# UNSTEADY MOTION OF A SECOND-ORDER MAGNETOHYDRODYNAMIC FLOW BETWEEN TWO PARALLEL PLATES

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الخلاصـــة :

تمَّ دراسة حلول معادلات الانسياب المضطرب ( غير الثابت ) للسوائل غير النيوتونية ـ والتي نخضع لمعادلة ( كوليمان ونول ) ـ بين لوحين متوازيين احدهما ( العلوى ) يتحرك بسرعه ثابته $U$ والاخر ( السفلى ) يتذبذب خطـيّاً بحركه توافقيه بسيطه $Ve^{i\omega t}$ فى وجود مجال مغناطيسى منتظم عمودى على اللوحين . . وأمكن الحصول على الصورة التحليلية للسرعة باستخدام تحويلات لابلاس ، ومنها أمكن الحصول على الاحتكاك السطحى عند اللوح السفلى . كذلك تمت دراسه تأثير كـــلٍ من المجال المغناطيسى $H_a$ والنسبه بين سرعتى اللوحين ($\beta$) لحالات $Nt$ المختلفة على الحركة .

## ABSTRACT

An exact solution is obtained for the unsteady motion of a non-Newtonian electrically conducting, incompressible fluid under the action of a transverse magnetic field between two infinitely extended parallel plates, when the upper plate is moving with uniform velocity and the lower plate is performing linear oscillations in its own plane. The results are interpreted with the aid of graphs. The skin friction at the lower plate is obtained.

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## INTRODUCTION

The exact solutions of the Navier–Stokes equations for the flow near a plate with impulsive and simple harmonic motion have been discussed by Pai [1]. The unsteady motion of a viscous incompressible fluid due to periodic pressure gradients in different geometries has been investigated by Sexel [2], Uchida [3], Verma [4], Drake [5], Dube [6], and Gupta and Goyal [7].

The problem of unsteady flow and temperature distribution of a viscous incompressible fluid between parallel plates has recently been considered by Verma and Gaur [8], who have improved the results of Gupta and Goyal. Unsteady flow of non-Newtonian fluids, which is of vital interest, has also been investigated by several researchers, following Rosenblat [9], who investigated the flow induced in a viscous fluid from small torsional oscillations of two infinite discs when (*a*) one disc is oscillating and the other is at rest and (*b*) both discs oscillate with same frequency but with phase difference of 180°. Rajeswari [10] extended the analysis for Reiner–Rivlin fluid. Bhatnagar and Rajeswari [11] and Srivastava [12] have studied the same problem for a special class of the Rivlin–Ericksen 'second order' fluids, while Frater [13] has discussed only the first case for an Oldroyd fluid. Bhatnagar and Rajeswari have found that a reversal of the direction of the steady secondary flow is a characteristic feature of the Rivlin–Ericksen fluid. This phenomenon has also been predicted by Frater for an Oldroyd fluid with elastic parameter $\sigma < 1/3$ and critical range of fluid parameter. S. Verma and Rajvanshi [14] have studied the same problem (both cases being discussed) for a Maxwell fluid and they have observed no reversal of flow, contrary to the reversal of flow as established by Frater for an Oldroyd fluid.

Rudraiah [15] studied the velocity distribution and the skin friction for a semi-infinite incompressible conducting Newtonian fluid bounded by a non-conducting flat plate in the presence of a uniform transverse magnetic field. Also, Sachet and Bhatt [16] have studied the flow of second order fluid through parallel discs. In a similar manner, Soundalgekar [17], Soundalgekar and Desai [18], and Rajagopal and Gupta [19–21] have studied some problems in this field.

In the present work we consider the unsteady motion of a second order magnetohydrodynamic flow between two infinitely extended parallel plates when the upper plate is moving with uniform velocity and the lower plate is performing linear oscillations in its own plane. It is assumed that a magnetic field perpendicular to the plates is present, that no external electric field is imposed, and that the magnetic Reynolds number is very small. The technique of Laplace transform has been employed to obtain the velocity distribution, which has been shown graphically in different cases to show the effects of magnetic field and oscillations; so also the coefficient of skin friction at the lower plate is obtained.

