# A BASIS FOR GENERATING TEST WAVEFORMS FOR DETERMINING THE PERFORMANCE LIMITS OF COMPUTER DISTANCE RELAYS

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## 1. INTRODUCTION

Significant progress has been made over the past few years in the development of digital simulators for the testing of protective relays [1–4]. Prior to the development of digital simulators, the performance of relays was generally evaluated using low- and high-power analog model power systems [5]. In such cases, the testing was limited to the configurations which could be realized using the existing models. With the advent of digital simulators, it is now possible to test the relays for a variety of system configurations. The testing process is not only more comprehensive, but it is also relatively easier and requires considerably less time.

Modern digital simulators provide different options for the generation of waveforms for testing the relays. The test waveforms can be generated in one of the following three ways:

1. By using actual or staged power system faults data recorded by digital fault recorders or other devices.
2. By using programs such as electromagnetic transient program (EMTP) for generating fault waveforms for different system configurations.
3. By manipulating an existing waveform or by generating a completely artificial waveform.

The first method is most appropriate when a sufficient amount of recorded data is available. In the absence of such data or for further evaluation of the relay, the other two methods can be used for generating the test waveforms. When using the second method, however, the question naturally arises as what should be the minimum number of system configurations considered for generating the test waveforms. Obviously, it is not practical to consider a large set of possible configurations to ensure thorough evaluation of the relay under test. With regards to artificially generated waveforms for testing a computer distance relay, there exists no basis for generating such waveforms. Completely random waveforms do not give a realistic evaluation of the relay. Also, as in the case of the second method, it is difficult to determine the required minimum number of test waveforms as well as the nature of such waveforms for proper evaluation of the relay.

The problem of determining the most appropriate waveforms for a limited number of tests is further complicated in the case of computer relays which can implement a variety of algorithms for the same hardware. In order not to increase the number of required test waveforms, the generated waveforms should be a function of both the power system and the specific algorithm used by the relay under test. With this objective, a basis for generating test waveforms for evaluating the performance limits of computer distance relays is presented in this paper. In this study, three different types of simulated digital distance relays are considered. The algorithms used by these simulated relays are based on Fourier-Series, Kalman Filtering, and single PI section transmission line model. Simulation results are presented to demonstrate the effectiveness of the presented approach in determining the performance limits of these three computer distance relays for a specific application. It is shown that, for determining such performance limits, the nature of the required test waveforms is dependent on both the power system and the relay algorithm.

## 2. PROBLEM FORMULATION

The objective of the presented study is to determine the waveforms that will cause maximum error in the estimated distance to fault by the computer distance relay under test. The value of the maximum error will quantitatively determine the performance limits of the tested relay. Since it is not possible to consider all possible system configurations and fault conditions for determining such worst-case waveforms, it is imperative that the problem is to be formulated as a non-linear optimization problem. This requires that all possible system configurations could be represented by a set of "design variables" and infeasible configurations are automatically eliminated by imposing certain "constraints".

Let $\mathbf{X}$ be a vector of design variables. Let $F(\mathbf{X})$ be a function of these design variables which is to be optimized. This function is commonly known as the objective function. The optimization problem in its general form is then stated as follows:

Minimize ( or maximize) $F(\mathbf{X})$

subject to :

$$g_i(\mathbf{X}) \le 0 \qquad i = 1,2,....,m \tag{1}$$

$$X_j^L \le x_j \le X_j^U \qquad j = 1,2,....,n$$

where $g_i$ are '$m$' number of inequality constraints, and $X_j^L$ and $X_j^U$ are the lower and upper bounds on the '$n$' number of design variables. Since in the presented study, a change in the values of design variables will result in a change in the shapes of the generated waveforms, these variables will be referred to as the shape variables. A description of the objective function, the shape variables, and the constraints for the problem under consideration is given below.

