DETERMINATION OF OPTIMUM SECOND DERIVATIVE WEIGHT COEFFICIENT SETS FOR VARIOUS GRID SYSTEMS

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الخلاصـة :

يصفُ هذا البحث إمكانية التقدير الأمثل للمشتقة الثانية لمجموعة معاملات الوزن لعدة أنظمة حلقات ذات استجابات السِّـعة مقارب للسعة النظرية لعمليات المشتقة الثانية . وقد تـمَّ اشتقاق مجموعات معاملات الوزن لأنظمة الحلقات على النحو التالي : اقتراض قيمة الجاذبية الدائرية لكل دائرة $(1/r_m^n)$ حيث (r_m) هي نصف قطر الدائرة ، و (n) هي عدد حقيقي لكل من قيم (n) .

وتـمَّ حساب معامل المضاهاة بين سعة كلٍّ مجموعة معاملات وسعة المشتقة الثانية النظري . باستخدام أعلىٰ معامل مضاهاة كفيصلٍ لتحديدِ أمثل مجموعة معاملات لكلٍّ نظام حلقات على حدة وعلاوة على ذلك تـمَّ استنتاج مجموعة معاملات جديدة لحساب المشتقة الثانية باستخدام أقل عددٍ من الدوائر لحساب متوسط قيمة معامل الوزن وفي نفس الوقت تُـعطي أحسن النتائج ، وتفوقُ هذه المجموعة الجديدة على المجموعات السابقة موضحٌ في هذا البحث . هذا وقد تـمَّ تطبيق الطريقة بنجاح على خريطة (بوجير) التثاقلية لجسم مِلْحي بمنطقة خليج السويس بمصر .

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ABSTRACT

This paper describes a possible approach to determine the optimum second derivative weight coefficient sets for various ring systems with amplitude responses approximating the theoretical response of the second derivative operation.

Weight coefficient sets for many ring systems are derived as follows. In each system, the average radial gravity for each circle is given a weight of $1/r_m^n$, where r_m is the radius of circle and n is a real number. For each n value, the overall similarity between the calculated amplitude response of each derived set and the theoretical response of the second derivative operation is determined by computing the correlation coefficient between the mapped variables. Similarity between the calculated and the theoretical responses measured by the highest correlation may generally be considered a criterion for determining the optimum n and consequently the optimum weight coefficient set for second derivative. For a given ring system, the derived coefficient set by this technique is considered the best one.

Moreover, to calculate the second vertical derivative, a new weight coefficient set, which uses the least number of circles to obtain average gravity values and at the same time yields best results has been developed. To test its validity, the proposed method has been applied to the Bouguer gravity anomaly in the Central Salt Province of the Gulf of Suez region, Egypt.

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INTRODUCTION

In gravity interpretation, second derivative maps play an important role in delineating the structures of interest. Generally, all such methods are based on potential theory to calculate the second derivative of a gravity anomaly [1 – 16]. Practical applications involve summation of a number of products of the average gravity values over circles of different radii with the corresponding weight coefficients.

The general expression for calculation of the second vertical derivative can be written as:

$$s^2\left(\frac{\partial^2 g}{\partial z^2}\right) = \sum_{m=0}^{M} c_m\, \bar{g}(r_m), \tag{1}$$

with

$$\sum_{m=0}^{M} c_m = 0,$$

where $\bar{g}(r_m)$ is the average gravity value over a circle of radius r_m and s is the grid spacing. The function $\bar{g}(r_m)$ can be expanded into a Taylor series

$$\bar{g}(r_m) = a_0 + a_2 r_m^2 + a_4 r_m^4 + a_6 r_m^6 + \ldots \tag{2}$$

where

$$a_0 = \bar{g}(0),\ a_2 = -\frac{1}{4}\left(\frac{\partial^2 g}{\partial z^2}\right),$$

$$a_4 = \frac{1}{64}\left(\frac{\partial^2 g}{\partial z^2}\right).$$

For numerical computation of the second vertical derivative, the infinite series is truncated after a certain number of terms and the resulting expression is solved by the method of least-squares and the corresponding coefficient for a_2 is multiplied by -4 to yield coefficients for second derivative.