## FORMULATION OF THE PROBLEM

We consider the flow of a second order magnetohydrodynamic fluid contained between two infinite parallel plates at a distance of $y_0$ from each other. The origin is considered to be on the lower plate along which the $x$-axis is taken and perpendicular to it and through the origin the $y$-axis is considered. The motion of the second order magnetohydrodynamic fluid is brought about by the forces given to the two plates. Initially the upper plate is assumed to be moving with a constant velocity $U$, and then from a certain instant, the lower plate starts executing simple harmonic motion, in its own plane, of the type $V\,e^{int}$ with amplitude $V$ and frequency $n$.

The constitutive equation of an incompressible second order fluid has been given by Coleman and Noll [22] as

$$\tau_{ij} = \mu A_{(1)ij} + \mu_1 A_{(2)ij} + \mu_2 A_{(1)ik} A_{(1)kj} \tag{1}$$

where

$$A_{(1)ij} = v_{i,j} + v_{j,i} \tag{2}$$

$$A_{(2)ij} = a_{i,j} + a_{j,i} + 2 v_{m,i} v_{m,j} \tag{3}$$

and

$$S_{ij} = \tau_{ij} - P g_{ij} \tag{4}$$

such that $S_{ij}$ is the stress tensor; $g_{ij}$ the metric tensor; $v_i$ and $a_i$ the velocity and acceleration vectors respectively; $P$ the pressure; and $\mu$, $\mu_1$, and $\mu_2$ material constants (the coefficient of ordinary viscosity, the coefficient of viscoelasticity and the coefficient of cross viscosity respectively). The momentum equations for the unsteady incompressible flow are:

$$\rho\left(\frac{\partial v_i}{\partial t}+v^j v_{i,j}\right)=-P_{,i}+\tau^j_{i,j}+\epsilon_{ijk}J_jB_k \qquad (5)$$

and the equation of continuity is

$$v^i_{,i}=0 \qquad (6)$$

where $\tau^j_i$ is the stress tensor, $\rho$ the density, $\epsilon_{ijk}$ is the alternating tensor, comma denotes covariant differentiation, $J_j$ ($J_x$, $J_y$, $J_z$) current density, and $B_k$ ($B_x$, $B_y$, $B_z$) magnetic flux density. Since the motion is a plane one and the plates are infinite we may assume all physical quantities to be independent of $x$ and $z$. Also we assume that the electrical conductivity $\sigma$ is small so that the perturbation in the magnetic field may be neglected. Similarly, we assume that there is no external electric field. Finally from all the above assumptions the equations of motion take the following formula:

$$\rho\frac{\partial u}{\partial t}=-\frac{\partial p*}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2}+\mu_1\frac{\partial^3 u}{\partial t\partial y^2}-\sigma B_0^2\,u \qquad (7)$$

and

$$0=-\frac{\partial p*}{\partial y} \qquad (8)$$

where $p*$ is the modified pressure (Bhatnagar [23]) given by

$$p*=p-(2\mu_1+\mu_2)\left(\frac{\partial u}{\partial y}\right)^2 \qquad (9)$$

Equation (7) shows that $\frac{\partial^2 P*}{\partial x^2}=0$, hence $\frac{\partial P*}{\partial x}=$ a constant. Now $\frac{\partial P*}{\partial x^2}=0$, only because the motion is purely due to the shear of the moving plate, and hence Equation (7) reduces to:

$$\rho\frac{\partial u}{\partial t}+\mu\frac{\partial^2 u}{\partial y^2}+\mu_1\frac{\partial^3 u}{\partial t\partial y^2}-\sigma B_0^2\,u \qquad (10)$$

Introducing the following non-dimensional variables:

$$u*=u/U,\; y*=y/y_0,\; t*=tU/y_0, \qquad (11)$$

Equation (10), after dropping the asterisk, becomes

$$R_e\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+\alpha\frac{\partial^3 u}{\partial t\partial y^2}-H_a^2\,u \qquad (12)$$

where

$R_e=\rho U y_0/\mu$, Reynolds number

$\alpha=\mu_1 U/\mu y_0$, non-dimensional parameter governing elastico-viscosity of the fluid.

$H_a=B_0 y_0\sqrt{(\sigma/\mu)}$, Hartmann number (13)

## METHOD OF SOLUTION

In order to solve Equation (12), we assume the expression for the velocity as:

$$u=u_0(y)+u_1(y,t) \qquad (14)$$

where $u_0(y)$ and $u_1(y,t)$ are respectively the steady and unsteady part of the solution of Equation (12).