### Objective Function

Consider a transmission line of length $L$ protected by a distance relay at one of its ends. Let $L_F$ define the length of the line from the relay location to the fault point, whereas $L_C$ defines the length of the line to the fault point as calculated by the relay. Let $L_R$ define the length of the line covered in the first zone, *i.e.* first-zone reach of the relay. The objective function is then defined as follows:

$$F(\mathbf{X}) = L_C(\mathbf{X}) - L_R\,. \tag{2}$$

In case of internal faults, the value of the objective function shall be maximized so that, depending upon the distance to the fault point, the relay may underreach and the internal fault would then appear to the relay as an external fault. For different values of $L_F$, worst-case waveforms, corresponding to the maximum value of the objective function, could therefore be determined. On the other hand, in case of external faults, the value of the objective function is minimized so that the relay may overreach and the external fault would then appear to the relay as an internal fault. Once again, worst-case waveforms, corresponding to the minimum value of the objective function, could be determined for external faults.

### Shape Variables

These are independent variables which affect the shapes of the generated test waveforms. The following shape variables were considered:

1. Distance to fault location from the relay terminal.
2. Time of fault initiation.
3. Time difference between the fault initiation and the first sample taken.
4. Fault resistance.
5. Values of system parameters for different configurations.

Each of these variables can acquire a value within a certain range specified by its lower and upper bounds in accordance with Equation (1).

### System Constraints

These constraints are functions of the shape variables and must not exceed specified limits. The nature of these constraints depends on the particular power system under consideration. The constraints considered in this study will be discussed later in Section 5 of the paper.

## 3. GENERAL PROCEDURE

The procedure for the generation of waveforms for determining the performance limits of a computer distance relay is explained with the help of a block diagram shown in Figure 1. The control program module shown in this figure supplies the most recent values of various shape variables to the transient analysis module. On the basis of these values, the transient analysis program generates sampled (discretized) data for pre- and post-fault current and voltage waveforms. These waveforms are filtered by a low-pass digital filter and applied to the simulated computer relay. The simulated relay estimates the fault distance and passes this information back to the control program which calculates the values of various constraints required by the optimization program. The optimization program will then update the values of shape variables in a way that waveforms generated in the next iteration cycle will increase the error in the calculated value of fault distance. This iterative process continues until convergence is achieved. At convergence, the sampled values of the voltage and current waveforms are output to a file. A more detailed description of various blocks shown in Figure 1 is given below.

### Control Program

This program interacts with the optimization program, the transient analysis program, and the simulated computer relay. It obtains the updated values of the shape variables from the optimization program and passes them on to the transient analysis program. It also receives the estimated value of fault distance from the simulated relay, calculates the values of various constraints, and passes them back to the optimization program.

### Transient Analysis Program

This program models the power system and, based upon the most recent values of shape variables received from the control program, it determines the transient response of the system at the relay terminal considering errors introduced by CT's and CCVT's. The sampled values of the relay current and voltage waveforms are then passed on to the digital low-pass filter.

### Digital Low-Pass Filter

The purpose of this filter is to remove high frequency components beyond the cut-off frequency of the relay under test. The cut-off frequency of the relay is based on the sampling rate used by the relay.

