RATIONAL EQUIVALENT LINEAR MODEL FOR SIMPLE YIELDING SYSTEMS

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الخلاصـــة :

في هذه الدراسة تَمَّ البحث في مفهوم المضاءلة اللزجة لنظام (مرن – لدن) وقدَّم حلًّا دقيقاً متكاملاً (في صورة معادلات محددة) للصلادة والتضاؤل المكافئين نتيجة إثارة جيبية. وتَمَّتْ مقارنة هذا الحل الدقيق بخمسة نماذج خطية مكافئة شائعة. وتَمَّ التوصل إلى أن طريقة الصلادة الهندسية تمثيلٌ ملائم للصلادة المكافئة نتيجة إثارة متوافقة. كما وجد أنه في حالة القيم الكبيرة المطاوعة فإن نسبة التضاؤل المكافئة والمرافقة لها تأثير صغير على الإستجابة نتيجة سيطرة الصلادة المكافئة المرافقة ذات القيم المخفضة.

ABSTRACT

The concept of equivalent viscous damping for an elasto–plastic system is investigated in this study. An exact closed form solution of the equivalent stiffness and damping is presented for sinusoidal excitation. The exact solution is compared to five commonly used equivalent linear models. It is argued that for harmonic excitation, the geometric stiffness method is an appropriate representation of the equivalent stiffness. For large values of ductility the associated equivalent damping ratios have little effect on the response as a result of the dominancy of the low value of the associated equivalent stiffness.

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1. INTRODUCTION

Linearization of non-linear dynamic systems is an attempt to simplify the response calculations of certain classes of problems [1]. Linearization refers to the replacement of the non-linear system with an equivalent linear system having similar energy dissipating characteristics [2, 3].

Non-linearity of dynamic systems involves both damping and restoring forces. This paper deals with the latter type of non-linearity; and more specifically, the linearization of an elasto-plastic system is investigated in this paper.

Despite the fact that many "hysteretic" constitutive models have been proposed [4–6] to simulate the steady state response of a yielding system, the elasto-plastic model is generally acceptable and provides reliable results for modeling steel structures behavior [7–9].

Several equivalency algorithms have been proposed [1, 10, 11] for the elasto-plastic system. However, almost all of these proposed methods do not predict exactly the steady state response of the yielding system. In the present study a closed form solution of the equivalent viscous damping and the equivalent stiffness is established for the yielding system subjected to a sinusoidal excitation.

2. ELASTO-PLASTIC SYSTEM

2.1. General

The general equation of motion of a yielding single-degree-of-freedom system (SDOF) with a "linear" viscous damping can be written as:

$$m\ddot{x} + c\dot{x} + P(x) = F_0 \sin \omega_f t, \qquad (1)$$

in which m = the mass of the oscillator, c = the viscous damping coefficient; $P(x)$ = the nonlinear restoring force in terms of the relation describing the hysteresis loop of steady state response of the yielding oscillator; F_0 = the amplitude of the excitation; ω_f = the frequency of the excitation; and x = the relative displacement of mass. The dots denote derivatives of displacement with respect to time. See Figure 1.

The "linear" natural frequency of the yielding system is defined as:

$$\omega_0 = \sqrt{\frac{k_0}{m}}, \qquad (2)$$

where k_0 is the initial stiffness of the system. A typical hysteresis loop of such system is shown in Figure 2. The yielding displacement is denoted as x_y; F_y is the yielding force; and x_0 is the maximum displacement of the mass relative to the ground.

2.2. Equivalent Viscous Damping

The principal advantages of the concept of equivalent viscous damping are that it allows approximate solutions to be found to problems of interest in design and analysis, using mathematical techniques already developed and used for linear systems, and that it describes certain features of a generally complex

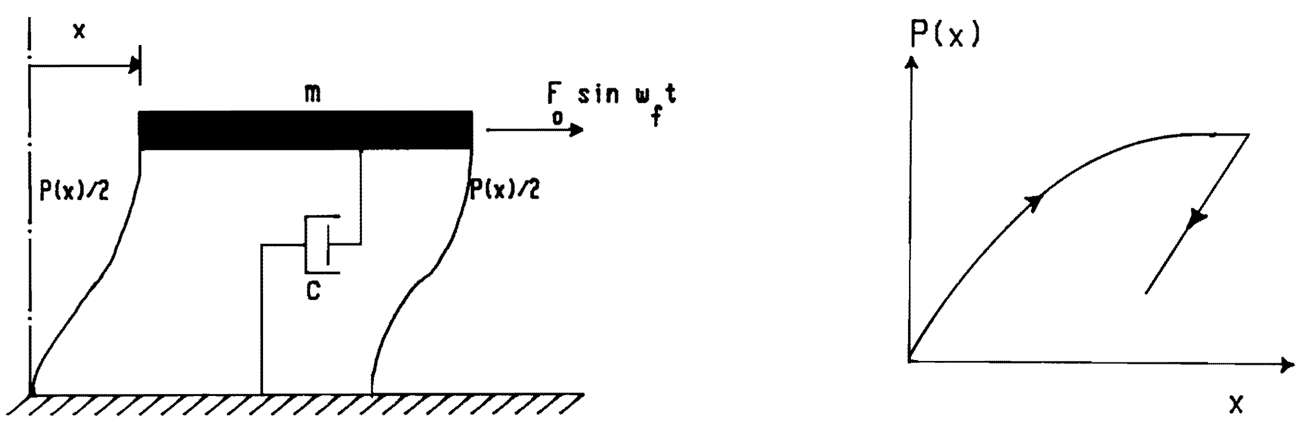


Figure 1. Single-degree-of-freedom Yielding Model.

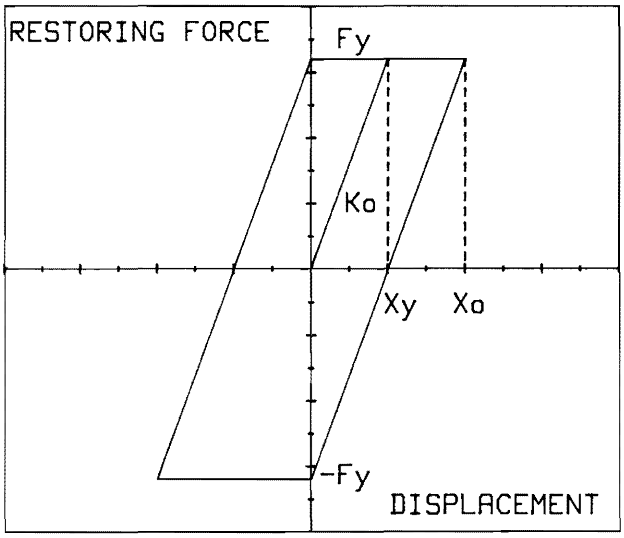


Figure 2. Typical Hysteresis Loop for Elasto-Plastic System.

dynamic problem in more familiar terms and methodologies [12]. Hower, it should be realized that the description of the yielding system response by an equivalent viscous damping linear system is only approximate. In addition, it should be realized that the utilization of such simplified approach does not provide permanent deformations and residual stresses.

