λ_m and V_m are assumed to be 1.09 and 12.5 percent, respectively [4].

Table 3. Statistical Characteristics of Strength Parameters.

Variable	Nominal Value	Mean Value	V%	CDF
Concrete				
f_c (RM) (MPa)	24	24	20	Normal
f_c (SM) (MPa)	20	17	40	Log-Normal
Steel				
f_y (MPa)	413	554	4.3	Normal
E_s (MPa)	200 000	214 505	2.1	
E_{sh} (MPa)	--	2920	16.6	
ϵ_{sh}	--	0.02	20	
Depth to Steel				
d (mm)	570	570	2	Normal
d' (mm)	50	50	20	

Monte-Carlo Simulation Technique

Monte-Carlo technique is employed for simulation of the random variables and the behavior of the beam sections. A rectangular section with depth $h=600$ mm and width $b=300$ mm are selected for the simulation process. Based on the statistics given in Table 3, the program simulates the parameters f'_c , ϵ_{cu} , f_y , ϵ_{sh} , E_s , E_{sh} , d , and d' whereas A_s , A'_s , h , and b are assumed to be deterministic parameters where A_s , d , A'_s , and d' are the area and effective depth of the tension and compression reinforcements.

Flexural Strength

The variation of the mean-to-nominal ratio of flexural strength λ_{FS} with the tension and compression reinforcement ratio, $(\rho - \rho')/\rho_b$, is investigated for beams with RM and SM concretes, denoted as BRM and BSM, respectively, as shown in Figure 2 where ρ and ρ' are the tension and compression reinforcement ratios and ρ_b is the balanced reinforcement ratio according to ACI-318 [17]. The relationship between flexural strength coefficient of variation, V_{FS} , and reinforcement ratio is also investigated as shown in Figure 3. The reinforcement ratio is taken between 0.3 and 0.75.

The types of the distribution functions of the flexural strength are investigated for BRM and BSM at reinforcement ratios of 0.3 and 0.75. Results obtained from the simulation process are plotted on normal probability papers, NPP, as shown in Figure 4. The NPP is a special scale which can be used to check the normality of a distribution function. If the distribution function is normal, it appears as a straight line. There are two tests usually employed to check the normality of a distribution at a specified level of significance which are the Chi-Square Test and the K-S Test. Details about these methods are available in reference [18].

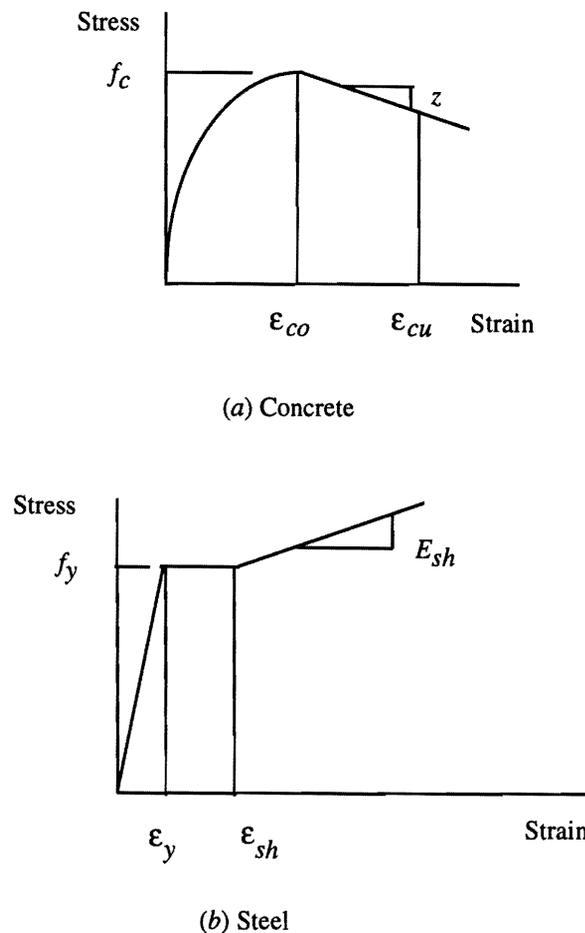


Figure 1. Constitutive models of concrete and steel.