Substituting Equation (14) into Equation (12) and equating steady and unsteady terms on both sides, we get the following differential equations:

$$\frac{d^2u_0}{dy^2}=H_a^2\,u_0 \qquad (15)$$

and

$$R_e\frac{\partial u_1}{\partial t}=\frac{\partial^2 u_1}{\partial y^2}+\alpha\frac{\partial^3 u_1}{\partial t\partial y^2}-H_a^2\,u_1 \qquad (16)$$

## SOLUTIONS OF EQUATIONS

Equation (15) is to be solved subject to the following boundary conditions:

$$u_0=0;\ \text{at } y=0$$
$$u_0=1;\ \text{at } y=1 \qquad (17)$$

Its solution is:

$$u_0(y)=\frac{\sinh yH_a}{\sinh H_a} \qquad (18)$$

Now in order to solve Equation (16), we define the Laplace transform

$$\bar{u}_1(y,\lambda)=\int_0^\infty e^{-\lambda t}u_1(y,t)\,\partial t,\ \lambda>0 \qquad (19)$$

Equation (18) is to be solved subject to the following boundary conditions

$u_1=\beta\,e^{iNt}$; at $y=0$, ($\beta=V/U$ and $N=ny_0/U$)

$u_1=0$; at $y=1$, $t>0$

$u_1=0$; at $t=0$, for all $y$ (20)

where the real part of $\beta\,e^{iNt}$ is to be considered.

Multiplying both sides of Equation (16) by a $e^{-\lambda t}$

and integrating between the limits 0 to ∞, we have the Laplace transform of Equation (16) as

$$\frac{\partial^2 \bar{U}_1}{\partial y^2} - \frac{R_e\lambda + H_a^2}{\alpha\,\lambda + 1}\,\bar{u}_1 = 0 \tag{21}$$

and the transformed boundary conditions are

$$\bar{u}_1 = \frac{\beta}{\lambda - iN} \quad \text{at} \quad y = 0$$

$$\bar{u}_1 = 0 \quad \text{at} \quad y = 1 \tag{22}$$

Now the solution of Equation (21) subject to boundary conditions (22) is:

$$\bar{u}_1(y,\lambda) = \frac{\beta \sinh\left\{\left[\frac{R_e\lambda + H_a^2}{\alpha\lambda + 1}\right]^{1/2}(1-y)\right\}}{(\lambda - iN)\sinh\left[\frac{R_e\lambda + H_a^2}{\alpha\lambda + 1}\right]^{1/2}} \tag{23}$$

The inverse Laplace integral of Equation (23) is evaluated by transforming the path of integration into a closed contour and applying the calculus of residues [24]. Hence we obtain Equation (24).

$$u_1(y,t) = \frac{\beta\, e^{iNt} \sinh \lambda_0(1-y)}{\sinh \lambda_0} + 2\beta\pi \sum_{k=1}^{\infty} \frac{k(-1)^k(R_e - H_a^2\alpha)\sin k\pi(1-y)\exp\left[-t\left(\frac{k^2\pi^2 + H_a^2}{R_e + \alpha k^2\pi^2}\right)\right]}{(R_e + \alpha k^2\pi^2)[k^2\pi^2 + H_a^2 + iN(R_e + k^2\alpha\pi^2)]} \tag{24}$$

where

$$\lambda_0 = \left(\frac{(\alpha R_e N^2 + H_a^2) + i(R_e N - H_a^2\alpha N)}{(1 + \alpha^2 N^2)}\right)^{1/2} \tag{25}$$