Three physical properties of the equivalent linear system are to be determined. Those properties are the mass, stiffness, and damping. The determination of these properties is based on two criteria; namely, energy dissipation and response amplitude. It is understood that the dissipated energy per cycle is the same for the yielding system and the equivalent linear system provided that both systems have identical steady state response [2].

Five commonly used equivalent models with harmonic excitation are briefly discussed below. The linearization characteristcs for these models are given in Table 1.

1. Dynamic Equivalence [10]: In this model the difference in response between the non-linear and linear systems is minimized. The equivalent mass of the linear model is assumed to be the same as the original mass of the yielding system.
2. Resonant Amplitude [2]: In this approach, the equivalent stiffness of the linear model is equal to the initial stiffness of the yielding system for small-amplitude frequency. Both masses are the same.
3. Geometric Stiffness [2]: In this method, the stiffness of the equivalent linear model is taken equal to the secant stiffness of the yielding system.
4. Dynamic Mass [2]: For this model, the small-amplitude stiffness(initial) of the yielding system is taken to be the equivalent stiffness of the linear model and an effective dynamic mass is defined

Table 1. Linearization of Elasto-plastic Hysteretic System.

Method	Damping ratio	Frequency shift	Stiffness
1. Dynamic equivalence	$\frac{2}{\pi}\frac{(\mu-1)}{\mu^2}\left(\frac{\omega_o}{\omega_e}\right)^2$	$\frac{\omega_o}{\omega_e} = \left[\frac{1}{\pi}\left(\theta - \frac{\sin 2\theta}{2}\right)\right]^{-\frac{1}{2}}$ $\theta = \cos^{-1}\left(\frac{\mu-2}{\mu}\right)$	$k_e = k_o\left(\frac{\omega_e}{\omega_o}\right)^2$
2. Resonant amplitude	$\frac{2}{\pi}\frac{(\mu-1)}{\mu^2}$	$\omega_o = \sqrt{\frac{k_o}{m_o}}$	k_o
3. Geometric stiffness	$\frac{2}{\pi}\frac{(\mu-1)}{\mu^2}\left(\frac{\omega_o}{\omega_e}\right)^2$	$\frac{\omega_o}{\omega_e} = \sqrt{\mu}$	$k_e = \frac{k_o}{\mu}$
4. Dynamic mass	$\frac{2}{\pi}\frac{(\mu-1)}{\mu^2}$	$\frac{\omega_o}{\omega_e} = \left[\frac{1}{\pi}\left(\theta - \frac{\sin 2\theta}{2}\right)\right]^{-\frac{1}{2}}$	k_o
5. Constant critical damping	$\frac{2}{\pi}\frac{(\mu-1)}{\mu^2}\left(\frac{\omega_o}{\omega_e}\right)$	$\frac{\omega_o}{\omega_e} \doteq \left[\frac{1}{\pi}\left(\theta - \frac{\sin 2\theta}{2}\right)\right]^{\frac{1}{2}}$	$k_e = k_o\frac{\omega_o}{\omega_o}$

leading to the same frequency shift as that in Model 1.

5. Constant Critical Damping [2]: This model is based on assuming the same critical damping for both linear and non linear models.

2.3. Equivalency Algorithm

Basically all five models are derived by making an assumption on one of the parameters of each model and then deducing the shift in frequency and equivalent damping values. The latter is obtained by equating the energy dissipated per cycle in the non-linear system due to the inelastic action (yielding) to the energy dissipated by the equivalent viscous damping in the linear system, or:

$$\Delta E \text{ per cycle} = \pi c_e \omega x_0^2; \ \pi = 3.14 \qquad (3)$$

where c_e is the equivalent viscous damping coefficient, ω is the frequency of the steady state response, and x_0 is the maximum attained displacement. The damping ratio can be defined as:

$$\xi_e = \frac{c_e}{2\sqrt{k_e m}} \qquad (4)$$

where k_e is the equivalent stiffness of the linear system. The variation of ξ_e, with the ductility ratio, is shown in Figure 3. As can be seen different definitions of ξ are obtained due to the various definitions of the equivalent stiffness of the linear system. However, since the basic concept for the linearization is identical for all models, the product $k_e\xi_e$ is the same for all models [1].

The variation of the ratio between the equivalent stiffness for the linear system and the initial stiffness of the yielding system is shown in Figure 4. It is obvious that the stiffness due to yielding is reduced (stiffness degradation). However this is not considered in the dynamic mass and resonant amplitude models [1]. The variation of the frequency ratio is shown in Figure 5. Despite the fact that dynamic mass method

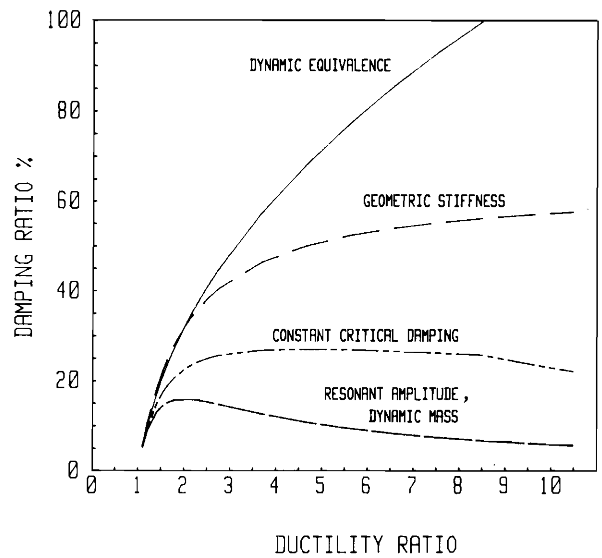


Figure 3. Equivalent Viscous Damping for Elasto-Plastic System.

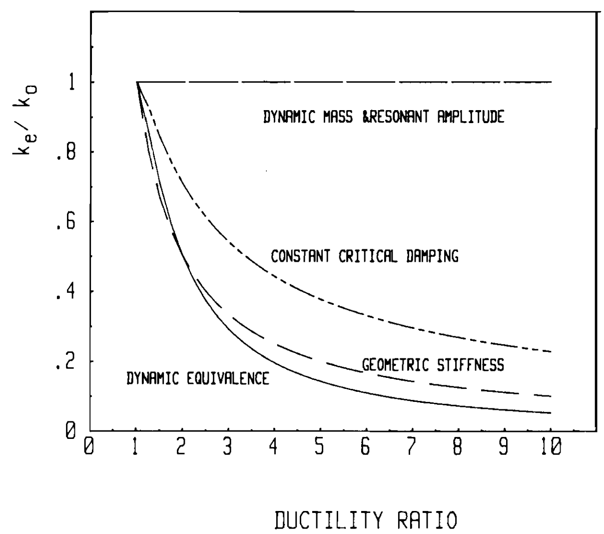


Figure 4. Variation of the Ratio of Equivalent Stiffness to Initial Stiffness.

