HARMONIC MODEL OF HOURLY FRACTIONS OF DAILY GLOBAL AND DIFFUSE SOLAR RADIATION

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الخلاصـــة :

لقد تَـمَّ في هذا البحث تطوير نموذج رياضي متناغم لحساب نسبة الإشعاع الكلي لكل ساعة ، والإشعاع المبعثر من الإشعاع الساقط للطاقة الشمسية . وقد حددت معاميل (فورير) للنموذج باستعمال قراءات ساعوية للطاقة الشمسية . وللتأكد من عمومية هذا المنهج تم حساب معاميل فورير لقراءات أخذت في خمس مدن في أقطار مختلفة . وقد بَـيَّـنَـتْ النتائج ثبات مستوى دقة النموذج . فالنتائج المحسوبة باستعماله جاءت في توافق ممتاز مع القراءات . وفي عدد كبير منها كان الاختلاف المئوي أقل من (١ ٪) . وتم كذلك إخراج معدلات لمعاميل فورير يجعل النموذج عاماً . وقد بقي الاختلاف بين النتائج المحسوبة والقراءات في حدود المقبول ، خاصة إذا طُـبِّـق النموذج في فترة العشرة ساعات حول منتصف النهار . وهكذا فإن حساب الكميات المرغوبة باستعمال النموذج المتناغم قد أدخل تحسيناً ملحوظاً على النتائج المحسوبة باستعمال النماذج الأخرى .

ABSTRACT

A harmonic model for the hourly global and diffuse solar radiation fractions is developed. The Fourier coefficients of the model are determined by using measured data. In testing the generality of this approach, similar analysis is applied to data collected in five cities in different countries. The results demonstrate a consistency in the level of accuracy of the model. Computations by the model are in excellent agreement with the data. The percentage error between the data and the computed values are in many cases negligible — less than 1%. The averaged Fourier coefficients are used in the model to make it general. The errors of the general model remain within acceptable limits, especially for time ranges of about ten hours around the solar noon. Computations of the hourly global and diffuse solar radiation fractions by the harmonic model developed in this research demonstrate a significant improvement over computations by existing models.

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INTRODUCTION

Mathematical models of solar radiation intensities are used by design and simulation engineers and scientists in the development of engineering systems that use solar energy as an input. In Jordan, research and development in the area of solar energy has been undertaken for almost two decades. The first models for the computation of solar energy in Jordan was developed by the author more than a decade ago [1].

Hourly solar radiation is needed for the optimization of the processes of collection and utilization of available solar energy. Several attempts have generated a number of models to compute the hourly global fraction (HGF) and the hourly diffuse fraction (HDF). An hourly fraction is defined as the monthly average of measured solar radiation during a given hour of a day divided by the monthly average of the measured daily solar radiation. Both HGF and HDF include values for cloudy as well as clear sky days. These models and others are reviewed by Alsaad and Audi [2].

Liu and Jordan [3] proposed the following model for estimating HDF and HGF.

$$\xi = \frac{\pi}{24}\left[\frac{\cos\omega - \cos\omega_s}{\sin\omega_s - \omega_s\cos\omega_s}\right], \tag{1}$$

where ω is the desired hour angle for the day, measured from solar noon and ω_s is the sunset hour angle for the day of interest. Obviously, this model does not depend on the characteristics of local climates. It has been used for computing both the HGF and the HDF. See, for example, Garg *et al.* [4].

However, this model was modified by Collares-Pereira and Rabl [5] and by Garg and Garg [4] to include parameters that are based on local climatological conditions. The model proposed by Collares-Pereira and Rabl [5] for the HGF is given in the following relation:

$$\xi_g = \frac{\pi}{24}(a + b\cos\omega)\left[\frac{\cos\omega - \cos\omega_s}{\sin\omega_s - \omega_s\cos\omega_s}\right], \tag{2}$$

where both a and b are constants given by the following relations:

$$a = 0.4090 + 0.5016\sin(\omega_s - 1.047)$$
$$b = 0.6609 - 0.4767\sin(\omega_s - 1.047)\ ,$$

where ω_s is in radians.

Garg *et al.* [4] sought to adapt these two models (Equations (1) and (2)) to data collected in five cities in India. Their research, however, introduced a new modification to the original Liu and Jordan model, Equation (1). The following are the two models produced by Garg *et al.* [4] for the HGF and the HDF, respectively:

$$\xi_g = \frac{\pi}{24}\left[\frac{\cos\omega - \cos\omega_s}{\sin\omega_s - \omega_s\cos\omega_s}\right] - 0.008\sin 3(\omega - 0.65) \tag{3}$$

$$\xi_d = \frac{\pi}{24}\left[\frac{\cos\omega - \cos\omega_s}{\sin\omega_s - \omega_s\cos\omega_s}\right] + 0.010\sin 3(\omega - 0.65)\ , \tag{4}$$

where the numerical coefficients in these models exist to represent local conditions and ω is to be given in radians.

These four models (Equations (1)–(4)) were tested for the HGF and HDF data collected in Amman, Jordan. The results produced were unsatisfactory, especially for the summer months, for both the global and the diffuse fractions. Figures 1–4 illustrate the discrepancy between the observed data and computations by these models. In addition to the values computed by the models, these Figures also include data collected in Amman and values computed by the harmonic model. For example, the observed HGF for the hour 8:00–9:00 am (−3) in Figure 1, of the month of January is 0.078, while the values computed by Equations (2) and (3) are 0.1047 and 0.0906, respectively. Thus the percentage error in the first is 34% and the error in the second is 16%. When the computed curves of the summer months are inspected, (Figures 2 and 4), it is seen that the discrepancy is more prominent.