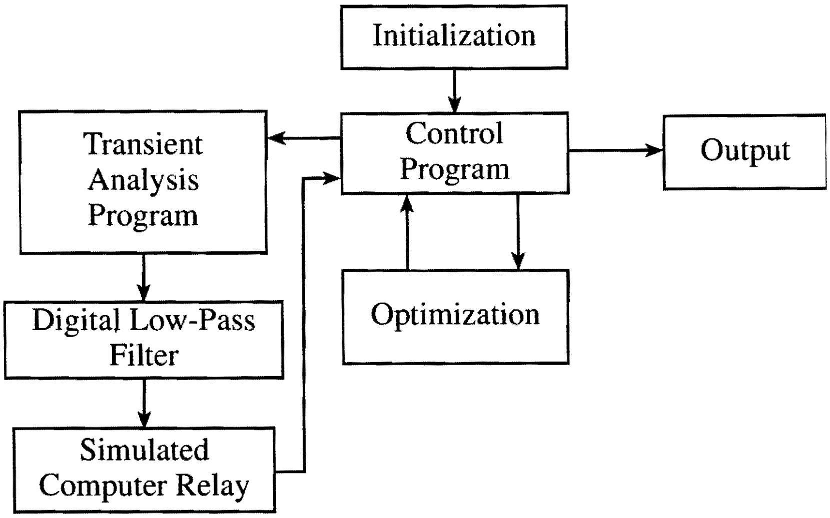


*Figure 1. Block Diagram Representation of Present Approach.*

### Simulated Computer Relay

This module basically simulates the algorithm used by the computer relay under test. The values of cut-off frequency, data-window, and sampling rate are identical to those of the actual relay. On the basis of sampled values of relay voltage and current signals, as received from the transient analysis program, it calculates the apparent distance to fault and passes this value back to the control program.

### Optimization Program

This program receives the following information from the control program:

1. Error in the calculated value of fault distance (objective function).
2. Values of various system constraints.

Based on this information, the optimization program updates the values of shape variables in order to further increase the error in the calculated value of fault distance. The updated values are then supplied to the control program which passes them on to the transient analysis program.

### Output

Depending upon the termination criterion, the optimization program will terminate when the desired fault and system conditions have been determined which result in maximum error in the calculated value of fault distance by the simulated relay. At convergence, the sampled values of relay voltage and current signals are stored in an output file. This data can be later used by a digital simulator to test the performance of the actual relay for which the worst-case waveforms were generated using a representative simulated relay.

## 4. ARTIFICIALLY GENERATED WAVEFORMS

The procedure described above is quite general and depending upon the model of the power system used in the transient analysis program, it can provide realistic worst-case waveforms for determining the performance limits of a computer distance relay. The same procedure can also be used for artificial generation of worst-case waveforms by replacing the "transient analysis program" with a "transient generation program" described in the following.

### Transient Generation Program

The input to this program consists of shape variables as specified above in the general procedure with the only difference that some of the shape variables related to the model of the power system will be replaced by new shape variables specifying the nature of transient components in the artificially generated current waveforms and the values of percentage errors introduced by CT's and CCVT's. Each of the transient components is considered as a damped sinusoid specified in terms of its amplitude, frequency, phase angle, and time constant [6]. For instance, the $i$th transient component $\Psi_i$ can be represented in the general form as follows:

$$\Psi_i = A_i e^{-t/\tau_i} \sin\left(w_i t + \theta_I\right) , \tag{3}$$

where $A_i$ is the amplitude, $w_i$ is the frequency (radians per second), $\theta_I$ is the phase angle (radians), and $\tau_i$ is the time constant of the $i$th transient component. The above equation is in a general form which can also represent a decaying DC-offset component for $w_i = 0$ and $\theta_I = \pi/2$, as given below:

$$\Psi_i = A_i e^{-t/\tau_i} . \tag{4}$$

The transient components are added to the post-fault fundamental frequency current component. Using the resultant current signal, the relay voltage waveforms are determined. The sampled values of the generated waveforms are then supplied to the simulated computer relay for determining the apparent distance to fault point.

Since the amplitude, frequency, phase angle, and time constant values for each of the transient components are specified independently, it is important to impose a constraint on the total harmonic distortion distortion [7] of the resultant current signal defined as follows:

$$\xi = \frac{1}{I_0}\sqrt{\sum_{j=1}^{N} I_j^2}\ , \tag{5}$$

where, $N$ is the total number of transient components in the current signal, $I_0$ is the amplitude of the fundamental component, and $I_j$ is the amplitude of the $j$th transient. The constraint on total harmonic distortion is then given as,

$$\xi \le \xi_{MAX} \tag{6}$$

where $\xi_{MAX}$ is the maximum possible total harmonic distortion. In addition to the constraint on total harmonic distortion, other constraints are imposed based on the representation of the system model in the transient generation program so that infeasible system configurations are not considered in the optimization process. All these constraints are defined as functions of the shape variables.