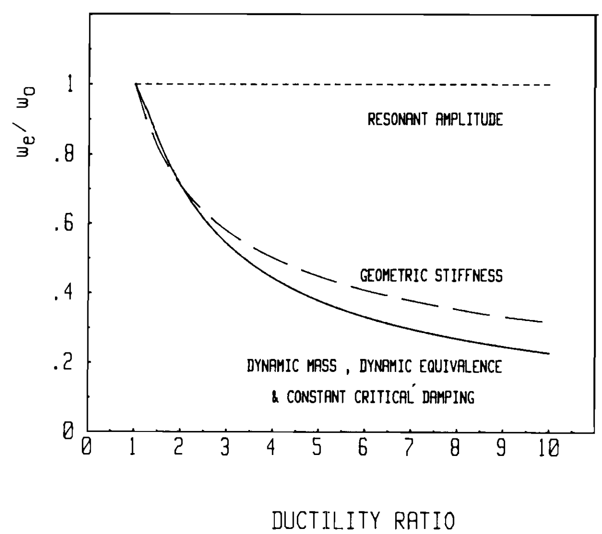


Figure 5. Variation of the Ratio of Equivalent Natural Frequency to Initial Natural Frequency.

assumes a constant equivalent stiffness for the linear system, the natural frequency decreases as the ductility ratio μ increases due to the frequency shift given in Table 1.

3. THE PROPOSED METHOD

As mentioned before, the equivalency concept is applied by considering two parameters of the response; the dissipated energy per cycle and the steady state amplitude. The equivalent viscous damping coefficient, c_e, can be found by equating the energy dissipated by the linear and the yielding systems as follows [2]:

$$4k_o x_y(x_o - x_y) = \pi c_e \omega_f x_o^2 \tag{5}$$

or

$$c_e = \frac{4k_o(\mu - 1)}{\pi \omega_f \mu^2}$$

$$\text{and } \mu = \frac{x_o}{x_y}$$

where ω_f is the frequency of the motion which is equal to the excitation frequency, μ is the ductility ratio, and x_y is the yielding displacement.

If viscous damping, c_o, is available in the yielding system, then the total equivalent viscous damping of the linear system is given by:

$$c = c_0 + c_e \tag{6}$$

The second constraint involves the steady state response, x_o, of the two systems, which should be identical.

$$x_o^2 = \frac{F_o^2}{m^2} \frac{1}{\left[(\omega_e^2 - \omega_f^2)^2 + \frac{c^2\omega_f^2}{m^2}\right]} \tag{7}$$

$$x_o^2 = \frac{F_o^2}{m^2} \frac{1}{\left[\left(\frac{k_e}{m} - \omega_f^2\right)^2 + \left(\frac{4k_o(\mu - 1)}{\pi\omega_f\mu^2} + c_o\right)^2 \frac{\omega_f^2}{m^2}\right]} \tag{8}$$

where ω_e is the "equivalent" natural frequency for the linear system and it is defined as:

$$\omega_e = \sqrt{\frac{k_e}{m}} \tag{9}$$

Equation (8) can be written in the form:

$$x_o^2 = \frac{F_o^2}{\left[(k_e - \omega_f m)^2 + \left\{\frac{4k_o(\mu - 1)}{\pi\mu^2} + c_o\omega_f\right\}^2\right]} \tag{10}$$

solving for k_e:

$$k_e = \omega_f^2 m \pm \sqrt{\left(\frac{F_o}{x_o}\right)^2 - \left\{\frac{4k_o(\mu - 1)}{\pi\mu^2} + c_o\omega_f\right\}^2} \tag{11}$$

$$\text{But since } m = \frac{k_o}{\omega_o^2},\ \mu = \frac{x_o}{x_y} \tag{12}$$

then Equation (11) can be written as:

$$k_e = k_o\left(\frac{\omega_f}{\omega_o}\right)^2 \pm k_o\sqrt{\left(\frac{F_o}{F_y\mu}\right)^2 - \left\{\frac{4(\mu - 1)}{\pi\mu^2} + \frac{c_o\omega_f}{k_o}\right\}^2} \tag{13}$$

or

$$k_e = k_o\left[\left(\frac{\omega_f}{\omega_o}\right)^2 \pm \sqrt{\left(\frac{F_o}{F_y\mu}\right)^2 - \left\{\frac{4(\mu - 1)}{\pi\mu^2} + \frac{c_o\omega_f}{k_o}\right\}^2}\right] \tag{14}$$

as can be seen from Equation (14), two solutions are obtained for the equivalent stiffness. It might be possible to obtain two values of k_e less than k_o for the frequency ratio below resonance. Both solutions are right. Accordingly, the equivalent damping ratio is given as:

$$\xi_{\text{total}} = \xi_o + \frac{4k_o(\mu - 1)}{\pi\omega_f\mu^2} \frac{1}{2\sqrt{k_e}\sqrt{m}} \tag{15}$$

or

$$\xi_{\text{total}} = \xi_o + \frac{2}{\pi}\frac{(\mu - 1)}{\mu^2}\left(\frac{\omega_o}{\omega_f}\right) \Big/$$

$$\left[\left(\frac{\omega_f}{\omega_o}\right)^2 \pm \sqrt{\left(\frac{F_o}{F_y\mu}\right)^2 - \left\{\frac{4(\mu - 1)}{\pi\mu^2} + \frac{c_o\omega_f}{k_o}\right\}^2}\right]^{\frac{1}{2}} \tag{16}$$

and when $\xi_o = 0$, and so $c_o = 0$, then

$$\xi_{\text{total}} = \frac{2}{\pi}\frac{(\mu-1)}{\mu^2}\left(\frac{\omega_o}{\omega_f}\right) \Big/$$

$$\left[\left(\frac{\omega_f}{\omega_o}\right)^2 \pm \sqrt{\left(\frac{F_o}{F_y\mu}\right)^2 - \left(\frac{4(\mu-1)}{\pi\mu^2}\right)^2}\right]^{\frac{1}{2}} \tag{17}$$

For resonance case, $\omega_f = \omega_o$, then

$$c_e = \frac{4k_o(\mu-1)}{\pi\omega_o\mu^2} \tag{18}$$

or

$$c_e = \frac{4\sqrt{k_o}(\mu-1)\sqrt{m}}{\pi\mu^2} \tag{19}$$

then

$$\xi_e = \frac{c_e}{c_{cr}} = \frac{c_e}{2\sqrt{k_e}\sqrt{m}} = \frac{2}{\pi}\frac{(\mu-1)}{\mu^2}\sqrt{\frac{k_o}{k_e}} \tag{20}$$

or

$$\xi_e = \frac{2}{\pi}\frac{(\mu-1)}{\mu^2}\left(\frac{\omega_o}{\omega_e}\right) \tag{21}$$

so that:

$$k_e = k_o\left[1 \pm \sqrt{\left(\frac{F_o}{F_y\mu}\right)^2 - \left(\frac{4(\mu-1)}{\pi\mu^2}\right)^2}\right] \tag{22}$$