In general, the magnitude of the percentage error varied from month to month and from hour to hour. However, the error was greater in computing values for times in the summer months than in winter. Moreover, the percentage error in computing the diffuse fraction is greater than the percentage error in computing the global fraction.

Obviously, other models must be sought to compute the hourly global and diffuse fractions of solar radiation, in Jordan.

Three simple models for computing HGF were tested by Audi and Alsaad [6]. These are (*i*) a normal distribution model as proposed by Jain [7], (*ii*) a polynomial model, and (*iii*) a half-sine wave model. The first of these models was tested by Jain [7] and found to be satisfactory for two cities in Canada. The other two were selected for the general shape they exhibit. The results published in reference [6] affirm that none of these models could by itself adequately represent the data collected in Jordan. Thus, a combination of them was found to solve the accuracy problem [6].

In analyzing the HDF data Audi and Alsaad [8] proposed the introduction of a control function Ψ which could be turned on or off depending on the hour of the day and the month of the year. This control function model is particularly useful in computer simulation of solar energy engineering systems. However, the procedure of developing these control functions, accurate as it is in computing the desired fraction, is tedious.

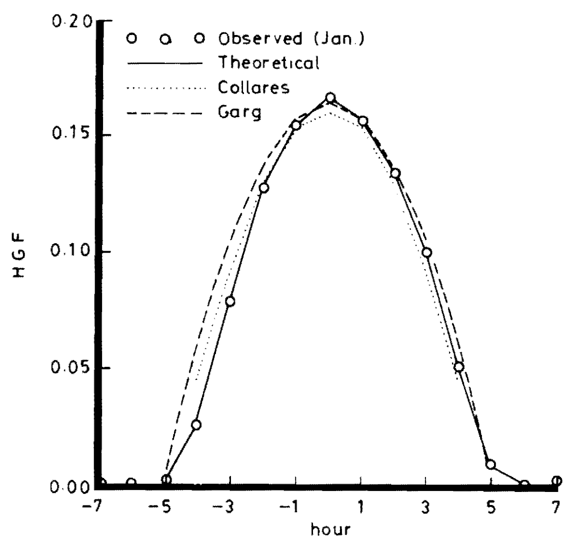

Figure 1. Data and Prediction for HGF.

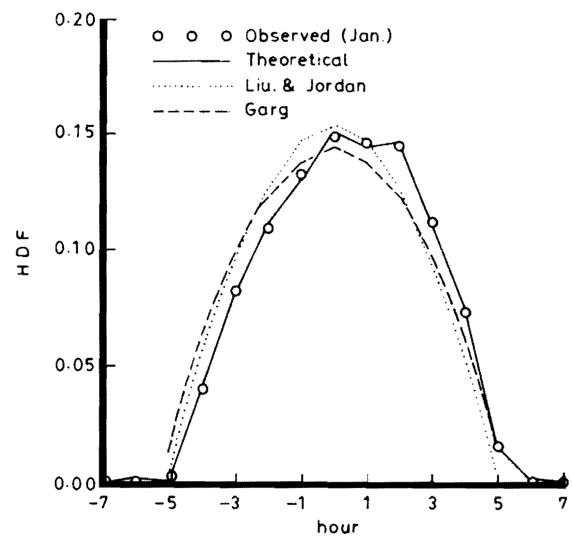

Figure 3. Data and Prediction for HDF.

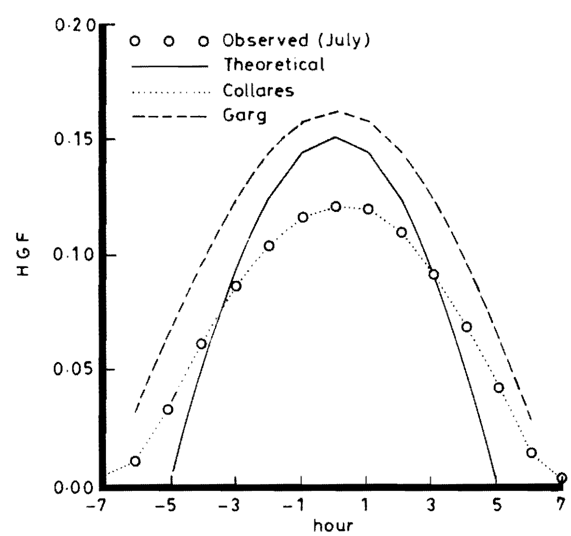

Figure 2. Data and Prediction for HGF.

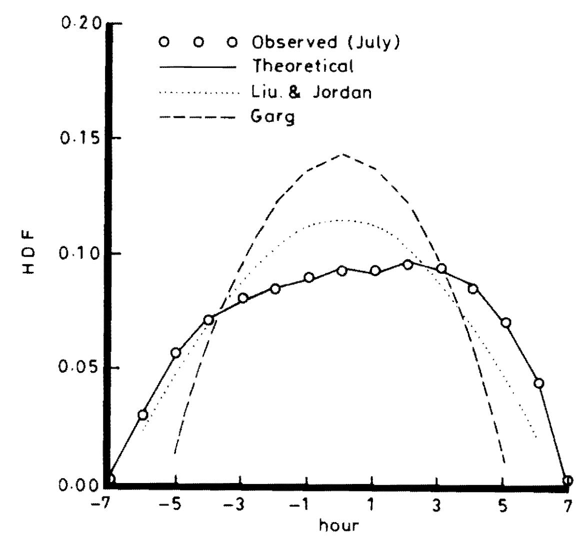

Figure 4. Data and Prediction for HDF.