Careful study of the Tables published by Pick and others [17], Nettleton [18], and Meskó [14] indicates that there are several weight coefficient sets derived by different approaches using the same geometry of circles and grid pattern. As an example, Rao and others [10] used 5 different ring systems and for each system they obtained six different sets of coefficients, using Peters' [1] and Elkins' [3] approaches. Thus, it is evident that any number of coefficient sets can be developed for the same ring system, depending on the allocation of weights. Using Fourier transformation, it is possible to compare operators by their frequency characteristics and accordingly to accept or reject the coefficients [11, 14, 19 – 21].

However, an unescapable question that may need to be answered is: what is the optimum weight coefficient set which can be derived, following a certain approach for a ring system; whose amplitude response is the best approximation to the theoretical amplitude response of the second derivative operation $h_t(u,v)$ (Figure 1); and which, when applied to the Bouguer gravity anomaly data, yields the best results. The aim of the present paper is to find an objective criterion for the selection of such an optimum weight coefficient set.

SOLUTION BY CORRELATION FACTOR DETERMINATION BETWEEN THE CALCULATED AND THE THEORETICAL AMPLITUDE RESPONSES

The grid systems considered in the present study consist mainly of circles as defined by many authors including those given by Peters [1], Henderson and Zietz [2], Elkins [3], Baranov [4], Rosenbach [5], Paul [7], Darby and Davies [8], and Rao and others [10]. They differ in the number of the circles and their radii. The first ring system S1 consists of the central point and the circles of radii 1, $\sqrt{2}$, and 2 while the second ring system S2 is defined by the central point and the circles of radii 1, $\sqrt{2}$, and $\sqrt{5}$; and similarly the other systems, S3, S4, ..., S12, are also defined in Table 1.

The average radial gravity for each circle, in each ring system, is also given a weight of $1/r_m^n$ except in the case of $\bar{g}(0)$, the anomaly at the origin, which is given a unit weight. If we use Peters' approach [1], then the equation obtained with a weight of $1/r_m^n$ to all circles giving preference to the central point is given as

$$\frac{\bar{g}(r_m)}{r_m^n} = \frac{\bar{g}(0)}{r_m^n} + \frac{a_2}{r_m^{n-2}} + \frac{a_4}{r_m^{n-4}}, \tag{3}$$

where n is a given real number.

The unknown a_2 and a_4 in Equation (3) can be obtained by direct minimization of

