ON THE ESTIMATION OF SHEAR PLANE ANGLE AND MEAN STRAIN RATE IN TWO-DIMENSIONAL MACHINING

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الخلاصـــة :

يعنىٰ هذا البحث بعملية التشغيل المستقرة ذات البعدين والتي يكون فيها اتجاه حركة آلة القطع عمودياً على حانة القطع . ولمّـا كان التغير البلاستيكي يتواجد بصفة عامة في منطقة ضيفة ذات إجهادٍ عالٍ تحت ظروف القطع العادية ؛ افـتُرحن نموذج قَـصٍّ بسيط في التحليل النظري . وقد ظهر لمعظم المواد الهندسية أنَّ منطقة أسفل مستوى القص تصل إلى نقطة الخضوع بفعل القطع . ويستخدم النموذج النظري في تقدير السماكة المتوسطة لمنطقة القص ومعدل الانفعال المتوسط .

ABSTRACT

This paper is concerned with the two-dimensional steady state machining process in which the direction of motion of the tool is perpendicular to its cutting edge. Since plastic deformation is generally confined in a narrow zone of intense shear under normal cutting conditions, a simple shear plane model is assumed in the theoretical analysis. It is shown that, for most engineering materials, a region below the shear plane is brought to the yield point by the cutting action, while the chip material is non-plastic due to its elevated yield stress. The theoretical model is used to estimate the mean thickness of the shear zone and the associated mean strain rate, a knowledge of which is essential for checking the validity of the shear angle formula.

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INTRODUCTION

In the orthogonal machining of metals, a wedge-shaped tool having a straight cutting edge moves perpendicular to this edge and parallel to the surface of the workpiece. If the depth of cut is small compared to its width, the process is effectively one of plane strain. Depending upon the material properties and cutting conditions, the chip can be either continuous or discontinuous. For the continuous chip formation, the problem can be considered as one of steady state, which is by no means unique. It is mainly this two-dimensional steady state process that has been frequently discussed in the literature for theoretical treatments.

The simplest theoretical asssumption for the cutting process consists of a simple shear deformation across a straight shear plane AB extending from the tool to the surface of the workpiece (Figure 1). Assuming that the work done per unit volume is a minimum, Merchant [1] derived the formula.

$$\phi = \frac{\pi}{4} - \tfrac{1}{2}(\lambda - \alpha)\,, \tag{1}$$

where ϕ is the shear plane angle, α the rake angle of the tool, and λ an average angle of friction between the chip and the tool. Unfortunately, the shear angle predicted by (1) does not agree well with experimental results. Lee and Shaffer [2] assumed that the chip is stressed to the yield point in a triangular region adjacent to the shear plane, the deformation still being caused by a simple shear across AB. Using the theory of slipline fields, they obtained the shear angle relation

$$\phi = \frac{\pi}{4} - (\lambda - \alpha)\,, \tag{2}$$

which has also been found to be in poor agreement with experiment. Hill [3] has shown that for an ideally plastic material, the shear plane angle must lie within a certain range in order that the yield limit is nowhere exceeded. Merchant's solution lies outside this range (except for $\phi = \pi/4$), while Lee and Shaffer's solution forms one boundary of it. The last-named authors also proposed a built-up nose solution that involves a certain coefficient of friction between the bottom of the nose and the machined surface. Since an independent determination of this coefficient is lacking, no satisfactory comparison of the theory can be made with experiment.

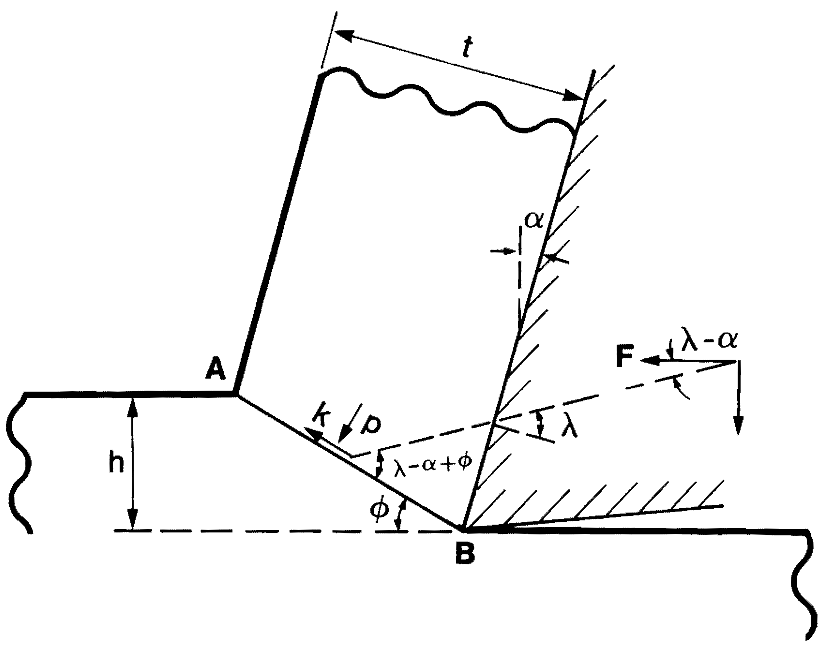

Figure 1. Geometry of Orthogonal Machining Involving a Single Shear Plane.