In this paper, harmonic analysis of both the HGF and the HDF data produced a considerably improved model for the computation for these quantities of Amman, Jordan and for the other four cities, including Montreal, Canada, for which Jain [7] developed an accurate normal distribution model of the desired fractions.

The sources of data for the cities of Montreal and New Delhi have already been noted above. The data for the city of Kuwait, Kuwait was taken from a report on using solar energy to operate air conditioning systems [9], and the data for Baghdad, Iraq is from reference [10].

The rest of the paper presents the harmonic model, the results and discussion of the model coefficients, and a conclusion.

THE HARMONIC MODEL

The harmonic model is based on the Fourier time series for periodic data. The coefficients of this series are given by the Euler formulas as follows:

$$A_o = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)\,\mathrm{d}t\,, \tag{5}$$

$$A_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos nt\,\mathrm{d}t \tag{6}$$

$$B_o = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin nt\,\mathrm{d}t\,, \tag{7}$$

where $f(t)$ is the function under consideration — the HGF or the HDF — at time t.

These integrals were numerically evaluated using the trapezoidal rule. The 2π cycle was considered to span the $2k$ time intervals (hours) of the measured data. The results of these approximations are given in the following equations:

$$A_o = \frac{1}{2k}\sum_{i=o}^{k}[f(t_i)+f(-t_i)] \tag{8}$$

$$A_n = \frac{1}{k}\sum_{i=1}^{k}[f(t_i)+f(-t_i)]\cos nt_i \tag{9}$$

$$B_n = \frac{1}{k}\sum_{i=1}^{k-1}[f(t_i)-f(-t_i)]\sin nt_i\,, \tag{10}$$

where $f(t_i)$ and $f(-t_i)$ are the values of the function at symmetrical times with respect to the solar noon.

The harmonic model is the Fourier time series with the coefficients given by Equations (8)–(10).

$$\xi = \sum_{n=o}^{k}\left[A_n\cos n\frac{\pi t}{k} + B_n\sin n\frac{\pi t}{k}\right], \tag{11}$$

where ξ could be either ξ_g, the computed HGF; or ξ_d, the computed HDF.

RESULTS AND DISCUSSION

In this section, the results of the computations for both of the HGF and HDF data are included. But only the coefficients of the model that applies to Amman, Jordan, are listed. However, in computing the averaged coefficients for the general model which is also discussed in this section, all the computed coefficients of other cities were used.

Fourier Coefficients for Amman

The first set of data used in this analysis is comprised of monthly averages of hourly radiation for five years. The daily records of these data are comprised of fourteen time intervals. Thus, the value of k used in the harmonic analysis is 7. All reference to hours in this discussion is based on time equal to zero at solar noon; it is negative for periods before noon and positive for periods in the afternoon.

The Fourier coefficients computed on the basis of the data of Amman, Jordan are listed in Table 1 for the computation of HGF and in Table 2 for the computation of the HDF. These coefficients can be used directly in Equation (11) to generate the harmonic model for computing HGF or HDF for any desired month. The specific value of HGF or HDF for a certain hour is obtained by taking a specific value for t, such as -3 for the time three hours before solar noon.

In comparing the values of HGF or HDF which are computed by using the harmonic model (with the coefficients listed in Tables 1 and 2) with the observed data are almost in perfect agreement between the two values is noted for hours of the day from -5 to 5. Some deviation is noticed for time ranges wider than this one. Most of the differences in the values of ξ_g and ξ_d take place in the fourth decimal places of the fractions.

To demonstrate this outstanding achievement of the model, the percentage error between the observed and the computed values for two representative month of the year are given in Tables 3 and 4. Table 3 shows that the maximum error in HGF for January is 0.67% at the fourth hour from solar noon, and for July the maximum error of 0.78% occurs at the sixth

Table 1. Fourier Coefficients for HGF for Amman.

