DEPTH DETERMINATION USING THE CHARACTERISTIC POINTS OF THE LEAST-SQUARES RESIDUAL GRAVITY ANOMALY PROFILES

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الخلاصـــة :

نقدم في هذا البحث طريقة عددية بسيطة لتقدير عُمق التراكيب الجيولوجية المدفونة تحت سطح الأرض من النقط الخاصة بالمقاطع الجانبيَّة الشّـاذة المتبقيات التثاقلية الناتجة عن تطبيق طريقة أقل المربعات وهي المسافات التي عندها تكون قيمة الشاذة تساوى نصف قيمتها العظمى وصفر . وتَـمَّ تحويل مشكلة تعيين العمق من هذه المسافات إلى مشكلة حل معادلة واحدة غير خطية في صورة $z = f(z)$. وقد أُجرى تطبيق الطريقة على العديد من البيانات التثاقلية النظرية ووُجـد أن جميع النتائج جيدة . أما في حالة إضافة خطأ عشوائي بنسبة (٥٪) الى البيانات النظرية فقد وجُـد أن الخطأ في تقدير العمق لا يزيد عن (٧٪) وقد أثبت أن هذه الطريقة قادرة على تقدير العمق من البيانات التثاقلية المتاحة على جزء صغير من البروفيل (المقطع الجانبي) .

وقد أمكن أيضاً اقتراح برنامج سهل لتعيين العمق الحقيقى للتركيب الجيولوجى ورتبة مجال الأرض الأقليمي التثاقلي في حالة تطبيقها على مقاطع جانبية بوجير Bouguer .

هذا وقد تَـمَّ أيضاً تجربة الطريقة على مثالين حقليين من الولايات المتحدة الأمريكية ومصر . وفي كلتا الحالتين وجُـد أن العمق المحسوب يتفق مع العمق الحقيقي للتراكيب الجيولوجية والمؤكد من بيانات الحفر .

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ABSTRACT

The present paper deals with a simple numerical approach to estimate the depth of a buried structure from the characteristic points of the least-squares residual gravity anomaly profile, namely, the points at which the anomaly attains its half-maximum and zero. The problem of depth determination from the characteristic distances is transformed into the problem of finding a solution of a nonlinear equation of the form $z = f(z)$. Formulas are derived for simple geometrically shaped causative sources for different orders of fit. The procedure has been applied to synthetic data with and without random errors. The method can be applied not only to residuals but also to the Bouguer anomaly profile consisting of the combined effect of a residual component due to a purely local structure and a regional component represented by a polynomial of low order. The method is easy to apply and may be automated if desired. The procedure has been tested on two field examples from the United States and Egypt. In both cases, the depth obtained is consistent with the actual depth.

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INTRODUCTION

Simple geometrically shaped models are frequently used in gravity interpretation to find the depth of most geological structures. Numerical and graphical techniques have been developed by many authors for interpreting the residual gravity anomalies due to these models. An excellent review is given in Bowin *et al.* [1]. Numerical methods include, for example, least-squares minimization approaches [2–4]; ratio techniques [5]; Mellin transform [6]; Fourier transform [7, 8]; and Walsh transforms techniques [9]. On the other hand, the methods of characteristic points for solving the inverse gravity problem include, for example, the use of the distances at which the gravity anomaly attains its maximum, half-maximum, and three quarters-maximum [10–15]. However, effective quantitative interpretation using iterative procedures based on the characteristic points of the least-squares residual gravity anomaly profiles are yet to be developed. The characteristic points of the least-squares residual gravity anomaly profile are the points at which the anomaly attains its half-maximum and zero.

The problem of depth determination from the characteristic points has been transformed into the problem of finding a solution of a non-linear equation of the form $z = f(z)$. Formulas have been derived for spheres, cylinders, and the first horizontal gradient of the gravity effect of a 2-D thin faulted layer for different orders of fit. The procedure has been applied to synthetic data with and without random errors. The depth obtained agrees with model depth within 7 percent in the case of the sphere and horizontal cylinder when the gravity data contain random errors of 5 percent. The method can be applied not only to residuals but also to the Bouguer anomaly profile consisting of the combined effect of a residual component due to a purely local structure and a regional component represented by low order polynomial. The validity of the method has been tested on two field examples: (1) the Humble Dome gravity anomaly near Houston and (2) the Abu Roash Dome gravity anomaly west of Cairo.

THEORY OF THE METHOD

The gravity effects of the sphere, the infinitely long horizontal cylinder, and the vertical cylinder (semiinfinite vertical line-element approximation) are expressed as [6, 11]:

$$g(x,z) = Az^m\, H(x,z), \tag{1}$$

where

$$H(x,z) = 1/(x^2 + z^2)^q.$$

In Equation (1), z is the depth; x is a position coordinate; and A, m, and q are defined in Table 1.

Table 1. Definition of *A*, *m*, and *q* used in Equation (1). *G* is the Universal Gravitational Constant; σ is the Density Contrast; *R* is the Radius.