For boundary condition of $\mu = 1$ or $x_o = x_y$ and $F_y = k_o x_y$ or $F_y = k_o x_o$ then Equation (14) can be written as

$$k_e = k_o\left[\left(\frac{\omega_f}{\omega_o}\right)^2 \pm \sqrt{\left(\frac{F_o}{k_o x_y}\right)^2}\right] \tag{23}$$

or

$$k_e = k_o\left[\left(\frac{\omega_f}{\omega_o}\right)^2 \pm \frac{F_o}{k_o x_y}\right]. \tag{24}$$

For a SDOF undamped system, the maximum steady state response is given by:

$$x_o = \pm\frac{F_o}{k_o}\frac{1}{1-\left(\frac{\omega_f}{\omega_o}\right)^2} \tag{25}$$

or

$$F_o = \pm k_o x_o\left(1 - \left(\frac{\omega_f}{\omega_o}\right)^2\right) \tag{26}$$

substitute in Equation (24) the following equation is obtained

$$k_e = k_o\left[\left(\frac{\omega_f}{\omega_o}\right)^2 \pm \left\{\mp\left(1 - \left(\frac{\omega_f}{\omega_o}\right)^2\right)\right\}\right] \tag{27}$$

which reduced to

$$k_e = k_o. \tag{28}$$

4. DISCUSSION OF RESULTS

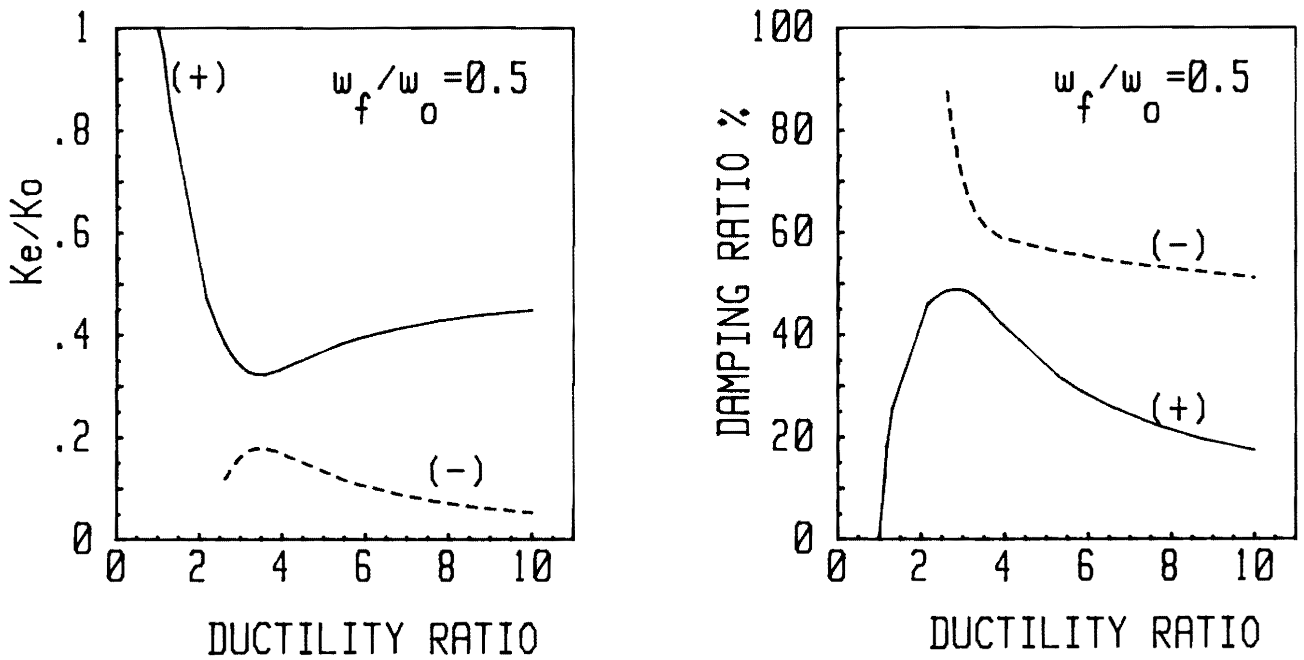
4.1. Numerical Verification

A step-by-step numerical integration is used to calculate the yield response of a SDOF elasto-plastic system subjected to a sinusoidal excitation. Fourth Order Rung–Kutta [12] method is employed.

In order to eliminate the phenomena of response drift and to obtain a steady state response (the same maximum positive and maximum negative) the excitation is applied in such a way to attain the maximum amplitude over 20 cycles [13].

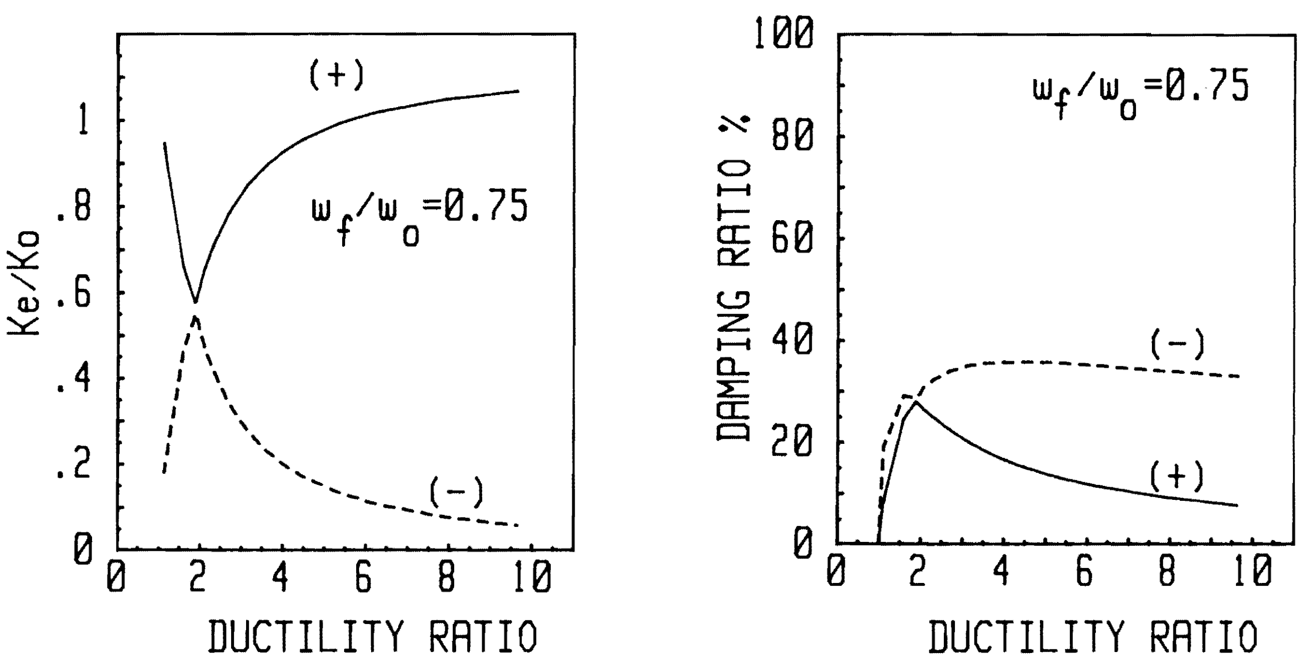
The equivalent viscous damping for the linearized system is shown in Figure 6 together with the equivalent linear stiffness for several frequency ratios (ω_f/ω_o). It can be noticed that damping is a function of the frequency ratio, and it decreases as this ratio increases for the same ductility ratio. The variations of the ratio of equivalent stiffness to the initial stiffness are also shown in the same figure. The (+) and (−) signs in the figure are consistent with the application of the plus and minus signs in Equation (14) and (17).