	Jan.	Feb.	March	April	May	June
A_0	0.0714	0.0714	0.0714	0.0714	0.0714	0.0714
A_1	0.0893	0.0824	0.0751	0.0663	0.0602	0.0565
A_2	0.0102	0.0046	−0.0014	−0.0046	−0.0080	−0.0085
A_3	−0.0078	−0.0055	−0.0014	0.0002	0.0015	0.0022
A_4	0.0016	0.0024	0.0026	0.0005	−0.0005	−0.0007
A_5	0.0013	0.0019	−0.0004	−0.0006	−0.0002	−0.0002
A_6	−0.0002	0.0004	0.0014	0.0002	0.0002	0.0006
A_7	0.0007	0.0000	0.0014	−0.0008	−0.0003	−0.0004
B_1	0.0083	0.0151	0.0082	0.0104	0.0003	0.0025
B_2	−0.0001	0.0003	0.0000	−0.0012	−0.0011	−0.0006
B_3	−0.0014	−0.0042	0.0010	0.0001	0.0006	0.0006
B_4	0.0008	0.0004	0.0022	0.0001	0.0005	0.0001
B_5	0.0012	−0.0003	−0.0005	−0.0009	0.0002	−0.0005
B_6	−0.0003	0.0000	0.0006	0.0007	0.0006	0.0001
	July	**Aug.**	**Sept.**	**Oct.**	**Nov.**	**Dec.**
A_0	0.0714	0.0714	0.0714	0.0714	0.0714	0.0714
A_1	0.0579	0.0632	0.0698	0.0743	0.0869	0.0900
A_2	−0.0080	−0.0072	−0.0054	−0.0033	0.0056	0.0108
A_3	0.0016	0.0008	−0.0012	−0.0031	−0.0097	−0.0082
A_4	−0.0004	0.0002	0.0009	0.0021	0.0034	0.0017
A_5	−0.0002	−0.0004	−0.0010	−0.0006	0.0033	0.0018
A_6	0.0001	0.0002	0.0006	0.0007	−0.0017	0.0006
A_7	−0.0003	−0.0002	−0.0006	0.0005	−0.0034	0.0018
B_1	0.0045	0.0038	0.0006	−0.0039	−0.0135	−0.0021
B_2	−0.0010	−0.0014	0.0005	−0.0002	−0.0037	−0.0032
B_3	0.0007	0.0013	0.0005	0.0018	0.0063	−0.0024
B_4	0.0003	−0.0000	−0.0002	0.0001	0.0023	−0.0006
B_5	−0.0005	−0.0004	−0.0001	0.0001	−0.0020	0.0000
B_6	0.0001	0.0005	0.0003	0.0004	−0.0016	−0.0006

hour. The minimum percentage error for January is 0.20% at solar noon and for July it is 0.10% at the solar noon, also. For the seventh hour there was no readings for the month of January. For July, the error produced is about 20%.

Table 4 shows that for computing HDF a maximum percentage error of 11.54% occurs during the month of January at the fifth hour. But during the month of July, this error is only 2.35% which occurs at the sixth hour. The minimum percentage error of 0.122% for the month of January occurs at solar noon, but that for July is 1.1% and occurs at the second hour.

One can also see that the percentage error is a little higher in computing HDF than computing HGF; however, the computed values are still in very good agreement with the data. Moreover, the computed values of both the HGF and the HDF are better for summer than winter, probably, because of the continuity of sunshine during the summer months compared with the cloudiness of the winter months.

Figures 1–4 show the closeness of the observed data and the computed values of HGF and HDF for a winter month, January, and a summer month, July. These figures are representative of the results of analyzing the data of Amman, Jordan. They also include computations using published models. The solid lines in these Figures represent the values computed by the harmonic model. These lines seem to pass through the data points because the computations were made for hours at which data existed.

Table 2. Fourier Coefficients for HDF for Amman.

	Jan.	Feb.	March	April	May	June
A_0	0.0714	0.0714	0.0715	0.0716	0.0718	0.0716
A_1	0.0802	0.0716	0.0686	0.0517	0.0412	0.0285
A_2	−0.0021	−0.0081	−0.0062	−0.0151	−0.0122	−0.0120
A_3	−0.0073	−0.0030	0.0009	0.0048	0.0056	0.0070
A_4	0.0048	0.0040	0.0023	−0.0001	−0.0034	−0.0051
A_5	−0.0004	−0.0012	−0.0014	−0.0006	0.0017	0.0041
A_6	−0.0000	0.0007	0.0009	0.0012	0.0000	−0.0021
A_7	0.0036	0.0013	−0.0004	−0.0018	−0.0011	0.0009
B_1	0.0148	0.0146	0.0094	0.0033	0.0042	0.0070
B_2	0.0043	−0.0020	−0.0008	−0.0018	−0.0030	−0.0019
B_3	−0.0020	−0.0016	0.0016	0.0010	0.0004	0.0017
B_4	0.0009	0.0031	0.0016	−0.0006	0.0005	−0.0011
B_5	−0.0014	−0.0034	−0.0015	−0.0006	0.0001	0.0003
B_6	−0.0021	−0.0009	0.0008	0.0003	0.0007	0.0009
	July	**Aug.**	**Sept.**	**Oct.**	**Nov.**	**Dec.**
A_0	0.0710	0.0712	0.0714	0.0714	0.0714	0.0714
A_1	0.0318	0.0403	0.0519	0.0642	0.0769	0.0798
A_2	−0.0155	−0.0163	−0.0150	−0.0143	−0.0064	−0.0026
A_3	0.0076	0.0079	0.0072	0.0005	−0.0078	−0.0082
A_4	−0.0043	−0.0038	−0.0010	0.0036	0.0054	0.0034
A_5	0.0035	0.0018	−0.0009	−0.0024	0.0003	−0.0014
A_6	−0.0022	−0.0011	0.0018	0.0021	0.0006	−0.0010
A_7	0.0021	0.0001	−0.0018	0.0011	0.0032	0.0021
B_1	0.0080	0.0044	−0.0002	−0.0085	−0.0005	−0.0011
B_2	−0.0019	−0.0024	0.0001	0.0019	0.0026	0.0017
B_3	0.0011	0.0009	−0.0001	0.0036	0.0026	0.0006
B_4	−0.0014	−0.0014	−0.0001	−0.0011	0.0017	−0.0009
B_5	0.0002	0.0014	0.0010	−0.0004	0.0021	−0.0002
B_6	0.0002	0.0001	0.0002	0.0001	0.0006	−0.0011

Smoother curves could have been obtained if more points were plotted.

The results of the harmonic analysis of the data of the other cities are similar to those of Amman. The model seems to be applicable to all hourly solar radiation data. The specific Fourier coefficients for each of the cities incorporate local climatological and geographical factors. For example, the Fourier coefficient A_0 of the model is uniformly equal to $1/\omega_s$, ω_s is a local geographical parameter.