## 5. IMPLEMENTATION

The optimization problem defined by Equation (1) is a constrained non-linear optimization problem and can be solved by a number of techniques. For this study, the method of feasible direction has been used. A general purpose computer program [8] based on this method was utilized for optimization. The control program, the transient generation program, and programs simulating the three computer relays were all written in FORTRAN 77 and linked together to form an integrated package. A brief description of the three distance relays simulated for this study is given below.

### Fourier-Series Based Distance Relay (FS Relay)

This relay uses one-cycle Fourier-series algorithm [9] for calculating the amplitude and phase angle of the fundamental frequency components of current and voltage signals. On the basis of the calculated values it finds the magnitude and phase angle of the impedance to fault point, and accordingly determines the apparent distance to fault point. The algorithm assumes that all transient components are periodic. It does not take into account the time-decaying attribute of the transient components. As a result of this assumption, unexpected relay operations may occur if the transient components have shorter time constants.

### Kalman Filter Based Distance Relay (KF Relay)

The simulated Kalman Filter (KF) distance relay utilizes a two-state KF for voltage estimation and a three-state KF for current estimation [10,11]. The initial estimate of the state vector and its initial error covariance matrix are considered to be known. The relay determines the minimum mean-squared error estimate of the state vector for the fundamental frequency components of both voltage and current signals. It then calculates the estimated value of distance to fault point.

A Kalman filter based relay may not perform satisfactorily if unmodeled transient components are present in the relay input signals. Although, higher order KF relay [12] could be utilized to model more transient components, this was not considered necessary, because the objective of this study is not to compare the performance of the three relays, but to demonstrate that the nature of test waveforms are dependent on the type of computer relay under test.

### PI-Model Based Distance Relay (PM Relay)

The algorithm for this relay assumes that the transmission line is modeled as a lumped single $\pi$ section for the portion of the line from the relay location to the fault point [13]. Furthermore, the far-end capacitance at the fault point is ignored by assuming that the fault resistance is negligible. Both these assumptions will affect the performance of the relay.

### Generation of Test Waveforms

The results presented in the following section are for simulated relays only, because the actual relays were not available for real-time testing. For the generation of artificial test waveforms, a 400 kV, 250 km long two-terminal transmission line was considered [14]. At each end of the line, the system behind the bus was represented by its Thevenin equivalent. The voltage magnitudes and phase angles of the equivalent Thevenin sources as well as the equivalent resistances and reactances were considered as shape variables with specified lower and upper bounds. In addition, since the equivalent resistance and reactance values were allowed to vary independently to represent different system configurations, bounds were specified for the resultant $X$ to $R$ ratios in accordance with the system under considerations. A constraint was also imposed on the maximum possible total harmonic distortion. For the computer distance relays, values of the sampling rate, data-window, and percentage reach of the first zone were specified.

## 6. RESULTS

The results of three simulation experiments are described in the following. These results are for a sampling rate of 32 samples per cycle and a data-window of one cycle. Also, the cut-off frequency of the digital low-pass filter was taken as 720 Hz. For each simulation experiment, the non-linear optimization was carried out a number of times starting with different initial values of shape variables in order to circumvent the problem of getting stuck in a local optimum.

### Simulation Experiment #1

For each of the three distance relays the first-zone reach was set to 85% of the line length. An external single-line-to-ground (SLG) solid fault was applied just beyond the end of the line. Test waveforms were generated artificially considering a damped third harmonic component added to the fundamental current component. The amplitude of the third harmonic was allowed to vary between 10% to 30% of the post-fault fundamental current component. The time constant of this exponentially decaying component could take any value between 15 and 30 ms. The fault initiation time $\Gamma_f$ was considered to be in the range of one to two cycles. The value of the lower bound was selected to ensure that at least one cycle of pre-fault data is available. The upper bound of two cycles would allow the coverage for fault initiation at any point on the waveform. The values of lower and upper bounds on the phase angle $\theta$ were –3.14 and +3.14 radians, respectively, thus covering all possible values of the phase angle.