Rowe and Spick [4] used a kinematic approach to derive the shear angle relationship without reference to a coefficient of friction. Their analysis is equivalent to minimizing the cutting force with respect to ϕ under the condition that the frictional force transmitted by the tool is a constant. Work-hardening has been considered by Palmer and Oxley [5], and later extended by Roth and Oxley [6], using a modified form of the Hencky equations and an experimentally-determined velocity field. A finite element solution has been recently put forwarded by Iwata, Osakada, and Terasaka [7].

Curled chips have been considered by Christopherson [8], Kudo [9], and Dewhurst [11] among others. Assuming that the chip is curled at the outset and the curled chip rotates as a rigid body after the material is sheared across a cylindrical surface, Christopherson derived a set of equations from which the relevant parameters of the cutting process can be evaluated. Of greater theoretical interest is Kudo's simple slipline field solution for an ideally plastic material. The work–chip interface is taken as a curve, a part of which is a circular arc. The material in the region covered by the field deforms in such a way that the chip is forced to curl as it leaves the tool. The solution requires a variation of the frictional stress along the tool face. A slightly modified slipline field has been proposed by Dewhurst [11], who employed the matrix method for analyzing the problem.

One of the main reasons for the discrepancy between theory and experiment is the total neglect of strain-hardening in the customary treatment of the machining process. Most of the published solutions including strain-hardening are either semiempirical, or far too complicated for practical purposes. The present investigation is based on a solution of the shear plane type given by Chakrabarty [12], in which the ability of the material to work-harden is allowed for in an implicit manner. The resulting shear angle relationship is presented in this paper for completeness, and used to obtain the mean strain rate in the shear zone of the machining process.

SHEAR ANGLE FORMULA

It is assumed, as usual, that the cutting speed is sufficiently high, so that the deformation can be reasonably approximated by a single shear plane. Since the engineering shear strain experienced by the material is usually greater than 2.0, it is evident that the fully hardened material above the shear plane is likely to be everywhere below the yield limit. It is reasonable, therefore, to look for the possibility of a portion of the workpiece immediately below the shear plane being stressed to the yield point. The material, on crossing the shear plane, partially unloads so as to become non-plastic. Both the yield stress and the maximum shear directions must be regarded as discontinuous across the assumed shear plane, which coincides with a slipline for the workpiece material. Indeed, if the maximum shear directions were to be continuous, the different values of the shear yield stress across the shear plane would violate the condition of tangential equilibrium. In a real metal, the yield stress would vary continuously through a narrow zone of intense shear, the limit of which coincides with the shear plane.

The proposed slipline field (Figure 2) contains a stress discontinuity AD which separates two regions of constant stress. In the triangular region ADE, the sliplines meet the stress-free surface AE at 45°, while in the remainder of the field, the sliplines are parallel and perpendicular to the shear plane. It follows from the Geiringer equations and the boundary conditions that no deformation occurs in $ABCDE$, although it is at the yield limit. Hence there is no strain-hardening in this region either. Since the sliplines are reflected in the discontinuity AD, it follows from geometry that the angle which the α-lines make with AD is $\theta = \pi/8 - \phi/2$. The jump in the hydrostatic pressure across AD is

$$2k \sin 2\theta = 2k \sin\left(\frac{\pi}{4} - \phi\right),$$

where k is the initial shear yield stress of the material at the appropriate temperature. Since the hydrostatic pressure along the free surface AE is equal to k, the normal pressure acting on the shear plane is:

$$p = k\left[1 + 2\sin\left(\frac{\pi}{4} - \phi\right)\right]. \qquad (3)$$

Although a centered fan field seems more appropriate for $\phi \le \pi/4$, a slightly better agreement with experiment is provided by the discontinuous field, and consequently (3) will be used for all values of the shear angle.

The line of action of the resultant tool force is inclined at an angle $\lambda - \alpha + \phi$ with the shear plane (Figure 1). Since the tangential stress across the shear plane is equal to k, the normal pressure on this plane is

$$p = k \tan(\lambda - \alpha + \phi) . \tag{4}$$

From (3) and (4), the required shear angle relationship can be expressed in the form

$$\tan(\lambda - \alpha) = \frac{1 + \sqrt{2} \cos\phi}{\tan(\pi/4 + \phi) + \sqrt{2} \sin\phi} , \tag{5}$$

which shows that ϕ depends only on the difference $\lambda - \alpha$, in accord with the experimental observation. The variation of ϕ with $\lambda - \alpha$ predicted by (5) is evidently non-linear, but the departure from linearity is very small. The shear angle ϕ would be mariginally increased for a given $\lambda - \alpha > 0$ if the discontinuous stress field is replaced by a continuous one involving a centered fan in the range $\phi \leq \pi/4$.