An example of the harmonic model according to Equation (11) is given below for the month of January. Equation (12) is for the HGF and Equation (13) is for HDF. The coefficients are taken from Tables 1 and 2, respectively.

$$\begin{aligned}\xi_g = {} & 0.0714 + 0.8930\cos\frac{\pi t}{k} + 0.0102\cos\frac{2\pi t}{k} \\ & - 0.0078\cos\frac{3\pi t}{k} + 0.0016\cos\frac{4\pi t}{k} \\ & + 0.0013\cos\frac{5\pi t}{k} - 0.0002\cos\frac{6\pi t}{k} \\ & + 0.0007\cos\frac{7\pi t}{k} + 0.0083\sin\frac{\pi t}{k} \\ & - 0.0001\sin\frac{2\pi t}{k} - 0.0041\sin\frac{3\pi t}{k} \\ & + 0.0008\sin\frac{4\pi t}{k} + 0.0012\sin\frac{5\pi t}{k} \\ & - 0.0003\sin\frac{6\pi t}{k}, \qquad (12)\end{aligned}$$

Table 3. Percentage Error in the HGF (Amman).

Hour	0	1	2	3	4	5	6
Measured (Jan)	0.1662	0.1564	0.1337	0.0996	0.0505	0.0084	0.0000
Computed	0.1666	0.1560	0.1340	0.0993	0.0509	0.0081	0.0003
% Error	0.20	0.22	0.25	0.34	0.67	0.40	–
Measured (July)	0.1223	0.1211	0.1114	0.0933	0.0708	0.0444	0.0163
Computed	0.1222	0.1212	0.1113	0.0935	0.0707	0.0445	0.0161
% Error	0.10	0.11	0.11	0.14	0.18	0.29	0.78

Table 4. Percentage Error in the HDF (Amman).

Hour	0	1	2	3	4	5	6
Measured (Jan)	0.1484	0.1448	0.1435	0.1110	0.0724	0.0157	0.0000
Computed	0.1502	0.1429	0.1454	0.1092	0.0742	0.0139	0.0018
% Error	1.22	1.25	1.26	1.63	2.50	11.54	–
Measured (July)	0.0929	0.0929	0.0960	0.0939	0.0856	0.0710	0.0449
Computed	0.0940	0.0918	0.0971	0.0929	0.0867	0.0699	0.0459
% Error	1.14	1.14	1.10	1.13	1.23	1.49	2.35

$$\begin{aligned}\xi_d = {} & 0.0714 + 0.0802 \cos \frac{\pi t}{k} - 0.0021 \cos \frac{2\pi t}{k} \\ & - 0.0073 \cos \frac{3\pi t}{k} + 0.0048 \cos \frac{4\pi t}{k} \\ & - 0.0004 \cos \frac{5\pi t}{k} + 0.0000 \cos \frac{6\pi t}{k} \\ & + 0.0036 \cos \frac{7\pi t}{k} + 0.0148 \sin \frac{\pi t}{k} \\ & + 0.0043 \sin \frac{2\pi t}{k} - 0.0020 \sin \frac{3\pi t}{k} \\ & + 0.0009 \sin \frac{4\pi t}{k} - 0.0014 \sin \frac{5\pi t}{k} \\ & - 0.0021 \sin \frac{6\pi t}{k} . \end{aligned} \tag{13}$$

As for the data for the other cities, the percentage error between the observed and computed values are within the range of values presented above for the city of Amman, Jordan. In general, however, this error increases for the computed values of both the HGF and the HDF as the hour of interest moves away from solar noon. The least error produced in computing HGF and HDF values for these cities by the harmonic models is for Kuwait, although the data available to the author for this city is very limited. This error is negligibly less than 0.1% for both the global and the diffuse fractions.

Comparison of Fourier Coefficients

In comparing the coefficient A_o of the harmonic model, it is found that the values are nearly equal to 0.0714 for four of the five sets of data considered in this research. This value is equal to $1/\omega_s$, which was taken as the same for all of this research. The deviation of the value of A_o for the fifth set of data is about 13%. These values are presented graphically in Figure 5 for the hourly global fraction and in Figure 6 for the hourly diffuse radiation fraction.

The coefficients of the first and other harmonics vary from relatively high values during the winter months to lower values during the summer months. Figures 7 and 8 demonstrate this observation and the considerable scatter in the values of the coefficients of the first harmonic, but the general trend noted above is well illustrated.

Averaged Fourier Coefficients

As explained above, the harmonic computation model for any desired location is completely determined by computing the Fourier coefficients of all the harmonics using available data for that particular location. It is also known, from statistical considerations, that the greater the extent of the data the more stable the model becomes.

However, two simplifying assumptions could be made which help produce a general computation model. The first assumption involves consideration of the symmetry of the data about solar noon. This assumption is inherent in the data used in this paper for both New Delhi, India [4]; and Baghdad, Iraq [10]. If this is done, the sine (odd functions) terms of the model disappear.

The second assumption involves the averaging of the coefficients of the harmonics of the model. When this simplification is tested it was found that the error generated by this model is less than that produced by the models given in Equations (1)−(4).

Representative (for January and July) comparison curves of the computed results based on the averaged coefficients with data for four cities are given in Figures 9 and 10, for the hourly global fraction, HGF, and in Figures 11 and 12, for the HDF. It is noticed that the percentage error increases with the increase in the time range. In most cases, however, the error is less than 10% for a time range of 10 hours or less around the solar noon.

