A MODEL REFERENCE ADAPTIVE CONTROL SCHEME WITH DIRECT CONTROLLERS

Riyadh Al-Salman, R. Nagarajan, and Dakhil H. Jerew

Department of Electrical Engineering, University of Basrah, Iraq

الخلاصـــة :

يبين هذا البحث تحقيق تهيئة بارمترات عالية السرعة بإدخال سلسلة كسب دارة معينة سهلة الإنتقاء في منظومات مهايئة بواسطة النموذج المرجعي . استخدمت طريقة التقدير المتدرج وطريقة لبانوف المباشرة على التوالي وذلك لإعداد خوارزم التهيئة ولتأمين الاتزان المقارب .

كما تم تبيان أن بالإمكان تحسين معدل التهيؤ بدمج تغذية مرتدة مناسبة لخطأ الحالة الخطية .

ABSTRACT

It is shown that a high speed of parameter adaptation is achieved by introducing certain easily selectable loop-gain sequences in model reference adaptive systems. The gradient estimation technique and the Lyapunov direct method are employed respectively for the development of the adaptation algorithm and for assuring asymptotic stability. It is also shown that the rate of adaptation can be further improved by incorporating suitable linear state error feedback.

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1. INTRODUCTION

The Model Reference Adaptive Control (MRAC) technique has been proved to be a successful approach for the design of control systems when drift variations in the parameters of a plant occur. The parameters of the plant or its controllers are adaptively adjusted so that the characteristics of the plant are always maintained close to those of a known reference model.

The unknown plant in the MRAC system can be controlled by a set of controllers linked to each of the plant parameters. Such controllers are referred to as 'direct controllers', and an adaptive system that employs direct controllers is known as a direct parameter MRAC system.

The popularity of direct parameter MRAC is due to the method of design originally proposed by Lindor et al. [1, 2], and later modified, improved, and applied to continuous and discrete time plants [3, 4, 5, 6]. Lyapunov's second method has been extensively employed to insure asymptotic stability of the overall adaptive control system.

It is essential that the plant parameters be accessible for on-line modification if the direct controllers are to be incorporated in the adaptive system. However, the algorithms developed for direct parameter MRAC systems are readily implemented in on-line identification and adaptive observer schemes [6, 7].

Recently, a computationally simple algorithm was proposed for direct parameter MRAC [8] in which a set of time-invariant loop gains having certain properties was employed to assure asymptotic stability. However, this scheme was subject to possible high overshoots and large settling times in parameter adaptive responses. Suggestions have also been made to increase the speed of adaptation [6, 9]. The higher the speed of adaptation, the smaller is the computation time, and the control computer can be effectively time-shared with a greater number of real time processes.

The present paper considers the problem of achieving high speed parameter adaptation using time-varying loop gains (gain sequences) in direct parameter discrete time MRAC systems. The gradient estimation approach is employed to develop the necessary adaptation algorithm. The effect of gain sequences in increasing the speed of parameter adaptation is studied, and methods for selecting the optimum set of gain sequences for rapid adaptation are also presented.

A linear state error feedback is proposed in this paper for the direct parameter MRAC systems. The effect of the state error feedback in further increasing the speed of parameter adaptation is investigated.

The suggestions of employing gain sequences and linear state error feedback in the MRAC scheme are illustrated by an example.

2. STATEMENT OF THE MRAC PROBLEM

A linear discrete time multivariable plant, incorporated with a set of direct controllers, is described by

$$x_p(k+1)=[A_p+\Gamma(k+1)]x_p(k) + [B_p+\Omega(k+1)]u(k) \quad (1)$$

where $x_p(k)\in \mathbb{R}^n$ and $u(k)\in \mathbb{R}^m$ are respectively the state and input vectors. The appropriately dimensioned system matrices A_p and B_p contain unknown elements. $\Gamma(k+1)$ and $\Omega(k+1)$ are the controllers to be adjusted on-line. It is assumed that $x_p(k)$ is accessible either directly or through an adaptive state observer.

The desirable characteristics of the plant are specified in a known time-invariant reference model of the same dimension as that of the plant, and are described by

$$x_m(k+1)=A_m x_m(k)+B_m u(k). \quad (2)$$

It is assumed that the matrix A_m has all its eigenvalues inside the unit circle of the complex plane.