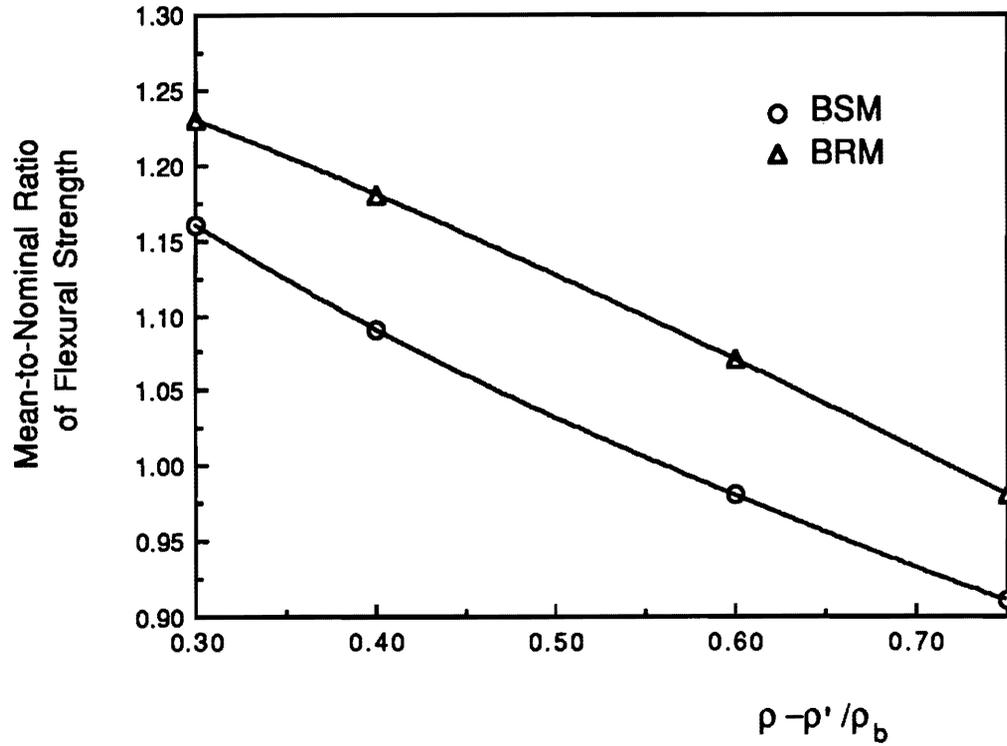


Figure 2. Variation of mean-to-nominal ratio of flexural strength with reinforcement ratio for BRM and BSM.

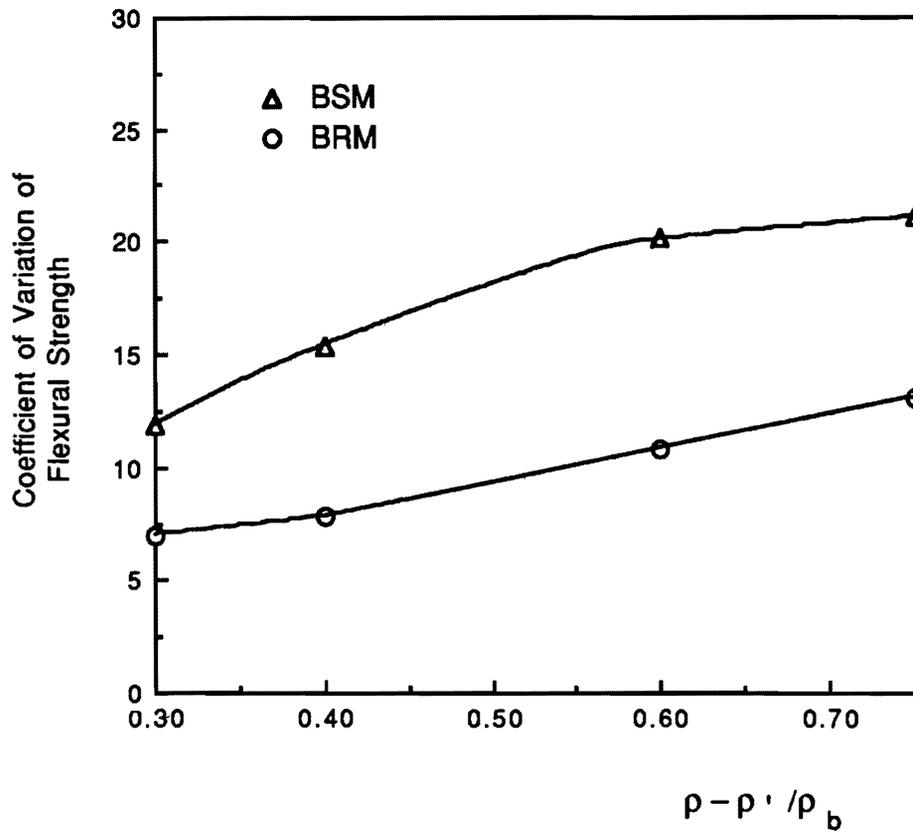


Figure 3. Variation of coefficient of variation of flexural strength with reinforcement ratio for BRM and BSM.

Shear Strength

The variability in shear strength of reinforced concrete beams, employing the statistics listed in Table 3, is investigated. The shear strength mean to nominal ratio, λ_{SS} , and coefficient of variation, V_{SS} , are evaluated for various shear reinforcements. The study is limited to beams with a/d greater than or equal to 2.5. The ACI-318 equations (11-2), (11-3), and (11-17) are employed for the shear strength. The mean-to-nominal ratio and coefficient of variation of the model for shear strength were assumed as 1.09 and 12.5 percent respectively. The variations of λ_{SS} and V_{SS} with shear reinforcement expressed as the number of stirrups per effective depth of the beam, d/s , were shown in Figures 5 and 6. The distribution functions of shear strength for BRM and BSM with 8 mm stirrups and $d/s=3$ are shown in Figure 7.