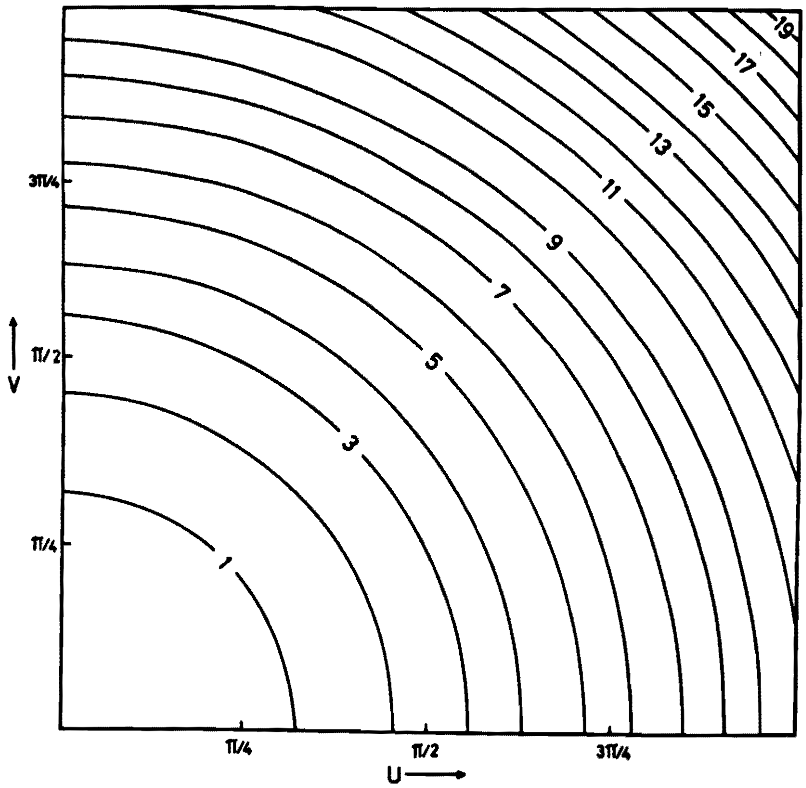

Figure 1. Theoretical Amplitude Response $h_r(u, v)$ *where* $u/2\Pi$ *and* $v/2\Pi$ *are the Frequencies in Cycles per Unit of Length in the* x *and* y *Directions, Respectively.*

$$\varphi = \sum_{m=1}^{M} \left[\frac{\bar{g}(r_m)}{r_m^n} - \frac{\bar{g}(0)}{r_m^n} - \frac{a_2}{r_m^{n-2}} - \frac{a_4}{r_m^{n-4}} \right]^2 = \min. \tag{4}$$

Alternatively, the minimization of Equation (4), *i.e.*, $(\partial\varphi/\partial a_2) = 0$ and $(\partial\varphi/\partial a_4) = 0$ will lead to the solution of two linear equations:

$$\left(\frac{\partial\varphi}{\partial a_2}\right) = \sum_{m=1}^{M} \left[\frac{\bar{g}(r_m)}{r_m^n} - \frac{\bar{g}(0)}{r_m^n} - \frac{a_2}{r_m^{n-2}} - \frac{a_4}{r_m^{n-4}} \right] \frac{1}{r_m^{n-2}} = 0 \tag{5}$$

and

$$\left(\frac{\partial\varphi}{\partial a_4}\right) = \sum_{m=1}^{M} \left[\frac{\bar{g}(r_m)}{r_m^n} - \frac{\bar{g}(0)}{r_m^n} - \frac{a_2}{r_m^{n-2}} - \frac{a_4}{r_m^{n-4}} \right] \frac{1}{r_m^{n-4}} = 0 \tag{6}$$

From Equations (5) and (6), the following two equations are derived after simplification:

$$a_2 \sum_{m=1}^{M} r_m^{4-2n} + a_4 \sum_{m=1}^{M} r_m^{6-2n} = \sum_{m=1}^{M} \bar{g}(r_m)\, r_m^{2-2n} - \bar{g}(0) \sum_{m=1}^{M} r_m^{2-2n}, \tag{7}$$

and

$$a_2 \sum_{m=1}^{M} r_m^{6-2n} + a_4 \sum_{m=1}^{M} r_m^{8-2n} = \sum_{m=1}^{M} \bar{g}(r_m)\, r_m^{4-2n} - \bar{g}(0) \sum_{m=1}^{M} r_m^{4-2n}, \tag{8}$$

respectively.

By solving these two equations for a_2 and then multiplying by -4, the second vertical derivative is determined.

Table 1. Optimum Weight Coefficient Sets for Various Ring Systems SI.

Grid system	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12
Optimum n	3.25	3.75	4.00	4.50	4.25	4.25	4.50	4.75	4.25	4.50	4.50	4.50
Correlation	0.96929	0.95412	0.96127	0.95922	0.95411	0.95427	0.95834	0.95728	0.95423	0.95437	0.95441	0.95455
Radii of the Circles	Optimum Coefficient Sets											
0	4.92457	4.82593	4.86998	4.79612	4.65548	4.63405	4.65015	4.59886	4.42261	4.43370	4.37484	4.30874
s	−5.13218	−5.06482	−5.10109	−5.05290	−4.82908	−4.80121	−4.79459	−4.74078	−4.44007	−4.33592	−4.43237	−4.31785
$s\sqrt{2}$	−0.15087	0.04321	−0.03110	0.06928	0.05203	0.05138	−0.00920	0.01713	−0.06839	0.02686	−0.00836	−0.05061
$2s$	0.35848		0.14774	0.09732			0.07248	0.05620				
$s\sqrt{5}$		0.19568	0.11446	0.06545	0.08810	0.08752	0.04997	0.03623	0.05734	0.05325	0.04622	0.03815
$s\sqrt{8}$				0.02472			0.01936	0.01234				0.01512
$s\sqrt{8.5}$					0.03347				0.02287	0.01781	0.01569	
$3s$								0.00925				
$s\sqrt{9.23}$						0.02826						
$s\sqrt{10}$							0.01183	0.00711				
$s\sqrt{13}$								0.00365				0.00513
$s\sqrt{17}$									0.00563	0.00361	0.00321	
$5s$												0.00110
$s\sqrt{34}$										0.00068	0.00061	
$s\sqrt{50}$												0.00020
$s\sqrt{58}$											0.00016	
$s\sqrt{136}$												0.00002
$s\sqrt{274}$												0.00000