The component of the tool force in the direction of cutting, known as the cutting force, is most conveniently obtained by considering the forces acting on the shear plane. Since there is a shear stress k and a normal pressure p acting uniformly on AB, the horizontal force (per unit width) transmitted across AB is

$$F = kh(p + k \cot\phi) ,$$

where h is the depth of cut. Substituting for p from Equation (3), we obtain

$$\frac{F}{kh} = 1 + 2 \sin\left(\frac{\pi}{4} - \phi\right) + \cot\phi , \tag{6}$$

which is again a function of ϕ. From (5) and (6), the dimensionless cutting force F/kh can be calculated for any given λ and α. For a given depth of cut, the cutting force rapidly increases as the shear plane angle decreases. From the geometry of Figure 1, the ratio of the chip thickness t and the depth of cut h is

$$\frac{t}{h} = \frac{\cos(\phi - \alpha)}{\sin\phi} . \tag{7}$$

Equations (5) and (7) give the ratio t/h as a function of λ and α. The length BG in Figure 2 may be considered as approximately equal to the length l of the chip–tool contact in the machining process, and an expression for l/h may be written down from simple geometry.

RANGE OF VALIDITY

It is easy to show that the rigid workpiece material near corner B remains non-plastic. The condition under which the yield criterion is not violated within the chip will now be examined. The stress will be discontinuous across AB, since the yield stress itself is discontinuous, which means that AB is not a slipline

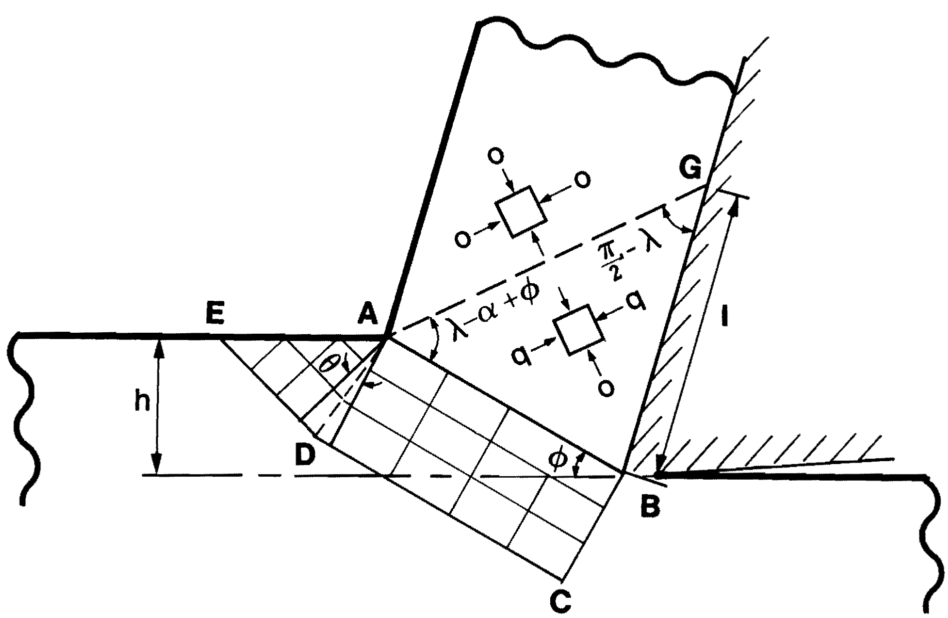

Figure 2. Slipline Field for Orthogonal Machining of a Work-Hardening Material.

for the material immediately above the shear plane. The stress is assumed to be a uniform compression q in the triangular region ABG, where AG is a stress discontinuity inclined at an angle $\pi/2-\lambda$ to the tool face. AG is therefore parallel to the resultant tool force. The material beyond AG is assumed stress-free. It is easy to see that the frictional condition on the tool face is then automatically satisfied. The fact that the shear stress must be continuous across AB gives

$$q = 2k \operatorname{cosec} 2(\lambda-\alpha+\phi), \tag{8}$$

the stress state in ABG will be statically admissible if $q \leq 2k'$, which is the plane strain uniaxial yield stress in this region. Evidently, k' depends on the mean strain rate and temperature in the shear zone, as well as on the total engineering shear strain γ having the magnitude [14]

$$\gamma = \frac{\cos\alpha}{\sin\phi\cos(\phi-\alpha)}. \tag{9}$$

Since the validity of the solution requires the right-hand side of (8) not to exceed $2k'$, the inequality

$$k' \geqslant k \operatorname{cosec} 2(\lambda-\alpha+\phi)$$

must be satisfied for all possible values of λ and α. Using (5), the above inequality can be written entirely in terms of ϕ as:

$$\frac{k'}{k} \geqslant 1 + \frac{1-\sin 2\phi}{1+2\sin(\pi/4-\phi)}, \tag{10}$$

As ϕ decreases from 45°, the right-hand side of (10) slowly increases from unity, approaching the value $\sqrt{2}$ as ϕ tends to zero. The right-hand side also increases when ϕ increases from 45°, but the value of this expression is less than 1.5 for $\phi<63°$ or $\lambda-\alpha>-42.1°$. The inequality will therefore be satisfied in most cases of practical interest.