Model	A	m	q
Sphere	$\frac{4}{3}\pi G\sigma R^3$	1	$\frac{3}{2}$
Horizontal cylinder	$2\pi G\sigma R^2$	1	1
Vertical cylinder	$\pi G\sigma R^2$	0	$\frac{1}{2}$
Vertical fault (FHD)	$2G\sigma t$	1	1

The gravity effect of a thin faulted layer is expressed as [11]:

$$g(x,z) = A(\pi/2 + \tan^{-1} x/z).$$

The first horizontal derivative (FHD) of the gravity effect of this structure is given by Equation (1) for the horizontal cylinder, where $A = 2G\sigma t$ and t is the thickness of the fault [6].

From Equation (1), we obtain the following equation at $x = x_i$, $i = 1,2,3,\ldots,N$:

$$g^o(x_i,z) = Az^m\, H(x_i,z), \tag{2}$$

where x_i are the discrete points on the ground surface at which $g^o(x_i,z)$ is observed.

Polynomial fitting by the least-squares method is an important technique for the separation of the gravity anomalies into residual and regional components. An excellent review was given in Nettleton [11]. In all cases, the condition of the least-squares solution is

$$\sum_{i=1}^{N} [R_p(x_i,z)]^2 = \text{minimum}, \tag{3}$$

where $R_p(x_i,z)$ is the residual component given as:

$$R_p(x_i,z) = g^o(x_i,z) - g_p^c(x_i,z), \tag{4}$$

and where $g^o(x_i, z)$ is the observed gravity given by Equation (1) and $g_p^c(x_i, z)$ is the computed regional gravity, which can be represented by

$$g_p^c(x_i, z) = \sum_{n=o}^{p} a_n x_i^n , \quad (5)$$

where a_n are $(p+1)$ coefficients and p is the order of the one-dimensional (1-D) polynomial.

Condition (3) is fulfilled when the partial derivatives with respect to each a_n are zero. This gives $(p+1)$ simultaneous linear equations from which the $(p+1)$ different values of a_n can be determined. As a result, the least-squares method will produce both positive and negative residuals even when the $g^o(x_i, z)$ values are only positive (or negative). The total sum of the residuals is zero. The least-squares profile is usually balanced between positive and negative values.

Let the sample observed gravity values be taken for symmetrical values of x_i and the interval for x be symmetric about the origin. In this case, when regional components of first-order are fit to the observed gravity field $g^o(x_i, z)$ the simplest first-order least-squares residual gravity expression is defined as [5]:

$$R_1(x_i, z) = g^o(x_i, z) - a_o, \quad (6)$$

and when regional components of second or third-order are fit to the same observed gravity data given by Equation (1), the second or third-order least-squares residual gravity anomaly expression is defined as [5]:

$$R_{2\&3}(x_i, z) = g^o(x_i, z) - a_o - a_2 x_i^2 , \quad (7)$$

and similarly, the fourth or fifth-order expression is given as

$$R_{4\&5}(x_i, z) = g^o(x_i, z) - a_o - a_2 x_i^2 - a_4 x_i^4 , \quad (8)$$

and so on.

Now, two detailed cases are presented to demonstrate the use of these expressions at the characteristic points in determining the depth to the buried structure.

Case 1: First-order Fit

Figure 1 shows the gravity effect due to horizontal cylinder having a positive density contrast, whereas Figure 2 shows the first-order least-squares residual effect of this structure. From Figure 2, we can immediately identify three significant points, the maximum positive ($x = 0$), the half-maximum positive ($x_{½}$), and the value of x where $R_1(x_i, z) = 0$ (denoted by x_c). These are the characteristic points of the first-order residual anomaly profile. The points $x_{½}$ and x_c are sufficient to determine the depth to the buried structure, as shown in the following paragraph.

Using Equation (1), Equation (6) may be written in the form

$$R_1(x_i, z) = Az^m/(x_i^2 + z^2)^q - a_o . \quad (9)$$

At the characteristic points, Equation (9) gives the following equations

$$R_1(\max) = Az^{m-2q} - a_o, \ x = 0 , \quad (10)$$

$$R_1(½\max) = Az^m/(x_{½}^2 + z^2)^q - a_o , \ x = x_{½} , \quad (11)$$

$$Az^m/(x_c^2 + z)^q = a_o , \ R_1(x_i, z) = 0, \ x = x_c . \quad (12)$$

Multiplying Equation (11) by 2 and equating the resultant with Equation (10), we have

$$2Az^m/(x_{½}^2 + z^2)^q = Az^{m-2q} + a_o . \quad (13)$$

Finally, from Equations (12) and (13), we obtain,

$$z = \left[\frac{(x_c^2 + z^2)^q \, (x_{½}^2 + z^2)^q}{2(x_c^2 + z^2)^q - (x_{½}^2 + z^2)^q} \right]^{1/2q} . \quad (14)$$

Case 2: Second or Third-order Fit

Figure 3 shows the second or third-order least-squares gravity anomaly due to a two-dimensional horizontal cylinder having a positive density contrast. From this Figure, four characteristic points, namely, the maximum, the half-maximum, and the values of x where $R_{2\&3}(x_i, z) = 0$ can be identified. They are denoted in Figure 3 by $x = 0$, $x_{½}$, x_{c1}, and x_{c2}, respectively. In this case, x_{c2} is greater than x_{c1}. The $x_{½}$, x_{c1}, and x_{c2} values are sufficient to determine the depth. Applying the method just described above to the second or third-order least-squares residual gravity expression shown in Equation (7), we obtain the following four equations

$$R_{2\&3}(\max) = Az^{m-2q} - a_o , \ x = 0, \quad (15)$$

$$R_{2\&3}(½\max) = Az^m/(x_{½}^2 + z^2)^q - a_o - a_2 x_{½}^2, \ x = x_{½}, \quad (16)$$

$$Az^m/(x_{c1}^2 + z^2)^q = a_o + a_2 x_{c1}^2, \ R_{2\&3}(x_i, z) = 0, \quad (17)$$

$$Az^m/(x_{c2}^2 + z^2)^q = a_o + a_2 x_{c2}^2, \ R_{2\&3}(x_i, z) = 0. \quad (18)$$