$$\frac{\partial^2 g}{\partial z^2} = -4a_2 = 4\left[\sum_{m=1}^{M} \bar{g}(r_m) \times \frac{(r_m^{4-2n}C - r_m^{2-2n}D)}{(B.D - C^2)}\right] + 4\bar{g}(0)\frac{(A.D - B.C)}{(B.D - C^2)} \quad (9)$$

where

$$A = \sum_{m=1}^{M} r_m^{2-2n}, \quad B = \sum_{m=1}^{M} r_m^{4-2n},$$

$$C = \sum_{m=1}^{M} r_m^{6-2n}, \quad D = \sum_{m=1}^{M} r_m^{8-2n}.$$

Comparing Equation (9) with Equation (1), we can calculate the required coefficients for computing the second derivative for any value of n and for any ring system. They are given as

$$c_0 = 4\frac{A.D - B.C}{B.D - C^2},$$

and

$$c_m = 4\frac{r_m^{4-2}.C - r_m^{2-2n}.D}{B.D - C^2}. \quad (10)$$

For each ring system of circles r_m, several weight coefficient sets are computed for second derivative, using Equation (10), and for different n values ranging from 2 to 5.5 with 0.25 steps. This provided a total number of 15 coefficient sets.

Each weight coefficient set thus obtained is subjected to frequency analysis, using the average of finite number of points [19]. Here, the amplitude response of each coefficient set is computed using 169 points on a square grid with a spacing of $\Pi/12$.

The overall similarity, in the form of correlation factor, between the calculated amplitude response of each set and the theoretical amplitude response of the second derivative operation is computed. The correlation factors between the calculated and the theoretical responses are determined using a formula given by Davis [22]. For each ring system, the highest correlation factor, between the calculated and the theoretical responses observed in the series of the correlations thus obtained, determines the optimum n and consequently the best coefficient set. For instance, Figure 2 shows the variation of the computed correlation factors with increasing value of n in 0.25 steps for S1, S2, and S3 systems.

Also, for each ring system, it is found that the numerical value of c_0 increases with increasing value of n, whereas the numerical value of c_1 decreases. Other coefficients generally approach zero with increasing value of n. Table 1 shows the highest correlations, the optimum values of n, and the corresponding weight coefficient sets for the various ring systems examined in this work.

It can be seen that the derived weight coefficient sets by this technique are generally different from one system to another but their responses are more or less similar to the theoretical response of the second derivative operation as verified by the highest correlations (Table 1). To the authors' knowledge, all the coefficient sets shown in Table 1 are new.

On the other hand, it is clear that the highest correlation coefficient among the correlations shown