To examine the reliability of the five mentioned methods of equivalency, the maximum steady state response of the yielding system is compared with the response of the equivalent linear system. This is shown in Figure 7 for three different frequency ratios. Figure 7 shows that the best representation is obtained by using the geometric stiffness method.



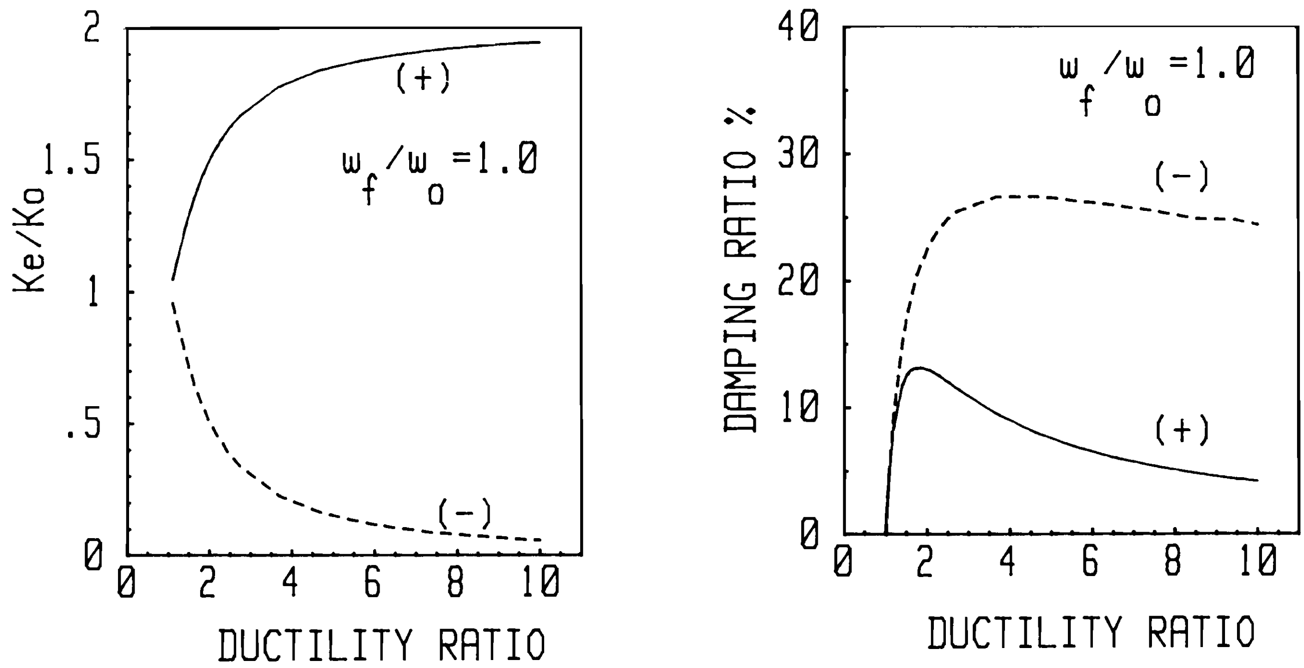
Ke/Ko
(+)
(-)
wf/wo=0.5
DUCTILITY RATIO
DAMPING RATIO %
(-)
(+)
wf/wo=0.5
DUCTILITY RATIO

(a)