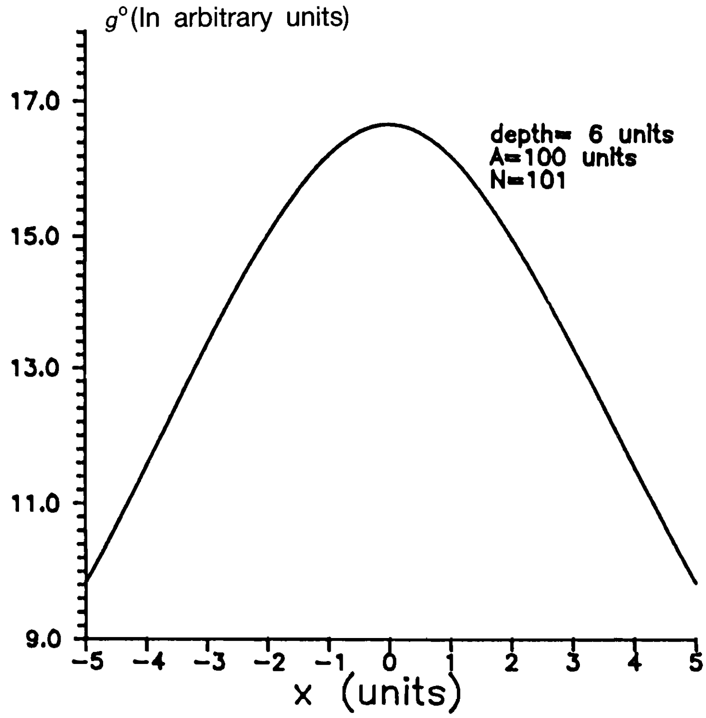

Figure 1. Residual Gravity Anomaly for a Buried Horizontal Cylinder.

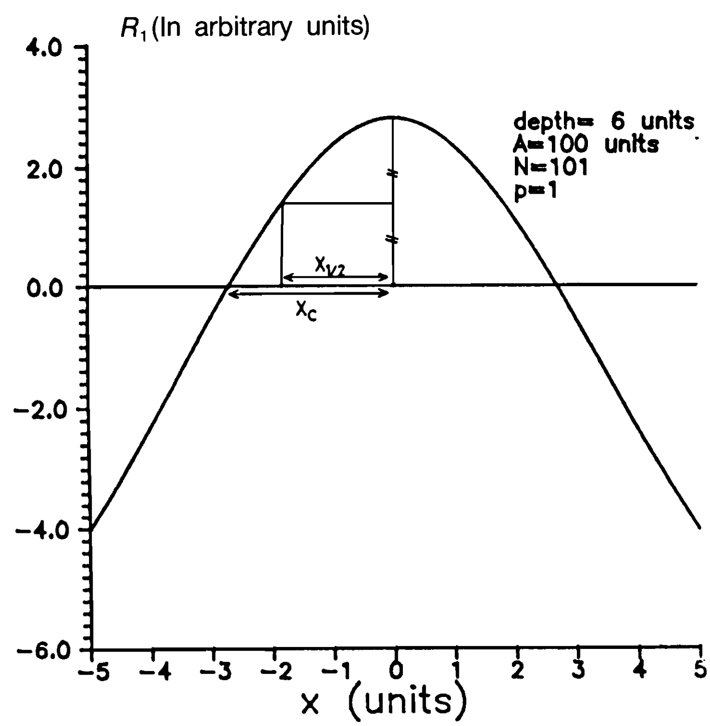

Figure 2. Least-Squares Residual Anomaly Obtained by Applying First-Order Fit to the Data of Figure 1.

Solving the above four equations and after simplifications the following nonlinear equation in z is obtained,

$$z = \left[\frac{B(z)\,C(z)\,D(z)}{2B(z)D(z) - C(z)B(z) - fC(z)(D(z) - B(z))} \right]^{1/2q}, \tag{19}$$

where

$B(z) = (x_{c2}^2 + z^2)^q$, $C(z) = (x_{½}^2 + z^2)^q$,

$D(z) = (x_{c1}^2 + z^2)^q$, and $f = (2x_{½}^2 - x_{c1}^2)/(x_{c2}^2 - x_{c1}^2)$.

Equations (14) and (19) can be solved by a simple iteration method [16]. Formulas for higher orders fit can be developed very easily, if required. In all cases the simple linear interpolation technique [17] can be used to estimate the values of the characteristic distances from the least-squares residuals.