If the ratio k'/k is such that the inequality (10) is violated, the region ABG will be stressed to the yield point, and the workpiece will be entirely non-plastic. The α−lines in the plastic region ABG must be inclined at an angle $\pi/4-\lambda$ to the rake face of the tool. The normal pressure across AB is no longer given by (3), but can be determined from the yield condition $q=2k'$ in ABG, and the fact that the shear stress across AB is still equal to k. The elimination of $\lambda-\alpha+\phi$ between (4) and (8) then furnishes [13]

$$\tfrac{1}{2}\left(\frac{p}{k}+\frac{k}{p}\right) = \frac{k'}{k}.$$

This is a quadratic in p/k, and the solution is:

$$\frac{p}{k} = \frac{k'}{k} \pm \sqrt{\left\{\left(\frac{k'}{k}\right)^2 - 1\right\}}. \tag{11}$$

When p/k is known from (11), the shear plane angle is obtained from the formula:

$$\phi = \tan^{-1}\frac{p}{k} - (\lambda-\alpha), \tag{12}$$

in view of (4). It may be noted in passing that (1), (2), and (5) are contained in (12) as special cases. Since k'/k depends on the mean strain rate and temperature, both of which are functions of ϕ, a process of trial and error will be necessary to obtain consistent values using (11) and (12). For a given $\lambda-\alpha$, the computed value of ϕ will be lower than (5), but higher than (2). For a given value of k'/k, the shear angle relationship furnished by (11) and (12) represents a straight line parallel to that given by (2), and terminating at a point on the curve represented by (5).

The shear plane angle ϕ predicted by the present theory is shown by the solid curve, Figure 3, as a function of $\lambda-\alpha$ for $\phi\leq 45°$. The curve lies between the broken straight lines which correspond to the shear angle relations due to Merchant and Lee and Shaffer. The predicted curve is tangential to the upper broken line at $\phi=45°$, where $\lambda=\alpha$ according to all the three theories. The experimental results of Merchant [1] and Thomsen *et al.* [15], covering a wide range of values of λ and α at various cutting speeds, are also plotted for comparison. It is evident that the agreement between the calculated and observed values of angle ϕ is generally good for the present shear angle relationship. Figure 4 shows the variations of the specific cutting force F/kh and the specific chip thickness t/h with the rake angle α and the angle of friction λ.

STRAIN RATE AND TEMPERATURE

The shear plane may be considered as the limit of a narrow transition region of variable thickness, through which the strain-hardening of the material and the subsequent unloading take place rapidly in a continuous manner. Since the direction of the maximum shear stress in the region ABG makes angle of $\pi/4$ with AG, the shear plane AB is inclined at an angle

$$\beta = \lambda-\alpha+\phi-\pi/4$$

to the maximum shear direction in the chip material adjacent to the shear plane. The numerical computation based on Equation (5) reveals that β varies from 0 to 16° as $\lambda-\alpha$ increases from 0 to 40°. The thickness of the transition zone does not therefore vary significantly according to the present theoretical model.

It is reasonable to suppose that the path of the particles crossing the transition zone rotates through an angle $\beta/2$ during the loading, while the remaining angle $\beta/2$ is required for the partial unloading. In other words, the upper boundary of the shear zone is inclined at an angle $\beta/2$ to the shear plane AB (Figure 5). Hence, the mean thickness of the shear zone is

$$\delta = (a + \tfrac{1}{4}\beta h)\operatorname{cosec}\phi = \left\{a + \frac{h}{4}\left(\lambda-\alpha+\phi-\frac{\pi}{4}\right)\right\}\operatorname{cosec}\phi\,, \tag{13}$$

where a is the projection of the shear zone thickness at point A along to the direction of cutting. The mean strain rate in the zone of shear is approximately equal to v/δ, where v is the magnitude of the velocity discontinuity across the shear plane AB. Since $v = U\cos\alpha/\cos(\phi-\alpha)$, where U is the cutting speed, we obtain

$$\frac{v}{\delta} = \frac{4U\cos\alpha\sin\phi\sec(\phi-\alpha)}{4a+h(\lambda-\alpha+\phi-\pi/4)}\,, \tag{14}$$