It is essential for the proposed scheme that the dimensions of plant and reference model are to be the same, since any plant-model order mismatch can affect the stability of the overall MRAC scheme. Any desirable characteristics can be assigned to the n^{th} order reference model, and the model can be readily incorporated in the software of the control computer.

The dynamics of the state error derived from Equations (1) and (2) are represented by

$$e(k+1)=A_m e(k)+[A_m-A_p-\Gamma(k+1)]x_p(k) + [B_m-B_p-\Omega(k+1)]u(k) \quad (3)$$

where the state error is given by

$$e(k)=x_{m}(k)-x_{p}(k). \tag{4}$$

It is now required to modify $(n \times f)$ dimensioned direct controllers $[\Gamma(k) \vdots \Omega(k)]$ by an adaptation mechanism so that they converge to the ideal, but unknown, matrices $[\Gamma^* \vdots \Omega^*]$ such that

$$[\Gamma^* \vdots \Omega^*]=[A_m-A_p \vdots B_m-B_p] \tag{5}$$

where $f=n+m$.

Defining the (controller) parameter alignment error matrix $\theta(k)$ as

$$\begin{aligned}\theta(k)&=[\alpha(k) \vdots \beta(k)]\\&=[\Gamma^*-\Gamma(k) \vdots \Omega^*-\Omega(k)]\end{aligned} \tag{6}$$

and the information vector

$$z^{T}(k)=[x_{p}^{T}(k) \vdots u^{T}(k)], \tag{7}$$

Equation (3) is rewritten as

$$e(k+1)=A_m e(k)+\theta(k+1)z(k). \tag{8}$$

Let

$$s(k)=e(k)-A_m e(k-1) \tag{9}$$

so that Equation (8) is given by

$$s(k+1)=\theta(k+1)z(k). \tag{10}$$

The objective is to develop a set of parameter adaptation algorithms so as to achieve:

$$\operatorname*{Lt}_{k\to\infty} e(k)=0 \tag{11}$$

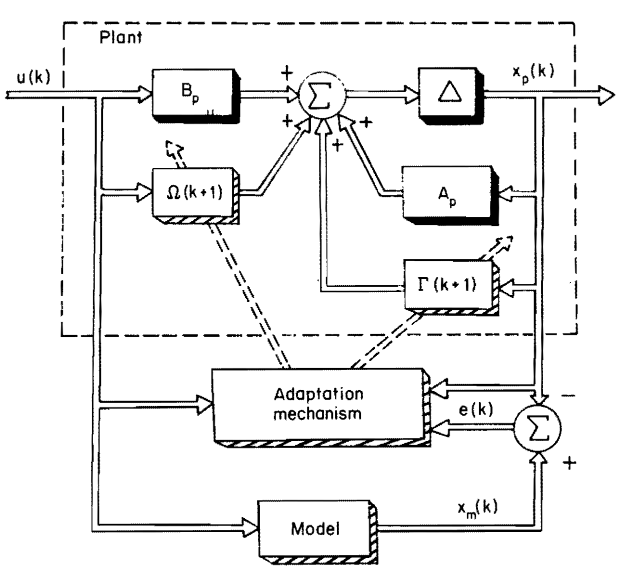


Figure 1. Direct Parameter MRAC Scheme

and

$$\operatorname*{Lt}_{k\to\infty} \theta(k)=0. \tag{12}$$

The proposed adaptive control scheme is illustrated in Figure 1.

3. DEVELOPMENT OF THE ADAPTATION ALGORITHM

In order to force the adaptive system errors $e(k)$ and $\theta(k)$ towards zero, it is proposed that the elements of the controller matrices are modified by the algorithm as

$$\begin{aligned}[\Gamma(k+1) \vdots \Omega(k+1)]&=[\Gamma(k) \vdots \Omega(k)]\\&\quad+[\Delta\Gamma(k) \vdots \Delta\Omega(k)]\end{aligned} \tag{13}$$

where $\Delta\Gamma(k)$ and $\Delta\Omega(k)$ are the first forward differences of $\Gamma(k)$ and $\Omega(k)$ respectively. These first forward differences are to be computed recursively by the algorithm.

