FIELD DEPENDENCE OF MOBILITY AND TEMPERATURE FOR QUASI-HOT ELECTRONS IN SEMICONDUCTING DEVICES

Vijay K. Arora and Muhammad A. Al-Mass'ari

Department of Physics, King Saud University, P.O. Box 2455, Riyadh 11451, Saudi Arabia

Recent developments in VLSI and VHSIC programs have indicated an ever-increasing importance of high-field effects in limiting the mobility in these devices. Considerable effort has been made to increase the speed of the devices using the collision-free ballistic transport phenomenon. These studies are based on classical concepts which treat the electron as a classical particle being accelerated by an electric field. The mobility is then limited by collision mechanisms dictated by one or more of several scattering interactions. For isotropic scattering interactions, e.g. acoustic-phonon scattering, the collision broadening of this classical electron is $\hbar\tau^{-1}$, where τ is the relaxation time. In n-Germanium [1] for example, at 77 K $\hbar\tau^{-1}=0.35$ meV for $\tau\sim1.9\times10^{-12}$s, and at 300 K, $\hbar\tau^{-1}=2.69$ meV if $\tau^{-1}\sim T^{3/2}$ is assumed for acoustic-phonon scattering [2]. $\hbar\tau^{-1}\ll k_BT$ at both these temperatures ($k_BT=6.7$ meV at 77 K and $=$ 26 meV at 300 K). This means collisions do not appreciably change the unperturbed distribution function. This is the low-field (Ohmic limit) regime of the scattering transport, where the energy gained during the mean free path $\ell=v\tau$ (v is the velocity of an electron) is small compared to the thermal energy of an electron.

In the other extreme is the hot-electron regime, when the electric field is very high in a semiconducting device. In this regime, the wave character of an electron cannot be ignored. Then, the position of an electron is uncertain by an amount $\lambda_D=h/p$, where $p=m^*v$ is the momentum of an electron with effective mass m^* and velocity v. In an electric field ε, this gives an energy uncertainty $\Delta E=e\varepsilon\bar{\lambda}_D$. The corresponding uncertainty in time is now $\tau_F\sim\hbar/\Delta E=\hbar/e\varepsilon\bar{\lambda}_D$. This gives a field broadening $\hbar\tau_F^{-1}=e\varepsilon\lambda_D$ which may suppress the collision broadening at sufficiently high electric fields, and hence limit the mobility. For n-Germanium at room temperature (with $p=\sqrt{2m^*k_BT}$), $\bar{\lambda}_D\sim27$ Å if an average mass $m^*\approx0.2m_0$ (where m_0 is the free electron mass) is taken. Therefore $\hbar\tau_F^{-1}=\hbar\tau^{-1}$ at a critical electric field $\varepsilon^*\sim$ 1 200 V/cm, where the onset of nonlinearity takes place [3] and field broadening becomes very important. At sufficiently high electric fields, this field broadening may suppress the collision broadening.

Taking these considerations into account, a radical transformation in our thinking of high-field transport is required [4]. By an extension of the formalism in [4] and using the Chamber's intuitive method [5] of

0377–9211/85/020203–03$01.00

obtaining the distribution function for acoustic-phonon scattering, the mobility and the relative change $\Delta T/T$ in electron temperature are obtained [6] as follows:

$$\mu = \mu_0 (3/\delta)\mathscr{L}(\delta), \tag{1}$$

$$\Delta T/T = (2\delta/3)\mathscr{L}(\delta), \tag{2}$$

with

$$\delta = \varepsilon/\varepsilon^*, \quad \varepsilon^* = k_B T/e\ell_a, \tag{3}$$

$$\mu_0 = 4e\ell_a/3(2\pi m^* k_B T)^{1/2}, \tag{4}$$

$$\mathscr{L}(\delta) = \coth(\delta) - \delta^{-1}, \tag{5}$$

where ℓ_a is the mean-free-path of an electron.