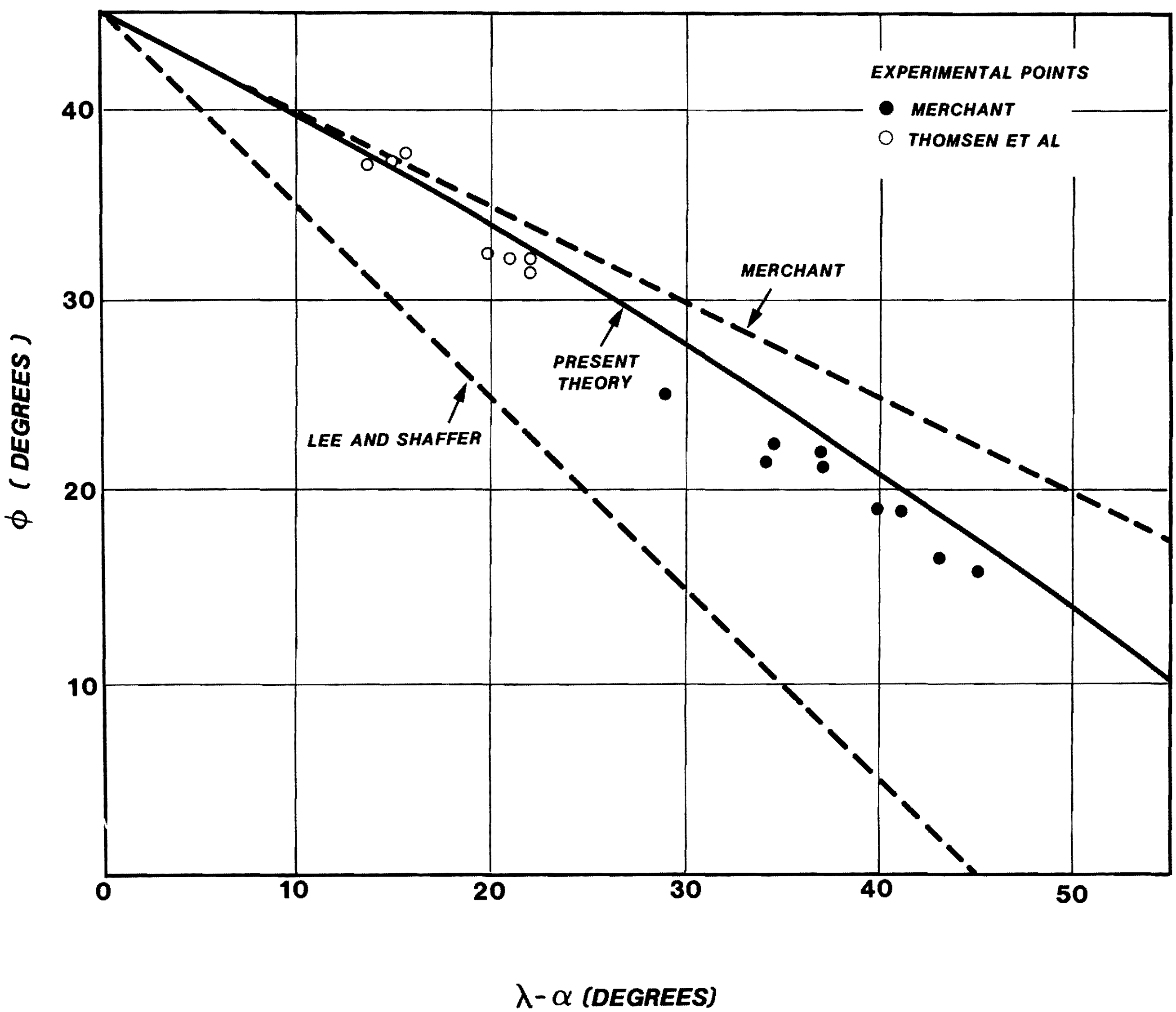

Figure 3. Shear Angle Relationship in Orthogonal Machining and Comparison with Experiment.

in view of (13). Available experimental results tend to suggest that the ratio a/h has an approximately constant value over a wide range of cutting conditions. Assuming $a = h/8$, which makes the above formula agree with Stevenson and Oxley's empirical formula [16] when $\lambda - \alpha = 0$, the expression for the mean strain rate is finally obtained as

$$\dot{\gamma} = \frac{4U \cos\alpha \sin\phi \sec(\phi - \alpha)}{h(0.5 + \lambda - \alpha + \phi - \pi/4)} . \tag{15}$$

It would be interesting to consider the significance of the above formula in relation to the assumption made by Stevenson and Oxley that the mean thickness δ of the shear zone is proportional to its length b. According to the present theory,