Equation (13) can be rewritten using Equation (6) as

$$\theta(k+1)=\theta(k)+\Delta\theta(k) \tag{14}$$

where

$$\Delta\theta(k)=[\Delta\Gamma(k) \vdots \Delta\Omega(k)]. \tag{15}$$

The required algorithm for determining $\Delta\theta(k)$ is derived using the gradient estimation technique.

Let the $(nf \times 1)$ parameter alignment error vector be written from the elements of $\theta(k)$ as

$$\begin{aligned}\phi^{T}(k)=[&\theta_{11}(k)\theta_{12}(k)\ldots\theta_{1f}(k)\\&\vdots \theta_{21}(k)\theta_{22}(k)\ldots\theta_{2f}(k)\\&\vdots \ldots \vdots \theta_{n1}(k)\theta_{n2}(k)\ldots\theta_{nf}(k)]\end{aligned} \tag{16}$$

where $\theta_{ij}(k)$ is the i^{th} row j^{th} column element of $\theta(k)$.

A quadratic index on $\phi(k)$ is defined as

$$J(\phi)=\phi^{T}(k)\phi(k). \tag{17}$$

The gradient vector of $J(\phi)$ is obtained from

$$\nabla J(\phi)=\frac{\partial J(\phi)}{\partial \phi(k)}=2\phi(k). \tag{18}$$

Computation of $J(\phi)$ is not possible since $\phi(k)$ is unknown. However, an estimated value of the gradient is determined by computing an estimate on $\phi(k)$.

Equation (10) is rewritten as

$$\theta(k)z(k-1)=\delta(k)[s(k)z^{T}(k-1)]z(k-1) \tag{19}$$

where

$$\delta(k) = [z^T(k-1)z(k-1)]^{-1}. \tag{20}$$

The estimate on $\theta(k)$ denoted by $\tilde{\theta}(k)$ is then obtained as

$$\tilde{\theta}(k) = \delta(k)s(k)z^T(k-1). \tag{21}$$

The estimate $\tilde{\phi}(k)$ on the vector $\phi(k)$ is thus derived to be

$$\tilde{\phi}(k) = \delta(k)m(k) \tag{22}$$

where the $(nf \times 1)$ vector $m(k)$ is given by

$$m(k) = \begin{bmatrix} s_1(k) \quad z(k-1) \\ \hline s_2(k) \quad z(k-1) \\ \hline \vdots \\ \hline s_n(k) \quad z(k-1) \end{bmatrix} \tag{23}$$

and $s_i(k)$ is the i^{th} component of $s(k)$.

From Equations (18) and (22), an estimated value of the gradient $\nabla\tilde{J}(\phi)$ is obtained from

$$\nabla\tilde{J}(\phi) = 2\delta(k)m(k). \tag{24}$$

Application of the gradient estimation approach [10,11] yields an algorithm for modifying the parameter alignment error vector. The algorithm is given by

$$\phi(k+1) = \phi(k) - 2\delta(k)P(k)m(k) \tag{25}$$

where $P(k)$ is a $(nf \times nf)$ positive definite symmetric matrix. $P(k)$ is chosen as

$$P(k) = \text{diag}[p_1(k), p_2(k), \ldots, p_{nf}(k)] \tag{26}$$

where $p_i(k) > 0, i = 1, 2, \ldots, nf$, for all k.

Thus the required algorithm for modifying $\theta(k)$ can be written from Equations (16) and (25) as

$$\theta(k+1) = \theta(k) - 2\delta(k)R(k) \circledast [s(k)z^T(k-1)]. \tag{27}$$

$R(k)$ is an $(n \times f)$ matrix derived from the elements of $P(k)$ such that

$$R(k) = \begin{bmatrix} p_1(k) & p_2(k) & \cdots & p_f(k) \\ p_{f+1}(k) & p_{f+2}(k) & \cdots & p_{2f}(k) \\ \vdots & \vdots & & \vdots \\ p_{(n-1)f+1}(k) & p_{(n-1)f+2}(k) & \cdots & p_{nf}(k) \end{bmatrix} \tag{28}$$

$$= [r_{ij}(k)], \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, f. \tag{29}$$

The symbol $\circledast$ stands for the element-by-element matrix product operator [9] such that, given any two matrices $V = [v_{ij}]$ and $W = [w_{ij}]$, then $V \circledast W = [v_{ij}w_{ij}]$.