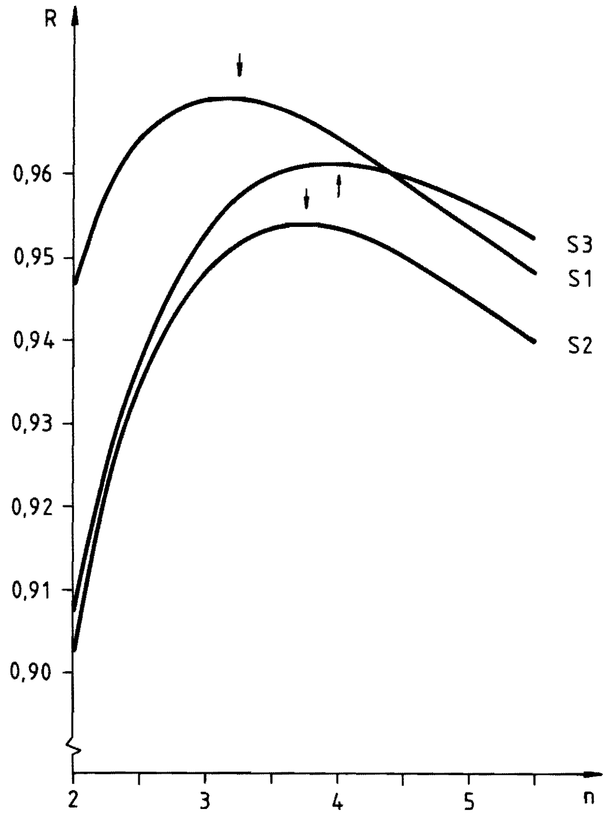

*Figure 2. Characteristic Curves of Variation in Correlation Factor (*R*) versus* n *for S1, S2, and S3 Ring Systems. Arrows show the location of the highest correlation.*

in Table 1, belongs to the derived coefficient set for S1 system. Consequently the S1 coefficient set can be considered generally the best coefficient set for second derivative. It uses the least number of rings to obtain average gravity values. The second derivative formula in this case is given by:

$$s^2\left(\frac{\partial^2 g}{\partial z^2}\right) = 4.92457\, g(0) - 5.13218\, \bar{g}(s) - 0.15087\, \bar{g}(s\sqrt{2}) + 0.35848\, \bar{g}(2s). \quad (11)$$

Figure 3 shows the corresponding amplitude response of Equation (11). Distortion of contours of the response of the derived set near the cutoff region is usually expected [11, 13, 16, 19, 21].

For the sake of comparison, the correlation factor between the previously proposed set of weights and the theoretical response of the second derivative using the same number of data points with the same spacing are computed. Results in this case are given in Table 2. For a given ring system, it is clear from Table 2 that the derived sets provide an improvement over the previously proposed sets. At the same time, the S1 coefficient set provides an improvement, in the sense of its close fit to the theoretical second derivative response, over all other coefficient sets. When Equation (11) is applied to the Bouguer anomaly data, it may give the best results among the other weight coefficient sets thus obtained.

APPLICATION

Figure 4 shows the Bouguer gravity anomaly map of the central area of the Gulf of Suez region, Egypt. The regional gravity field masks the gravity anomaly of a shallow NW – SE salt body of Miocene age, delineated from drill-hole and seismic information. Figure 5 shows the second vertical derivative anomaly of this area, calculated by convolving Equation (11) with the observed gravity field using a grid of 0.5 km spacing. The derivative field emphasizes the gravimetric effect of the salt body very clearly. The axis of the salt body interpreted from seismic data correlates with the axis of the elongated NW–SE central second vertical gravity low.