Collecting the real part of Equation (24) and using Equation (18), Equation (14) becomes:

$$\begin{aligned} u = {} & \frac{\sinh yH_a}{\sinh H_a} + \beta[\sinh A \cos B\{\sinh A(1-y)\cos B(1-y)\cos Nt \\ & - \cosh A(1-y)\sin B(1-y)\sin Nt\} + \cosh A \sin B\{\cosh A(1-y) \\ & \times \sin B(1-y)\cos Nt + \sinh A(1-y)\cos B(1-y)\sin Nt\}]/\{\sinh^2 A \cos^2 B + \cosh^2 A \sin^2 B\} \\ & + 2\beta\pi \sum_{k=1}^{\infty} \frac{k(-1)^k(R_e - H_a^2\alpha)(k^2\pi^2 + H_a^2)\sin k\pi(1-y)\exp\left[-\left(\frac{k^2\pi^2 + H_a^2}{R_e + \alpha k^2\pi^2}\right)t\right]}{(R_e + \alpha k^2\pi^2)[(k^2\pi^2 + H_a^2)^2 + N^2(R_e + k^2\alpha\pi^2)^2]} \end{aligned} \tag{26}$$

where

$$A = \left[\frac{(\alpha R_e N^2 + H_a^2) + [(R_e^2 N^2 + H_a^4)(1 + \alpha^2 N^2)]^{1/2}}{2(1 + \alpha^2 N^2)}\right]^{1/2}$$

$$B = \frac{R_e N - H_a^2 \alpha N}{(2(1 + \alpha^2 N^2)\{(\alpha R_e N^2 + H_a^2) + [(R_e^2 N^2 + H_a^4)(1 + \alpha^2 N^2)]^{1/2}\})^{1/2}}$$

Some values of $u$ are shown in Table 1.

**Table 1. Values of $u$, Calculated from Equation (26) For $\alpha = -1$, $R_e = 2$ and $t = 0.5$**

| | $Nt=\pi/2$ | | | | $Nt=3\pi/4$ | | | | $Nt=\pi$ | | | | $Nt=5\pi/4$ | | $Nt=3\pi/2$ | | $Nt=7\pi/4$ | | $Nt=2\pi$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\beta$=0.5 | $\beta$=1 | $\beta$=0.5 | $\beta$=1 | $\beta$=0.5 | $\beta$=1 | $\beta$=0.5 | $\beta$=1 | $\beta$=0.5 | $\beta$=1 | $\beta$=0.5 | $\beta$=1 | $\beta$=0.5 | $\beta$=1 | $\beta$=0.5 | $\beta$=1 | $\beta$=0.5 | $\beta$=1 | $\beta$=0.5 | $\beta$=1 |
| | $H_a=1$ | | $H_a=4$ | | $H_a=1$ | | $H_a=4$ | | $H_a=1$ | | $H_a=4$ | | $H_a=4$ | | $H_a=4$ | | $H_a=4$ | | $H_a=4$ | |
| $y$ | $u\times10^{-2}$ | | | | | | | | | | | | | | | | | | | |
| 0 | 0 | 0 | 0 | 0 | −35 | −70 | −35 | −71 | −50 | −100 | −50 | −100 | −35 | −71 | 0 | 0 | 35 | 71 | 50 | 100 |
| 0.2 | 20 | 23 | 14 | 23 | −14 | −44 | −20 | −44 | −29 | −74 | −29 | −74 | −35 | −71 | −8 | −19 | 31 | 58 | 43 | 82 |
| 0.4 | 39 | 43 | 23 | 37 | 11 | −13 | −2 | −14 | −3 | −40 | −18 | −44 | −22 | −53 | −8 | −24 | 29 | 49 | 42 | 75 |
| 0.6 | 58 | 62 | 32 | 43 | 37 | 20 | 6 | −8 | 27 | 1 | 1 | −18 | −3 | −26 | 4 | −11 | 33 | 46 | 44 | 67 |
| 0.8 | 78 | 80 | 52 | 58 | 67 | 59 | 43 | 41 | 62 | 48 | 35 | 25 | 33 | 21 | 36 | 26 | 49 | 52 | 57 | 70 |
| 1.0 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |

## COEFFICIENT OF FRICTION

The coefficient of friction is given by the following expression:

$$C_f=\frac{\partial u}{\partial y}+\alpha\frac{\partial^2 u}{\partial t\partial y} \tag{27}$$