APPLICATION TO THEORETICAL DATA

A theoretical residual gravity field due to a simple geometrically-shaped body buried at different depths is generated using Equation (1) so that the interval for x is symmetric about the origin and the sample values $g(x_i, z)$ are taken for symmetrical values of x_i. Each theoretical field is subjected to a separation technique using the least-squares method. Regional components of first and second orders are fitted to the input data using the algorithm described by Gangi and Shapiro [18]. In this way two residual profiles are obtained for each input profile. A search algorithm is used to estimate the values of $x_{½}$ and x_c for the first-order profile and $x_{½}$, x_{c1}, and x_{c2} for the second-order profile using a linear interpolation method [17]. Considering the values of these characteristic points and making use of Equations (14) and (19), the depth to the buried structure can be determined from the first and the second-order residual profiles, respectively. Numerical results obtained for two geometries of various depths, profile lengths and sampling intervals are shown in Tables 2–4, respectively.

It is verified numerically that Equations (14) and (19) give accurate value of z when using synthetic

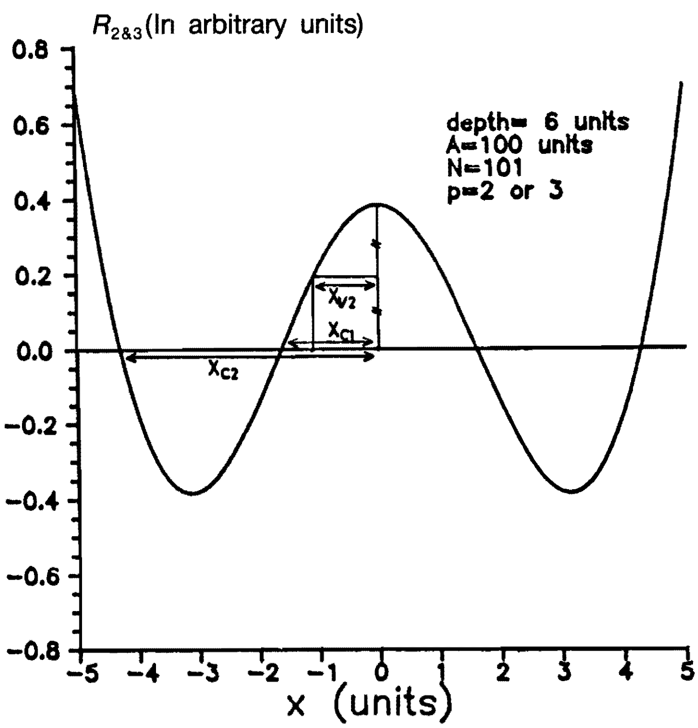

Figure 3. Least-Squares Residual Anomaly Obtained by Applying Second or Third-Order Fit to the Data of Figure 1.

Table 2a. Effect of Random Errors on Depth Determination from First-Order Least-Squares Residual Gravity Anomaly Profiles of Actual Length 100 Units and a Sampling Interval of 1 Unit Due to a Spherical and a Horizontal Cylindrical Source at Various Depths.

	Computed depth (units)							
	Sphere				Horizontal cylinder			
Model depth (unit)	Using synthetic data	% error	Using synthetic data with random errors 5%	% error	Using synthetic data	% error	Using synthetic data with random errors 5%	% error
2.0	2.0897	4.49	2.1045	5.22	2.0195	0.97	2.0698	3.49
2.5	2.5246	0.98	2.5736	2.94	2.5520	2.08	2.6280	5.12
3.0	3.0458	1.53	3.1390	4.63	3.0304	1.02	3.1023	3.41
3.5	3.5527	1.51	3.6458	4.17	3.5229	0.65	3.6264	3.61
4.0	4.0146	0.37	4.1019	2.55	4.0361	0.90	4.1624	4.06
4.5	4.5311	0.69	4.6383	3.07	4.5033	0.07	4.6673	3.72
5.0	5.0361	0.72	5.1663	3.32	5.0277	0.55	5.1670	3.34
5.5	5.5158	0.28	5.6750	3.18	5.5202	0.37	5.5844	1.53
6.0	6.0131	0.22	6.1805	3.01	6.0027	0.05	5.9895	0.17
6.5	6.5298	0.46	6.6528	2.35	6.5207	0.32	6.6527	2.35
7.0	7.0152	0.22	7.0830	1.19	7.0168	0.24	7.3390	4.84
7.5	7.5012	0.02	7.5101	0.13	7.4998	0.00	7.8875	5.17
8.0	8.0176	0.22	8.1311	1.64	8.0168	0.23	8.3416	4.27

Table 2b. Effect of Random Errors on Depth Determination from Second-Order Least-Squares Residual Gravity Anomaly Profiles of Actual Length 100 Units and a Sampling Interval of 1 Unit Due to a Spherical and a Horizontal Cylindrical Source at Various Depths.