Compressive Strength

The variability in the compressive capacity of short columns is investigated employing the statistics listed in Table 3. The compressive strength mean to nominal ratio of columns, λ_{CS} , and coefficient of variation, V_{CS} , are evaluated for various longitudinal reinforcements, ρ_l . The study is limited to tied columns with longitudinal reinforcement ratio between 0.01 and 0.08 which are the minimum and maximum permissible reinforcement ratios according to ACI-318. The ACI code equation (10-1) was employed for the compressive strength. The mean-to-nominal ratio and coefficient of variation of the model for axial strength were assumed as 1.01 and 4.6 percent, respectively.

Variation of λ_{CS} and V_{CS} with the longitudinal reinforcement ratio was investigated for columns with RM and SM concretes, denoted as CRM and CSM, respectively, as shown in Figures 8 and 9. The types distribution functions of axial strength were investigated for CRM and CSM at longitudinal reinforcement ratios of 0.02 and 0.06 as shown in Figures 10 and 11.

RESULTS, ANALYSIS, AND DISCUSSION

Flexural Strength of Beams

Mean-to-Nominal Ratio

Figure 2 presents the variation of λ_{FS} with reinforcement ratio for BRM and BSM. Results indicate that λ_{FS} decreases with increasing reinforcement ratio. This is attributed to two main reasons (1) at low reinforcement ratios, there is a high

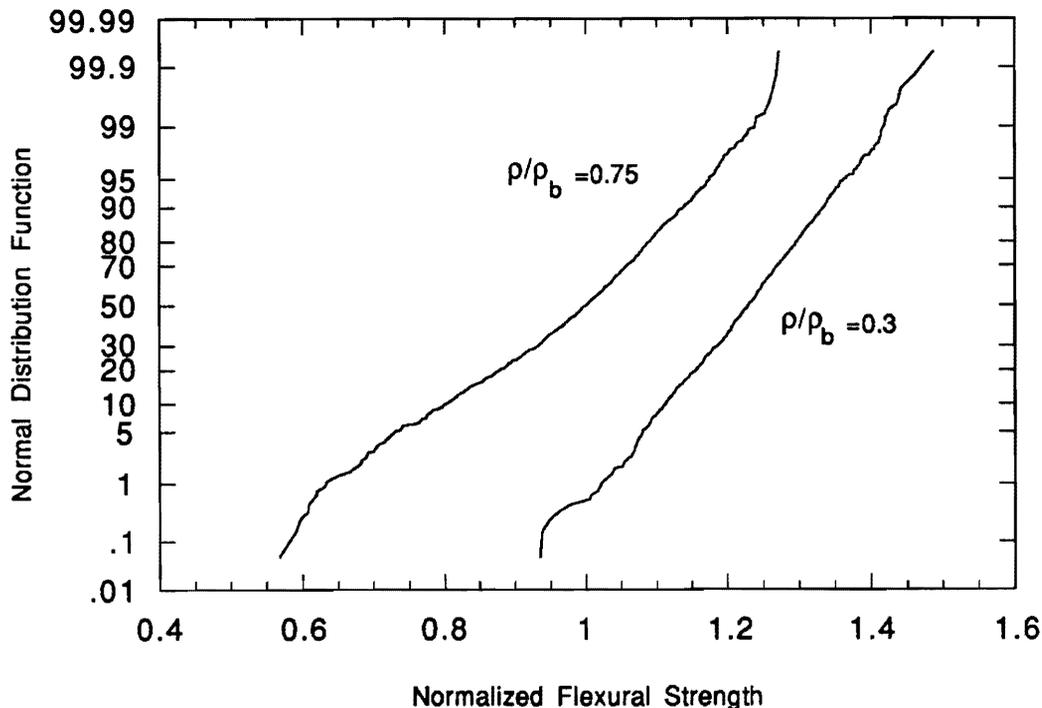


Figure 4. Distribution functions of flexural strength for BRM.