From Equations (14), (15), and (27), the resulting adaptation algorithm is given by

$$[\Gamma(k+1) \vdots \Omega(k+1)] = [\Gamma(k) \vdots \Omega(k)] - 2\delta(k) \times R(k) \circledast [s(k)z^T(k-1)]. \tag{30}$$

The algorithm Equation (30) is implemented on-line and the controller parameters are modified at every k from their values at $(k-1)$. If the overall system is stable, Equation (3) forces the controller parameters towards their respective ideal values.

It is noted that the updating algorithm depends on the estimated value of $\theta(k)$ which in turn depends on the estimation procedure. There could therefore be a possibility of biased estimation. However, since the direct parameter MRAC is a class of parallel model reference adaptive system [2], such a biased estimation can be reduced if the system is proved to be stable and the adaptive gains are low [12].

4. ASYMPTOTIC STABILITY OF THE MRAC SCHEME

Application of Lyapunov's direct method for assuring asymptotic stability of the proposed scheme leads to the conditions under which $R(k)$ is to be chosen.

A candidate Lyapunov function for the parameter misalignment is selected as

$$V(k) = T_r[R^*(k)^T\theta(k) \circledast \theta(k)] \tag{31}$$

where $R^*(k)$ is the element-by-element inverse such that $R^*(k) \circledast R(k) = E$ and all elements of E are unity for all k. $T_r(\cdot)$ is the trace of the matrix.

The first forward difference $\Delta V(k)$ defined by

$$\Delta V(k) = V(k+1) - V(k) \tag{32}$$

is derived as

$$\Delta V(k) = h(k) + g(k) \tag{33}$$

where

$$h(k) = T_r\{[R^*(k+1) - R^*(k)]^T \times [\theta(k+1) \circledast \theta(k)]\} \tag{34}$$

and

$$g(k) = T_r\{[R^*(k)]^T \times [[\theta(k+1) \circledast \theta(k+1)] - [\theta(k) \circledast \theta(k)]]\}. \tag{35}$$

The speed of adaptation is related to the rate with which $V(k)$ goes to zero. Hence it is advantageous if $h(k)$ and $g(k)$ are made individually negative for all k.

The term $h(k)$ can be made negative definite for all $\theta(k)\neq 0$ when the elements of $R(k)$ are selected as

$$r_{ij}(k)<r_{ij}(k+1). \tag{36}$$

Using Equation (27), Equation (35) is simplified to be

$$g(k)=4\delta(k)\,[s^{\mathrm{T}}(k)[\delta(k)\,T(k)-I]s(k)] \tag{37}$$

where

$$T(k)=\mathrm{diag}[t_1(k),\ t_2(k),\ \ldots,\ t_n(k)] \tag{38}$$

$$t_i(k)=\sum_{j=1}^{f} z_j(k-1)r_{ij}(k), \quad i=1,2,\ldots,f. \tag{39}$$

The term $g(k)$ can be negative for all $s(k)\neq 0$ if

$$r_{ij}(k)<1, \quad i=1,2,\ldots,n; \quad j=1,2,\ldots,f. \tag{40}$$

Consequently $\Delta V(k)<0$ for all $\theta(k)$, $s(k)\neq 0$ and $V(k)$ is a Lyapunov function. That is, the adaptive scheme is proved to be asymptotically stable in θ-space.

It can be easily verified [13] that the adaptive scheme is asymptotically stable in (s,θ)-space if

(i) the input $u(k)$ is sufficiently general, and
(ii) the plant is completely controllable.

It still remains to be proved that $e(k)\to 0$ as $k\to\infty$. Rewriting Equation (8) as

$$e(k+1)=A_{\mathrm{m}}e(k)+w(k), \tag{41}$$

it can be readily shown [8] that $\theta(k)\to 0$ implies $w(k)\to 0$ and hence $e(k)\to 0$ since A_{m} has its eigenvalues within the unit circle of the complex plane.

Thus the proposed MRAC system fulfills the conditions stipulated in Equations (11) and (12) and assures asymptotic stability in (e,θ)-space.

It is to be noted that the developed algorithm Equation (30) is similar to the one given by Sebakhy [8], except we use here the gain sequence $R(k)$ for improved convergence. Let Equation (30) be called the algorithm with improved convergence.