Thus at the lower plate, we have from Equation (26) and (27)

$$\begin{aligned}(C_f)y=0 &= \frac{H_a}{\sinh H_a}+\beta[\sinh A\cos B\{\acute{p}(\cos Nt-\alpha N\sin Nt)\\ &\quad+\acute{Q}(\sin Nt+\alpha N\cos Nt)\}+\cosh A\sin B\{\acute{p}(\sin Nt+\alpha N\cos Nt)\\ &\quad+\acute{Q}\alpha N\sin Nt-\cos Nt)\}]/\{\sinh^2 A\cos^2 B+\cosh^2 A\sin^2 B\}\\ &\quad-2\beta\pi^2\sum_{k=1}^{\infty}\frac{K^2(R_e-H_a^2\alpha)^2(k^2\pi^2+H_a^2)\exp\left[-t\left(\frac{k^2\pi^2+H_a^2}{R_e+\alpha k^2\pi^2}\right)\right]}{(R_e+\alpha k^2\pi^2)^2[(k^2\pi^2+H_a^2)^2+N^2(R_e+k^2\alpha\pi^2)^2]}\end{aligned} \tag{28}$$

where

$$\acute{p}=B\sinh A\sin B-A\cosh A\cos B,$$

$$\acute{Q}=A\sinh A\sin \boldsymbol{B}+\boldsymbol{B}\cosh A\cos B.$$

Some values of $C_f$ are shown in Table 2.

**Table 2. Values of $C_f$, Calculated from Equation (28) For $\alpha=-1$, $H_a=1$, $\beta=0.2, 0.5, 1, 1.5, 2$**

| $Nt$ | $C_f$ | | | | |
|---|---|---|---|---|---|
| | $\beta=0.2$ | $\beta=0.5$ | $\beta=1$ | $\beta=1.5$ | $\beta=2$ |
| $\pi/2$ | 0.69 | 0.45 | 0.05 | −0.35 | −0.75 |
| $3\pi/4$ | 0.91 | 0.99 | 1.13 | 1.1 | 1.41 |
| $\pi$ | 1.18 | 1.67 | 2.48 | 3.29 | 4.11 |
| $5\pi/4$ | 1.33 | 2.04 | 3.23 | 4.42 | 5.61 |
| $3\pi/2$ | 1.27 | 1.89 | 2.95 | 4 | 5.1 |
| $7\pi/4$ | 0.98 | 1.17 | 1.5 | 1.8 | 2.1 |
| $2\pi$ | 0.54 | 0.08 | −0.68 | −1.4 | −2.2 |

## DISCUSSION

On putting $H_a=0$, the velocity expression (26) reduces to the solution of the same problem without magnetic field obtained by Sacheti and Bhatt [25]. The following particular case may be derived from Equation [26]. When the lower plate is kept at rest, i.e. $\beta=0$, we get from Equation (26), $u=\sinh yH_a/\sinh H_a$. Also in the absence of magnetic field ($H_a=0$), the well-known Couette flow is obtained ($u=y$). The velocity profiles (26) have been shown graphically in Figures 1–4 for $H_a=1$, 4 and $\beta=0.5$, 1. Figure 1, represents the velocity profiles plotted against $y$ for different values of $Nt$. The oscillations set up by the lower plate are affected by the motion of the upper plate. It is clear from Figure 2 that the curve decreases as the Hartmann number increases.

The coefficient of skin friction at the lower plate has also been plotted against $Nt$ (Figure 5) and the curve increases as the amplitude of the oscillations of the lower plate increases.