	Computed depth (units)							
	Sphere				Horizontal cylinder			
Model depth (unit)	Using synthetic data	% error	Using synthetic data with random errors 5%	% error	Using synthetic data	% error	Using synthetic data with random errors 5%	% error
2.0	2.1031	5.16	2.1109	5.55	2.0378	1.89	2.0885	4.42
2.5	2.5321	1.29	2.5832	3.33	2.5292	1.17	2.6374	5.50
3.0	3.0089	0.30	3.1428	4.76	3.0469	1.56	3.1531	5.10
3.5	3.5469	1.34	3.6758	5.02	3.5147	0.42	3.6312	3.75
4.0	4.0442	1.10	4.1620	4.05	4.0154	0.38	4.1880	4.70
4.5	4.4936	0.14	4.6206	2.68	4.5255	0.57	4.7566	5.70
5.0	5.0159	0.32	5.2003	4.01	5.0204	0.41	5.3314	6.63
5.5	5.5298	0.54	5.7614	4.75	5.4895	0.19	5.8529	6.42
6.0	6.0059	0.10	6.3022	5.04	6.0056	0.09	6.2492	4.15
6.5	6.4940	0.09	6.8763	5.79	6.5166	0.26	6.6018	1.57
7.0	7.0053	0.08	7.4230	6.04	7.0070	0.10	6.9858	0.20
7.5	7.5019	0.03	7.7949	3.93	7.4826	0.23	7.3574	1.90
8.0	7.9983	0.02	8.1424	1.78	7.9894	0.13	7.8988	1.26

data. After adding 5 percent random error in the theoretical data, the depth obtained is within 7 percent. Generally, the method does not depend on the profile length, the geometry of the body, and the order of fit (Tables 3 and 4). It is evident from Table 4 that the percentage of error in depth decreases as the number of measurements made around $g(\max)$ and within the restricted length of profile increases. In this particular case, the depth obtained is in a very good agreement with the model depth.

OPTIMUM-ORDER REGIONAL DETERMINATION

Polynomial fitting by least-squares method is an effective technique for separation of gravity anomalies into residual and regional components. However, the method depends on selecting the optimum degree of regional polynomial to fit the Bouguer anomalies. Abdelrahman *et al.* [19] presented a procedure to select the optimum polynomial order based on the correlation factors between residuals of successive orders. Zeng [20] showed that the optimum order of the regional can be estimated from the point of discontinuity of the gradient on a graph of variance against the polynomial degree. That graph is obtained by fitting polynomials of different orders to an upward continuation of the Bouguer anomaly at a proper height, where the shape of the anomaly is similar to that of the regional anomaly. However, because all the least-squares residual anomalies are

Table 3. Effect of Profile Length on Depth Determination from Least-Squares Residual Gravity Anomalies Due to a Spherical and a Horizontal Cylindrical Source at Depth of 6 Units. Sampling Interval is 1 Unit.

		Computed depth (units)							
		Sphere				Horizontal cylinder			
		First-order fit		Second or third-order fit		First-order fit		Second or third-order fit	
No. of data points	Profile length (units)	Computed depth	% error	Computed depth	% error	Computed depth	% error	Computed depth	% error
41	40	6.0242	0.40	5.8615	2.31	5.9876	0.21	5.9447	0.92
51	50	6.0100	0.17	5.9451	0.91	6.0195	0.32	5.9560	0.73
61	60	6.0000	0.00	5.9896	0.17	6.0222	0.37	5.9705	0.49
71	70	6.0012	0.02	6.0003	0.01	6.0154	0.26	6.0019	0.03
81	80	5.9960	0.07	6.0080	0.13	6.0068	0.11	5.9755	0.41
91	90	6.0081	0.14	6.0215	0.36	6.0215	0.36	6.0076	0.13
101	100	6.0131	0.22	6.0059	0.10	6.0027	0.05	6.0056	0.09

Table 4. Effect of Sampling Interval on Depth Determination from Least-Squares Residual Gravity Anomaly Profiles of Actual Length of 40 Units due to a Spherical and a Horizontal Cylindrical Source at a Depth of 6 Units.

		Computed depth (units)							
		Sphere				Horizontal cylinder			
		First-order fit		Second or third-order fit		First-order fit		Second or third-order fit	
No. of data points	Sampling interval (units)	Computed depth	% error	Computed depth	% error	Computed depth	% error	Computed depth	% error
21	2.000	5.9247	1.25	5.6789	5.34	6.0007	0.12	5.5427	7.62
41	1.000	6.0242	0.40	5.8615	2.31	5.9876	0.21	5.9447	0.92
81	0.500	5.9963	0.06	5.9830	0.28	5.9962	0.06	5.9721	0.46
161	0.250	5.9987	0.02	5.9946	0.09	5.9989	0.02	5.9916	0.14
321	0.125	5.9998	0.00	5.9983	0.03	6.0000	0.00	5.9982	0.03

distorted in shape [21], Abdelrahman *et al.*, [5] showed that polynomial regional terms can be treated simultaneously with the anomaly in a least-squares inversion of gravity data. By examining the results obtained using regionals of $p+3$ ($p = 1,3,5,...$) or $p+2$ ($p = 2,4,6,...$) of successive orders, the optimal order for the regional field can be determined simultaneously with the interpretation. Here, we formulate a simple procedure to determine the optimum order of the regional gravity field along the Bouguer anomaly profile and to obtain simultaneously the true depth to the buried structure, assuming that the Bouguer-anomaly profile consists of the combined effect of a residual component due to a purely local structure and a regional component represented by polynomial of any order.

The data from a small segment of the Bouguer-anomaly profile Δg of order p are subjected to a separation technique using the least-squares method. Regional components of successive orders k are fitted to the input data using, for example, the algorithm described by Gangi and Shapiro [18] to obtain k-residual profiles. Each k-residual profile is subjected to a search algorithm to determine the corresponding characteristic distances from the observed data using a simple linear interpolation method [17]. The depth to the buried structure is then determined from the corresponding non-linear equation $z = f(z)$, For example, using Equation (14) for $k = 1$, and Equation (19) for $k = 2$ and 3, the z_k values are obtained.