5. SELECTION OF OPTIMUM WEIGHTAGES

Optimum Scalar Weightage

In Equation (30), the gain sequence $R(k)$ is weighted by the factor $\delta(k)$. If this weighting factor is a free sequence, then it can be selected to make $g(k)$, Equation (37), a minimum.

Minimizing $g(k)$ with respect to $\delta(k)$ results in an algorithm for optimum convergence with scalar weightage. The algorithm is given by

$$[\Gamma(k+1)\,\vdots\,\Omega(k+1)]=[\Gamma(k)\,\vdots\,\Omega(k)] -2\delta^{*}(k)R(k)\circledast s(k)z^{\mathrm{T}}(k-1) \tag{42}$$

where the optimum value of $\delta(k)$ is

$$\delta^{*}(k)=\frac{s^{\mathrm{T}}(k)s(k)}{s^{\mathrm{T}}(k)\,T(k)s(k)} \tag{43}$$

and $T(k)$ is given by Equation (38).

Optimum Matrix Weightage

Let $R(k)$ be weighted by an $(n\times n)$ matrix sequence $\Delta(k)$ such that the algorithm is written as

$$[\Gamma(k+1)\,\vdots\,\Omega(k+1)]=[\Gamma(k)\,\vdots\,\Omega(k)] -2\Delta(k)R(k)\circledast[s(k)z^{\mathrm{T}}(k-1)] \tag{44}$$

where

$$\Delta(k)=\mathrm{diag}[\delta_1(k),\delta_2(k),\ldots,\ \delta_n(k)]. \tag{45}$$

Conditions for selecting $\Delta(k)$ and $R(k)$ can be derived as in Section 4 using $V(k)$ given by Equation (31). This leads to $\Delta V(k)=h(k)+g(k)$, where $h(k)$ is as in Equation (34). The factor $g(k)$ is derived as

$$g(k)=\sum_{i=1}^{n} g_i(k) \tag{46}$$

where

$$g_i(k)=4[\delta_i^2(k)t_i(k)-\delta_i(k)]s_i^2(k). \tag{47}$$

From Equation (47), the optimum value of $\delta_i(k)$ is obtained as

$$\delta_i^{*}(k)=[2t_i(k)]^{-1}=\left[2\sum_{j=1}^{f} r_{ij}(k)z_j^2(k-1)\right]^{-1} \quad i=1,2,\ldots,n. \tag{48}$$

The algorithm for optimum convergence with matrix weightage is then given by

$$[\Gamma(k+1)\,\vdots\,\Omega(k+1)]=[\Gamma(k)\,\vdots\,\Omega(k)] -2\Delta^{*}(k)R(k)\circledast s(k)z^{\mathrm{T}}(k-1) \tag{49}$$

where

$$\Delta^{*}(k)=\mathrm{diag}[\delta_1^{*}(k),\ \delta_2^{*}(k),\ldots,\ \delta_n^{*}(k)]. \tag{50}$$

6. MRAC SYSTEM WITH STATE ERROR FEEDBACK

The error dynamics, Equation (8), are considered to have two parts — a linear part $A_{\mathrm{m}}e(k)$ and a nonlinear part $\theta(k)z(k-1)$. It has been shown in Section 4 that $\theta(k)\to 0$ if $z(k)$ satisfies certain conditions. When $\theta(k)=0$, $e(k)$ approaches zero at the rate specified by the eigenvalues of A_m.

This section investigates the possibility of assigning a desirable set of new locations to the eigenvalues for the linear part of the error model using state error feedback. Such a set of desirable closed loop poles ensures a higher rate of decay of $e(k)$ and hence a faster adaptation.

The MRAC scheme with the proposed state error feedback is indicated in Figure 2.