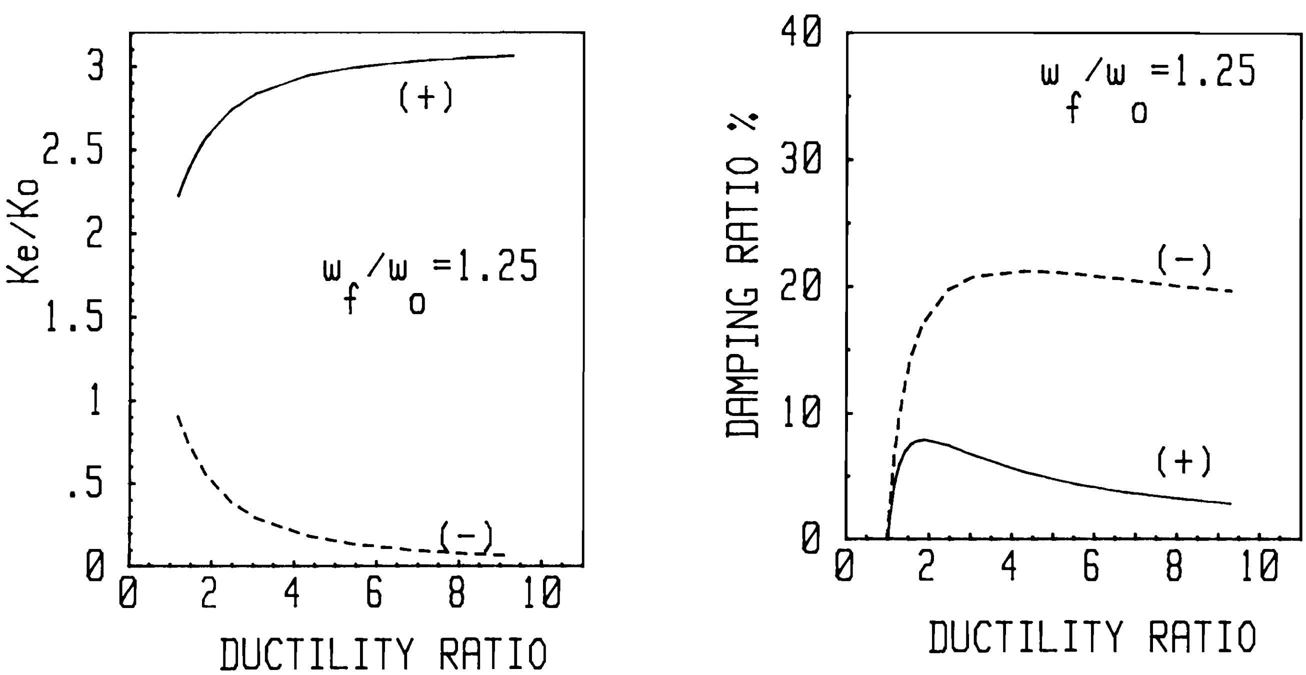


Ke/Ko
(+)
(-)
wf/wo=0.75
DUCTILITY RATIO
DAMPING RATIO %
(-)
(+)
wf/wo=0.75
DUCTILITY RATIO

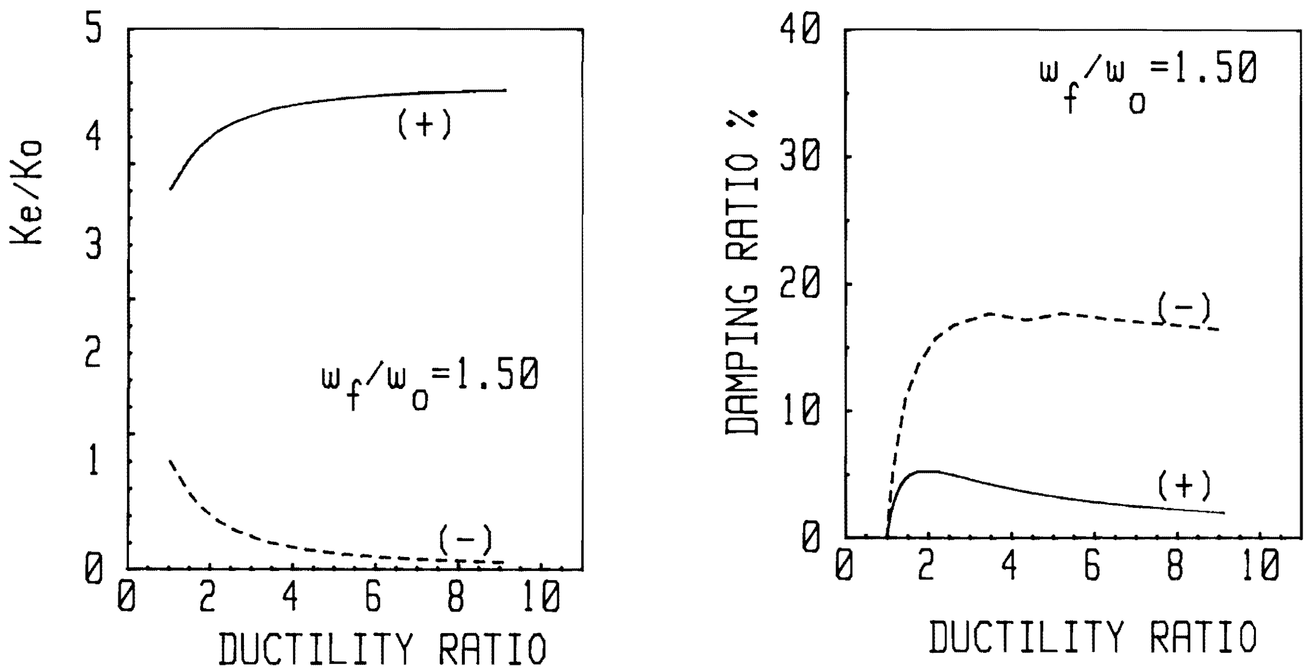
(b)


Ke/Ko
DUCTILITY RATIO
(+)
(−)
w_f/w_o=1.0
DAMPING RATIO %
DUCTILITY RATIO
w_f/w_o=1.0
(−)
(+)

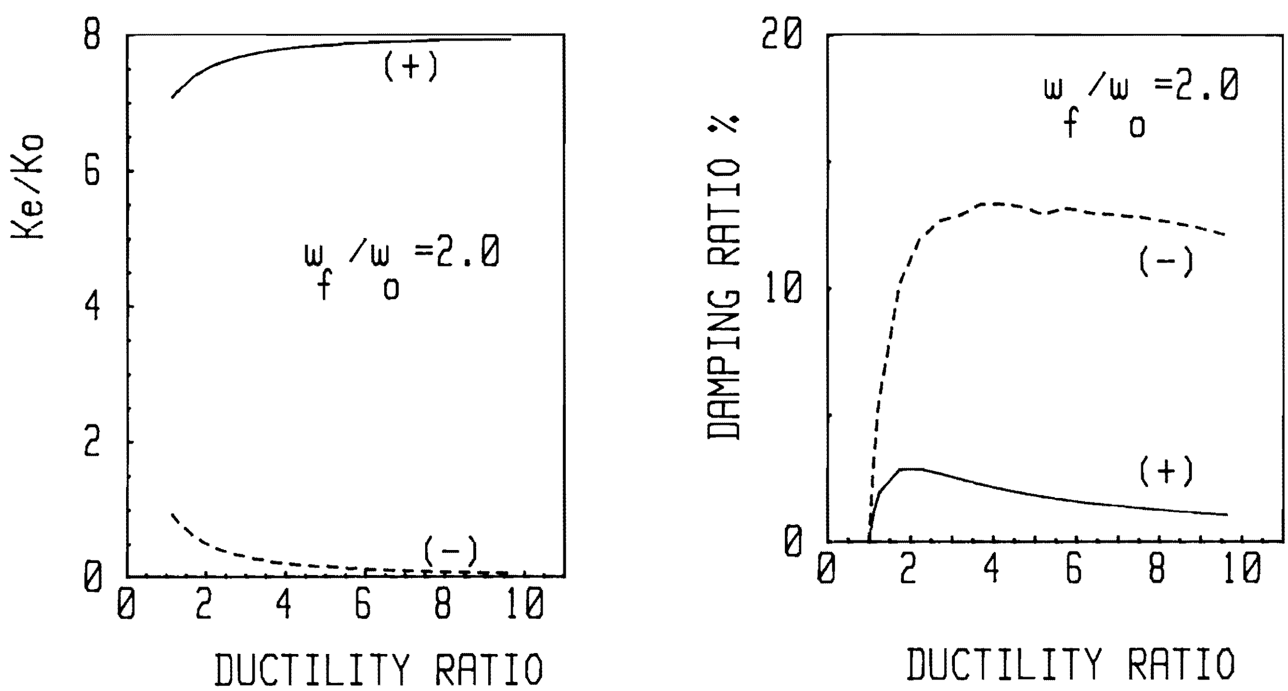
(c)


Ke/Ko
DUCTILITY RATIO
(+)
(−)
w_f/w_o=1.25
DAMPING RATIO %
DUCTILITY RATIO
w_f/w_o=1.25
(−)
(+)

(d)



(e)



(f)

Figure 6. Variation of the stiffness and damping for several frequency ratios. (a) $\omega_f/\omega_o = 0.5$; *(b)* $\omega_f/\omega_o = 0.75$; *(c)* $\omega_f/\omega_o = 1.0$; *(d)* $\omega_f/\omega_o = 1.25$; *(e)* $\omega_f/\omega_o = 1.5$; *(f)* $\omega_f/\omega_o = 2.0$.

The ratio of the equivalent stiffness of the linear system to the initial stiffness of the yielding system is shown in Figure 8 for the resonance condition. As can be seen, the geometric stiffness and the dynamic equivalence methods provide the best representation of the equivalent stiffness to the proposed method.

Considering the equivalent damping ratio obtained by Equation (2), it can be seen that this ratio is identical to the damping ratio given by the constant critical damping method. This is shown in Figure 9 for the resonance condition. However, the equivalent stiffness obtained by this method is not well compared with the equivalent stiffness obtained by the proposed method (Figure 8). As a result, the response shown in Figure 7*b*, which was obtained using this method, does not compare well with the actual response.