It is clear that the effect of an magnetic field on the

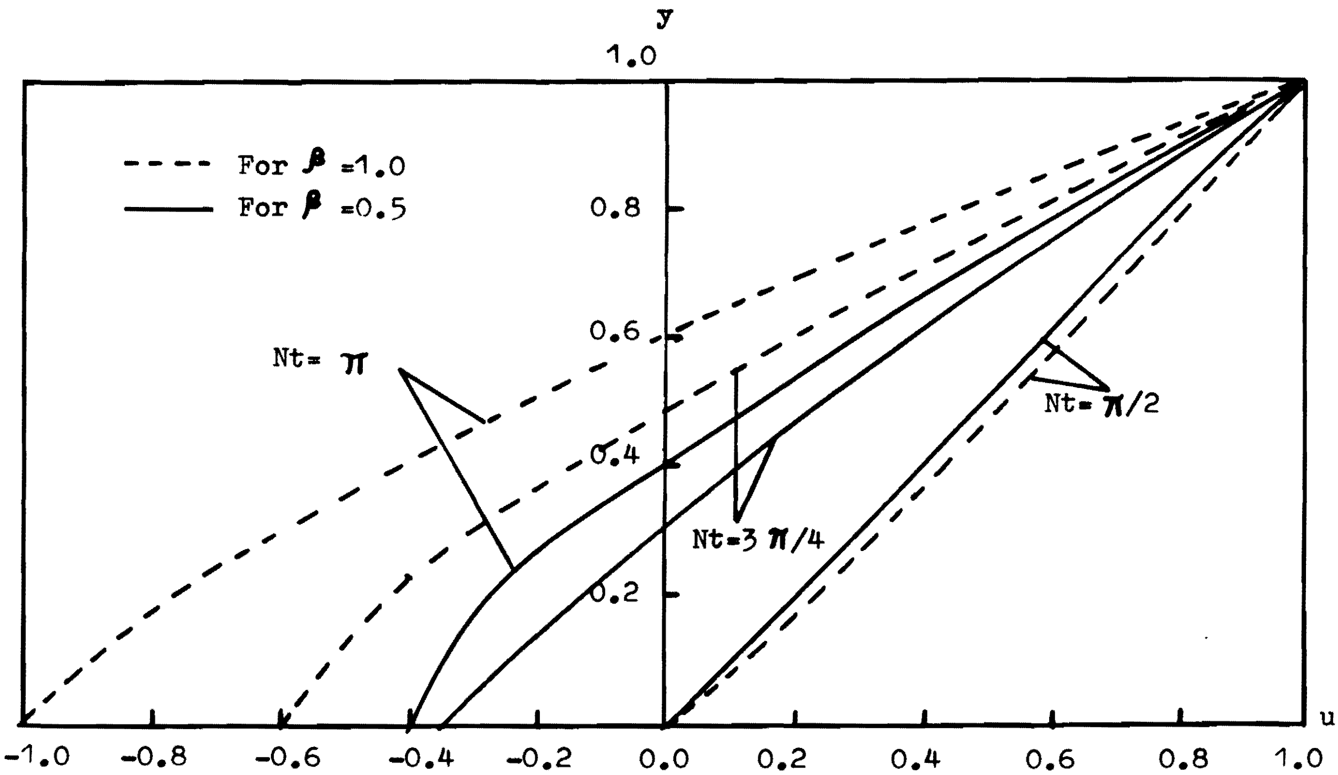


*Figure 1. Velocity Profiles Plotted Against* y *for* $\alpha=-1.0$, $R_e=2$, *and* $H_a=1$.