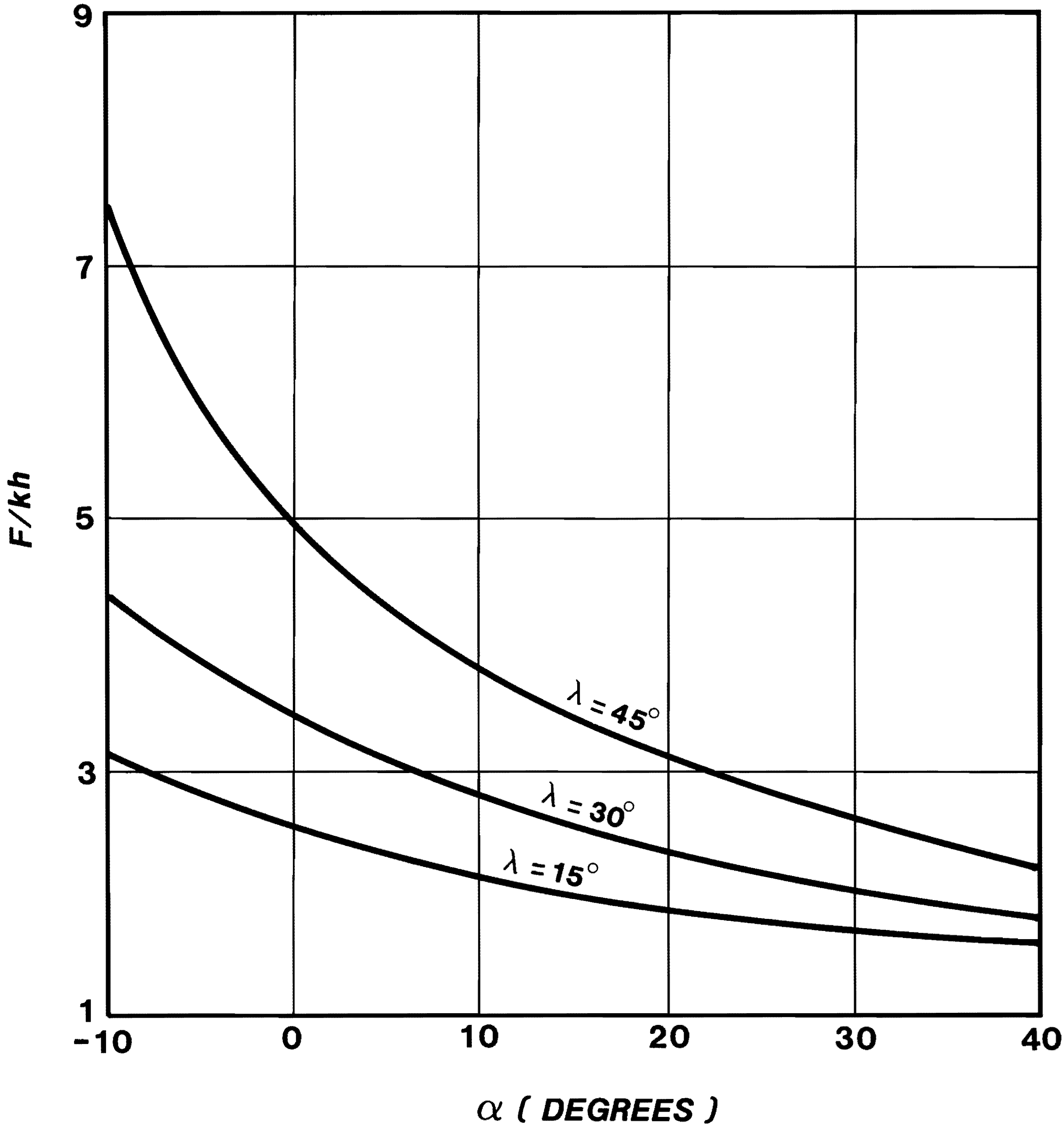

Figure 4. Variation of the Dimensionless Cutting Force with the Rake Angle for Different Angles of Friction.