Equation (1) is obtained from [6] by writing $\mu = \sigma/n_e e$ and using $\tau = \ell_a/v$. Similarly, Equation (2) is equivalent to Equation (11) of [6] where a factor T is missing in the second term. Then α in [6] has the same meaning as δ used here. The mobility expression of Equation (1) reduces to its Ohmic value μ_0 in the limit $\delta \to 0$ (or $\varepsilon \to 0$); is a quadratic function of δ in the warm-electron regime ($\delta \ll 1$); and is inversely proportional to the electric field in the hot-electron regime ($\delta \gg 1$). The transition between the low-field and the high-field regime takes place at $\delta = 1$ (which is equivalent to $\tau_F = \tau$ stated above). The hot-electron mobility obtained from Equation (1) can then be written as

$$\mu_H = e\langle\tau_F\rangle/m^*, \tag{6}$$

with

$$\langle\tau_F\rangle = 2\hbar/\sqrt{\pi}\, e\varepsilon \lambda_{DT}, \tag{7}$$

$$\lambda_{DT} = \hbar/(2m^* k_B T)^{1/2}. \tag{8}$$

It should be noted that λ_{DT} is the value of $\lambda_D = \hbar/p = \hbar/(2m^*\epsilon)^{1/2}$ evaluated at thermal energy $\epsilon = k_B T$. Thus, at high-fields, collision broadening is washed away by the field broadening. In Figure 1, we show the relative mobility μ/μ_0 as a function of δ. It is clear from the graph that the nonlinearity becomes appreciable only at δ-values greater than one, where field broadening is appreciable. Since $\delta \sim T^{-2}$, the field broadening becomes even more important at lower temperatures. The relative mobility $\mu/\mu_0 = 0.94$ when $\delta = 1$, and reduces to 0.5 at $\delta = 4.78$. Thus at low temperatures and high electric fields, the field broadening is very important.

In Figure 2, we show the relative change in the quasi-hot-electron temperature ($\Delta T/T$) as a function of δ. Again, it is seen that $\Delta T/T = 2\delta/3$ in the hot-electron regime and hence is proportional to the electric field.

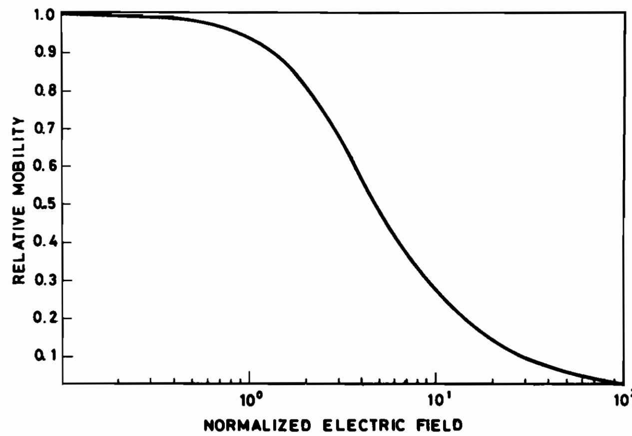

Figure 1. Relative Mobility vs Normalized Electric Field $\delta = \varepsilon/\varepsilon^*$

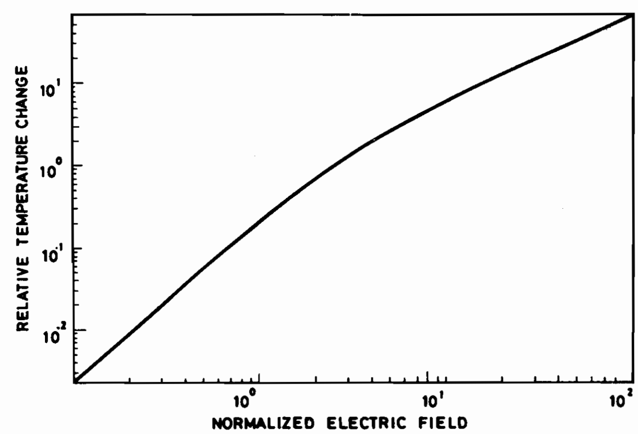

Figure 2. Relative Change in Temperature $\Delta T/T$ of Quasi-Hot Electrons as a Function of Normalized Electric Field $\delta = \varepsilon/\varepsilon^*$.