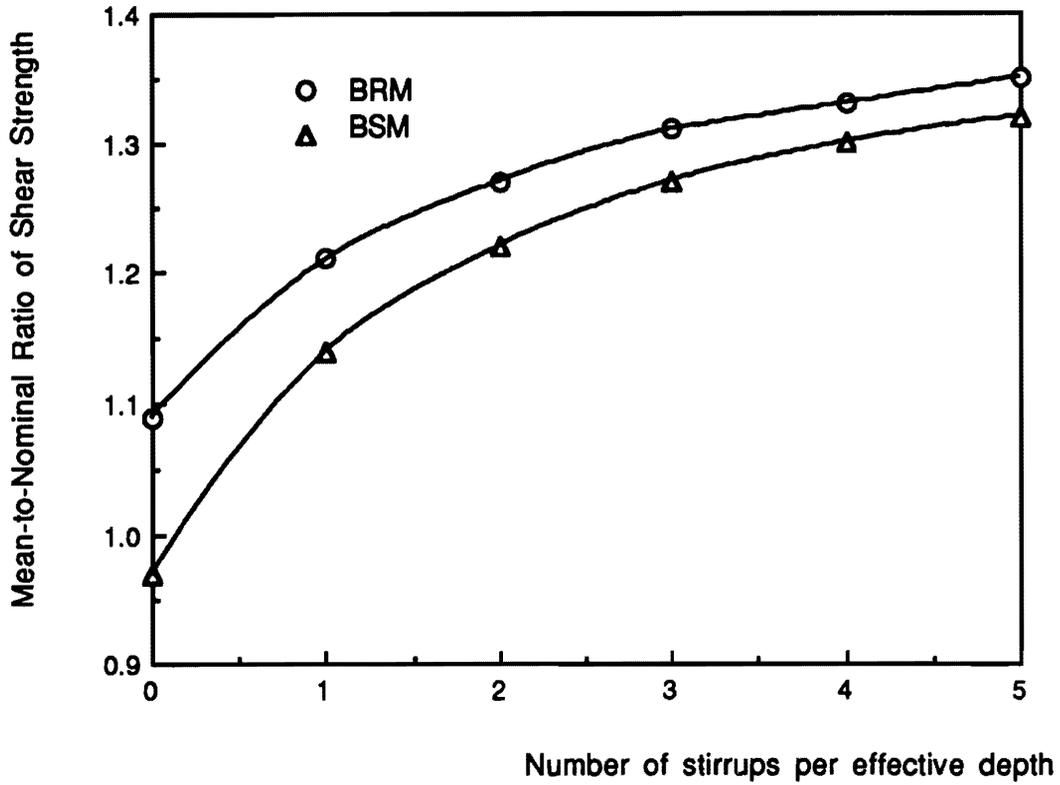


Figure 5. Variation of mean-to-nominal of shear strength with number of stirrups per effective depth for BRM and BSM.

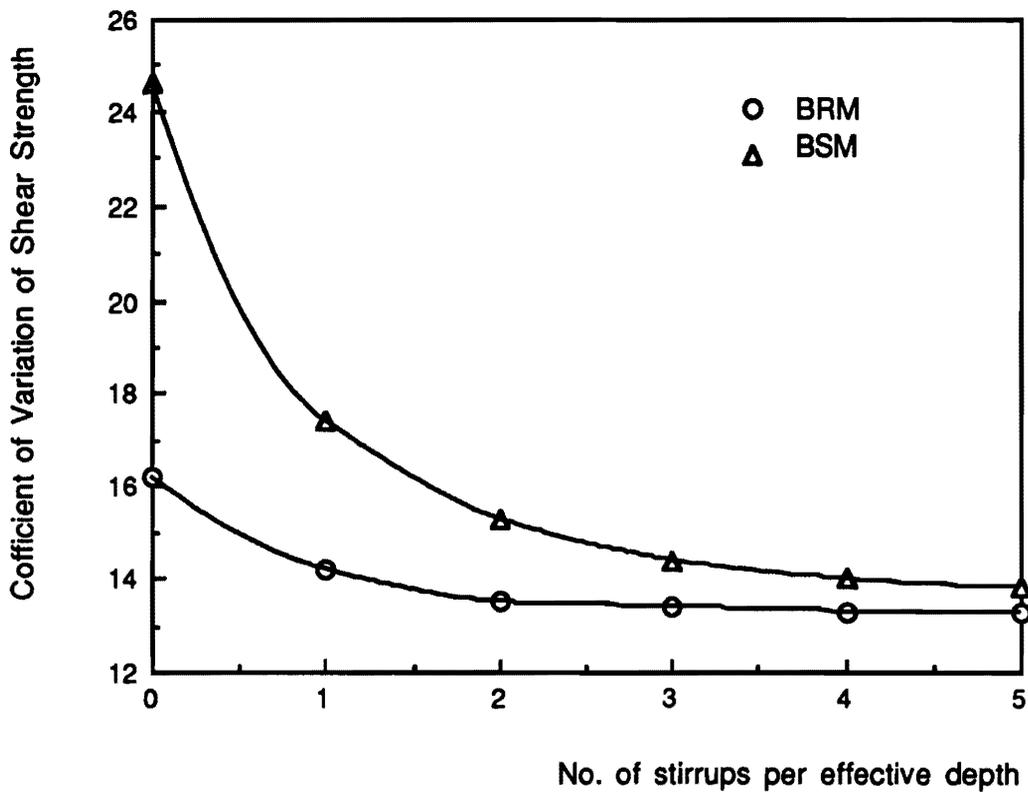


Figure 6. Variation of coefficient of variation of shear strength with number of stirrups per effective depth for BRM.

probability that strain in tension steel exceeds the strain at hardening and consequently stresses at failure are higher than the actual yield stress which results in high flexural capacity, and high values of λ_{FS} (2) at high reinforcement ratios, the probability of compression failure is high which adversely affects the flexural capacity and results in low values of λ_{FS} . Similar results were obtained for BSM, however, values of λ_{FS} are lower than those observed for BRM.