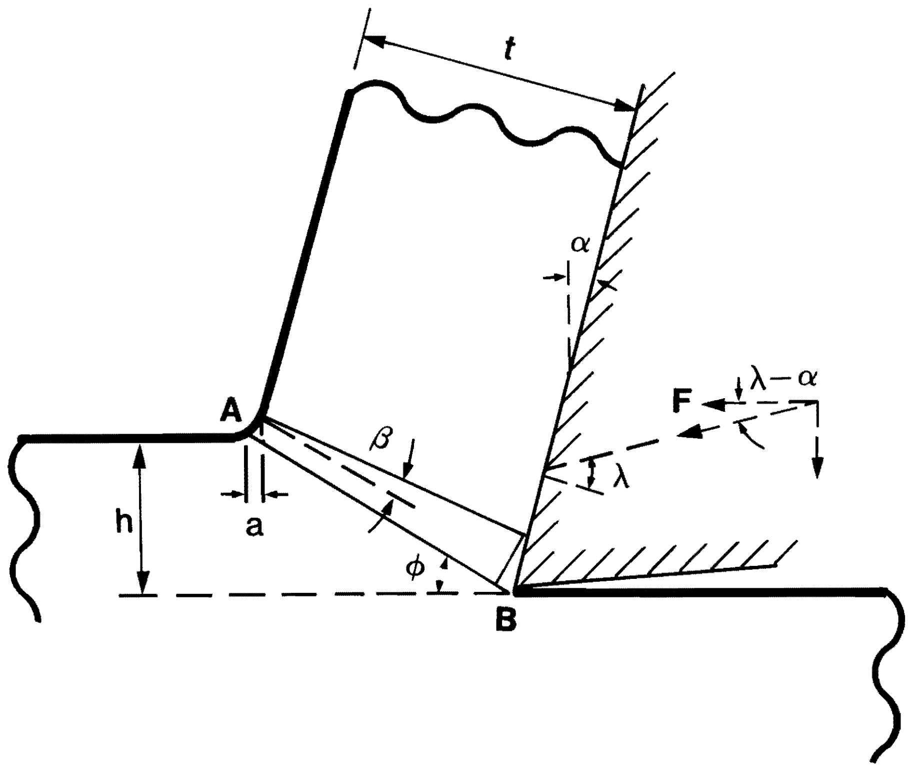

Figure 5. Geometry of the Assumed Shear Zone in Orthogonal Machining.

$$\frac{b}{\delta} = \frac{h}{\delta} \operatorname{cosec} \phi = \frac{4}{0.5 + \lambda - \alpha + \phi - \pi/4},$$

which shows b/δ varies with the parameter $\lambda - \alpha$. This result is in agreement with the experimental observation of Stevenson and Oxley [16], although these authors suggested a constant value C for the ratio b/δ. In Figure 6, the mean strain rate given by (15) is compared with that given by the empirical formula $\dot{\gamma} = Cv/b$ with $C = 6.5$, which is a suitable mean value of the multiplying factor. The mode of variation of the strain rate predicted by the present theory appears to be more realistic. Hastings, Oxley, and Stevenson [17] computed the mean strain rate in their machining theory on the basis of their empirical formula, which seems to be a rather poor approximation.

The yield stress of the material crossing the shear zone depends not only on the strain and the mean strain rate, but also on the mean temperature in the shear zone. This mean temperature exceeds the original temperature of the workpiece by the amount $0.5\,\Delta T$, where ΔT is the rise in temperature given by [13]:

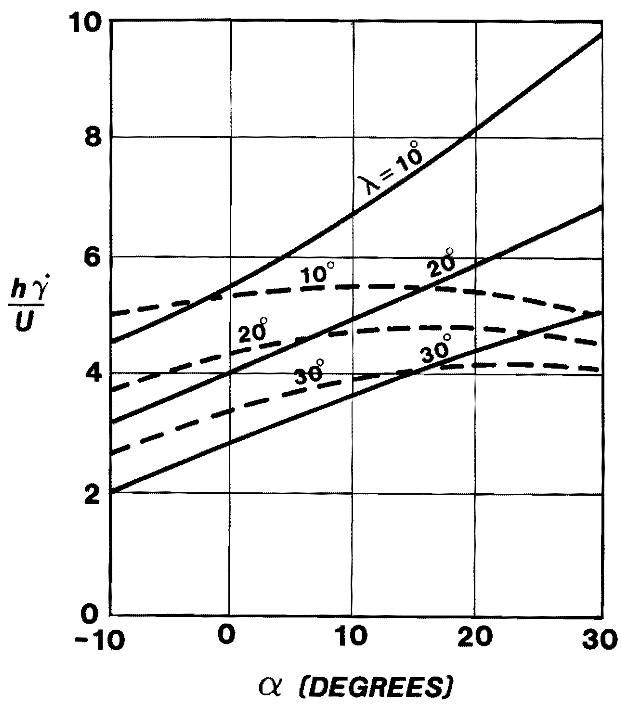

Figure 6. Influence of Rake Angle and Angle of Friction on the Mean Strain Rate i Orthogonal Machining.

$$\Delta T = \frac{\eta F}{h\rho c}\left\{1 - \frac{\sin\lambda \sin\phi}{\cos(\lambda-\alpha)\cos(\phi-\alpha)}\right\}, \quad (16)$$

where F is the cutting force, ρ the density of the workpiece material, c its specific heat, and η the proportion of the generated heat that is convected into the chip. The parameter η may be obtained from an experimental curve derived by Boothroyd [18]. The shear yield stress k' of the chip material may be computed from the power law $k' = B\gamma^n$ where the constants B and n are experimentally determined for the computed values of the mean strain rate and temperature.

Usui and Takada [10] calculated a range of values of a nominal shear angle ϕ using a slipline field proposed by Kudo [9]. The field is defined by several independent parameters which cannot be uniquely determined from an average angle of friction λ. Their theoretical results, which apply to a non-hardening material, form a wide band in the plot of ϕ against $\lambda - \alpha$. Although the available experimental points fall within this band, it is hard to see how the results may be used for a quantitative estimation of the shear plane angle in any particular situation.