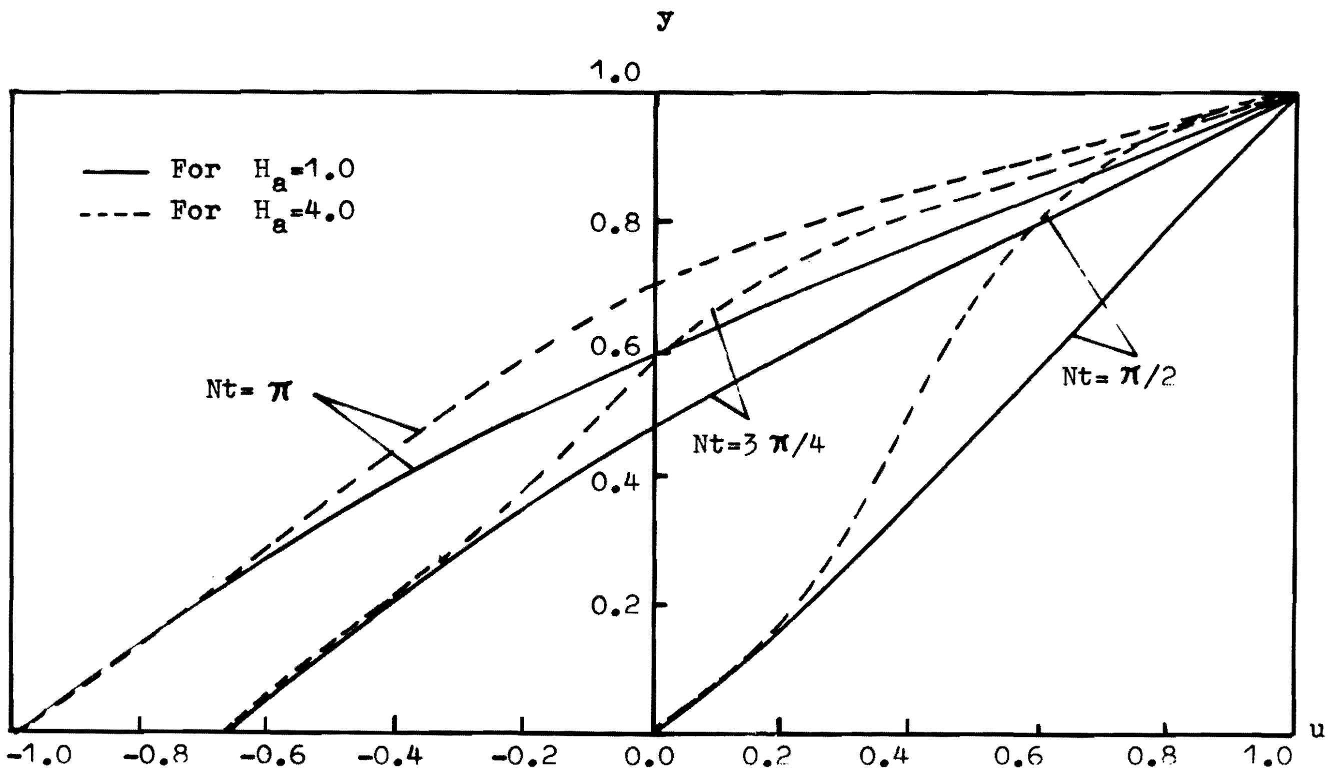


*Figure 2. Velocity Profiles Plotted Against* y *for* $\alpha=-1.0$, $R_e=2$, *and* $\beta=1$.

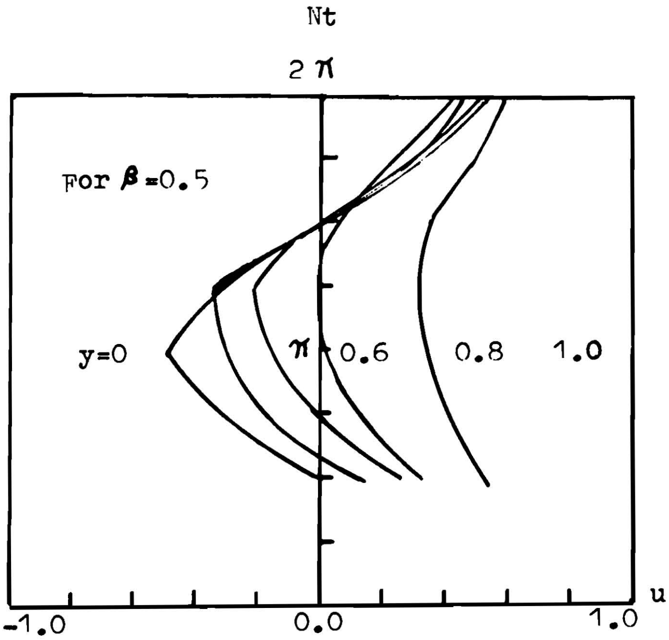


*Figure 3. Velocity Profiles Plotted Against* Nt *for* $\alpha=-1.0$, $R_e=2$, *and* $\beta=0.5$.