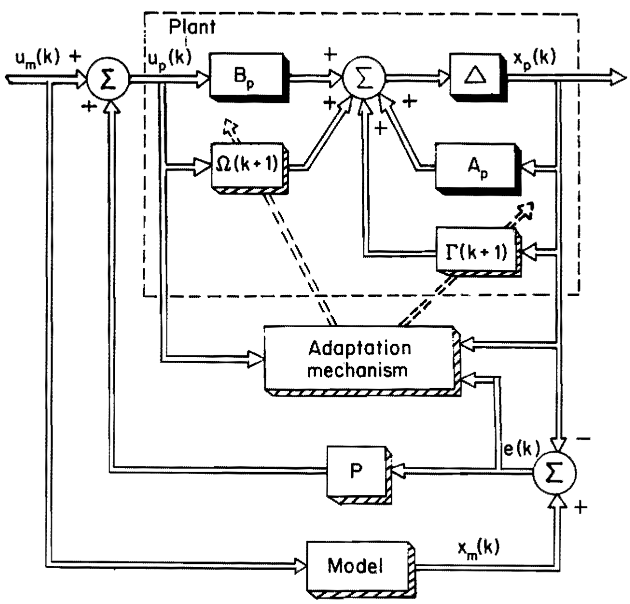


Figure 2. Direct Parameter MRAC Scheme with State Error Feedback.

The dynamics of the plant are now derived as

$$x_p(k+1)=[A_p+\Gamma(k+1)]x_p(k) + [B_p+\Omega(k+1)]u_p(k) \quad (51)$$

where

$$u_p(k)=u_m(k)+P^T e(k) \quad (52)$$

and P is the $(m\times n)$ state error feedback matrix. From Equations (2), (51), and (52), the error model is derived as

$$\begin{aligned} e(k+1)&=[A_m-B_mP^T]e(k)+[A_m-A_p-\Gamma(k+1)]x_p(k) \\ &\quad+[B_m-B_p-\Omega(k+1)]u_p(k) \quad (53)\\ &=[A_m-B_mP^T]e(k)+\theta(k+1)\hat{z}(k) \quad (54)\end{aligned}$$

where

$$\hat{z}(k)=[x_p^T(k) \vdots u_p^T(k)]^T. \quad (55)$$

Since Equation (54) is similar to Equation (8), it is easy to prove that $\theta(k)\to 0$ as $k\to\infty$ if $[A_m-B_mP^T]$ has stable eigenvalues. When $\theta(k)=0$, $e(k)$ is forced to zero at the rate specified by the eigenvalues of $[A_m-B_mP^T]$. The eigenvalues of $[A_m-B_mP^T]$ can be placed at any desired locations in the z-plane. The error dynamics are thus controlled by the pole placement of the error system.

Equation (55) is written as

$$\hat{s}(k)=\theta(k)\hat{z}(k-1) \quad (56)$$

with

$$\hat{s}(k)=e(k)-[A_m-B_mP^T]e(k-1). \quad (57)$$

Equations (56) and (57) are respectively similar to Equations (10) and (9), and hence the proof of asymptotic stability is direct. In addition, the adaptation algorithms for improved convergence and for optimum convergence with scalar and matrix weightages can also be developed as detailed in Section 5.

7. ILLUSTRATIVE EXAMPLE

The usefulness of the developed MRAC scheme is demonstrated by the following example.

Plant (with controllers)

The plant dynamics are specified as in Equation (1) with

$$A_p+\Gamma(k)=\begin{bmatrix} a_{p11}+\Gamma_{11}(k) & a_{p12}+\Gamma_{12}(k) \\ a_{p21}+\Gamma_{21}(k) & a_{p22}+\Gamma_{22}(k)\end{bmatrix}$$

and

$$B_p+\Omega(k)=\begin{bmatrix} b_{p11}+\Omega_{11}(k) \\ b_{p21}+\Omega_{21}(k)\end{bmatrix}.$$

For the purpose of simulation, the plant matrices are assumed to be

$$A_p=\begin{bmatrix} 0.2 & 0.0 \\ -0.3 & 0.3\end{bmatrix} \text{ and } B_p=\begin{bmatrix} 0.2 \\ 0.4\end{bmatrix}.$$

The initial values of the direct controller parameters are taken as zeros.

Model

The model dynamics are as specified by Equation (2) with

$$A_m = \begin{bmatrix} 0.6 & 0.6 \\ 0.5 & 0.62 \end{bmatrix} \quad \text{and} \quad B_m = \begin{bmatrix} 0.8 \\ 1.6 \end{bmatrix}.$$

State Error Feedback Matrix

In order to compare the performances of the proposed algorithms, the following state error feedback matrices are considered:

(1) $P = 0$, so that the error model has poles at 0.893 and 0.327;
(2) $P^T = [0.562 \; -0.3]$, so that the error model has poles at 0.882 and 0.369.