The values of lower and upper bounds on various constraints are dependent upon the particular power system under consideration [14]. For this study, the $X$ to $R$ ratios at the source and load ends ($X_s/R_s$ and $X_L/R_L$ , respectively) were permitted to vary between 5 and 15. The maximum possible value of total harmonic distortion $\supset_{\mathrm{MAX}}$ was limited to 25%. Errors due to CT's and CCVT's were considered to be in the range of ±2%. The worst-case (optimized) test waveforms were generated for the three simulated relays. The results are summarized in Table 1. In this table, $A_1$, $\theta_1$, and $\tau_1$ represent the amplitude, phase angle, and time constant of the damped third harmonic component present in the post-fault current waveform. As shown in Table 1, the values of optimized shape variables for the three relays are not identical, except for the value of time constant $\tau_1$ which attained its lower bound value for all three relays. Also, for PM relay the optimal value of fault initiation time $\Gamma_f$, which is defined in terms of the number of prefault cycles, is notably different from the other two cases. It is to be noted that the values of the shape variables given in Table 1 for the three relays are their optimal values for the specified noise consisting of only the damped third harmonic. These optimal values, which define the shape of the worst-case waveforms for relay testing, will be different for other noise conditions such as those considered in simulation experiment #2. However, the technique presented here is quite general and optimal values of shape variables can be determined for any combination of noise components.

The optimal values of shape variables obtained for each relay are for different system configurations, as is clear from the $X/R$ ratios given in Table 1. The values shown in the table are the optimal values of $X/R$ ratios which correspond to system configurations for which the worst-case waveforms for the three relays were obtained for the specified noise.

**Table 1. Optimized Values of Shape Variables and Constraints for Three Relays.**

| Shape Variable/ Constraint | Lower Bound | Upper Bound | Optimized value for F.S. Relay | Optimized value for K.F. Relay | Optimized value for P.M. Relay |
|---|---|---|---|---|---|
| $\Gamma_f$(cycles) | 1.0 | 2.0 | 1.16 | 1.23 | 1.44 |
| $A_1$ (%) | 10.0 | 30.0 | 24.7 | 24.4 | 24.6 |
| $\theta_1$ (radians) | –3.14 | 3.14 | 0.47 | 0.50 | 0.55 |
| $\tau_1$ (msec) | 15.0 | 30.0 | 15.0 | 15.0 | 15.0 |
| $X_s / R_s$ | 5.0 | 15.0 | 13.33 | 13.86 | 12.25 |
| $X_L / R_L$ | 5.0 | 15.0 | 6.69 | 9.45 | 14.56 |

The operations of these relays for optimized test waveforms are summarized in Table 2, which clearly shows the importance of generating worst-case waveforms specific to the relay under test. When the test waveforms optimized for the worst-case performance of either FS or KF relay were applied to all three relays, both FS and KF relays operated incorrectly by tripping on an external fault, while the PM relay operated correctly by restraining on an external fault. On the other hand, when the test waveforms optimized for the worst-case performance of PM relay were applied to the three relays, FS and KF relays operated correctly while the PM relay operated incorrectly. This is because each of these relays is susceptible to incorrect operation for those noise components which cannot be adequately filtered out by the relay algorithm. For example, the FS relay algorithm is not appropriate for handling significant amount of dacaying harmonics present in the input waveforms, especially if the time constants have shorter values. Similarly, for noise components which are not modeled in the state-transition matrices of the KF relay, the relay will respond poorly if such components are present in significant amount in the input waveforms. On the other hand, the PM relay considered here is based on a simplified model of the transmission line consisting of a single lumped $\pi$ section and ignores the far-end capacitance. This assumption causes an error in the calculated distance to fault, especially when the fault is near the end of the line. The optimization program exploits the weakness of each relay algorithm and generates the test waveforms which will cause maximum error in the relay reach.