Based on a limited survey conducted as a part of the study, the reinforcement ratio $(\rho - \rho')/\rho_b = 0.3$ was selected as a representative value for current design in the Kingdom. At this reinforcement ratio, the values of λ_{FS} for BRM and BSM are 1.23 and 1.16, respectively as listed in Table 4.

Coefficient of Variation

Figure 3 presents the variation of V_{FS} with reinforcement ratio for BRM and BSM. Results indicate that V_{FS} increases with increasing reinforcement ratio. This is also attributed to the high probability of compression failure at high reinforcement ratios. It is worth mentioning that, the variation in compression failure is higher than that for tension failure. Values of V_{SF} for BSM are higher than those of BRM because the V_c of SM is higher than that of RM. At a reinforcement ratio of 0.3 the values of V_{FS} for BRM and BSM are 7 and 12 percent, respectively as listed in Table 4.

Distribution Function

Figure 4 presents the distribution functions, CDF, of the normalized flexural strength of BRM with reinforcement ratios of 0.3 and 0.75 plotted on a normal probability paper. The CDF with reinforcement ratio of 0.3 is very close to the straight line, therefore it has normal distribution as shown in Figure 4, whereas CDF at reinforcement ratio of 0.75 does not resemble the normal distribution. This is attributed to the high probability of compression failure at high reinforcement ratios which results in reduction of flexural strength, as compared with tension failure, as shown in Figure 4.

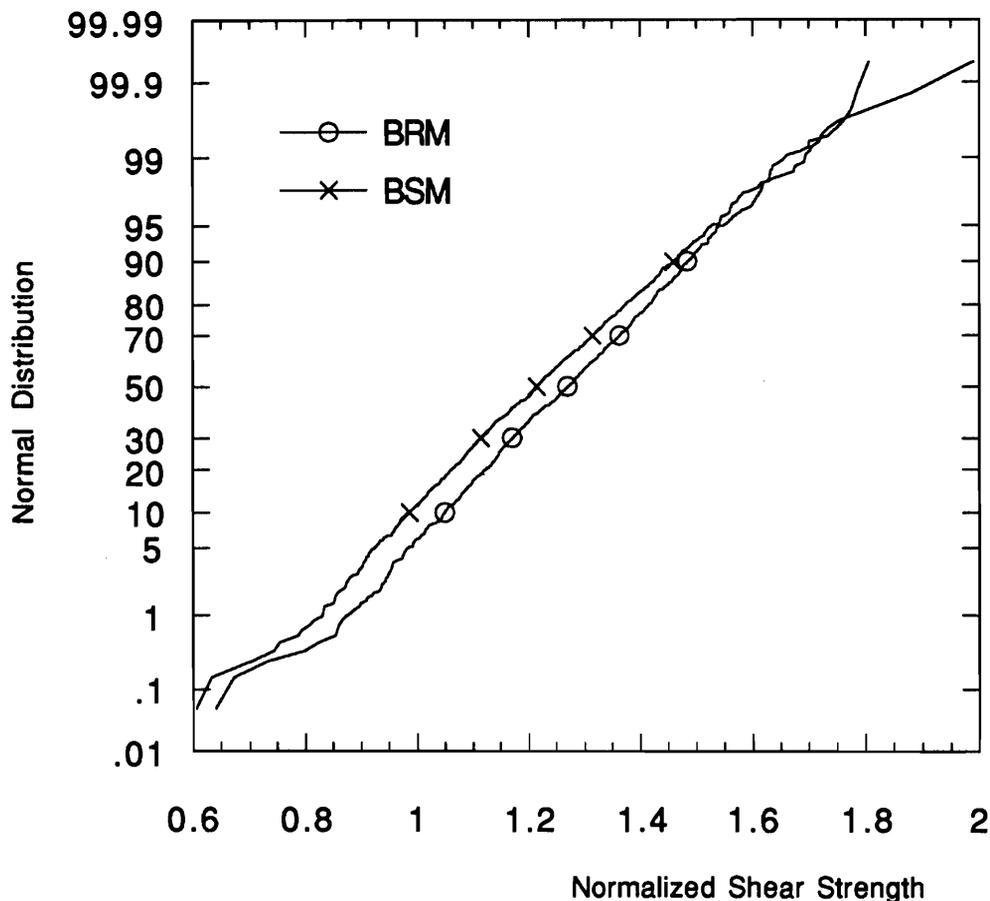


Figure 7. Distribution functions of shear strength for BRM and BSM ($d/s = 3$ and bar diameter = 8 mm).