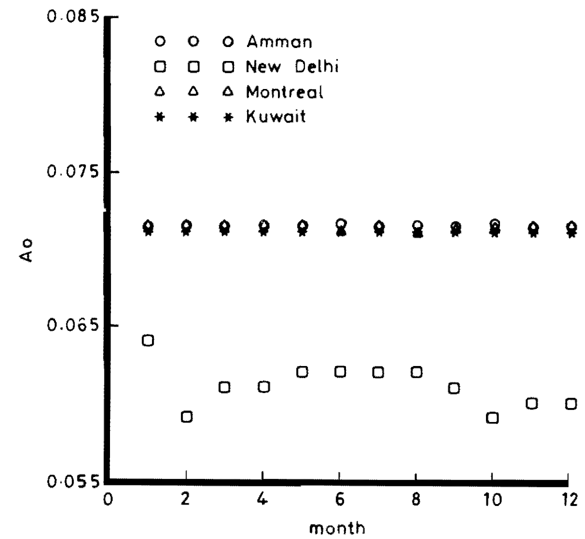

Figure 5. Harmonic Model Constants — HGF.

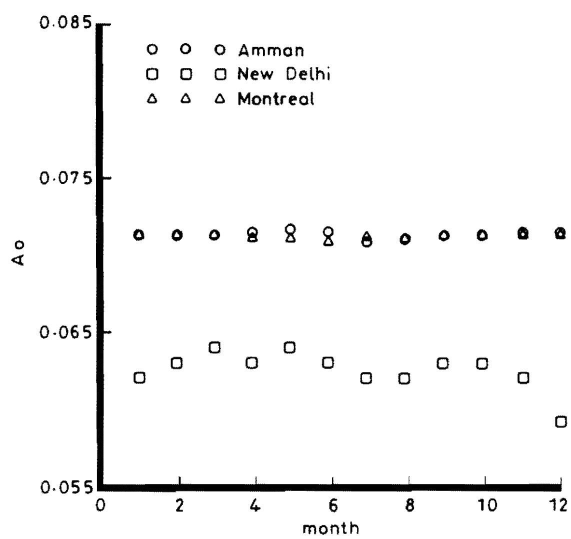

Figure 6. Harmonic Model Constants — HDF.

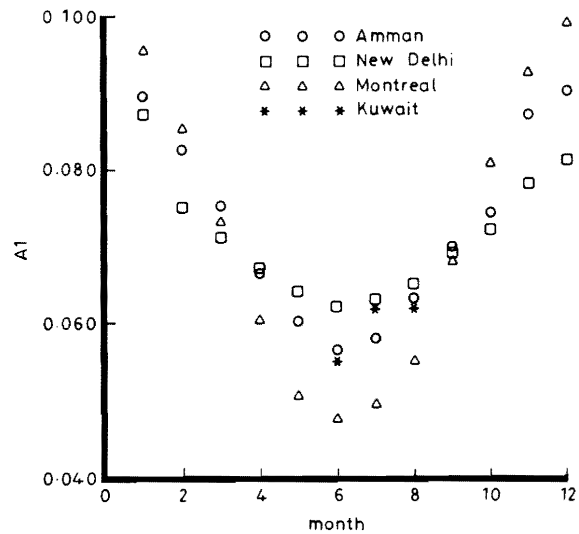

Figure 7. Coefficients of First Harmonic — HGF.

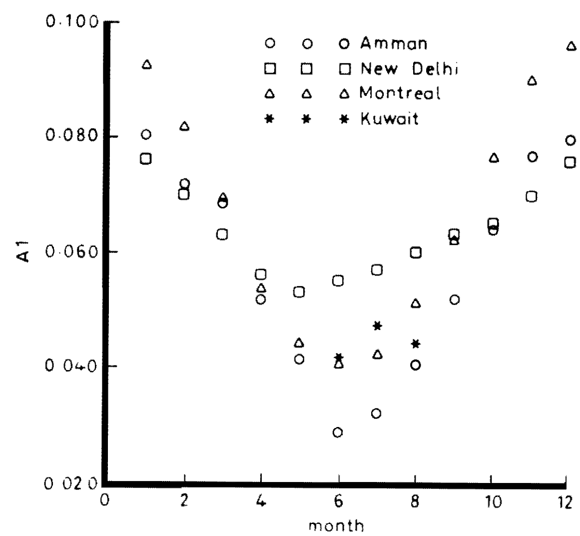

Figure 8. Coefficients of First Harmonic — HDF.

The computation formulas of the generalized model for both HGF and HDF are given by the following equations for the month of July:

$$\xi_g = 0.0714 + 0.057\cos\frac{\pi t}{k} - 0.007\cos\frac{2\pi t}{k} + 0.002\cos\frac{3\pi t}{k} - 0.001\cos\frac{4\pi t}{k}, \tag{14}$$

$$\xi_d = 0.0714 + 0.037\cos\frac{\pi t}{k} - 0.011\cos\frac{2\pi t}{k} + 0.006\cos\frac{3\pi t}{k} - 0.003\cos\frac{4\pi t}{k} + 0.002\cos\frac{5\pi t}{k} - 0.001\cos\frac{6\pi t}{k} + 0.002\cos\frac{7\pi t}{k}. \tag{15}$$