Test waveforms generated for one relay may also cause a maloperation for another relay, as can be seen from Table 2 where, for example, waveforms optimized for FS relay will also cause incorrect tripping when applied to the KF relay. This is possible because even though the reach error for the other relay is relatively small, it may be sufficient to cause overreaching. However, as noted from the recorded values of maximum errors in the calculated distance to fault, the reach error for a relay is maximum when the applied input waveforms are those optimized for that particular relay.

The results presented in Table 2 are for the simulated KF and PM relays as described in Section 5 above. Obviously, higher-order KF relays and multi-$\pi$ section PM relays will have different worst-case waveform shapes as compared to those obtained for the presently used KF and PM relays. The performance of those relays would accordingly be determined for their own worst-case waveforms. In general, for any digital relay, the performance limits are dependent on not only the power system but also on the particular type of the relay under test.

### Simulation Experiment #2

This simulation experiment was performed to study the response of the three relays for an internal SLG fault. The system data and the relay reach were kept the same as in simulation experiment #1. Also, there was no change in the values of various constraints. However, additional transient components were allowed to be included in the generated waveforms. The purpose of adding these additional noise components was to demonstrate that the presented approach could generate worst-case waveforms comprising of a number of noise components by determining the optimal values of amplitude, phase angle and time constant for each noise component. For this study, the additional components consisted of damped fifth and seventh harmonics. For each of these damped harmonics, bounds on amplitude, phase, and time constant were specified. Worst-case waveforms for the three relays were determined for a fault applied at 70% of the line length. The shapes of these waveforms correspond to the optimal values of shape variables as given in Table 3. It is observed that the shape variable values for the three relays are different from one another, especially for the case of PM relay as compared to the two other relays. This is due to the fact that both FS and KF relays make some assumptions regarding the fault waveforms, whereas the PM relay makes assumption for the transmission line model.

The worst-case waveforms of unfiltered current waveforms obtained using the optimal values of shape variables are shown in Figure 2 for FS and PM relays. The figure also shows the unfiltered voltage waveforms for these relays as determined from the generated worst-case current waveforms considering the corresponding system configuration behind the bus at the relay end. These shapes, especially those of voltage waveforms, clearly show that for a given system the worst-case waveforms are different for different relays.

**Table 2. Operation of Relays for Three Different Optimized Waveforms.**

| Waveforms optimized for | Relay response for an external fault | | |
|---|---|---|---|
| | FS Relay | KF Relay | PM Relay |
| FS Relay | Relay trips | Relay trips | Relay restrains |
| KF Relay | Relay trips | Relay trips | Relay restrains |
| PM Relay | Relay restrains | Relay restrains | Relay trips |

### Simulation Run #3

For most of the simulation runs performed in this study, it was observed that the time constants of the optimized transient components were restricted to values determined by their specified bounds. The objective of this simulation experiment was to analyze the sensitivity of the relay response to the time constant of a transient component. For this purpose, a simple case of damped third harmonic transient was considered. For different values of lower bound on the time constant, worst-case (optimum) waveforms were generated for internal faults at 70% of the line length for FS and KF relays. The results are plotted in Figure 3, which shows the variation of optimum value of the objective function with respect to the lower bound on time constant. In this figure, positive value of the objective function corresponds to incorrect operation (underreaching) of

**Table 3. Parameters of Optimized Transient Components.**

| Waveforms Optimized for | Frequency (Hz) | Amplitude (%) | Phase (Radians) | Time constant (msec) |
|---|---|---|---|---|
| FS Relay | 180 | 13.2 | 0.54 | 30 |
| KF Relay | 180 | 13.8 | 0.48 | 30 |
| PM Relay | 180 | 22.7 | 0.51 | 30 |
| FS Relay | 300 | 7.2 | 0.46 | 12 |
| KF Relay | 300 | 8.5 | 0.49 | 12 |
| PM Relay | 300 | 7.1 | 0.41 | 20 |
| FS Relay | 420 | 15.1 | 0.52 | 10 |
| KF Relay | 420 | 12.5 | 0.50 | 10 |
| PM Relay | 420 | 5.2 | 0.56 | 10 |