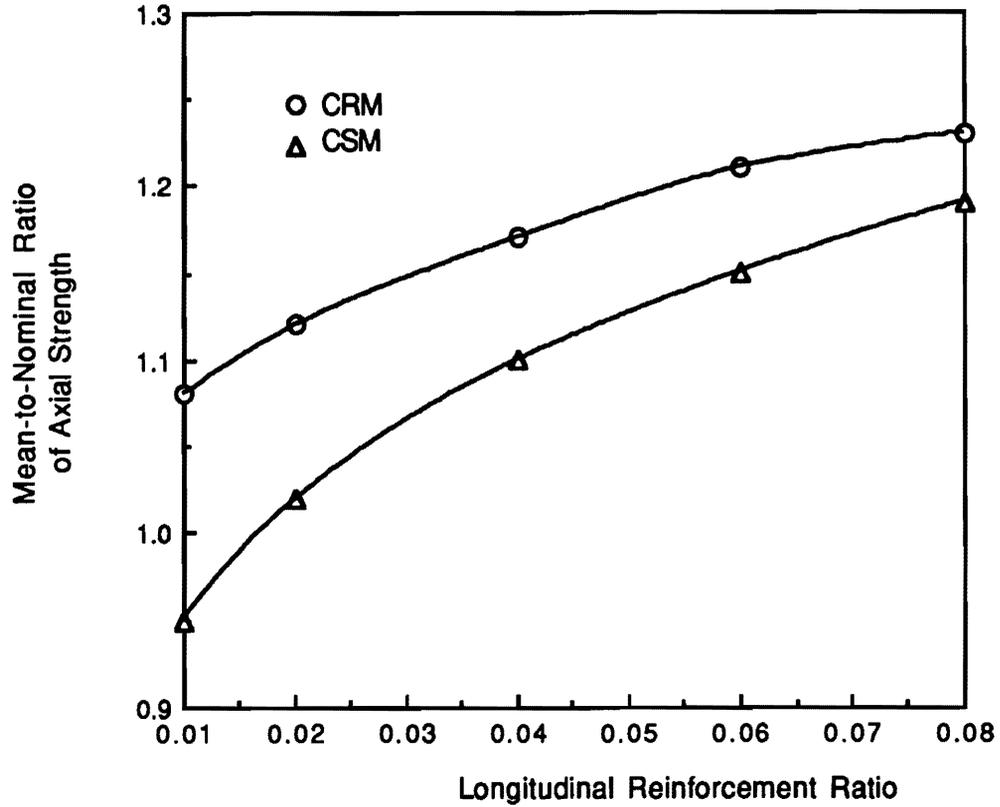


Figure 8. Variation of mean-to-nominal ratio of compressive strength with longitudinal reinforcement ratio.

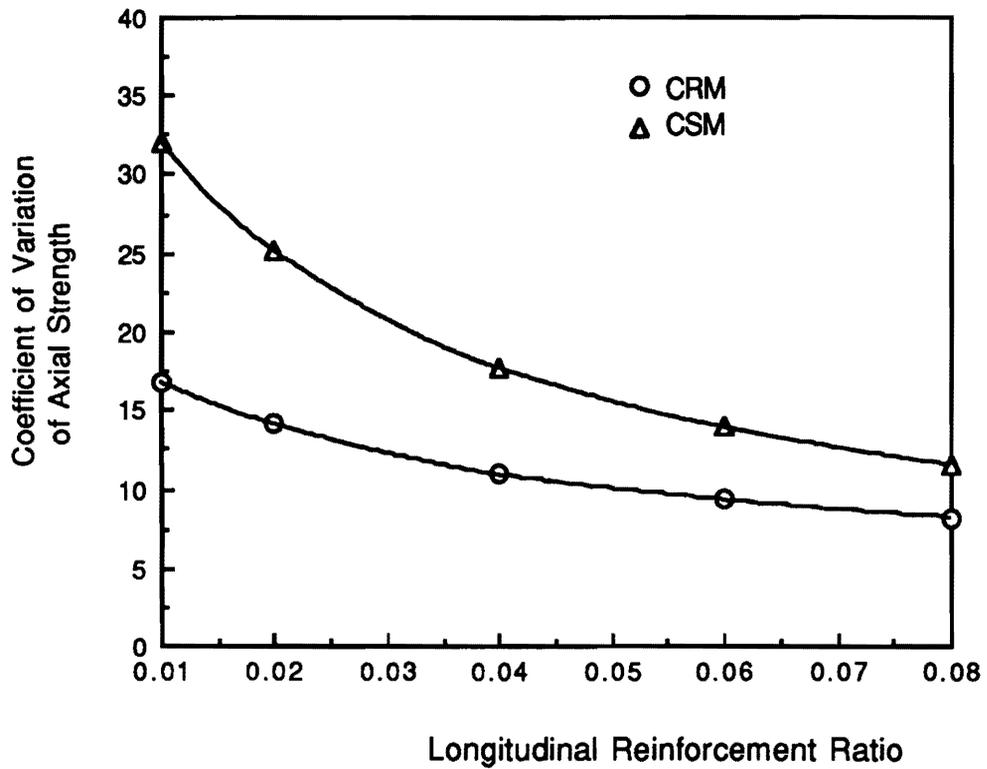


Figure 9. Variation of coefficient of variation of compressive strength for CRM and CSM.