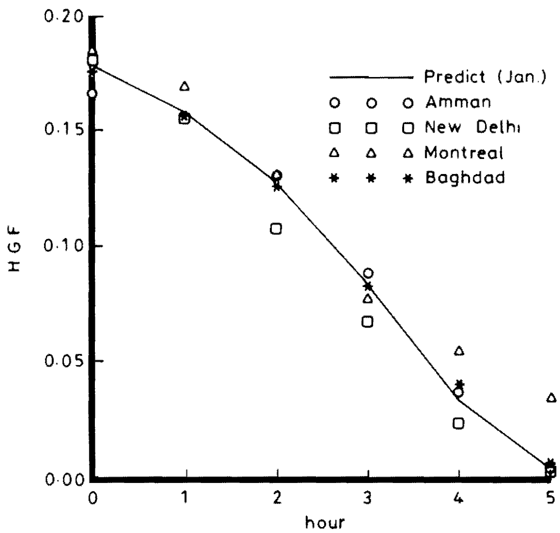

Figure 9. Model with Averaged Coefficients.

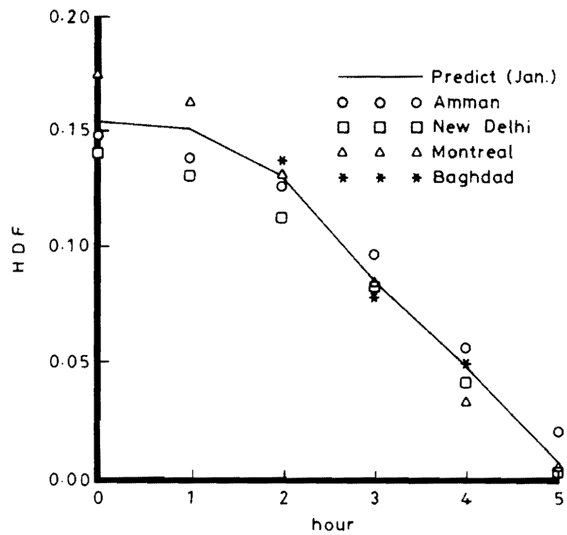

Figure 11. Model with Averaged Coefficients.

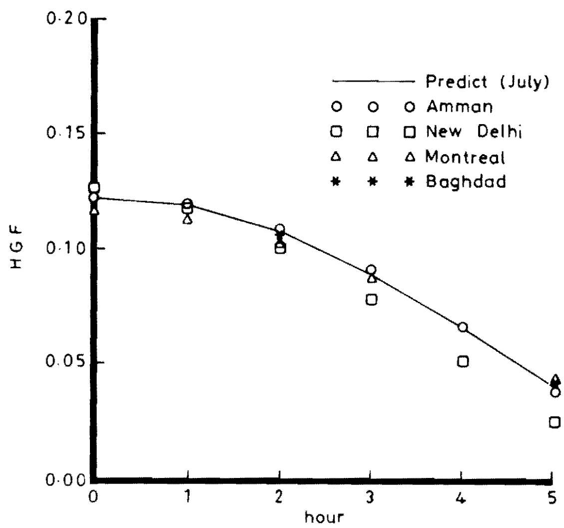

Figure 10. Model with Averaged Coefficients.

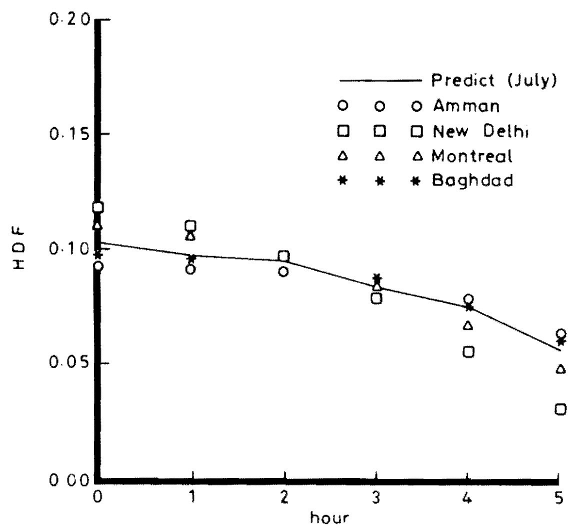

Figure 12. Model with Averaged Coefficients.

The averaged coefficients for HGF and HDF are given in Tables 5 and 6, respectively.

The harmonic model for any month can be produced by using these coefficients to generate equations similar to Equations (14) and (15).

CONCLUSION

In this research, harmonic analysis of HGF and HDF of four full sets of data and of a few months of a fifth set of data was undertaken. The data represent cities in Canada, Jordan, Kuwait, Iraq, and India. The conclusion of this study is outlined in the following statements.

1. The published models — presented in the introduction of this paper — for the computation of the hourly fractions did not produce satisfactory results in computing these quantities for Amman, Jordan. An example of the magnitude of deviation is a 34% error produced in the computation of HGF for the hour 8:00–9:00 am in January. Therefore, a model which incorporates local data was desirable. The harmonic model was developed in this paper to respond to this need.
2. In using actual data for the global and diffuse fractions to determine the Fourier coefficients of a harmonic series, almost exact models were developed. For time periods of twelve hours or less, the percentage error in computing the global fraction is less than 1%, and in computing the diffuse fraction is 2.5% or less.
3. For a general model that could apply to more than one locality, two assumptions were introduced. These are (*i*) the assumption that the data is symmetric about the solar noon — this is inherent in the data used in this research for Baghdad and New Delhi, — and (*ii*) the Fourier coefficients of the general harmonic are averages of those of the individual cities. This produced results that are acceptable — less than 10% error in many cases.
4. The general model produced by the averaged Fourier coefficients is limited in its application to climate which are symmetric with respect to solar noon. Furthermore, it is based on data for a limited number of cities. Thus, from a statistical point of view any additional data used to further develop the Fourier coefficients may improve the accuracy of the computed values.