Input

The input signal $u_m(k)$ is a train of rectangular pulses switching between -1 and $+1$ with a frequency corresponding to three sample periods.

Gain Matrices

With the aim of comparing the performance of the developed algorithms (containing matrix gain sequence $R(k)$), with that of conventional schemes (containing fixed gain matrices), the following gains are considered:

(1) $R(k) = R_L = \begin{bmatrix} 0.031 & 0.0067 & 0.0265 \\ 0.034 & 0.008 & 0.266 \end{bmatrix}$;

(2) $R(k) = R_H = \begin{bmatrix} 0.9999 & 0.21 & 0.9 \\ 0.9999 & 0.205 & 0.9 \end{bmatrix}$;

(3) $R(k)$ is a monotonically increasing sequence starting with its lowest value as R_L, and reaching its highest value R_H, with the following increments at every k

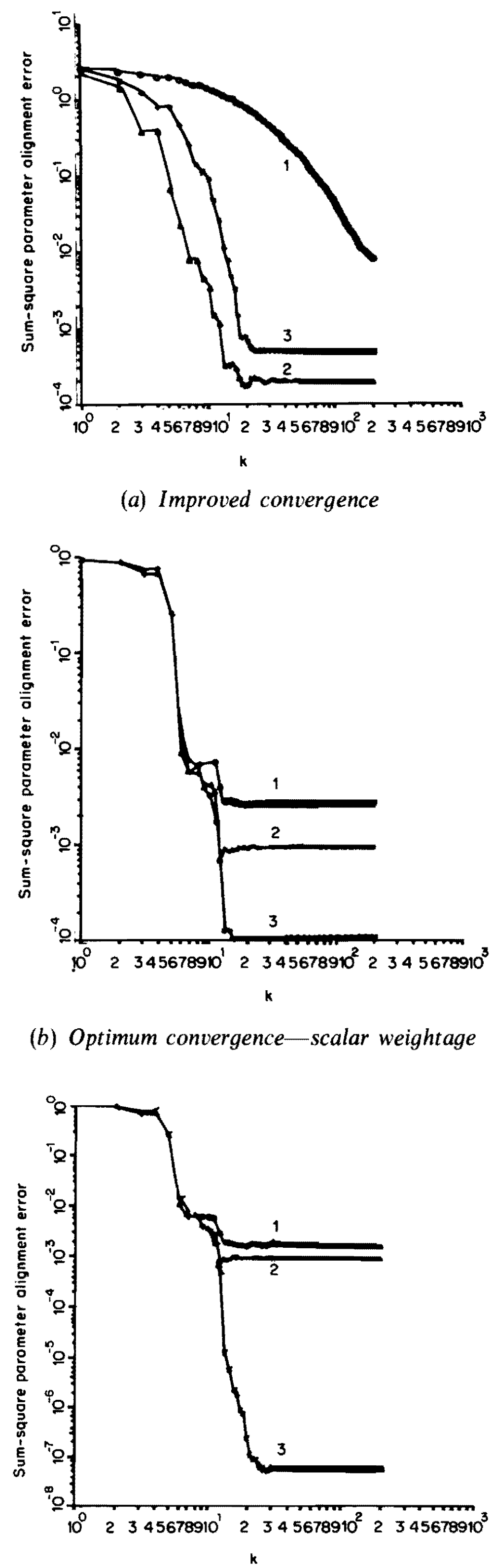
$$R(k) = R_L + kH$$

where

$$H = \begin{bmatrix} 0.03143 & 0.00664 & 0.0265 \\ 0.036 & 0.00746 & 0.02684 \end{bmatrix}.$$

With each of the above gain matrices, computer simulations are carried out to obtain adaptive responses using algorithms for