It is interesting to notice that even though the equivalent damping obtained by the geometric stiffness method does not compare well to the exact values, the yielding response is well represented. This is due to the fact that response is controlled more by stiffness than damping for high frequency ratio. This may be explained by considering the steady state response of the linearized system

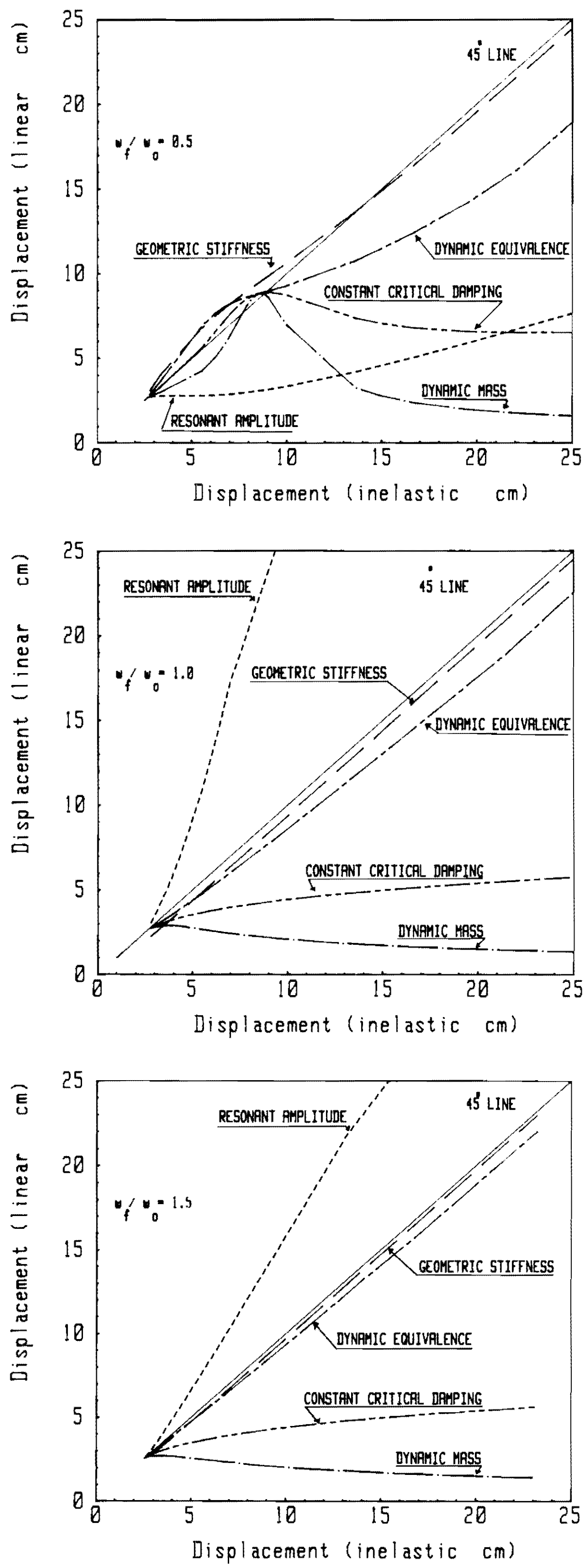


Figure 7. Comparison of Steady State Response for the Yielding and Equivalent Linear System. (a) $\omega_f/\omega_o = 0.5$; (b) $\omega_f/\omega_o = 1.0$; (c) $\omega_f/\omega_o = 1.5$.

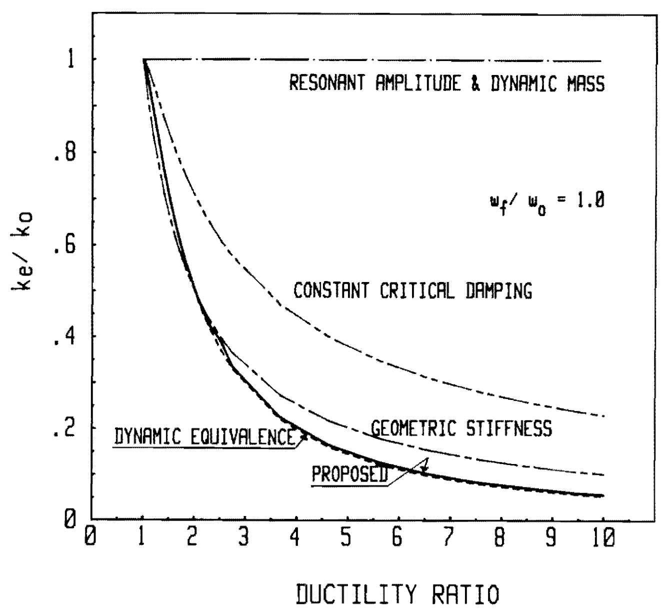


Figure 8. Variation of the Ratio of the Equivalent Stiffness to the Initial Stiffness ($\omega_f/\omega_o = 1.0$).

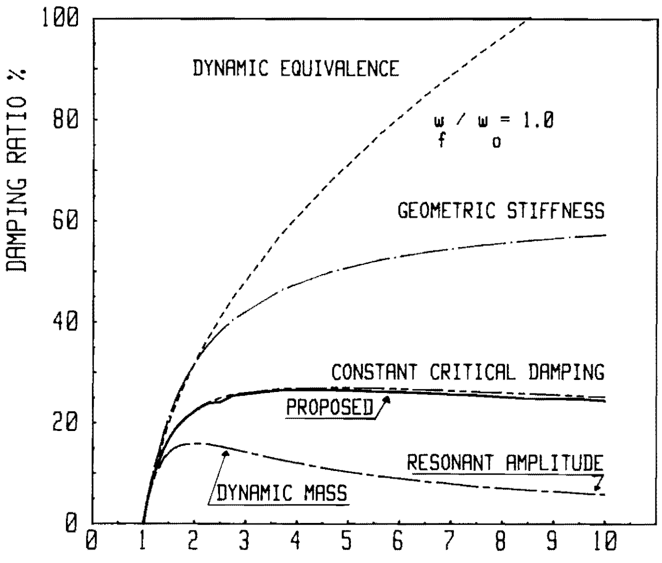


Figure 9. Variation of the Equivalent Viscous Damping by Several methods ($\omega_f/\omega_o = 1.0$).

$$x_o = \frac{F_o}{k_e} \frac{1}{\sqrt{\left\{1 - \left(\frac{\omega_f}{\omega_e}\right)^2\right\}^2 + \left\{2\xi_e \frac{\omega_f}{\omega_e}\right\}^2}}, \quad (29)$$

and by considering the extreme reduction in the stiffness (as μ increase in Figure 8); the first term under the root dominates. So the response is not very much affected by damping in that region.

4.2. Errors in the Equivalency Algorithm

The concept of equivalency is based on steady state response, or in other words, the non-linear system is assumed to yield symmetrically in the positive and negative senses. This situation is no longer available when excitation is applied to its full magnitude (amplitude) from the initial time. As a result, drift in the response will develop. Such drift, D, can be expressed as

$$D = \frac{X_{max} + X_{min}}{2} \quad (30)$$

where X_{max}, X_{min} are the extreme displacements in the positive and negative direction respectively. Taking into consideration the algebraic sign of the extreme response. The average response can be given as

$$X_{ave} = \frac{X_{max} - X_{min}}{2}. \quad (31)$$

It was found that the average response of a yielding system subjected to a sinusoidal excitation applied to its full amplitude is exactly the same as the response of the same yielding system subjected to the same excitation applied gradually to attain its full amplitude over some cycles (no drift).