Thus harmonic analysis of solar radiation data is an appropriate tool. The models generated using this approach could be more accurate if local data are used. If a general expression for the model is desired, statistically, the greater the number of sets of data used for averaging the Fourier coefficients the better the model would be.

Table 5. Averaged Fourier Coefficients for HGF.

	Jan.	Feb.	March	April	May	June
A_0	0.0714	0.0714	0.0714	0.0714	0.071	0.071
A_1	0.091	0.080	0.074	0.065	0.058	0.055
A_2	0.015	0.004	0.001	−0.004	0.007	−0.007
A_3	−0.005	−0.004	−0.001	0.001	0.001	0.002
A_4	0.002	0.002	0.001	0.000	−0.001	−0.001
A_5	0.003	0.002	−0.001	0.000	−0.001	0.000
A_6	0.000	0.004	−0.001	0.001	0.000	0.000
A_7	0.001	0.003	−0.001	0.000	0.000	0.000
	July	**Aug.**	**Sept.**	**Oct.**	**Nov.**	**Dec.**
A_0	0.071	0.071	0.071	0.071	0.071	0.071
A_1	0.057	0.175	0.069	0.077	0.087	0.091
A_2	−0.007	−0.007	0.003	0.001	0.010	0.016
A_3	0.002	0.002	−0.002	−0.003	−0.006	−0.007
A_4	−0.001	0.001	0.001	0.002	0.002	−0.001
A_5	0.000	0.000	0.000	−0.001	0.002	0.001
A_6	0.000	−0.001	0.001	0.000	−0.001	0.001
A_7	0.000	−0.001	−0.001	0.001	−0.001	0.002

Table 6. Averaged Fourier Coefficients for HDF.

	Jan.	Feb.	March	April	May	June
A_0	0.071	0.071	0.071	0.071	0.071	0.071
A_1	0.085	0.075	0.065	0.051	0.043	0.035
A_2	0.006	−0.001	−0.007	−0.013	−0.012	−0.010
A_3	−0.007	−0.003	0.001	0.004	0.004	0.007
A_4	0.001	−0.002	0.001	−0.001	−0.002	−0.003
A_5	−0.003	−0.005	−0.003	−0.001	0.001	0.002
A_6	−0.001	0.001	0.002	0.001	0.000	−0.001
A_7	0.002	0.003	0.000	0.000	−0.001	0.001
	July	Aug.	Sept.	Oct.	Nov.	Dec.
A_0	0.071	0.071	0.071	0.071	0.071	0.071
A_1	0.037	0.045	0.056	0.047	0.080	0.085
A_2	−0.011	−0.012	−0.012	−0.009	0.001	0.005
A_3	0.006	0.005	0.003	−0.002	−0.007	0.010
A_4	−0.003	−0.002	−0.001	0.002	0.003	0.002
A_5	0.002	0.001	−0.001	−0.002	−0.001	−0.001
A_6	−0.001	−0.001	0.001	0.001	0.001	−0.001
A_7	0.002	0.000	−0.001	0.001	0.002	0.001

REFERENCES

[1] M. S. Audi, "Estimation of Solar Energy Flux in Jordan", *Dirasat,* **6(2)** (1979), p. 121.

[2] M. A. Alsaad and M. S. Audi, "Solar Radiation Characteristics in Jordan", *Proceedings of the Fourth International Arab Solar Energy Conference, Royal Scientific Society, Amman, Jordan,* 1989.

[3] B. Y. H. Liu and R. C. Jordan, "The Interrelationship and Characteristic Distribution of Direct Diffuse and Total Solar Radiation", *Solar Energy,* **4** (1960), p. 1.

[4] H. P. Garg and S. N. Garg, "Improved Correlation of Daily and Hourly Diffuse Radiation with Global Radiation for Indian Stations", *Solar and Wind Technology,* **4(2)** (1987), p. 113.

[5] M. Collares-Pereira and A. Rabl, "The Average Distribution of Solar Radiation-Correlations Between Diffuse and Hemispherical and Between Daily and Hourly Insulation Values", *Solar Energy,* **22** (1979), p. 155.

[6] M. S. Audi and M. A. Alsaad, "Simple Hourly Global Solar Radiation Prediction Model", *Int. J. Renewable Energy,* **1(3/4)** (1991), p. 213.

[7] P. C. Jain, "Estimation of Monthly Average Hourly Global and Diffuse Irradiation", *Solar and Wind Technology,* **5(1)** (1988), p. 7.

[8] M. S. Audi and M. A. Alsaad, "A General Model for the Prediction of Hourly Diffuse Solar Radiation", *Int. J. Solar Energy,* **10(1-2)** (1991), p. 39.

[9] Ehab Sabrin, "Report on Using Solar Power to Operate Air Conditioning Systems for Automobiles, *B.Sc. Project, 1990.*

[10] I. Ahmad, A. E. A. Khashkool, S. M. Ali, and A. Z. Zahron, *Solar Energy Utilization in Developing Countries, Symposium, Baghdad, 1981,* p. 15.

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