These computed depths z_k can be used to determine the optimum order of the regional field and to estimate the true depth of the buried structure by making use of the fact that the depths z_k and z_{k+1}, $k = 1,2,3,...$, should be equal, provided that the computed regional of order k is equal to or higher than the order of the actual regional present in the input data p. The following cases are given for illustration:

(1) if $z_1 = z_2$, then $p = 1$, and the true depth is z_1;

(2) if $z_1 \neq z_2 = z_3$, then $p = 2$, and the true depth is z_2;

(3) if $z_1 \neq z_2 \neq z_3 = z_4$, then $p = 3$, and z_3 is the true depth, and so on.

FIELD EXAMPLES

To examine the applicability of the present method, the following two field examples are presented.

Humble Dome Anomaly

A gravity profile along the line AA' of the gravity map of the Humble Salt Dome, Houston, USA, [11] (shown in Figure 4) has been digitized at an interval of 1.09 km. The discrete data thus obtained have been subjected to a separation technique using the least-squares method. Regional components of first, second, and third-orders were fitted to the input data. In this way, three successive least-squares residual gravity anomaly profiles were obained (Table 5 and Figure 4). The average values of the characteristic distances have been determined from each residual anomaly profile thus obtained using a simple linear interpolation technique. The results are listed as follows:

$x_{½}$ = 2.266737 (unit);
x_c = 4.021195 (unit), for first-order fit;

$x_{½}$ = 1.686162 (unit);
x_{c1} = 2.711215 (unit);
x_{c2} = 8.521696 (unit), for second-order fit;

$x_{½}$ = 1.682477 (unit);
x_{c1} = 2.709037 (unit);
x_{c2} = 8.518064 (unit), for third-order fit;

and from Equations (14) and (19), the computed depths (assuming a spherical body approximation [11]) were obtained as:

$z_1 = 4.356 \times 1.09 = 4.748$ km;
$z_2 = 4.453 \times 1.09 = 4.854$ km; and
$z_3 = 4.418 \times 1.09 = 4.816$ km.

Using the criterion established above, the regional field along this profile can be represented by a second-order polynomial and the depth to the center of the salt body is 4.854. The depth determined by Nettleton [11] is 4.968 km.

Abu Roash Dome Anomaly

An east–west gravity profile of this area [19] is shown in Figure 5. This profile has been digitized at an interval of 0.85 km. Adopting the same procedure used in case of the Humble Dome anomaly, the results (Table 6 and Figure 5) were obtained as:

$x_{½}$ = 3.365836 (unit);
x_c = 5.335580 (unit), for first-order fit;

$x_{½}$ = 1.651681 (unit);
x_{c1} = 3.217214 (unit);
x_{c2} = 8.749662 (unit), for second-order fit;

$x_{½}$ = 1.641278 (unit);
x_{c1} = 3.232207 (unit);
x_{c2} = 8.748182 (unit), for third-order fit.

Table 5. Numerical Values of the Bouguer and the Least-Squares Residual Gravity Anomalies Over the Humble Salt Dome, Harris County, Texas, U.S.A.

x-Coordinate (in 1.09 km)	Bouguer values (in mGal)	$R_1(x_i, z)$ (in mGal)	$R_2(x_i, z)$ (in mGal)	$R_3(x_i, z)$ (in mGal)
−10	−13.60	3.08051	−3.01457	−2.77066
−9	−13.20	3.31932	−0.94724	−0.84968
−8	−12.80	3.55812	0.92762	0.91735
−7	−13.00	3.19693	2.01000	1.92612
−6	−13.40	2.63574	2.69990	2.57238
−5	−14.00	1.87454	2.99732	2.85184
−4	−15.60	0.11335	2.10227	1.96021
−3	−17.40	−1.84785	0.81474	0.69322
−2	−19.80	−4.40904	−1.26526	−1.35341
−1	−22.40	−7.17024	−3.73774	−3.78395
0	−22.90	−7.83143	−4.30270	−4.32070
1	−21.96	−7.05262	−3.62013	−3.57392
2	−19.50	−4.75382	−1.61004	−1.52189
3	−16.88	−2.29501	0.36758	0.48910
4	−14.60	−0.17621	1.81271	1.95478
5	−12.68	1.58260	2.70537	2.85086
6	−11.60	2.50140	2.56556	2.69308
7	−10.80	3.14021	1.95327	2.03714
8	−10.36	3.41901	0.78850	0.79877
9	−10.00	3.61782	−0.64874	−0.74630
10	−9.96	3.49663	−2.59846	−2.84237