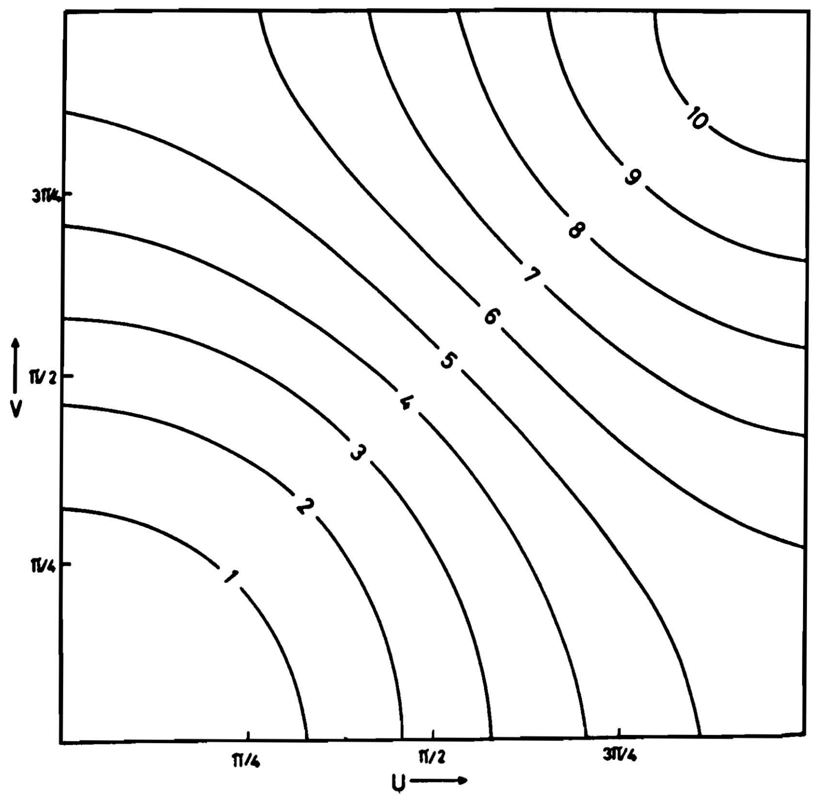

Figure 3. Amplitude Response of Equation (11) where u / 2Π and v / 2Π are the Frequencies in Cycles per Unit of Length in the x and y Directions, Respectively.

Table 2. Comparison of Correlation Values Obtained From the Present Study with Previous Studies. *Indicates that only the coefficients derived with weightage of $1/r^4$ are used for comparison.

Ring system (S1)	Source		Correlation Factor	Present Correlation
S1	Reference [2],	Equation (15)	0.88187	0.96929
	Reference [3],	Equation (13)	0.29900	
	Reference [3],	Equation (15)	−0.30000	
S2	Reference [4],		0.93135	0.95412
	Reference [5],	Equation (16)	0.86310	
	Reference [7],	Equation (25)	0.84643	
	*Reference [10],	Table 1	0.95348	
S3	Reference [7],	Equation (27)	0.10720	0.96127
S4	Reference [11],	Equation (26)	0.93929	0.95922
S5	*Reference [10],	Table 1	0.95387	0.95411
S6	Reference [1],	Equation (27)	−0.14892	0.95427
S7	Reference [12],	For $\lambda = 0.05$	0.59238	0.95834
S8	Reference [8],		0.85460	0.95728
	Reference [11],	Equation (27)	0.95684	
S9	*Reference [10],	Table 1	0.95308	0.95423
S10	*Reference [10],	Table 1	0.95216	0.95437
S11	*Reference [10],	Table 1	0.95139	0.95441
S12	Reference [6],		0.94823	0.95455

CONCLUSION

The method presented here is an attempt to find the optimum weight coefficient set for second derivative when a definite geometry of the circles for obtaining average gravity values are used. To calculate the second derivative, a weight coefficient set, which uses the least number of rings and at the same time yields the best results, is obtained. The derived set provides an improvement, in the sense of its close fit to the theoretical second derivative response, over many of the previously proposed sets. The results obtained may be of interest in the field of exploration geophysics, although further detailed examination of different ring system are required when other approaches are used.

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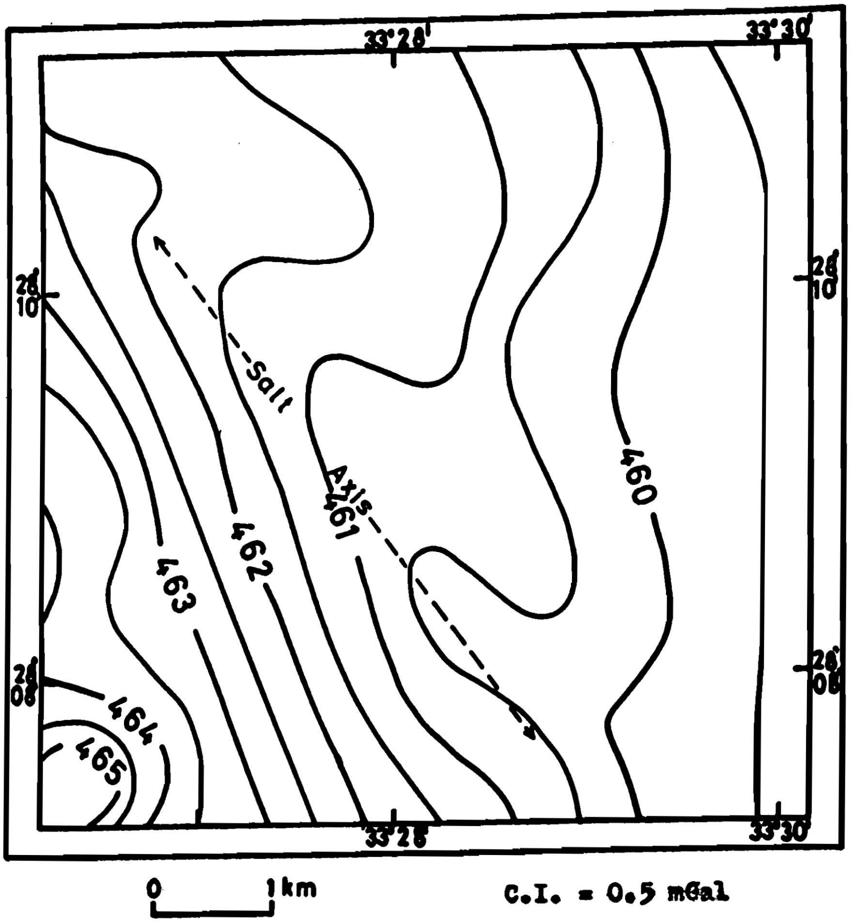