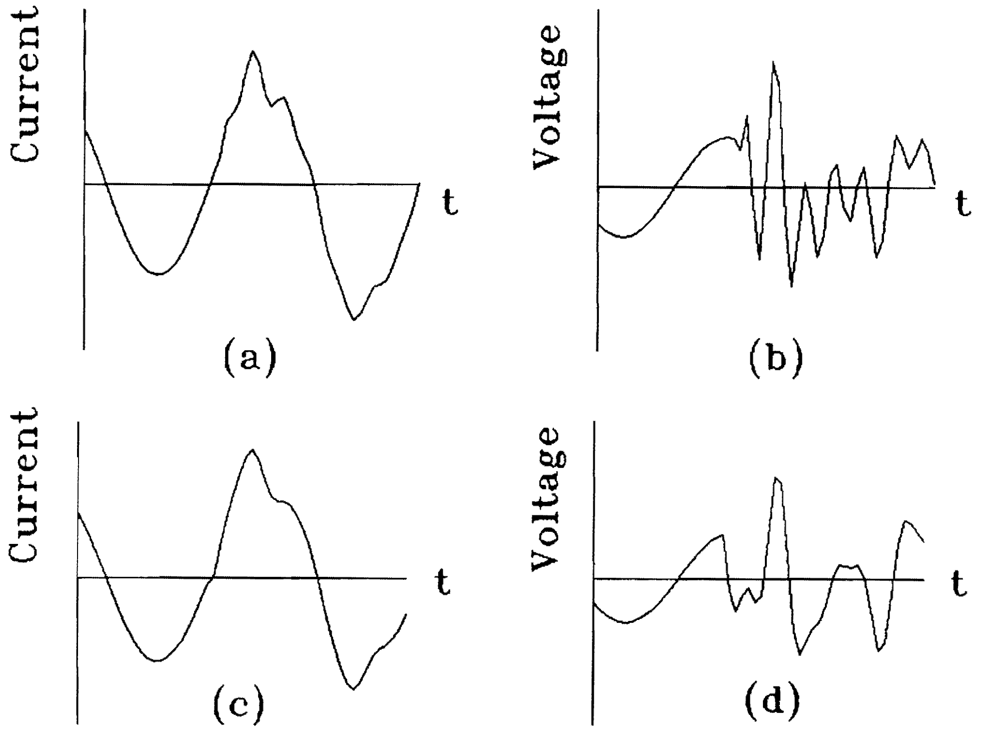


*Figure 2. Unfiltered Optimized Waveforms for FS and PM Relays.* (a), (b): *FS Relay;* (c), (d): *PM Relay.*