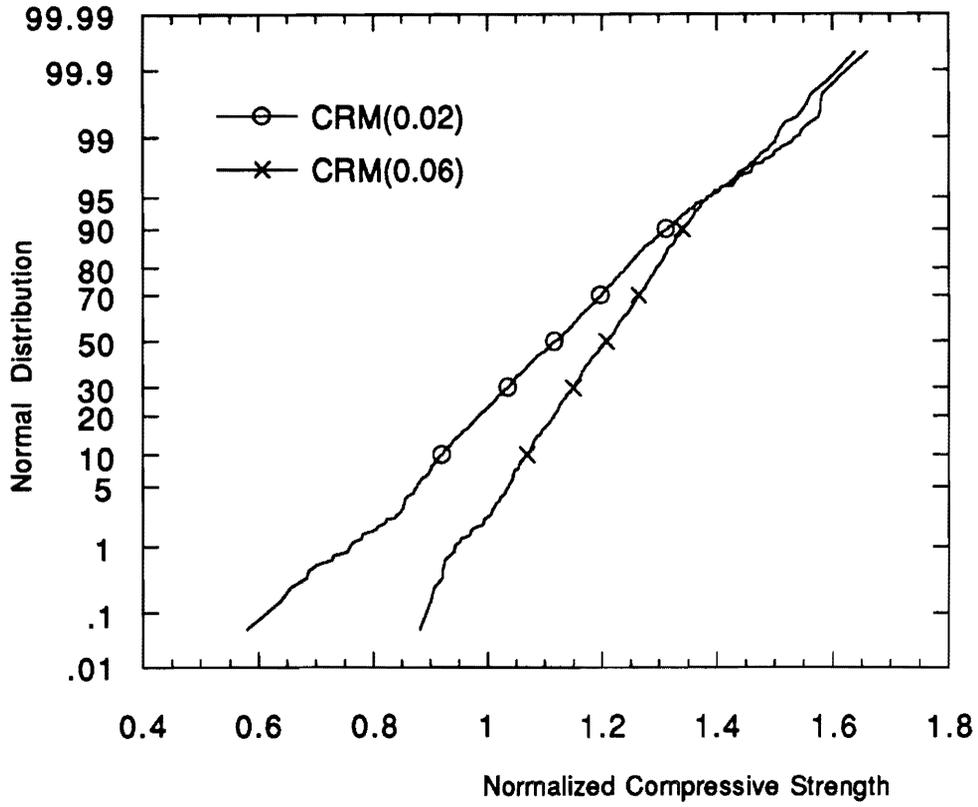


Figure 10. Distribution functions of normalized compressive strength of CRM.

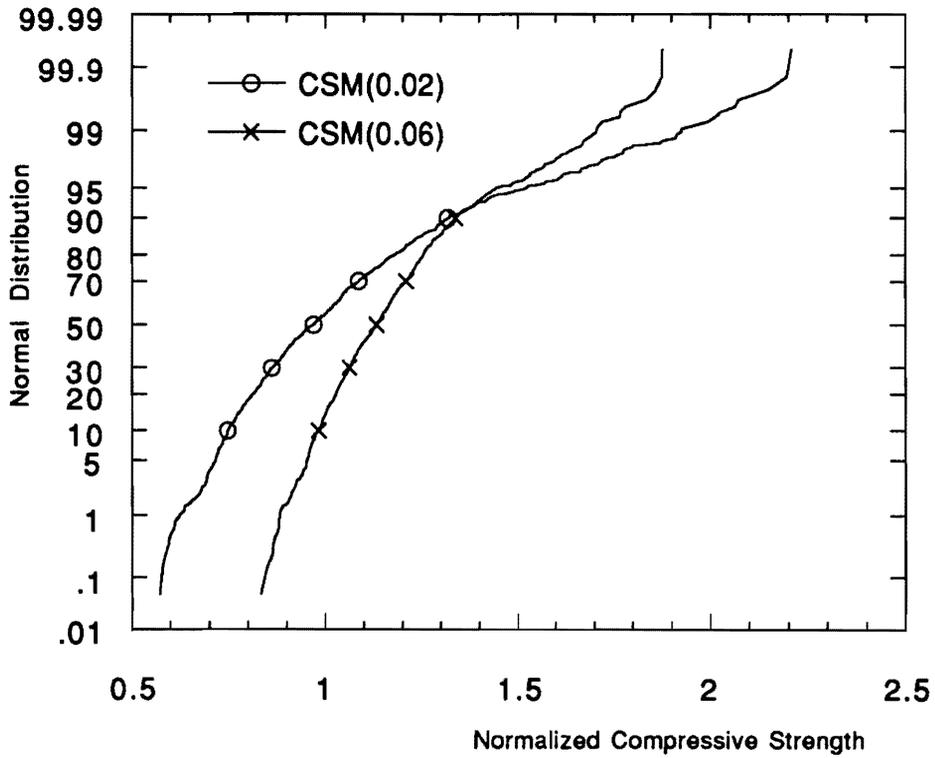


Figure 11. Distribution functions of normalized compressive strength of CSM.

Shear Strength of Beams

Mean-to-Nominal Ratio

Figure 5 presents the variation of λ_{SS} with shear reinforcement in terms of the number of stirrups per effective depth, d/s , for BRM and BSM. Results indicate that λ_{SS} increases with increasing d/s . This is due to the increasing contribution of shear reinforcement to shear strength with increasing d/s . Values of λ_{SS} for BRM are higher than those for BSM. The difference between the curves decreases with increasing d/s .