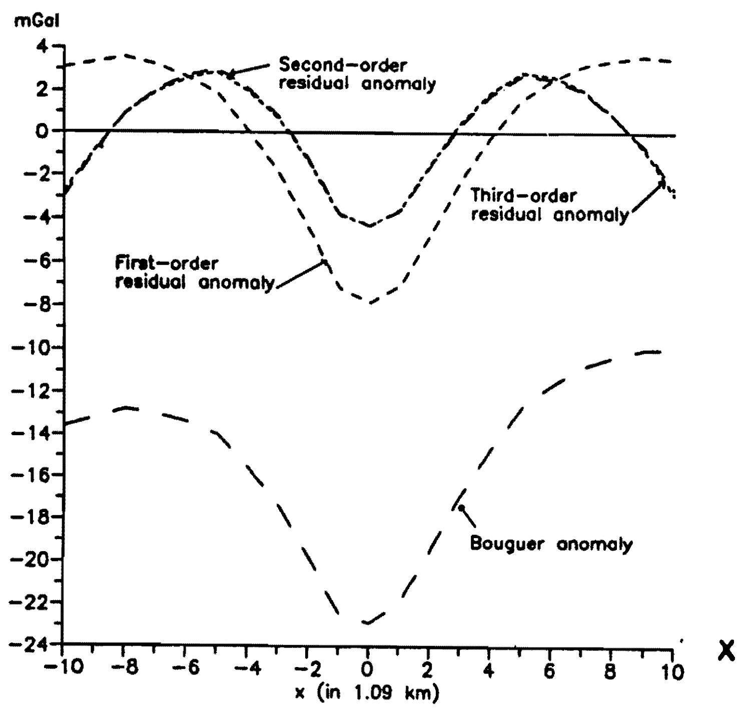

Figure 4. Observed Bouguer Gravity Anomaly Profile and Its Successive Least-Squares Residual Anomaly Profiles of the Humble Salt Dome, Harris County, Texas.

Table 6. Numerical Values of the Bouguer and the Least-Squares Residual Gravity Anomalies Over Abu Roash Dome, West Cairo, Egypt.

x-Coordinate (in 0.85 km)	Bouguer values (in mGal)	$R_1(x_i, z)$ (in mGal)	$R_2(x_i, z)$ (in mGal)	$R_3(x_i, z)$ (in mGal)
−10	−9.60	−3.25047	0.94381	1.02046
−9	−9.28	−2.94976	−0.01376	0.01690
−8	−8.75	−2.43904	−0.62888	−0.63211
−7	−7.81	−1.51833	−0.70155	−0.72791
−6	−6.75	−0.47762	−0.52177	−0.56184
−5	−5.80	0.45310	−0.31953	−0.36526
−4	−5.00	1.23381	−0.13485	−0.17950
−3	−4.20	2.01453	0.18229	0.14410
−2	−3.66	2.53524	0.37187	0.34417
−1	−3.12	3.05595	0.69391	0.67939
0	−2.90	3.25667	0.82840	0.82840
1	−3.20	2.93738	0.57534	0.58986
2	−3.75	2.36810	0.20473	0.23243
3	−4.30	1.79881	−0.03343	0.00476
4	−5.00	1.07952	−0.28914	−0.24449
5	−5.90	0.16024	−0.61239	−0.56667
6	−6.75	−0.70905	−0.75320	−0.71312
7	−7.40	−1.37833	−0.56155	−0.53519
8	−8.00	−1.99762	−0.18746	−0.18423
9	−8.72	−2.73691	0.19909	0.16843
10	−9.40	−3.43619	0.75809	0.68144

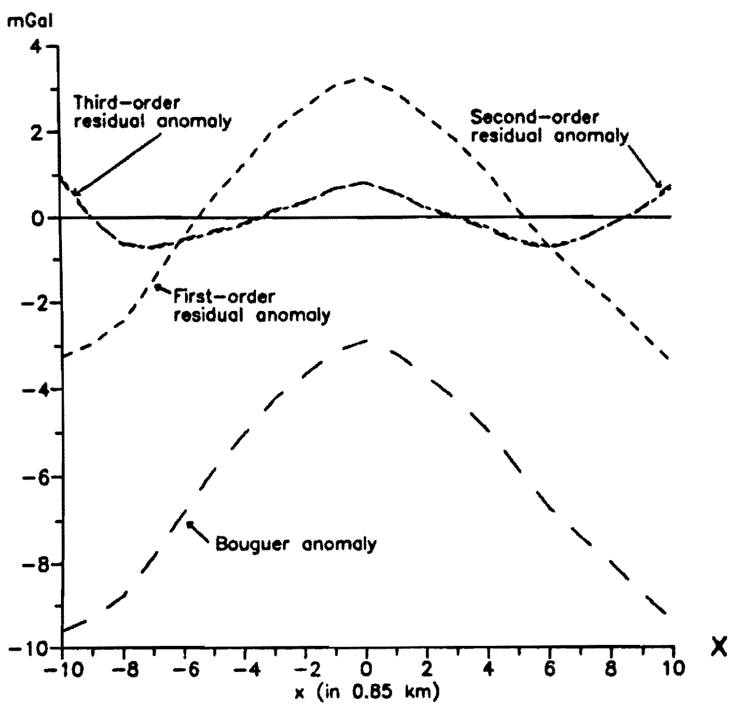

Figure 5. Observed Bouguer Gravity Anomaly Profile and Its Successive Least-Squares Residual Anomaly Profiles of Abu Roash Dome, West Cairo, Egypt.

From Equations (14) and (19), the computed depths (assuming a vertical cylinder approximation) were

$z_1 = 6.211 \times 0.85 = 5.279$ km;
$z_2 = 2.080 \times 0.85 = 1.768$ km; and
$z_3 = 2.033 \times 0.85 = 1.728$ km.

Since z_2 is approximately equal to z_3, the regional field can be represented by a second-order polynomial; the value of z to the top of the basement given by the present technique is 1.768 km. This value agrees with the drill hole data given by Said [22], where the depth to the basement was found to be 1.810 km in Abu Roash well #1.