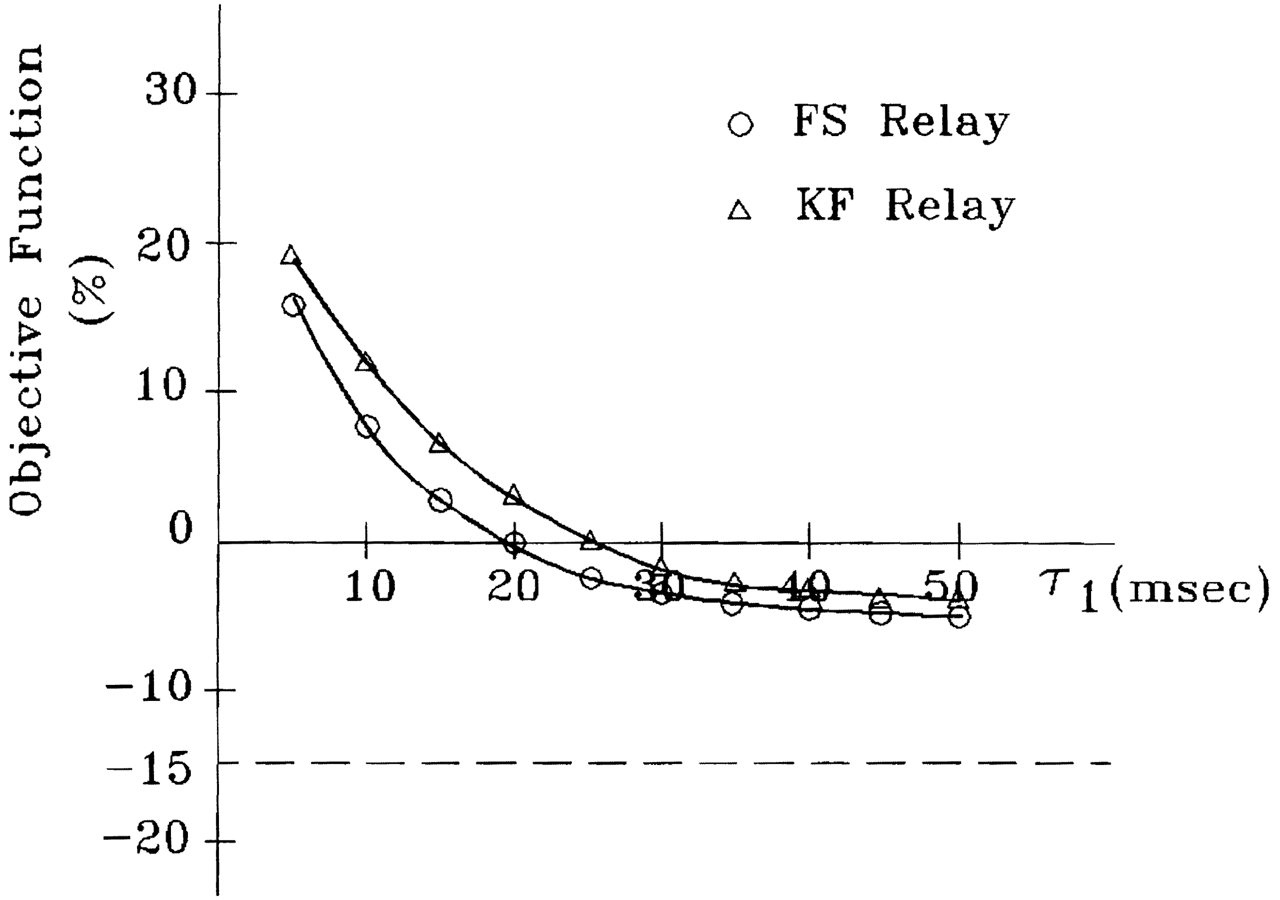


*Figure 3. Optimal Sensitivity Analysis for Time Constant of Damped Transient Component.*

the relay, while the negative value indicates correct operation (tripping on an internal fault) of the relay. The dotted horizontal line shown in the figure indicates the value of objective function for which the calculated distance to fault equals the actual distance. It is obvious that in this particular case of single transient component, the relay response shifts towards the boundary of incorrect operation (zero-reference line) when the value of lower bound on the time constant of the transient component is reduced. For FS relay, the curve cuts the zero-reference line at $\tau_1 = 20$ msec, whereas for KF relay this zero-crossing is for $\tau_1 = 25$ msec. This means that for FS relay there is no possibility of relay maloperation for $\tau_1 > 20$ msec. The same is true for KF relay for $\tau_1 > 25$ msec. Similar sensitivity curves can be drawn for time constants of other transient components present in the post-fault waveforms. It is to be noted that these sensitivities are of the optimum solution and not simply the response of individual parameters.

## 7. CONCLUSIONS

A methodology for generating test waveforms for quantitatively determining the performance limits of computer distance relays is presented. In contrast to the normal practice of generating a number of waveforms for various power system configurations for testing the relay response, the presented approach utilizes mathematical programming to determine the worst-case waveforms taking into account both the power system configurations for a particular application as well as the operating algorithm of the computer relay. The procedure is also applicable to the generation of artificial waveforms for evaluating the performance limits of a computer distance relay. Results are presented for three different types of simulated digital distance relays. The results confirm that the procedure for generating test waveforms for a given application must take into account the relay algorithm for determining the performance limits of the relay under test.

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