CONCLUSIONS

The shear angle formula (5) is independent of the precise strain-hardening characteristic of the material so long as inequality (10) is satisfied. The shear yield stress k' of the fully hardened material, under the mean strain rate and temperature existing in the neighborhood of the shear plane, must be determined in order to check the validity of (5). In exceptional cases, when inequality (10) is violated, Equations (11) and (12) should be used to compute the shear plane angle. The theory shows in a quantitative manner how an appropriate inclusion of the effects of work-hardening, strain rate, and temperature enables us to predict the shear plane angle that is in agreement with the experimental measurement.

It is important to note that the shear angle relationship (5) applies to most cases of practical metal cutting, since the intense shearing of the material crossing the shear plane would almost invariably elevate the yield stress of the chip to a value well above that of the workpiece. Only in exceptional cases, when the degree of work-hardening of the material is not able to outweigh the degree of thermal softening to the extent required by inequality (10), will the shear plane angle be given by equations (11) and (12). According to the present theory, the shear plane relation is always represented by a point lying between the solid curve and the lower broken line shown in Figure 3. This conclusion is supported, within experimental error, by the measured values of the shear plane angle published in the literature over a wide range of cutting conditions. The predicted value of the shear plane angle depends, however, on the accuracy of the value of k'/k determined from the computed strain rate and temperature of the material crossing the shear plane.

REFERENCES

[1] M. E. Merchant, "Mechanics of Metal Cutting Process", *J. Appl. Phys.*, **16** (1945), p. 267.

[2] E. H. Lee and B. W. Shaffer, "The Theory of Plasticity Applied to a Problem of Machining", *J. Appl. Mech., Trans. ASME,* **18** (1951), p. 405.

[3] R. Hill, "The Mechanics of Machining: A New Approach", *J. Mech. Phys. Solids,* **3** (1954), p. 47.

[4] G. W. Rowe and P. T. Spick, "A New Approach to Determination of the Shear Plane Angle in Machining", *J. Eng. Indust., Trans. ASME,* **89** (1967), p. 530.

[5] W. B. Palmer and P. L. B. Oxley, "Mechanics of Orthogonal Machining", *Proc. Instn. Mech. Engrs.*, **173** (1959), p. 623.

[6] R. N. Roth and P. L. B. Oxley, "Slipline Field Analysis for Orthogonal Machining Based upon Experimental Flow Fields", *J. Mech. Engng. Sci.*, **14** (1972), p. 85.

[7] K. Iwata, J. Osakada, and Y. Teresaka, "Process Modeling for Orthogonal Cutting by the Rigid/Plastic Finite Element Method", *J. Eng. Mat. Tech.*, **106** (1984), p. 132.

[8] D. G. Christopherson, "Contribution to the Theory of Orthogonal Cutting in a Non-Work-Hardening Material", *J. Mech. Engng. Sci.*, **4** (1962), p. 292.

[9] H. Kudo, "Some New Slipline Solutions for Two Dimensional Steady State Machining", *Int. J. Mech. Sci.*, **7** (1965), p. 43.

[10] E. Usui and K. Takada, "Study of Compatibility of Kudo's Slipline Solution for Two-Dimensional Flow Type Machining", *J. Japan Soc. Precision Mech.*, **33** (1967), p. 23.

[11] P. Dewhurst, "On the Non-Uniqueness of the Machining Process", *Proc. Roy. Soc. (London), Ser. A.*, **360** (1978), p. 587.

[12] J. Chakrabarty, "A New Slipline Field Solution for the Orthogonal Machining of Metals", *Proc. Int. Conf. Prod. Engng, New Delhi,* 1977.

[13] J. Chakrabarty, *Theory of Plasticity,* ch. 7. New York: McGraw–Hill, 1987.

[14] R. Hill, *The Mathematical Theory of Plasticity.* Oxford: Clarendon Press, 1950.

[15] E. G. Thomsen, J. T. Lapsely, and R. C. Grassie, "Deformation Work Absorbed by the Workpiece During Metal Cutting", *Trans. ASME,* **75** (1953), p. 591.

[16] M. G. Stevenson and P. L. B. Oxley, "An Experimental Investigation of the Influence of Speed and Length on Strain-Rate in a Zone of Intense Plastic Deformation", *Proc. Instn. Mech. Engrs.*, **184** (1969), p. 561.
[17] W. F. Hastings, P. L. B. Oxley, and M. G. Stevenson, "Predicting a Material's Machining Characteristics Using Flow Stress Properties Obtained from High Speed Compression Test", *Proc. Instn. Mech. Engrs.*, **188** (1974), p. 245.
[18] G. Boothroyd, "Temperatures in Orthogonal Metal Cutting", *Proc. Instn. Mech. Engrs.*, **177** (1963), p. 789.

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