DISCUSSION AND CONCLUSIONS

The problem of depth determination of a buried structure from the values of the characteristic distances of the least-squares residual gravity anomaly profile is transformed into the problem of finding a solution of a nonlinear equation of the form $z = f(z)$. The present approach is capable of determining the depth of a buried structure from gravity data given in a small area over the buried structure, *i.e.*, from the small segment of the gravity profile around $g^o(\max)$. The method does not depend on the profile length and the order of fit. It can be automated if desired.

Real data contain measurement errors which may be compounded by errors in computing the depth. In spite of this, high structural resolution may be achieved at the expense of decreasing tolerance to instrument readings. However, since the interpretation requires only a relatively short length of the profile, the problem may be overcome effectively and economically by increasing the number of measurements made within the restricted length of profile, provided that each local high and low is taken as a separate source. At the same time, using a relatively short length of profile results in a very high rejection of the neighboring disturbances.

A scheme for interpretation to obtain the depth parameter and the optimum-order of the regional gravity field along the Bouguer anomaly profile has been developed. In this case, data interpretation requires analysis of only $p+1$ successive least-squares residual profiles which is more advantageous than the scheme of Abdelrahman *et al.* scheme [5] which employed $p+2$ or $p+3$ successive residual profiles, to obtain the true depth and the optimum-order of the regional gravity field.

Finally, analysis of two field data has demonstrated the applicability of the present technique, which is very simple.

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REFERENCES

[1] C. Bowin, E. Scheer, and W. Smith, "Depth Estimates from Ratio of Gravity, Geoid, and Gravity Gradient Anomalies", *Geophysics,* **51** (1986), p. 123.
[2] O. P. Gupta, "A Least-Squares Approach to Depth Determination from Gravity Data", *Geophysics,* **48** (1983), p. 357.
[3] L. R. Lines and S. Treitel, "A Review of Least-Squares Inversion and Its Application to Geophysical Problems", *Geophys. Prosp.,* **32** (1984), p. 159.
[4] E. M. Abdelrahman, "Discussion on: 'A Least-Squares Approach to Depth Determination from Gravity Data, by O. P. Gupta'", *Geophysics,* **55** (1990), p. 376.
[5] E. M. Abdelrahman, A. I. Bayoumi, Y. E. Abdelhady, M. M. Gobashy, and H. M. El-Araby, "Gravity Interpretation Using Correlation Factors Between Successive Least-Squares Residual Anomalies", *Geophysics,* **54** (1989), p. 1614.
[6] N. L. Mohan, L. Anandababu, and R. Sheshagiri, "Gravity Interpretation Using Mellin Transform", *Geophysics,* **51** (1986), p. 114.
[7] M. E. Odegard and J. W. Borg, "Gravity Interpretation Using the Fourier Integral", *Geophysics,* **30** (1965), p. 424.
[8] B. Sharma and L. P. Geldart, "Analysis of Gravity Anomalies of Two-Dimensional Faults Using Fourier Transforms", *Geophys. Prosp.,* **16** (1968), p. 77.
[9] R. K. Shaw and S. N. P. Agarwal, "The Application of Walsh Transforms to Interpret Gravity Anomalies due to Some Simple Geometrically Shaped Causative Sources: A Feasibility Study", *Geophysics,* **55** (1990), p. 843.
[10] L. L. Nettleton, "Gravity and Magnetic Calculations", *Geophysics,* **7** (1942), p. 293.
[11] L. L. Nettleton, *Gravity and Magnetics in Oil Prospecting.* New York: McGraw–Hill, 1976.
[12] M. B. Dobrin, *Introduction to Geophysical Prospecting, 2nd edn.* New York: McGraw–Hill, 1962.

[13] M. Pick, J. Pícha, and V. Vyskočil, *Theory of the Earth's Gravity Field.* Prague: Academic Publ. House, Czechoslovakia, 1973.

[14] M. M. Telford, L. P. Geldart, R. E. Sherif, and D. A. Key, *Applied Geophysics,* Cambridge University Press, 1976.

[15] J. G. Saha, "Gravity Approximation and Interpretation of a Two-Dimensional Vertical Fault", *Geophysics,* **42** (1962), p. 1462.

[16] B. P. Demidovich and I. A. Maron, *Computational Mathematics.* Moscow: Mir Publishers, 1973.

[17] J. C. Davis, *Statistics and Data Analysis in Geology.* New York: Wiley, 1973.

[18] A. F. Gangi and J. N. Shapiro, "A Propagating Algorithm for Determining *N*th-Order Polynomial, Least-Squares Fit", *Geophysics,* **42** (1977), p. 1265.

[19] E. M. Abdelrahman, S. Riad, E. Refai, and Y. Amin, "On the Least-Squares Residual Anomaly Determination", *Geophysics,* **50** (1985), p. 473.

[20] H. Zeng, "Estimation of the Degree of Polynomial Fitted to Gravity Anomalies and Its Application", *Geophys. Prosp.,* **37** (1989), p. 959.

[21] D. C. Skeels, "What is Residual Gravity?", *Geophysics,* **32** (1967), p. 872.

[22] R. Said, *The Geology of Egypt.* Amsterdam: Elsevier, 1962.

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