The results obtained here are in direct contrast to those obtained by earlier theories, most prominent of which are expansion-method theories. A comprehensive review of these theories is given by Nag [7] who has compounded the experimental data on elemental semiconductors. The major difference in the outcome of these theories from that presented here is that velocity saturation cannot be obtained by acoustic phonon scattering. The critical field for the onset of nonlinearity given by Shockley [3] is $\varepsilon_S^* = 1.51\mu/\mu_0$, which differs from $\varepsilon^* = 2v_{rms}/3\sqrt{\pi}\mu_0$ obtained from Equation (3), where $v_{rms} = (2k_B T/m^*)^{1/2}$ is the thermal velocity corresponding to energy $\epsilon = k_B T$. The acoustic-phonon velocity in Shockley's condition is thus replaced by the random thermal velocity. The importance of random thermal velocity in describing high-field transport has been indicated most recently in the semi-empirical formulation of Schwarz and Russek [8]. In the high field limit, the drift velocity is shown to be proportional to v_{rms} multiplied by a temperature- and field-dependent factor. Similarly, the high-field limit of Equation (1) gives for the drift velocity

$$v_d = (2v_{ms}/\sqrt{\pi})(1-\delta^{-1}). \tag{9}$$

Since in practice δ is not very large (for example $\delta \approx 5$ near the saturation regime in *n*-Germanium) and is proportional to T^{-2}, Equation (9) gives a slight decrease in saturation drift velocity. The effect of other scattering mechanisms, at least phenomenologically, can be included by replacing ℓ_a in Equation (3) by an effective mean-free-path which can be obtained from empirical fits. As stated earlier, the electric field $\varepsilon_{1/2}$ at which mobility falls to half of its Ohmic value is equal to $4.78\varepsilon^*$. In *n*-Germanium at $T = 300$ K, $\varepsilon_{1/2} = 2.8$ kV/cm [7], gives $\varepsilon^* = 0.59$ kV/cm. With this value of ε^*, the relative mobilities obtained from Equation (1), $\mu/\mu_0 = 0.85$, 0.62, 0.47, 0.38, are in excellent agreement with the median experimental values of $\mu/\mu_0 = 0.82$, 0.62, 0.46, and 0.38 at $\varepsilon = 1,2,3,4$ kV/cm, respectively. Similar comparisons can be made for other semiconducting devices and at different temperatures. The simplified model considered here does not take into account the anisotropic nature of the band structure, nor the inter-valley transfer of electrons. Neither have we considered other complications of scattering mechanisms. Nevertheless, we hope that the results presented above will form a nucleus for more advanced studies of high-field transport in newly emerging VLSI and VHSIC devices, thereby contributing effectively to the modeling of these devices.

REFERENCES

[1] Vijay K. Arora, 'Quantum and Classical-Limit Longitudinal Magnetoresistance for Anisotropic Energy Surfaces', *Physical Review B*, **26** (1982), p. 7046.

[2] B. R. Nag, *Electron Transport in Compound Semiconductors*. Berlin–Heidelberg–New York: Sprinter–Verlag, 1980, p. 174.

[3] W. Shockley, 'Hot Electrons in Germanium and Ohm's Law', *Bell System Technical Journal*, **30** (1951), p. 990.

[4] Vijay K. Arora, 'Nonlinear Quantum Transport in Semiconductors, *Journal of Applied Physics*, **54** (1983), p. 824.

[5] Joseph Callaway, *Quantum Theory of the Solid State*. New York: Academic Press, 1976, p. 615.

[6] V. K. Arora, 'Non-Ohmic Conductivity for Acoustic-Phonon Scattering', *Physica Status Solidi(b)*, **115** (1983), p. K-107.

[7] B. R. Nag, 'Hot-Carrier D.C. Conduction in Elemental Semiconductors', *Solid State Electronics*, **10** (1967), p. 385.

[8] Steven A. Schwarz and Stephen E. Russek, 'Semi-Empirical Equations for Electron Velocity in Silicon. I. Bulk', *IEEE Transactions on Electron Devices*, **ED-30** (1983), p. 1629.

Paper Received 22 April 1984; Revised 7 November 1984.