Assuming the drift, D, is known, the geometric stiffness can be expressed as:

$$k_e = \frac{k_o}{\mu_m - \frac{D}{x_y}}, \quad (32)$$

where $\mu_m = (x_m/x_y)$, and x_m is maximum displacement (equal to x_{ave}).

The corresponding damping coefficient is given by:

$$c_e = \frac{\bar{c}\left(1 - \frac{D}{x_m - x_y}\right)}{\left[\frac{x_m - D}{x_m}\right]^2}, \quad (33)$$

where $\bar{c}$ = the damping coefficient obtained using Equation (5) with x_m as x_o, *i.e.* no drift is occurring.

Apparently, the geometric stiffness obtained by using Equation (32), considering drift to be zero, is higher than the stiffness calculated for a certain amount of drift.

An elasto-plastic system subjected to a harmonic excitation applied to its full amplitude from the initial condition is used to obtain the response. The maximum response is calculated numerically and it is shown in Figure 10 labeled "numerical inelastic" as a function of the ratio of the driving force amplitude to the yielding force. The driving frequency is not to be equal to the initial natural frequency. The drift is computed by using Equation (30). Using the geometric stiffness equivalency the equivalent stiffness and the equivalent damping were found using Equations (32) and (33). The linearized response is calculated and added to the drift. This is shown in Figure (9) and labeled "Geometric stiffness + drift". An excellent agreement between the two solutions is clear in Figure 10. The drift is a difficult quantity to find, since there is no relationship between the drift and any other parameter of the yielding system or driving

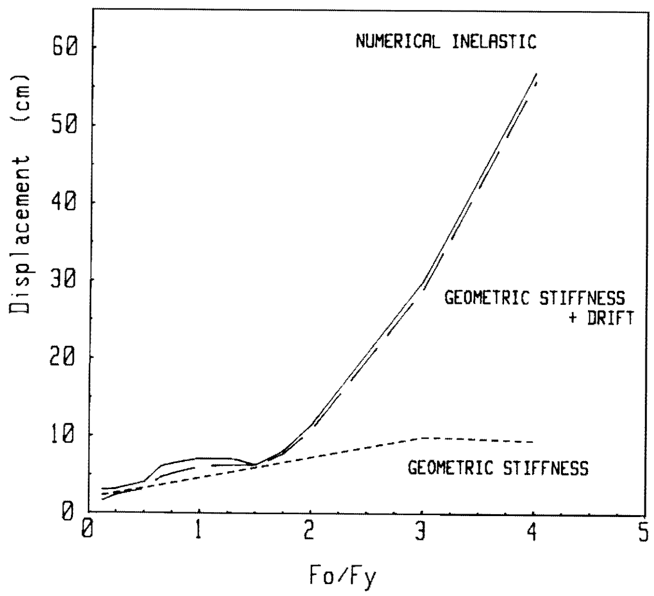


Figure 10. Response Approximation for Sinusoidal Excitation.

force. If the equivalent stiffness and equivalent damping coefficient are obtained from Equations (32) and (33) respectively, by setting D equal to zero, then the response would be as shown in Figure 10 which is labeled "geometric stiffness". It can be seen that high discrepancy in the response approximation exists in this case.

4.3. Earthquake Response

The application of the concept of equivalent viscous damping to earthquake response calculation of yielding systems is expected to involve more difficulties than the case of sinusoidal excitation. The application of the proposed method of equivalent viscous damping would require that some response representative amplitudes be determined. In addition some force amplitude representative is needed in Equation (14). Furthermore, the drift in the response introduces additional errors in the equivalency concept, and such equivalency can not be modeled effecively by any usual definition of equivalent damping.

However, since it was found that the best representation of equivalent viscous damping for elastoplastic system is by using the geometric stiffness method, the maximum and minimum response were used in Equation (30) to obtain the drift and Equation (32) to obtain the equivalent stiffness for a yielding system subjected to El-Centro, 1940, NS excitation. Using this equivalent stiffness the corresponding damping was obtained using Equation (33).

The yielding response and the equivalent linearized response are shown as a function of the initial natural frequency of the yielding system in Figure 11. Even though, the problem of drift has been eliminated, the two responses differ noticeably. The reason is due to the exact definition of the steady state response. The maxima of the yielding response may be reached only once and therefore, the equivalent damping obtained was over-estimated for the linearized system. Also, the resonant amplitude method was used to obtain an equivalent linearized response since it is well known that this method is the easiest and the most usable method. However, as can be seen from Figure 11, the response was not well represented by this method and less reliable for this excitation.

5. CONCLUSIONS

The application of the equivalent viscous damping concept to the steady-state response of yielding structures subjected to sinusoidal excitation has been discussed in this paper. The elasto-plastic oscillator was used as an example to illustrate as to how the equivalent viscous damping factor is dependent on the choice of equivalent stiffness.

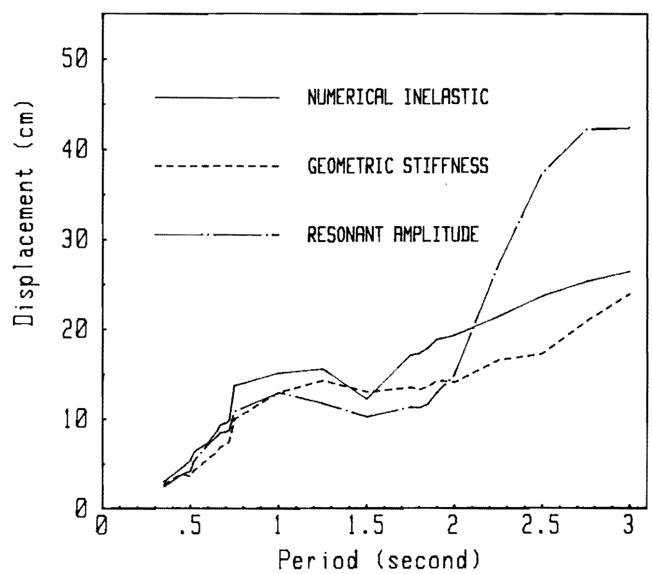


Figure 11. Response Approximation of a SDOF Elasto-Plastic System Subjected to El-Centro 1940 N-S excitation.

A proposed closed form solution based on the energy equivalency was introduced. The proposed solution has been applied for sinusoidal excitation condition. It has also been compard with the five well-known methods of equivalency in term of equivalent stiffness and equivalent viscous damping. It has been shown that the geometric stiffness is the best method to represent the equivalent stiffness of the linear system. For earthquake excitation, the problems of response drift and the steady state response makes it difficult to attain an equivalent linear system. It is concluded that even though the amplitude method is simple, clear, and conservative, the geometric stiffness method provides more reliable results for response approximation.

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