(a) improved convergence;
(b) optimum convergence with scalar weightage; and

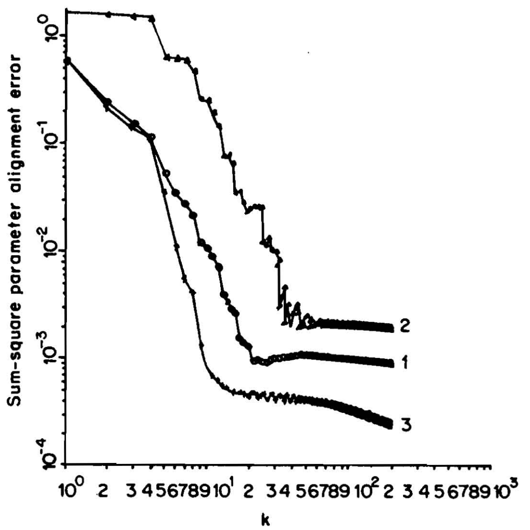


(a) Improved convergence

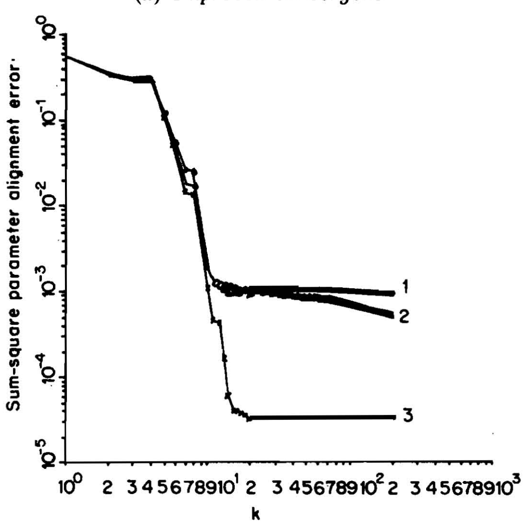
(b) Optimum convergence—scalar weightage

(c) Optimum convergence—matrix weightage

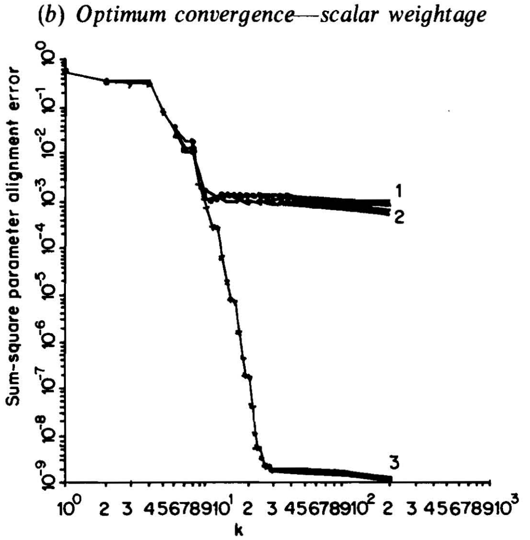
Figure 3. Response of Sum-Square Parameter Alignment Error (P = 0)



(*a*) *Improved convergence*



(*b*) *Optimum convergence—scalar weightage*



(*c*) *Optimum convergence—matrix weightage*

Figure 4. Responses of Sum-Square Parameter Alignment Error ($P \neq 0$).

(c) optimum convergence with matrix weightage.

Figures 3 and 4 demonstrate the adaptive responses of sum-square parameter alignment error (i.e. $T_r[\theta^T(k)\theta(k)]$) for the above three algorithms, with ($P \neq 0$) and without ($P = 0$) state error feedback. Responses for gain matrices R_L, R_H, and $R(k)$ are shown respectively as 1, 2, and 3 in the figures.

The adaptive responses indicate that the introduction of matrix gain sequences in the adaptive loops increases the speed of adaptation. The adaptive scheme with state error feedback and its algorithm with optimum matrix weightage offer the fastest response.

8. CONCLUSIONS

The proposed MRAC scheme introduces a certain gain matrix sequence and weighting factor in its adaptive loops. The elements of the gain matrix sequence can be selected with the simple restriction that they be positive monotonically increasing functions of k towards maximum values less than unity. They can be selected independently of each other. It has been shown that such a selection increases the rate of convergence of the controller parameters towards their respective ideal values. The rate of convergence can be made substantial if the gain sequence has matrix weightage.

It has also been shown that the poles of the error system can be arbitrarily located in the complex plane using state error feedback. The reallocation of poles is found to further increase the rate of adaptation.

The proposed MRAC scheme can be readily extended to on-line parameter identification and adaptive observer schemes.

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