The survey on design practices showed that stirrups with $d/s = 3$ and diameter of 8 mm can be selected as representative values for the current design practice. At these values λ_{SS} is equal to 1.27 and 1.22 for BRM and BSM, respectively, as listed in Table 4.

Coefficient of Variation

Figure 6 presents the variation of V_{SS} with d/s . Results indicate that V_{SS} decreases with increasing d/s . This is attributed to higher contribution of shear reinforcement in the beam shear strength with closer spacing of stirrups. Values of V_{SS} for BRM are lower than those for the BSM. The difference between the curves decreases with increasing d/s . At $d/s = 3$ and stirrup diameter of 8 mm, the representative values of V_{SS} for BRM and BSM are 13.4 and 15.3 percent, respectively, as listed in Table 4.

Distribution Function

Figure 7 presents the distribution functions of normalized shear strength for BRM and BSM with $d/s = 3$ and stirrup diameter of 8 mm. Results indicate that the distributions are normally distributed. It is clear that high percent of obtained results are higher than the nominal shear strength. This is mainly attributed to the high yield strength of shear reinforcement.

Compressive Strength of Columns

Mean-to-Nominal Ratio

Figure 8 presents the variation of λ_{CC} with ρ_l . Results indicate that λ_{CC} increases with increasing ρ_l which is mainly due to the increasing contribution of longitudinal reinforcement to the compressive strength. Values of λ_{CC} for CSM are less than those obtained for CRM. The difference between the two curves decreases as ρ_l increases. At $\rho_l = 0.02$, the value of λ_{CC} is selected as the representative value. Results are listed in Table 4.

Coefficient of Variation

Figure 9 presents the variation of V_{CC} with ρ_l . Results indicate that V_{CC} decreases with increasing ρ_l . The values of V_{CC} for CRM are lower than those for CSM indicating the effect of concrete quality, expressed in terms of V_c on V_{CC} . The

Table 4. Statistics of Member Limit States.

Limit State	Concrete Type	λ	V%	CDF
Flexure $\rho - \rho' = 0.3\rho_b$	RM	1.23	7	Normal
	SM	1.16	12	Normal
Shear $a/d > 2.5$ $A_v = 2\phi 8$ mm $d/s = 3$	RM	1.27	13.5	Normal
	SM	1.22	15.3	Normal
Axial tied short column $\rho = 2\%$	RM	1.12	14.1	Normal
	SM	1.02	25.1	lognormal

difference between the two curves decreases as ρ_l increases. At $\rho_l = 0.02$, the value of V_{CC} is selected as the representative value. Results are listed in Table 4.

Distribution Function

Figure 10 presents the distribution functions of normalized axial strength for CRM with reinforcement ratio equal to 0.02 and 0.06. Results indicate that the two distributions are normally distributed and V_{CC} with $\rho_l = 0.02$ is higher than that with $\rho_l = 0.06$.

Figure 11 presents the distribution functions of normalized compressive strength for CSM with $\rho_l = 0.02$ and 0.06. Results indicate that the two distributions do not have normal distribution. The Chi-Square test, at 5% significant level, show that the two functions are closer to the lognormal distribution. This is mainly attributed to the effect of the statistical characteristics of SM concrete on the compressive strength of columns.

CONCLUSIONS

In this paper, the basic statistics of reinforced concrete members (beams and short columns) at limit states are investigated. The basic statistics of concrete and reinforcing steel produced in the Kingdom are employed. Results, obtained from simulation process and listed in Table 4, indicate that:

- (1) the mean-to-nominal ratio of flexural, shear and axial strengths for BRM are higher than that for BSM whereas their coefficients of variation for BRM are lower than those BSM. This mainly attributed to the low quality of the SM concrete expressed in terms of low mean-to-nominal ratio and high coefficient of variation.
- (2) The obtained results are sensitive to the reinforcement ratio. High reinforcement ratios adversely affects the statistics of the flexural limit state. This mainly attributed to (a) high probability of brittle failure at high reinforcement ratios and (b) high yield strength of longitudinal reinforcement. For shear and axial limit states high reinforcement ratio improves the strength statistics because the variation in reinforcement yield strength is much lower than that for concrete.
- (3) The distribution functions of the limit states are normally distributed except in case of the compression limit state with SM concrete because (a) the distribution function of the SM concrete is lognormal and (b) the high contribution of concrete strength in the compression limit state of columns.

The statistics obtained in this study and listed in Table 4 are essential to calculate the resistance factors of the different limit states for the design of reinforced concrete structures in the Kingdom of Saudi Arabia.

ACKNOWLEDGEMENT

This paper is part of a study sponsored by King Abdul-Aziz City for Science and Technology under grant numbers AR-12-58. The author would like to express his thanks and appreciation for this support.

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Paper Received 3 August 1996; Revised 18 February 1998; Accepted 8 June 1998.