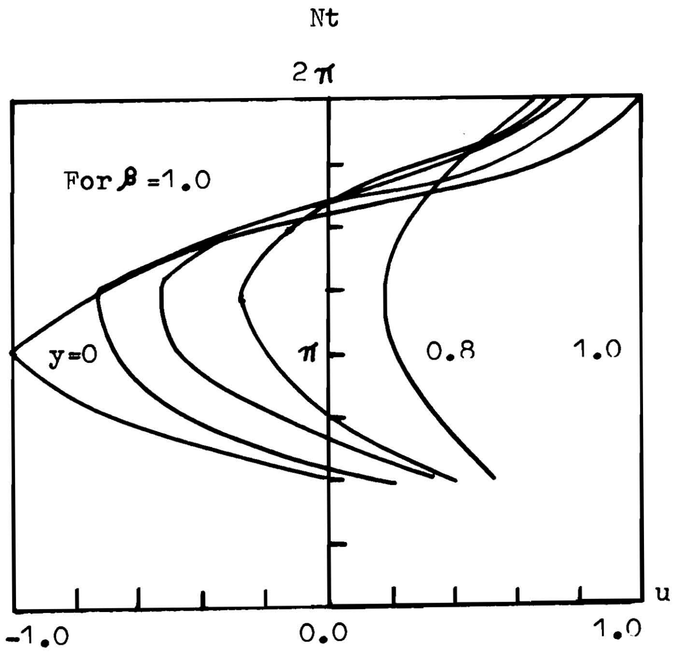


*Figure 4. Velocity Profiles Plotted Against* Nt *for* $\alpha=-1.0$, $R_e=2$, *and* $\beta=1.0$.

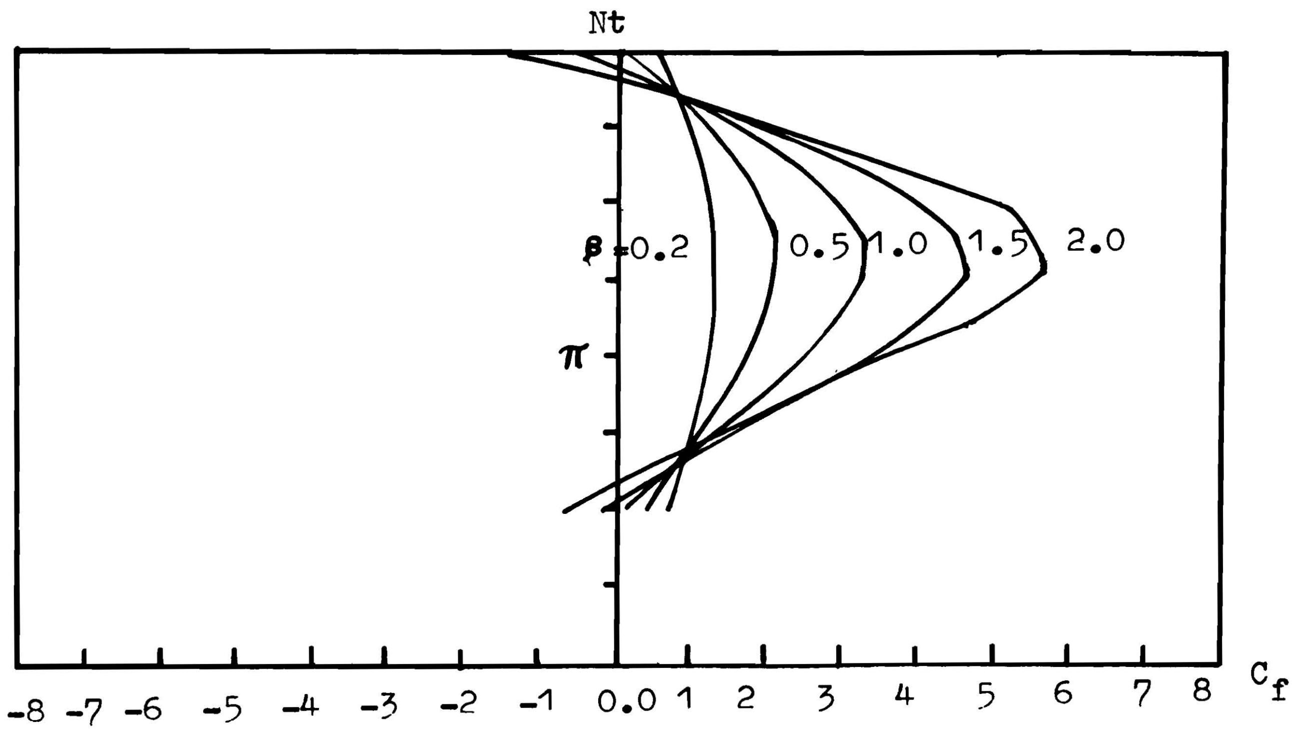


*Figure 5. Coefficients of Skin Friction Plotted Against* Nt *where* $\alpha=-1$, $H_a=1$, $\beta=0.2, 0.5, 1, 1.5, 2$.

flow of a viscoelastic fluid has been studied with increasing interest for its applications in astronautics, plasma physics, aerospace engineering, and petroleum engineering concerned with the movement of oil, gas, and water through the reservoir of an oil or gas field.

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