Figure 4. Bouguer Gravity Anomaly Map of an Area in the Gulf of Suez Region, Egypt.

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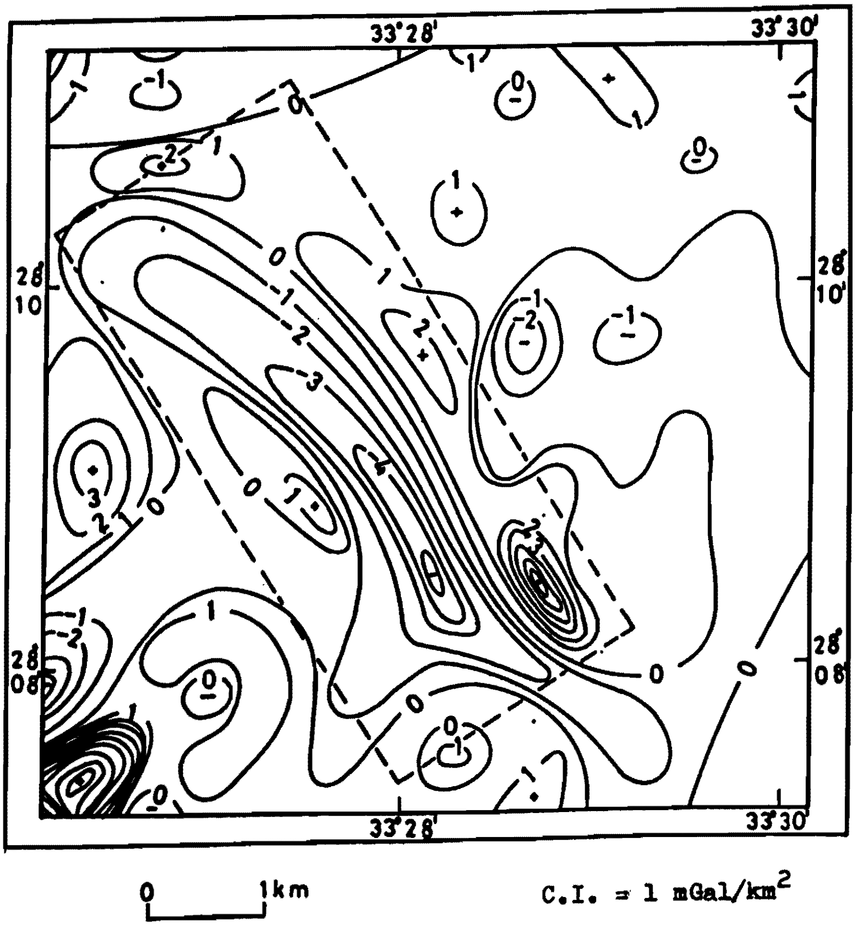

Figure 5. Second Vertical Derivative Anomaly Map of an Area in the Gulf of Suez Region, Egypt, as Calculated from Equation (11).

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